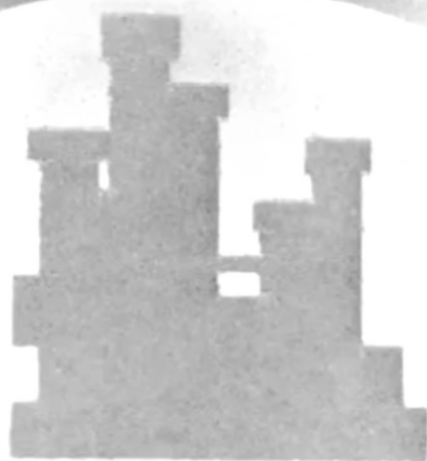


**EXAMPLES
AND PROBLEMS
TO THE COURSE
OF UNIT OPERATIONS
OF CHEMICAL
ENGINEERING**

**K.F. Pavlov
P.G. Romankov
A.A. Neskov**



**MIR
PUBLISHERS
MOSCOW**

ABOUT THE BOOK

The gaining of skill in solving practical engineering problems is very important for students of the course in unit operations. The Department of Unit Operations of Chemical Engineering of the Leningrad Technological Institute gives great attention to this aspect, and the present book contains many examples and problems in the course that are the result of many years of experience in instruction. Great attention is devoted to independent work of the students, and problems are given on all the sections of the course: fundamentals of applied hydraulics; pumps, fans, compressors; hydromechanical separation methods; hydrodynamics of a fluidized bed; agitation in a liquid medium; heat transfer in a chemical apparatus; evaporation, crystallization; mass transfer, absorption; distillation and rectification; extraction and leaching; adsorption; drying; moderate and deep refrigeration. The basic formulas and equations needed for solving the problems are given at the beginning of each chapter. The appendices contain numerous reference tables and diagrams. The present book, edited by Associate Member of the USSR Academy of Sciences P. G. Romankov, is intended as a training aid for students of chemical engineering and related specialities of day, evening and correspondence faculties. It will also be a useful aid in course and diploma designing.

ABOUT THE AUTHORS

KONSTANTIN F. PAVLOV (1895-1944) was an outstanding specialist in the separation of natural and industrial gases. In 1934 he received his doctor's degree and the rank of professor. He headed the departments of chemical engineering of inorganic substances and of general chemical engineering at the Leningrad Technological Institute, where in 1936 he founded the first training laboratory in unit operations of chemical engineering in the country (it is now named after him). He is the author of a number of training aids in chemical engineering and of original research work (the rule of linearity of chemical engineering functions, etc.). During the last years of his life he worked in Moscow at the Institute of Physical Problems of the USSR Academy of Sciences.

PYOTR G. ROMANKOV, D.Sc., professor, after working for a number of years as a chemical engineer and also as an instructor at several chemical institutes, since 1941 is head of the Department of Unit Operations of Chemical Engineering at the Leningrad Technological Institute. Together with his pupils and collaborators he has published over 250 scientific works including several monographs on the theory and application of hydromechanical, heat and mass-exchange processes of chemical engineering. In 1964 Pyotr Romankov was elected an associate member of the USSR Academy of Sciences. He is an Honoured Scientist of the RSFSR, and has also received a number of government awards. He is a Doctor *honoris causa* of a number of foreign higher educational establishments.

ANATOLI A. NOSKOV (1904-1977) worked for many years as an engineer in the field of gas separation under the guidance of Prof. Konstantin Pavlov. He was soon invited to work at the Department of Unit Operations of Chemical Engineering of the Leningrad Technological Institute, where he advanced from lecturer to professor. Anatoli Noskov is well known for his work in the field of improving rectification processes and the methods of calculating standard operations of chemical engineering.

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TO THE READER

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**К. Ф. Павлов
П. Г. Романков
А. А. Носков**

**ИЗДАТЕЛЬСТВО
«ХИМИЯ»**

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И ЗАДАЧИ
ПО КУРСУ
ПРОЦЕССОВ И АППАРАТОВ
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PREFACE

The part played at present by the general engineering course in "Unit Operations of Chemical Engineering" has grown quite considerably in the training of specialists with a broad range of knowledge for the chemical and related branches of industry. The science of unit operations forms the theoretical foundation of modern chemical engineering and makes it possible to solve numerous and diverse problems connected with the design, erection, and operation of chemical enterprises. Special attention is devoted to the development of new highly intensive production processes, and of new designs of chemical apparatuses and machines having a high output and economical in operation.

The gaining of skill in solving practical engineering problems is very important. The Department of Unit Operations of Chemical Engineering of the Leningrad Technological Institute named after the Leningrad Soviet has compiled examples and problems in the course in unit operations as a training aid.

Great attention is given to independent work of the students (problems are given on all the sections of the course). The present book is intended as a training aid for students of chemical engineering and related specialities of day, evening and correspondence faculties. It will also be a useful aid in course and diploma designing.

Most of the problems are typical ones; they are intended in their volume for seminars, but can also be used for individual assignments.

The present English edition is a translation of the new eighth Russian edition published in the USSR in 1976 to which new examples and problems were added with revision of the formulas used for calculations and the reference data.

This book has found great favour not only in the USSR, but also in other countries; it has been published in Bulgaria, Czechoslovakia, the German Democratic Republic, Hungary, Poland, Romania, Vietnam, and Yugoslavia.

The authors express their gratitude to their colleagues of the Department of Unit Operations for looking through separate chapters of the manuscript of the Russian edition and their additions to them. The authors would like to thank specially M. I. Kurochkin for her creative participation in preparing the new Russian edition.

The authors with great appreciation also note the creative approach and high skill of G. V. Leib in translating the book into English.

P. Romankov

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INTRODUCTORY METHODOICAL INSTRUCTIONS

Practical lessons in the course of unit operations of chemical engineering are aimed at training the students in the methods and procedures of the calculation and design of chemical apparatuses by considering examples and solving concrete problems.

Various physical quantities and data on the physical properties of substances (density, viscosity, thermal conductivity, etc.) have to be used in the calculation of apparatuses. All these quantities can be measured and expressed in various units.

The USSR has adopted the International System of Units (SI) worked out by the Conférence Générale des Poids et Mesures (the General Conference on Weights and Measures) as preferable in all fields of science, engineering, and the national economy, and also in education. The basic units of the SI system are the unit of length—the metre (m), of mass—the kilogram (kg), of time—the second (s), of current intensity—the ampere (A), of temperature—the kelvin (K), of luminous intensity—the candela (cd), and the unit of the amount of substance—the mole (mol). The SI system also has two supplementary units: the unit of plane angle—the radian (rad) and the unit of solid angle—the steradian (sr).

The units of all other quantities are found from the basic ones and are called derived units. Their dimensions are determined with the aid of the physical equations expressing the relationship between the relevant quantities. Some of the derived units have been given special names; for example, the unit of force in the SI system is called the newton ($1 \text{ N} = 1 \text{ kg} \cdot \text{m}/\text{s}^2$), the unit of pressure—the pascal ($1 \text{ Pa} = 1 \text{ kg}/\text{m} \cdot \text{s}^2 = 1 \text{ N}/\text{m}^2$), and the unit of energy is called the joule ($1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 1 \text{ N} \cdot \text{m}$).

A comparison of the most important (for our purposes) SI units with units of other systems and non-system units is given in the Appendix in Table A-58.

When beginning to solve a problem, draw a diagram of the installation or apparatus, designate all the dimensions and quan-

tities in it, show the directions of flows with arrows, and gain an understanding of the conditions of its operation.

Next write out all the initial data of the problem and the main equations for the calculations, plan the course of the solution, dividing the problem into a number of particular questions, and write out the constants needed.

After introducing numerical values into the equations, check whether this has been done correctly, and then commence the arithmetical calculations.

The answer should be assessed critically from the standpoint of correspondence of the result obtained to the actual conditions of operation of the installation or apparatus being calculated.

It is good to solve a problem in the general form and investigate the results by determining the influence of the various factors included in the initial conditions of the problem.

During group lessons, the students must learn how to use the fundamental reference books combining the experience of a great number of research workers and designers (see Bibliography), and also the relevant standards and catalogues.

The main task of the exercises in the course is to develop skill in the conducting of engineering calculations. The error usually tolerated in such calculations is about 5%.

The use of a slide rule or similar aid is obligatory. Some of the problems should be solved with the use of an electronic computer (draw up a programme)—see examples 4-20, 4-V, 6-8, 7-15.

For more efficient studying of the course, it is good to have two note-books: one for group lessons, the other for independent homework—the solution of problems and the drawing of sketches of the main apparatuses and their components.

Clear statements, a systematic course of calculations, and accurate records are conditions that must be observed. Otherwise the result will be an unproductive expenditure of time by the student and the instructor.

In conclusion, let us consider a few examples on the establishment of units of physical quantities and the relationships between them.

1. Find the SI unit of the dynamic viscosity.

Newton's equation for the force of friction F between parallel flowing layers of a liquid has the form

$$F = \mu A \frac{dv}{dy}$$

where μ = dynamic viscosity

A = friction area

dv/dy = velocity gradient.

Solving this equation with respect to μ , we get the following unit for it (the brackets enclosing the symbol of a quantity or quantities indicate that we are dealing with the dimension of

a unit of the relevant quantity or quantities):

$$[\mu] = \left[\frac{F dy}{A dv} \right] = \frac{\text{N} \cdot \text{s} \cdot \text{m}}{\text{m}^2 \cdot \text{m}} = \frac{\text{N} \cdot \text{s}}{\text{m}^2} = \text{Pa} \cdot \text{s} = \frac{\text{kg} \cdot \text{m} \cdot \text{s}}{\text{s}^2 \cdot \text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

2. Find the SI unit for the coefficient of thermal conductivity.

For a steady-state heat flux Q through a homogeneous flat wall we can write:

$$Q = \frac{\lambda}{l} A \Delta T$$

where λ — thermal conductivity coefficient

l = thickness of the wall

A = area of surface through which the heat is transmitted

ΔT = difference of temperatures at both sides of the wall.

Solving this equation with respect to λ , we get:

$$[\lambda] = \left[\frac{Ql}{A \Delta T} \right] = \frac{\text{J} \cdot \text{s}^{-1} \cdot \text{m}}{\text{m}^2 \cdot \text{K}} = \frac{\text{W}}{\text{m} \cdot \text{K}}$$

3. Find the relationship between the SI and CGS units of dynamic viscosity.

$$1 \text{ Pa} \cdot \text{s} = 1 \frac{\text{kg}}{\text{m} \cdot \text{s}} = \frac{1000 \text{ g}}{100 \text{ cm} \cdot \text{s}} = 10 \frac{\text{g}}{\text{cm} \cdot \text{s}} = 10 \text{ P} = 1000 \text{ cP}$$

$$1 \text{ cP} = 10^{-3} \text{ Pa} \cdot \text{s} = 1 \text{ mPa} \cdot \text{s}$$

4. Establish the relationship between the thermal conductivity coefficient units $\frac{\text{kcal}}{\text{m} \cdot \text{h} \cdot ^\circ\text{C}}$ and $\frac{\text{W}}{\text{m} \cdot \text{K}}$.

$$1 \frac{\text{kcal}}{\text{m} \cdot \text{h} \cdot ^\circ\text{C}} = \frac{4190 \text{ J}}{\text{m} \cdot 3600 \text{ s} \cdot \text{K}} = 1.163 \frac{\text{W}}{\text{m} \cdot \text{K}}$$

5. Bearing in mind that 1 British pound = 0.454 kg and $1 ^\circ\text{C} = 1.8 ^\circ\text{F}$, find the relationship between the British thermal unit (BTU), the kilocalorie (kcal) and the joule (J). 1 BTU is the quantity of heat needed to heat 1 lb of water by $1 ^\circ\text{F}$. The specific heat capacity of water in the British system of units is

$$c = 1 \frac{\text{BTU}}{\text{lb} \cdot ^\circ\text{F}}.$$

We find the required relationship with the aid of the well-known formula

$$Q = cM(t_2 - t_1)$$

where Q = quantity of heat spent for heating M kg of water

c = specific heat capacity of water

t_1 and t_2 = initial and final temperatures.

From the equations

$$Q = 1 \frac{\text{BTU}}{\text{lb} \cdot ^\circ\text{F}} 1 \text{ lb} \cdot 1 ^\circ\text{F} = 1 \text{ BTU}$$

and

$$Q = 1 \frac{\text{kcal}}{\text{kg} \cdot ^\circ\text{C}} 0.454 \text{ kg} \frac{1}{1.8} ^\circ\text{C} = 0.252 \text{ kcal}$$

it follows that $1 \text{ BTU} = 0.252 \text{ kcal}$.

Since $1 \text{ kcal} = 4190 \text{ J}$, then $1 \text{ BTU} = 0.252 \times 4190 \text{ J} = 1055 \text{ J}$.

6. Express the overall coefficient of heat transfer $K = 50 \frac{\text{BTU}}{\text{ft}^2 \cdot \text{h} \cdot ^\circ\text{F}}$ in $\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$.

Since $1 \text{ BTU} = 1055 \text{ J}$, $1 \text{ ft} = 0.305 \text{ m}$, and $1 ^\circ\text{F} = 0.56 \text{ K}$, then

$$K = 50 \frac{1055 \text{ J}}{(0.305 \text{ m})^2 \times 3600 \text{ s} \times 0.56 \text{ K}} = 50 \times 5.6 \frac{\text{J}}{\text{m}^2 \cdot \text{s} \cdot \text{K}} = 280 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}$$

FUNDAMENTAL RELATIONSHIPS
AND FORMULAS FOR CALCULATIONS

1. The unit or volume weight (the weight of a unit volume) γ and the density (the mass of a unit volume) ρ are related by the expression

$$\gamma = \rho g \quad (1-1)$$

where $g = 9.81 \text{ m/s}^2$ is the acceleration due to gravity.

Since the unit of mass in the SI system (kg) is the mass of the international prototype of the kilogram, while the unit of force in the MK (force) S system (kgf) is the weight of the same prototype, then the density in SI units (kg/m^3) is numerically equal to the unit weight in MK (force) S units (kgf/m^3).

The relative density (specific gravity) Δ is the ratio of the density (unit weight) of a substance to the density (unit weight) of water:

$$\Delta = \frac{\rho}{\rho_w} = \frac{\gamma}{\gamma_w} \quad (1-2)$$

Within the interval from 0 to 100°C , the density of water ρ_w can be assumed to equal 1000 kg/m^3 with an accuracy sufficient for engineering calculations.

The density of a mixture of liquids whose mixing is not attended by any appreciable physicochemical changes can be calculated approximately by assuming that the volume of the mixture equals the sum of the volumes of its components:

$$\frac{1}{\rho_{\text{mix}}} = \frac{\bar{x}_1}{\rho_1} + \frac{\bar{x}_2}{\rho_2} + \dots \quad (1-3)$$

where $\bar{x}_1, \bar{x}_2, \dots$ = mass fractions of the mixture components

$\rho_{\text{mix}}, \rho_1, \rho_2, \dots$ = densities of the mixture and its components.

The density of a suspension ρ_{susp} is calculated by a similar formula:

$$\frac{1}{\rho_{\text{susp}}} = \frac{\bar{x}}{\rho_c} + \frac{1-\bar{x}}{\rho_{\text{liq}}} \quad (1-4)$$

here \bar{x} = mass fraction of the solid phase in the suspension
 ρ_c and ρ_{liq} = densities of the solid (crystalline) and liquid phases.

2. On the basis of the Clapeyron equation*, the density ρ of a gas at the temperature T and the pressure p can be calculated by the formula:

$$\rho = \rho^\circ \frac{T^\circ p}{T p^\circ} = \frac{M}{22.4} \frac{273p}{T p^\circ} \quad (1-5)$$

where $\rho^\circ = M/22.4 \text{ kg/m}^3$ = density of the gas in standard conditions (denoted by the superscript $^\circ$), i.e. at $T^\circ = 273.15 \text{ K}$ (0°C) and $p^\circ = 760 \text{ mm Hg} = 1.013 \times 10^5 \text{ Pa}$

M = molar mass of the gas, kg/kmol

T = temperature, K .

The pressures p and p° must be expressed in the same units.

The density of a mixture of gases is:

$$\rho_{\text{mix}} = \varphi_1 \rho_1 + \varphi_2 \rho_2 + \dots \quad (1-6)$$

where $\varphi_1, \varphi_2, \dots$ = volume fractions of the components of the gas mixture

ρ_1, ρ_2, \dots = respective densities of the components.

3. The pressure p of a column of liquid having the height h at a density of the liquid of ρ equals

$$p = \rho g h \quad (1-7)$$

where p is expressed in Pa , ρ in kg/m^3 , g in m/s^2 , and h in m .

This equation gives us the following relationships between the various units of pressure:

$$1 \text{ atm} = 760 \text{ mm Hg} = \rho g h = 13\,600 \times 9.81 \times 0.76 = 1.013 \times 10^5 \text{ Pa} = 1.033 \times 10^4 \text{ mm H}_2\text{O} = 1.033 \times 10^4 \text{ kgf/m}^2 = 1.033 \text{ kgf/cm}^2$$

$$1 \text{ at (technical atmosphere)} = 1 \text{ kgf/cm}^2 = 10^4 \text{ kgf/m}^2 = 9.81 \times 10^4 \text{ Pa} = 735 \text{ mm Hg} = 10^4 \text{ mm H}_2\text{O}$$

4. The fundamental equation of hydrostatics is

$$p = p_0 + \rho g h \quad (1-8)$$

where p = hydrostatic pressure at a depth of h (in m) from the surface of a liquid, Pa

p_0 = pressure at the surface of the liquid, Pa .

The force due to the pressure of a liquid on a flat wall F (in N) is

$$F = (p_0 + \rho g h_w) A \quad (1-9)$$

* In the Clapeyron equation $pV = RT$ for 1 kmol of a gas, the molar gas constant $R = p^\circ V^\circ / T^\circ = 760 \times 133.3 \times 22.4 / 273 = 8310 \text{ J/kmol} \cdot \text{K}$.

where p_0 = pressure on the surface of the liquid, Pa
 h_w = depth of submergence of the centre of gravity of the wall under the level of the liquid, m
 ρ = density of the liquid, kg/m³
 A = area of the wall surface, m²
 g = acceleration due to gravity, $g = 9.81 \text{ m/s}^2$.

5. The units of the dynamic viscosity, often simply called the viscosity, are:

$$\text{in the SI system } [\mu] = \frac{\text{N} \cdot \text{s}}{\text{m}^2} = \text{Pa} \cdot \text{s} = \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

$$\text{in the MK(force)S system } [\mu] = \frac{\text{kgf} \cdot \text{s}}{\text{m}^2}$$

$$\text{in the CGS system } [\mu] = \frac{\text{dyn} \cdot \text{s}}{\text{cm}^2} = \frac{\text{g}}{\text{cm} \cdot \text{s}} = \text{P (poise)}$$

The relationship between the customarily used viscosity units is:

$$1 \text{ cP} = 10^{-3} \text{ Pa} \cdot \text{s} = \frac{1}{9810} \frac{\text{kgf} \cdot \text{s}}{\text{m}^2}$$

The kinematic viscosity ν (in m²/s) is related to the dynamic viscosity μ by the expression

$$\nu = \frac{\mu}{\rho} \quad (1-10)$$

6. The values of μ for liquids at different temperatures can be determined by means of the nomogram given in Fig. A-5 (see the Appendix).

In the absence of experimental data, the dynamic viscosity of many organic liquids (at 20 °C) can be calculated approximately according to the following empirical formula *

$$\log (\log \mu) = \left(\sum A_n + \sum B \right) \frac{\rho}{10^3 M} - 2.9 \quad (1-11)$$

where μ = dynamic viscosity of a liquid at atmospheric pressure and 20 °C, mP

ρ = density of the liquid, kg/m³

M = molar mass, kg/kmol

A = number of identical atoms in a molecule of the organic compound

n = numerical value of the atomic constant found in Table A-9 (see Appendix)

B = correction for the grouping of the atoms and the nature of the bond between them, given in Table A-10.

* Souders, M. J. *Am. Chem. Soc.*, 60: 154 (1938). Planovsky, A. N. and Kafarov, V. V. *Khim. prom.*, 8: 19 (1944).

It is convenient to use a nomogram (Fig. A-1) to determine μ on the basis of Eq. (1.11). The quantity $\log (\log \mu)$ in the nomogram is denoted by y . If y is positive, the right-hand scale should be used, and if it is negative, the left-hand scale. A definite value of μ —the dynamic viscosity at 20°C—corresponds to each value of y .

7. To determine the dynamic viscosity of a gas at different temperatures, the nomogram given in Fig. A-6 can be used.

The dynamic viscosity of a gas mixture can be calculated according to the approximate formula

$$\frac{M_{\text{mix}}}{\mu_{\text{mix}}} = \frac{\varphi_1 M_1}{\mu_1} + \frac{\varphi_2 M_2}{\mu_2} + \dots \quad (1-12)$$

where $M_{\text{mix}}, M_1, M_2, \dots$ = molar masses of the mixture of gases and the separate components

$\mu_{\text{mix}}, \mu_1, \mu_2, \dots$ = corresponding dynamic viscosities

$\varphi_1, \varphi_2, \dots$ = volume fractions of the components in the mixture.

The following empirical formula, which has been checked for a number of gas mixtures (coke-oven gas, producer gas, etc.) at atmospheric pressure, is also used to calculate μ_{mix} for gases:

$$\mu_{\text{mix}} = \frac{\varphi_1 \mu_1 \sqrt{M_1 T_{\text{cr}, 1}} + \varphi_2 \mu_2 \sqrt{M_2 T_{\text{cr}, 2}} + \dots}{\varphi_1 \sqrt{M_1 T_{\text{cr}, 1}} + \varphi_2 \sqrt{M_2 T_{\text{cr}, 2}} + \dots} \quad (1-13)$$

where $T_{\text{cr}, 1}, T_{\text{cr}, 2}, \dots$ are the critical temperatures of the components, K. The other symbols are the same as in Eq. (1-12). When using Eq. (1-13), all the values of the viscosities μ_1, μ_2, \dots must be taken for the same temperature T , and the viscosity of the mixture μ_{mix} is obtained for this temperature.

Table A-11 gives the values of $\sqrt{MT_{\text{cr}}}$ for selected gases.

The change in the dynamic viscosity of gases with the temperature is expressed by the formula

$$\mu_T = \mu_0 \frac{273 + C}{T + C} \left(\frac{T}{273} \right)^{3/2} \quad (1-14)$$

where μ_0 = dynamic viscosity at 0°C

T = temperature, K

C = Sutherland's constant [0-26].

8. For a mixture of normal (non-associated) liquids, the value of μ_{mix} can be calculated by the formula:

$$\log \mu_{\text{mix}} = x_1 \log \mu_1 + x_2 \log \mu_2 + \dots \quad (1-15)$$

where μ_1, μ_2, \dots = dynamic viscosities of the separate components

x_1, x_2, \dots = mole fractions of the components in the mixture.

In accordance with the additivity of the fluidities of the components*, the dynamic viscosity of a mixture of normal liquids is determined by the equation:

$$\frac{1}{\mu_{\text{mix}}} = \frac{\varphi_1}{\mu_1} + \frac{\varphi_2}{\mu_2} + \dots \quad (1-16)$$

where $\varphi_1, \varphi_2, \dots$ are the volume fractions of the components in the mixture.

The dynamic viscosity of dilute suspensions μ_{susp} can be calculated by the formulas:

$$\text{when the volume concentration of the solid phase is less than 10\%} \\ \mu_{\text{susp}} = \mu_{\text{liq}} (1 + 2.5\varphi) \quad (1-17)$$

when the volume concentration of the solid phase is up to 30% **

$$\mu_{\text{susp}} = \mu_{\text{liq}} \frac{0.59}{(0.77 - \varphi)^2} \quad (1-18)$$

where μ_{liq} = dynamic viscosity of the pure liquid

φ = volume fraction of the solid phase in the suspension.

9. The dynamic viscosity of a liquid can be found according to that of a standard substance by using the rule of linearity of single valued chemicochemical functions established by K. Pavlov***. On the basis of the rule of linearity, we get****:

$$\frac{t_{\mu_1} - t_{\mu_2}}{\theta_{\mu_1} - \theta_{\mu_2}} = K = \text{const} \quad (1-19)$$

where t_{μ_1} and t_{μ_2} = temperatures of the liquid

θ_{μ_1} and θ_{μ_2} = temperatures of the standard substance at which its dynamic viscosities equal the respective dynamic viscosities of the liquid μ_1 and μ_2 .

For many liquids, the relationship between $\log \mu$ and $1/T$ is virtually linear:

$$\log \mu = a + \frac{b}{T} \quad (1-20)$$

where a and b = individual constants of a liquid

T = temperature, K.

It follows from Eq. (1-20) that the relationship between the logarithms of the dynamic viscosities of two liquids A and B (at the same temperatures) will be identical:

$$\frac{\log \mu_{A, T_1} - \log \mu_{A, T_2}}{\log \mu_{B, T_1} - \log \mu_{B, T_2}} \quad (1-21)$$

* The fluidity is the reciprocal of the dynamic viscosity μ .

** Weinspach, P. M. *Chem. Ing. Techn.*, 41, 5-6: 260-265 (1969).

*** Pavlov, K. F. *Khim Prom.*, 6: 327 (1936). See also Pilař, A. *Lineární závislosti chemicko-inženýrských výpočtech*. Prague (1957).

**** This relationship was found experimentally by A. Porter.

Upon the approximate calculation of the dynamic viscosities of liquids according to the linearity rule, the results will be the more reliable, the closer is the physicochemical nature of the two liquids being compared (aqueous solutions of salts-water, saturated hydrocarbons-heptane, etc.).

10. Equations of rate of flow.

The volumetric rate of flow of a fluid (liquid or gas) Q_v (in m^3/s) is:

$$Q_v = vA \quad (1-22)$$

The mass rate of flow of a fluid Q_m (in kg/s) is:

$$Q_m = Q_v \rho = vA\rho \quad (1-23)$$

Here A = cross-sectional area of the flow, m^2

v = mean velocity of the flow, m/s

ρ = density of the fluid, kg/m^3 .

For a round pipe, Eq. (1-22) becomes:

$$Q_v = 0.785d^2v \quad (1-24)$$

where d is the internal diameter of the pipe, m .

With a given rate of flow Q_v and adopted velocity v , the diameter of a pipe is determined by the equation:

$$d = \sqrt{\frac{Q_v}{0.785v}} \quad (1-25)$$

This equation was used to construct a nomogram (Fig. A-4).

The approximate values of the velocities used in calculations of intraplant piping are given in Table 1-1.

TABLE 1-1

Flow	v , m/s
Gases with natural draught	2-4
Gases at atmospheric pressure or one close to it in ventilation gas ducts and pipes	5-20
Liquids flowing by gravity	0.1-0.5
Liquids in pressure pipes	0.5-2.5
Steam at absolute pressure of:	
$\geq 4.9 \times 10^4 \text{ Pa}$ (0.5 at)	15-40
$(1.96-4.9) \times 10^4 \text{ Pa}$ (0.2-0.5 at)	40-60

The equation of continuity for an incompressible fluid flowing through a pipe with a variable section is:

$$Q_v = v_1 A_1 = v_2 A_2 = v_3 A_3 = \dots \quad (1-26)$$

11. Principal dimensionless numbers for flows through pipes and channels.

The *Reynolds number* characterizing the hydrodynamic conditions and serving as a measure of the forces of inertia and internal friction (viscosity) in a flow is:

$$Re = \frac{vd\rho}{\mu} = \frac{vd}{\nu} \quad (7)$$

where v = mean velocity of the flow, m/s

d = diameter of the pipe, m

ρ = density of the liquid, kg/m³

μ = dynamic viscosity, Pa·s

ν = kinematic viscosity, m²/s.

Flows through straight pipes are characterized by the following values of the Reynolds number:

Laminar flow	$Re < 2300$
Transition region	$2300 < Re < 10\,000$
Developed turbulent flow	$Re > 10\,000$

Flows through bent pipes (coils) have a critical value of the Reynolds number Re_{cr} that is higher than for straight pipes. It depends on the ratio d/D , where d is the internal diameter of the coil pipe, and D is the diameter of the coil turns. This relationship is shown in Fig. 1-1.

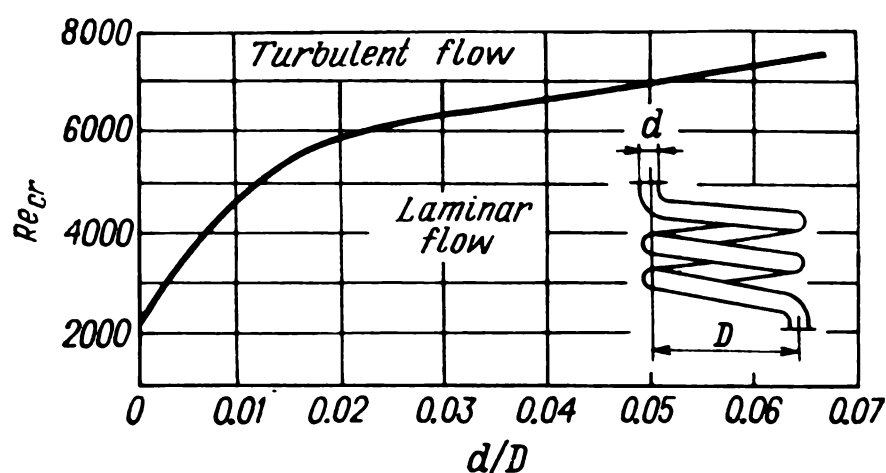


Fig. 1-1. Dependence of Re_{cr} in coils on the ratio d/D

For flows through pipes that are not round, the equivalent diameter equal to four times the hydraulic radius is introduced into the expression for Re .

The hydraulic radius (hydraulic mean depth) r_h is the ratio of the area of the flow cross section A to the wetted perimeter P

$$r_h = \frac{A}{P} \quad (1-28)$$

For a round pipe completely filled with a liquid, we have:

$$r_h = \frac{\pi d^2}{4\pi d} = \frac{d}{4} \quad (1-29)$$

Consequently, the equivalent diameter can be used instead of the diameter for flows through pipes whose cross section is not round:

$$d_{eq} = 4r_h = \frac{4A}{P} \quad (1-30)$$

The *Froude number*, which is a measure of the ratio of the forces of inertia and gravity in a flow, is: *

$$Fr = \frac{v^2}{gd} \quad (1-31)$$

where g is the acceleration due to gravity, m/s^2 .

The *Euler number*, which is a measure of the ratio of the forces of pressure and inertia in a flow, is:

$$Eu = \frac{\Delta p}{\rho v^2} \quad (1-32)$$

where Δp is the pressure difference (the loss of pressure spent for overcoming the hydraulic resistance), Pa.

12. The Bernoulli equation for a non-viscous (ideal) incompressible liquid is:

$$z_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad (1-33)$$

For a viscous (real) incompressible liquid, we have:

$$z_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} - h_1 = z_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad (1-34)$$

Here z = geometrical (position) head, m

$p/\rho g$ = piezometric (static) head, m

$v^2/2g$ = velocity (dynamic) head, m

h_1 = hydraulic loss head, m.

13. The relationship between the mean velocity v and the maximum (axial) velocity v_{max} in a pipe is:

(a) for laminar flow conditions $v = 0.5v_{max}$;

(b) for turbulent flow conditions the ratio v/v_{max} depends on the value of the Reynolds number $Re = v_{max}d\rho/\mu$ (Fig. 1-2).

We can assume for turbulent flow conditions that v approximately equals from $0.8v_{max}$ to $0.9v_{max}$, although the ratio v/v_{max} may exceed 0.9 at high values of Re .

14. The velocity of discharge of a liquid v (in m/s) from a small orifice in the bottom or a wall of a vessel with a con-

* In English publications, the Froude number is also expressed by the equation $Fr = v/(gd)^{1/2}$ — *Translator's note*.

stant level of the liquid in the vessel is calculated by the equation:

$$v = \varphi \sqrt{2gH} \quad (1-35)$$

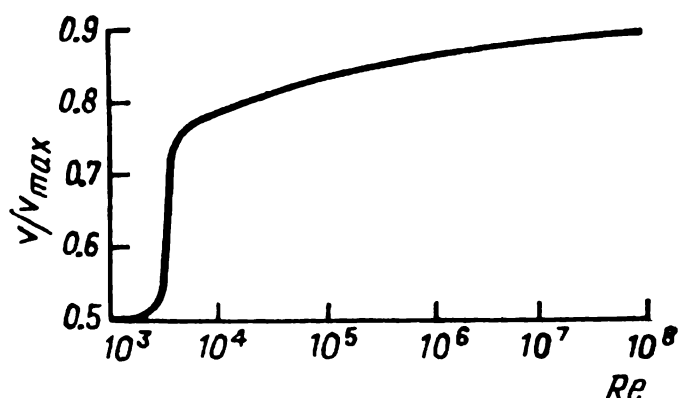
where φ = dimensionless velocity coefficient

g = acceleration due to gravity, m/s²

H = height of liquid level above centre of orifice, m.

If the pressure on the surface of the liquid in a vessel (p_0 , Pa) and the pressure in the space into which the stream is flowing

Fig. 1-2. Dependence of ratio v/v_{\max} on the Reynolds number Re



out of the vessel (p , Pa) are different, then the quantity $H' = H + (p_0 - p)/\rho g$, where ρ is the density of the flowing out liquid, kg/m³, should be substituted in Eq. (1-35) for H .

The volumetric rate of flow of a liquid Q_v (in m³/s) flowing through an orifice with the area A_o (in m²) at a constant level of the liquid in the vessel and at $p_0 = p$ is

$$Q_v = C_d A_o \sqrt{2gH} \quad (1-36)$$

where C_d is a dimensionless discharge coefficient. It is the product of the velocity coefficient φ and the coefficient of contraction κ :

$$C_d = \varphi \kappa \quad (1-37)$$

15. The time τ (in s) needed for emptying an open vessel with a constant cross-sectional area A through an orifice with the area A_o can be calculated by the equation:

$$\tau = \frac{2A\sqrt{H}}{C_d A_o \sqrt{2g}} \quad (1-38)$$

where H is the initial level of the liquid over the orifice, m.

16. **Measurement of the rate of flow of a fluid using a normal measuring orifice plate (Fig. 1-3).**

The volumetric rate of flow of a fluid Q_v (in m³/s):

$$Q_v = C_d k A_o \sqrt{2 \frac{\Delta p}{\rho}} = C_d k A_o \sqrt{2gH \frac{\rho_{\text{man}} - \rho}{\rho}} \quad (1-39)$$

where C_d = discharge coefficient of the normal measuring orifice plate in a smooth (not rough) pipe (Table A-15)

k = roughness coefficient allowing for the roughness of the pipe walls (the mean values of k for pipes are given in Table A-16); for hydraulically smooth pipes $k \approx 1$)

$A_o = 0.785d_o^2$ = cross-sectional area of the measuring plate orifice, m^2

d_o = diameter of the orifice, m

H = difference between the levels of the liquid in the differential manometer connected to the orifice plate, m

ρ_{man} = density of the liquid in the differential manometer, kg/m^3

ρ = density of the fluid flowing through the pipe, kg/m^3 .

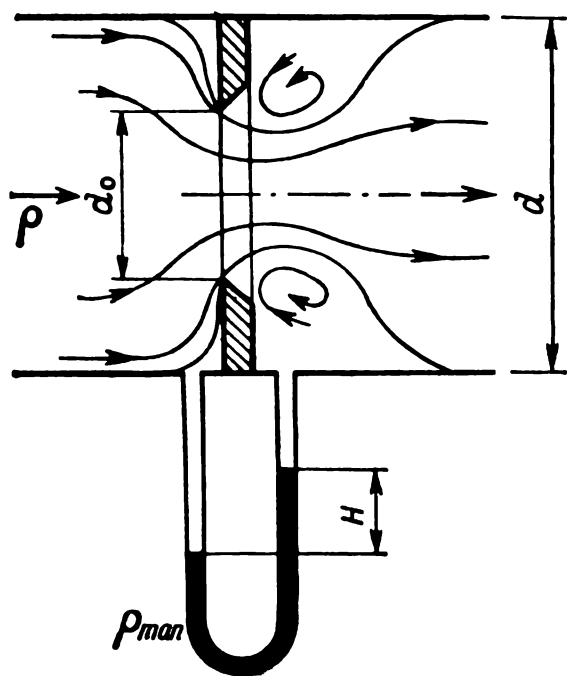


Fig. 1-3 Measuring the rate of flow with an orifice plate

Since the discharge coefficient of the measuring orifice plate C_d depends on $Re = vd/\nu$, and the value of Re is not known beforehand, then when measuring the rate of flow Q_v it is necessary to determine the mean value of C_d for the given $(d_o/d)^2$ from Table A-15. After calculating Q_v , we determine the value of Re , find the quantity C_d more precisely, and correct our calculations if necessary.

17. Measuring the rate of flow of a fluid with the aid of a Pitot-Prandtl tube (Fig. 1-4).

The Pitot-Prandtl tube is installed exactly along the axis of the pipe, and the differential manometer attached to it is used to find the value of $\Delta p = H(\rho_{man} - \rho)g = \Delta p_{vel}$. Next the maximum (axial) velocity of the flow is determined, i. e. $v_{max} = \sqrt{2gH \frac{\rho_{man} - \rho}{\rho}}$, the quantity $Re = \frac{v_{max} d \rho}{\mu}$ is calculated, and the graph in Fig. 1-2 is used to find the ratio v/v_{max} . The latter is used to calculate the mean velocity v .

The rate of flow of a fluid is determined by Eq. (1-22):

$$Q_v = vA$$

where A is the cross-sectional area of the pipe, m^2 .

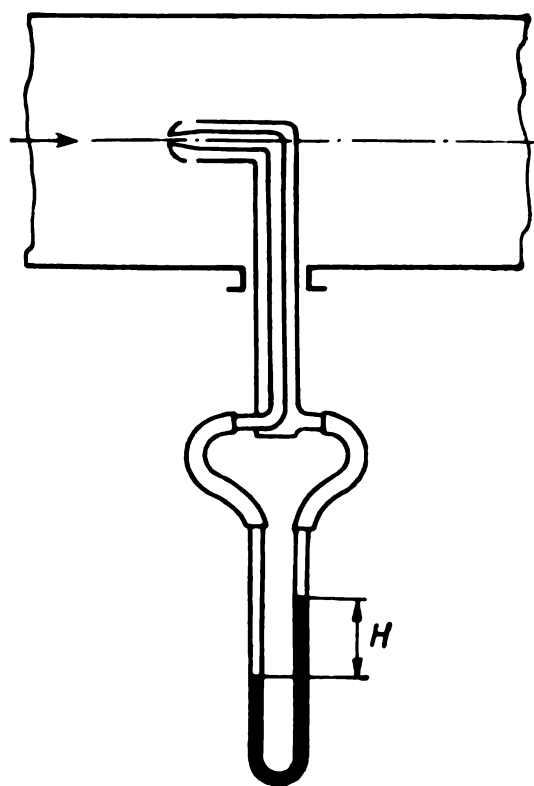


Fig. 1-4. Measuring the velocity pressure with a Pitot-Prandtl tube

18. The power P (in kW) consumed by the motor of a pump (or a fan) is calculated by the formula

$$P = \frac{Q_v \Delta p}{1000\eta} = \frac{Q_v \rho g H_p}{1000\eta} \quad (1-40)$$

where Q_v = volumetric rate of flow of the fluid, m^3/s

Δp = increase in pressure created by the pump (fan) in the flow being handled and equal to the total hydraulic resistance of the network*, Pa

H_p = head created by the pump, m

ρ = density of the fluid, kg/m^3

$g = 9.81 \text{ m/s}^2$ = acceleration due to gravity

η = overall efficiency of the pump (fan) plant — see Eq. (2-5).

For pumps handling liquids, Eq. (1-40) holds for any values of Δp , and for fans handling gases only when $\Delta p \leq 0.981 \times 10^4 \text{ Pa}$, or 0.1 at. When $\Delta p > 0.981 \times 10^4 \text{ Pa}$ for gases (blowers, compressors), the consumption of energy is calculated by thermodynamic formulas—see Chap. 2.

When using Eq. (1-40) to calculate the power consumed by a pump or fan, the quantity Δp equal to the total hydraulic resistance of the network is calculated as the sum of the following

* By a network is meant the system of pipes and apparatuses through which the pump (fan) is pumping the fluid. The pressure received by the fluid from the pump (fan) is completely used up for overcoming all the hydraulic resistances of the network.

addends:

$$\Delta p = \Delta p_{vel} + \Delta p_{fr} + \Delta p_{lr} + \Delta p_{lift} + \Delta p_{add} \quad (1-41)$$

where Δp_{vel} = expenditure of pressure for creating a velocity of the flow at the outlet from the network (the velocity in the suction space equals zero)

Δp_{fr} = loss of pressure for overcoming the friction resistance in the pipes

Δp_{lr} = loss of pressure for overcoming local resistances

$\Delta p_{lift} = \rho g h_{lift}$ = expenditure of pressure for lifting the fluid

$\Delta p_{add} = p_2 - p_1$ = difference between pressures in the delivery space (p_2) and in the suction space (p_1).

19. The expenditure of pressure for creating a velocity of flow is:

$$\Delta p_{vel} = \frac{v^2 \rho}{2} \quad (1-42)$$

where v = velocity of the flow in a pipe, m/s

ρ = density of the fluid, kg/m³.

20. The loss of pressure for overcoming friction resistance in straight pipes and channels is found as follows:

A. Isothermal flow

The temperature of the fluid flowing through a pipe is constant. The formula used for calculations is

$$\Delta p_{fr} = \lambda \frac{L}{d_{eq}} \frac{v^2 \rho}{2} \quad (1-43)$$

or in the dimensionless form (for a given roughness of the pipe wall)

$$Eu = C Re^m S_L \quad (1-44)$$

In Eqs. (1-43) and (1-44) the following notation is used:

λ = dimensionless pipe resistance coefficient * (its value in the general case depends on the flow conditions and on the roughness of the pipe wall e)

d_{eq} = equivalent diameter, m (for round pipes $d_{eq} = d$)

L = length of pipe, m

v = velocity of flow, m/s

ρ = density of fluid, kg/m³

$$Eu = \frac{\Delta p_{fr}}{\rho v^2}$$

$$Re = \frac{v d_{eq} \rho}{\mu}$$

$$S_L = \frac{L}{d_{eq}}$$

* In British and American technical publications, the coefficient $4F$ is customarily used instead of the pipe resistance coefficient λ .

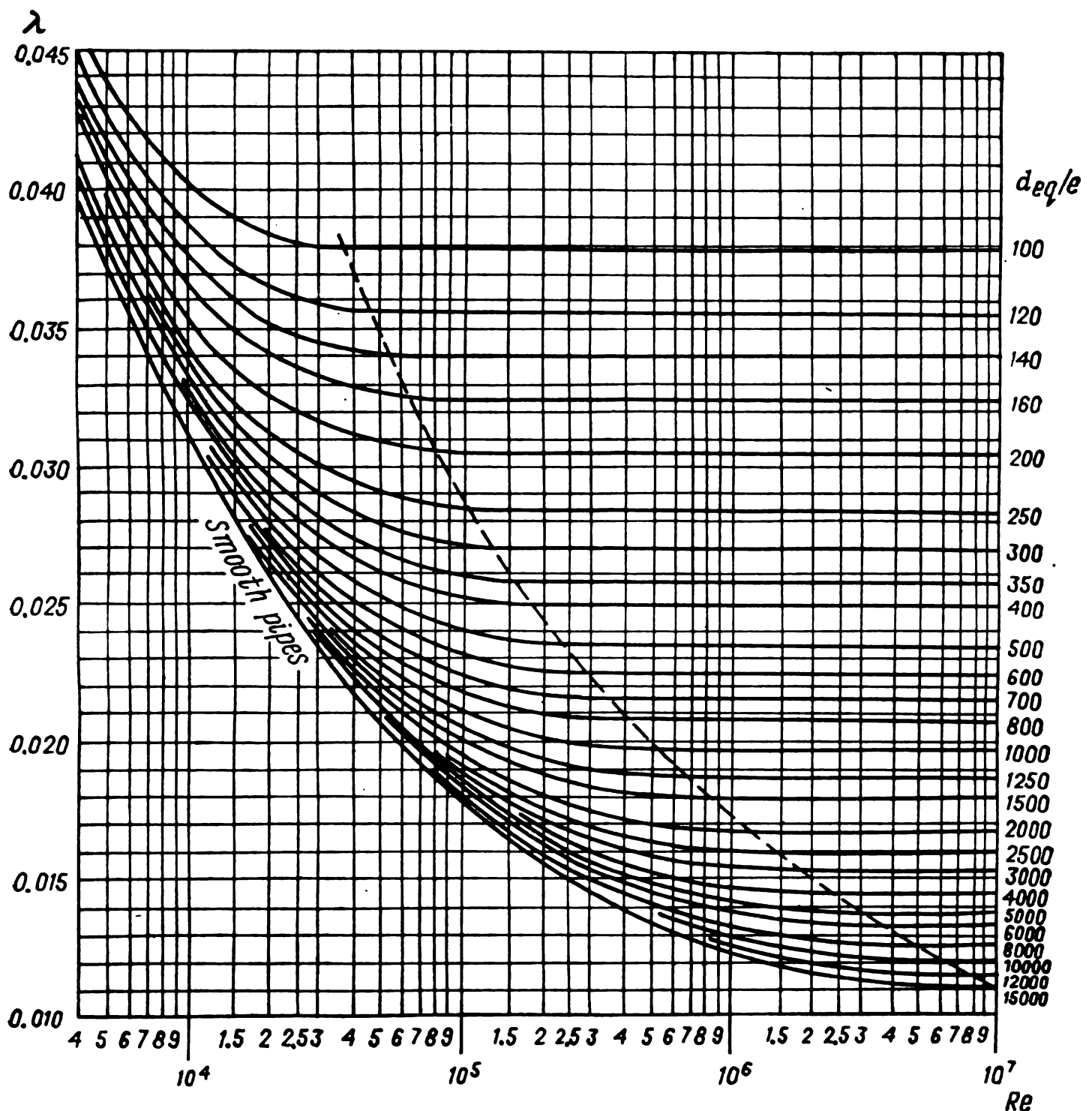


Fig. 1-5. Dependence of the pipe resistance coefficient λ on the Reynolds number Re and the degree of roughness d_{eq}/e :

d_{eq} = equivalent diameter, m; e = average height of roughness projections on inner surface of pipe, m

The value of the pipe resistance coefficient λ is determined according to the data of Table A-12 and from Figs. 1-5* and 1-6, or using the formulas given below.

1. Laminar flow ($Re < 2300$)

The pipe resistance coefficient λ does not depend on the roughness of the wall of a pipe, but depends only on Re :

* The dash line in Fig. 1-5 shows the boundary of the so-called self-similar region in which the pipe resistance coefficient λ does not depend on the Reynolds number Re and is determined only by the value of the ratio d_{3p}/e .

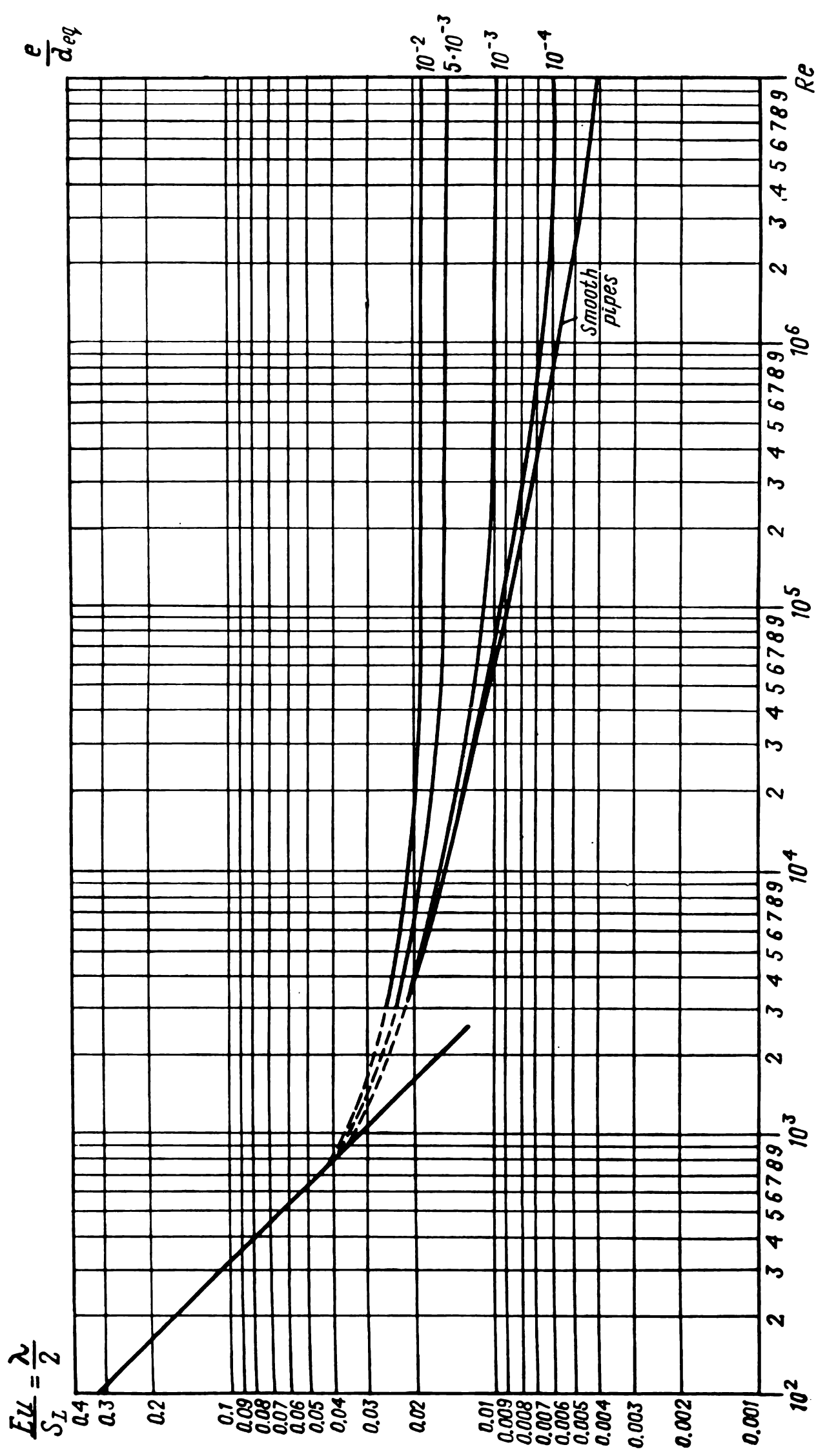


Fig. 1-6. Dependence of the ratio $\frac{Eu}{S_L} = \frac{\lambda}{2}$ on the Reynolds number Re and the relative roughness e/d_{eq}

for round pipes

$$\lambda = \frac{64}{Re} \quad (1-45)$$

for conduits that are not round

$$\lambda = \frac{A}{Re} \quad (1-46)$$

The values of A for sections having various shapes are given in Table A-14.

For the isothermal laminar flow of fluids through pipes, the loss of pressure due to friction can also be calculated according to the Hagen-Poiseuille law:

$$\Delta p_{fr} = 32 \frac{v\mu L}{d^2} \quad (1-47)$$

II. Turbulent flow ($Re > 2300$)

1. Hydraulically smooth pipes (made of glass, copper, lead):

$$\lambda = \frac{0.316}{Re^{0.25}} \quad (1-48)$$

Equation (1-48) holds for $Re < 100\,000$.

2. Hydraulically rough pipes (made of steel, iron).

A dimensionless geometrical characteristic of hydraulically rough pipes, apart from the ratio L/d_{eq} , is the relative roughness, i. e. the ratio of the average height of the projections (e) on the walls of a pipe to its equivalent diameter d_{eq} :

$$\varepsilon = \frac{e}{d_{eq}} \quad (1-49)$$

The reciprocal of this quantity (d_{eq}/e) is also used.

The approximate average values of the roughness of pipes e (in mm) are given in Table A-12.

The following formula is used for calculating the pipe resistance coefficient λ in rough pipes:

$$\frac{1}{\sqrt{\lambda}} = -2 \log \left[\frac{\varepsilon}{3.7} + \left(\frac{6.81}{Re} \right)^{0.9} \right] \quad (1-50)$$

This equation also holds for the self-similar region if the second addend in the brackets is equated to zero.

B. Non-isothermal flow

In non-isothermal flow, when the liquid flowing through a pipe becomes heated or cooled (the temperature of the pipe wall differs from that of the liquid), the right-hand sides of Eqs. (1-45), (1-46), and (1-48) should be multiplied by the dimensionless correction factor x [4-18]:

for laminar conditions:

$$x = \left(\frac{Pr_w}{Pr_{lq}} \right)^{1/3} \left[1 + 0.22 \left(\frac{Gr_{lq} Pr_{lq}}{Re_{lq}} \right)^{0.15} \right] \quad (1-51)$$

for turbulent conditions in hydraulically smooth pipes:

$$x = \left(\frac{Pr_w}{Pr_{lq}} \right)^{1/3} \quad (1-52)$$

where Re_{lq} , Pr_{lq} , Gr_{lq} = Reynolds, Prandtl, and Grashof numbers [see Eqs. (4-13), (4-12), and (4-15)] calculated for the average temperature of the liquid

Pr_w = Prandtl number calculated for the liquid at the temperature of the pipe wall.

Since for gases, the value of the Prandtl number Pr remains virtually constant with a change in the temperature, the correction factor x for gases determined by Eq. (1-52) equals unity.

With elevation of the temperature, the values of the Prandtl number Pr decrease for dropping liquids (Fig. A-13). Hence, the correction factor x determined by Eq. (1-52) is greater than unity when a liquid becomes cooled ($t_w < t_{lq}$) and is less than unity when a liquid is heated ($t_w > t_{lq}$).

21. The loss of pressure because of friction in a bent pipe (coil) Δp_{coil} is greater than in a straight pipe Δp_{str} :

$$\Delta p_{coil} = \Delta p_{str} \psi \quad (1-53)$$

The dimensionless correction factor $\psi > 1$ is calculated by the formula

$$\psi = 1 + 3.54 \frac{d}{D} \quad (1-54)$$

where d = internal diameter of the pipe

D = diameter of a turn of the coil (see Fig. 1-1).

22. The loss of pressure for overcoming the resistance of local obstacles (bends or elbows, membranes, shut-off fittings, sudden expansions or contractions, etc.) is determined as follows.

The loss of pressure in each local obstacle is the sum of two losses: the loss due to friction and the additional loss due to the change in direction or the cross-sectional area of the flow. Since the entire length of a pipe is taken into account when calculating the pipe friction resistance Δp_{fr} (including the local resistances), then Δp_{lr} in Eq. (1-41) is the sum of these additional pressure losses.

The loss of pressure due to the resistances of local obstacles is calculated by the formula

$$\Delta p_{lr} = \sum \zeta \frac{v^2 \rho}{2} \quad (1-55)$$

where ζ is a dimensionless local resistance coefficient (its values for selected local obstacles are given in Table A-13).

Sometimes a different way of calculating is used when the loss of pressure in a local obstacle is equated to the hydraulic resistance of a straight pipe of an equivalent length:

$$\Delta p_{lr} = \lambda \frac{L_{eq}}{d} \frac{\rho v^2}{2} = \lambda n \frac{\rho v^2}{2}$$
 (1-56)

where $L_{eq} = nd$ = equivalent length of a straight pipe having the same hydraulic resistance as the given local obstacles
 n = equivalent resistance (dimensionless) whose values are given in Table 1-2.

TABLE 1-2. EQUIVALENT RESISTANCE OF SCREWED FITTINGS AND VALVES *

Fittings	Equivalent resistance, n
45° elbows	15
90° elbows (standard-radius)	32
90° square elbows	60
180° close return bends	75
Tees (used as elbow, entering run)	60
Tees (used as elbow, entering branch)	90
Couplings	Negligible
Unions	Negligible
Gate valves (open)	7
Globe valves (open)	300
Angle valves (open)	170

* From Perry, J. H. (ed.) *Chemical Engineer's Handbook*, 3rd ed., p. 390, New York, McGraw-Hill (1950).

23. If a network consists of piping having a constant cross section, then the velocity of the fluid is constant along the length of the piping, and the total hydraulic resistance of the network Δp_n according to Eq. (1-41) is

$$\Delta p_n = \frac{\rho v^2}{2} \left(1 + \frac{\lambda L}{d_{eq}} + \sum \zeta \right) + \rho g h_{lft} + (p_2 - p_1)$$
 (1-57)

or using the second way of calculating the losses due to local obstacles:

$$\Delta p_n = \frac{\rho v^2}{2} \left[1 + \frac{\lambda (L + \sum L_{eq})}{d_{eq}} \right] + \rho g h_{lft} + (p_2 - p_1)$$
 (1-58)

24. The hydraulic resistance of a bank of pipes or tubes with the lateral flow of a stream around them (Fig. 1-7) is found as follows:

Square pitch:

$$Eu = b (3 + 4.5m) \left(\frac{s_1}{d} \right)^{-0.23} Re^{-0.26}$$
 (1-59)

Triangular pitch:

at $\frac{s_1}{d} < \frac{s_2}{d}$

$$Eu = b(2 + 3.3m) Re^{-0.28} \tag{1-60}$$

at $\frac{s_1}{d} > \frac{s_2}{d}$

$$Eu = b(2.7 + 1.7m) Re^{-0.28} \tag{1-61}$$

In the above equations the following notation is used:
 b = correction factor depending on the angle of attack
 φ (the angle between the axis of the pipe and the

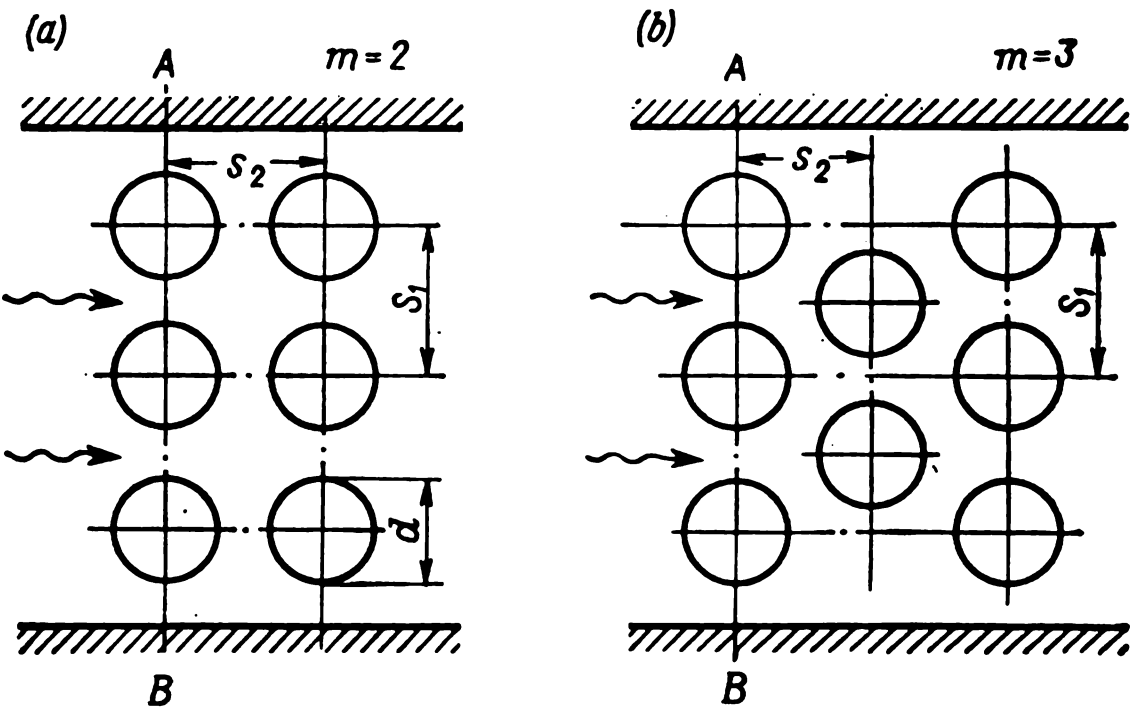


Fig. 1-7. Tube bank arrangements:
(a) square pitch; (b) triangular pitch

direction of the flow — see Fig. 4-3), determined from Table 1-3;
 m = number of pipe rows in a bank in the direction of the flow
 d = external diameter of a pipe
 s_1 and s_2 = lateral and longitudinal pipe pitches.

TABLE 1-3

φ , deg	90	80	70	60	50	40	30	10
b	1	1	0.95	0.83	0.69	0.53	0.38	0.15

The flow velocity is calculated for the narrowest section of a bank AB ; the values of the physicochemical constants are taken

for the mean temperature of the flow. The Reynolds number Re is calculated according to the external diameter of a pipe.

25. Hydraulic resistance of shell-and-tube heat exchangers.

For the tube space (tube side) of a heat exchanger, and also for the intertubular space (shell side) without lateral baffles (see Fig. 1-22), we have:

$$\Delta p = \lambda \frac{nL}{d_{eq}} \frac{\rho v^2}{2} + \sum \zeta \frac{\rho v^2}{2} \tag{1-62}$$

where L = length of one pass, m
 n = number of passes.

The remaining symbols are the same as in the preceding equations.

The following values are adopted for the coefficients of local resistances ζ in shell-and-tube heat exchangers:

Tube side	
Inlet or outlet header	1.5
180° bend between passes or sections	2.5
Tube inlet or outlet	1.0
Shell side	
Shell side inlet or outlet	1.5
180° bend through baffle in shell side	1.5
90° bend in shell side	1.0

When the shell side has baffles (see Fig. 4-4), the hydraulic resistance in it is determined by Eqs. (1-59)-(1-61)*.

The velocities of a fluid in pipe connections are usually close to those in the tubes or on the shell side. If the velocity in the pipe connections is greater, then the losses of pressure at the inlet to a heat exchanger and at the outlet from it are calculated according to the velocities in the connections.

26. Hydraulic resistance of scrubber packings.

The resistance of a layer of a dry (non-irrigated) packing with the height H is:

$$\Delta p_{dry} = \lambda \frac{H}{d_{eq}} \frac{v_g^2 \rho}{2} \tag{1-64}$$

where λ = dimensionless resistance coefficient when a gas passes through a layer of the packing

H = height of packing layer, m

d_{eq} = equivalent diameter, m

v_g = velocity of the gas through the voids of the packing (actual), m/s

ρ = density of the gas, kg/m³.

* The following formula is also used [0-20]:

$$\zeta = \frac{3m}{Re^{0.2}} \tag{1-63}$$

where m is the number of tube rows in the direction of the flow.

The equivalent diameter can be expressed through the characteristics of the packing—its free volume V_{free} in m^3 of voids per m^3 of packed volume (numerically equal to the unit void area in a column section in m^2/m^3) and the unit surface area σ in m^2 of dry packing per m^3 of packed volume:

$$d_{\text{eq}} = \frac{4V_{\text{free}}}{\sigma} \quad (1-65)$$

The actual velocity of a gas v_g is found through the fictitious velocity v (related to the total cross section of the apparatus) by the equation

$$v_g = \frac{v}{V_{\text{free}}} \quad (1-66)$$

According to N. Zhavoronkov, for a randomly dumped packing of rings:

$$\text{at } Re_g < 40 \text{ we have } \lambda = \frac{140}{Re_g} \quad (1-67)$$

$$\text{at } Re_g > 40 \text{ we have } \lambda = \frac{16}{Re_g^{0.2}} \quad (1-68)$$

$$\text{where } Re_g = \frac{v_g d_{\text{eq}} \rho}{\mu} = \frac{4v\rho}{\sigma\mu}.$$

The calculations of the hydraulic resistance of irrigated packings are more complicated—see the formulas and graphs given in [6-3 and 6-5].

27. Hydraulic resistance of plate column apparatuses.

The resistance of a bubble plate Δp is computed as the sum of three addends:

$$\Delta p = \Delta p_{\text{dry}} + \Delta p_{\sigma} + \Delta p_{\text{g-l}} \quad (1-69)$$

where Δp_{dry} = resistance of a dry plate

Δp_{σ} = resistance due to the forces of surface tension

$\Delta p_{\text{g-l}}$ = resistance of the froth layer on a plate.

The resistance of a dry plate is

$$\Delta p_{\text{dry}} = \zeta \frac{v_h^2 \rho_g}{2} \quad (1-70)$$

where v_h = velocity of the gas in the slots of the cap or the holes of a plate, m/s

ρ_g = density of the gas, kg/m^3

ζ = resistance coefficient equal to:

for bubble-cap plates	4.5 to 5.0
for perforated (sieve) plates	
with a free section of the holes of 7 to 10%	1.82
ditto, 11 to 25%	1.45
for grid trays	1.4 to 1.5

The resistance due to the forces of surface tension is

$$\Delta p_{\sigma} = \frac{4\sigma}{d_{eq}} \quad (1-71)$$

where σ = surface tension, N/m

d_{eq} = equivalent diameter of a hole, m; for bubble-cap plates $d_{eq} = 4A/P$ (A is the area of the clear cross section of a slot, P is the perimeter of the slot), for sieve and perforated plates and trays d_{eq} equals the diameter of a hole, and for grid trays—the double width of a slot.

The resistance of the froth layer (at the same velocities of the gas that are used in plate mass-exchange columns—see Chaps. 6 and 7) is determined as follows:

(a) on a bubble-cap plate

$$\Delta p_{g-lq} = 1.3k\rho_{lq} \left(L + \frac{e}{2} + \Delta h \right) g \quad (1-72)$$

where g = acceleration due to gravity, m^2/s

k = relative density of the froth layer (foam); it is assumed that $k \approx 0.5$

ρ_{lq} = density of the liquid, kg/m^3

L = distance from the top edge of the slots to the weir crest, m (Fig. 1-8)

e = height of a slot, m

Δh = height of the liquid level above the weir crest, m

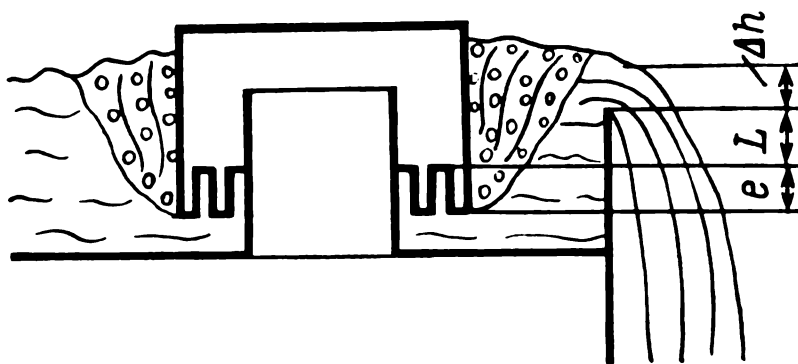


Fig. 1-8. Schematic view of bubble plate

(b) on a sieve plate

$$\Delta p_{g-lq} = 1.3gk\rho_{lq} (h_w + \Delta h) \quad (1-73)$$

where h_w is the height of the weir crest, m (see Fig. 7-18).

The quantity Δh is determined by the formula for outflow over a weir with account taken of the density of the froth:

$$\Delta h = \left(\frac{Q_{v, lq}}{1.85Pk} \right)^{2/3} \quad (1-74)$$

where $Q_{v,lg}$ = volumetric rate of flow of the liquid, m^3/s
 P = weir perimeter, m
 $k = 0.5$ (see above).

EXAMPLES

Example 1-1. The relative density of petroleum is 0.89. Determine the density of the petroleum in the SI and MK(force)S systems of units.

Solution. (1) SI units. By Eq. (1-2), $\Delta = \rho/\rho_w$. Hence

$$\rho = \Delta \rho_w = 0.89 \times 1000 = 890 \text{ kg/m}^3$$

(2) MK(force)S units. We have $\gamma = \Delta \gamma_w = 0.89 \times 1000 = 890 \text{ kgf/m}^3$. By Eq. (1-1)

$$\rho = \frac{\gamma}{g} = \frac{890}{9.81} = 90.6 \text{ kgf} \cdot \text{s}^2/\text{m}^4$$

Example 1-2. Determine the density of nitrogen dioxide in SI units at $p_{\text{gauge}} = 10 \text{ at}$ and $t = 20^\circ\text{C}$. Atmospheric pressure is 760 mm Hg (1.03 at).

Solution. By Eq. (1-5), we have

$$\rho = \frac{M}{22.4} \frac{273p}{T p^\circ} = \frac{46 \times 273 \times 11.03}{22.4 \times 293 \times 1.03} = 20.5 \text{ kg/m}^3$$

Example 1-3. Determine the density of air under a vacuum of 440 mm Hg at -40°C . Atmospheric pressure is 750 mm Hg.

Solution. The molar mass of air (79% of nitrogen and 21% of oxygen by volume) is $M = 0.79 \times 28 + 0.21 \times 32 = 28.8 \text{ kg/kmol}$. By Eq. (1-5), we have

$$\rho = \frac{M}{22.4} \frac{273p}{T p^\circ} = \frac{28.8 \times 273 (750 - 440)}{22.4 \times 233 \times 760} = 0.615 \text{ kg/m}^3$$

Example 1-4. Air having an average temperature of 50°C and a pressure (according to a manometer) of 2 at flows with a velocity of 9 m/s along the tubes of a single-pass shell-and-tube heat exchanger (the number of tubes $n = 121$, the external diameter of a tube is 38 mm, the thickness of its wall is 2 mm). The barometric pressure is 740 mm Hg. Find: (a) the mass rate of flow of the air; (b) the volumetric rate of flow of the air in operating conditions; (c) the volumetric rate of flow of the air in standard conditions.

Solution. The density of air in standard conditions is $\rho^\circ = 1.293 \text{ kg/m}^3$ (Table A-5).

The operating pressure (absolute) is:

$$p = p_{\text{bar}} + p_{\text{man}} = 740 \times 133.3 + 98\,100 \times 2 = 294\,800 \text{ Pa}$$

or

$$p = p_{\text{bar}} + p_{\text{man}} = 740 + 735 \times 2 = 2210 \text{ mm Hg}$$

For the relationships between the units, see Table A-58.

The density of the air in operating conditions is:

$$\rho = \rho^\circ \frac{p T^\circ}{p^\circ T} = 1.293 \times \frac{294\,800 \times 273}{101\,300 (273 + 50)} = 3.18 \text{ kg/m}^3$$

or

$$\rho = \rho^\circ \frac{p T^\circ}{p^\circ T} = 1.293 \times \frac{2210 \times 273}{760 (273 + 50)} = 3.18 \text{ kg/m}^3$$

The mass rate of flow of the air by Eq. (1-23) is:

$$\begin{aligned} Q_m &= Q_v \rho = v A \rho = v n \times 0.785 d^2 \rho = \\ &= 9 \times 121 \times 0.785 \times 0.034^2 \times 3.18 = 3.14 \text{ kg/s} \end{aligned}$$

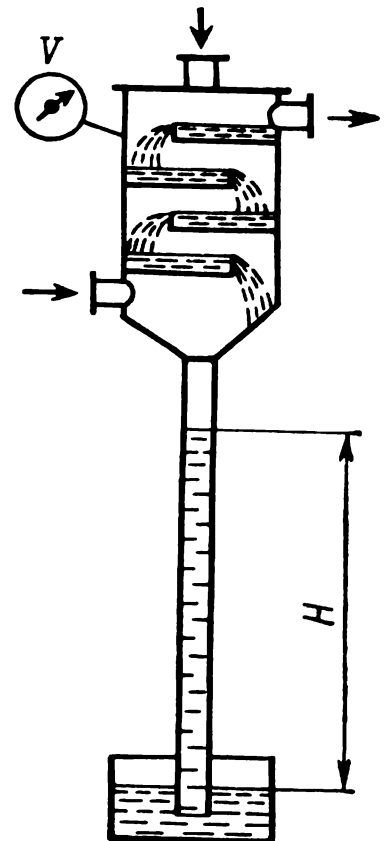


Fig. 1-9. To Example 1-6

The volumetric rate of flow of the air in operating conditions is:

$$Q_v = \frac{Q_m}{\rho} = \frac{3.14}{3.18} = 0.99 \text{ m}^3/\text{s}$$

The volumetric rate of flow of the air reduced to standard conditions is:

$$Q_v^\circ = \frac{Q_m}{\rho^\circ} = \frac{3.14}{1.293} = 2.43 \text{ m}^3/\text{s}$$

Example 1-5. An open reservoir contains a liquid having a relative density of 1.23. A manometer connected to a certain point of the reservoir wall shows a pressure of $p_{\text{gauge}} = 0.31 \text{ at}$. At what height above the given point is the level of the liquid in the reservoir?

Solution. The height of the level of the liquid in the reservoir above the point of connection of the manometer is determined with the aid of Eq. (1-8):

$$p = p_0 + \rho gh, \text{ whence } h = \frac{p - p_0}{\rho g}$$

According to the initial conditions $p - p_0 = 0.31 \text{ at} = 0.31 \times 10^4 \times 9.81 \text{ Pa}$.

The density of the liquid is $\rho = 1.23 \times 1000 = 1230 \text{ kg/m}^3$.

Hence

$$h = \frac{0.31 \times 10^4 \times 9.81}{1230 \times 9.81} = 2.52 \text{ m}$$

Example 1-6. A vacuum gauge on a barometric condenser shows a vacuum of 600 mm Hg. The barometric pressure is 748 mm Hg. Determine: (a) the absolute pressure in the condenser in Pa and in at, (b) the height H which water rises to in the barometric tube (Fig. 1-9).

Solution. The absolute pressure in the condenser is:

$$\begin{aligned} p &= 748 - 600 = 148 \text{ mm Hg} = 148 \times 133.3 = 19\,700 \text{ Pa} = \\ &= \frac{19\,700}{9.81 \times 10^4} = 0.201 \text{ at} \end{aligned}$$

We find the height of the column of water in the barometric tube from the equation

$$p + H\rho g = p_{\text{bar}}$$

whence

$$H = \frac{p_{\text{bar}} - p}{\rho g} = \frac{600 \times 133.3}{1000 \times 9.81} = 8.16 \text{ m}$$

Example 1-7. A U-shaped glass differential manometer filled with mercury is connected to two points of a horizontal pipe. The difference between the levels of the mercury in the limbs of the manometer is $h = 26 \text{ mm}$. What is the difference between the pressures at these points if the fluid flowing through the pipe is: (a) water, and (b) air at 20°C and atmospheric pressure?

Solution. From the condition of the equality of the pressures at the level $a-a$ in the left- and right-hand limbs (Fig. 1-10), we have:

$$p_1 + h_1\rho g = p_2 + h_2\rho g + h\rho_{\text{man}}g$$

Substituting $h_1 - h$ for h_2 , we get

$$p_1 - p_2 = h(\rho_{\text{man}} - \rho)g$$

where ρ = density of the liquid in the pipe (and in the connecting pipes)

ρ_{man} = density of the liquid in the differential manometer.

According to the initial conditions, we have $h = 0.026 \text{ m}$, and $\rho_{\text{man}} = 13\,600 \text{ kg/m}^3$.

(a) For water, $\rho = 1000 \text{ kg/m}^3$, and

$$p_1 - p_2 = 0.026(13\,600 - 1000)9.81 = 3220 \text{ Pa}$$

(b) For air, $\rho = \frac{29 \times 273}{22.4 \times 293} = 1.2 \text{ kg/m}^3$, and

$$p_1 - p_2 = 0.026 (13\,600 - 1.2) 9.81 \approx 0.026 \times 13\,600 \times 9.81 = 3470 \text{ Pa}$$

It follows from the last equation that when measuring the pressure difference in gas flows with the aid of liquid differential

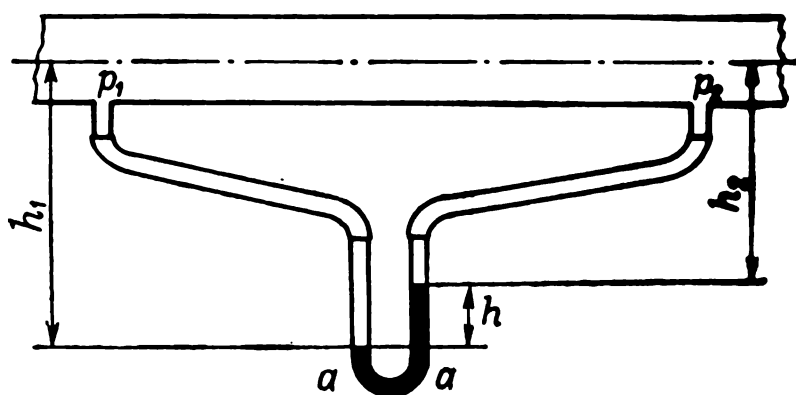


Fig. 1-10. To Example 1-7

manometers, the correction for the density of the gas may be disregarded because the density of the gas (at pressures close to atmospheric) is very small in comparison with that of the liquid.

Example 1-8. The bell of a wet gas-holder for nitrogen having a diameter of 6 m weighs 2900 kgf with its additional ballast (Fig. 1-11). Ignoring the loss in weight of the part of the bell submerged in the water, determine the gauge pressure of the gas in the filled gas-holder.

Solution. The area of the horizontal projection of the bell is

$$\frac{1}{4} \pi D^2 = 0.785 \left(\frac{6000}{1000} \right)^2 = 28.2 \text{ m}^2.$$

The pressure in the gas-holder is:

$$p_{\text{gauge}} = \frac{2900 \times 9.81}{28.2} = 1010 \text{ Pa} = \frac{1010}{9.81 \times 10^4} \approx 0.01 \text{ at}$$

Example 1-9. Determine the kinematic viscosity of carbon dioxide at $t = 30^\circ\text{C}$ and $p_{\text{abs}} = 5.28 \text{ at}$.

Solution. Ignoring the dependence of the dynamic viscosity on the pressure, we use the graph in Fig. A-6 to find its value for carbon dioxide at 30°C : $\mu = 0.015 \text{ cP} = 0.015 \times 10^{-3} \text{ Pa}\cdot\text{s}$.

We find the density of carbon dioxide:

$$\rho = \frac{44 \times 273 \times 5.28}{22.4 \times 303 \times 1.033} = 9.05 \text{ kg/m}^3$$

The kinematic viscosity is:

$$\nu = \frac{\mu}{\rho} = \frac{0.015 \times 10^{-3}}{9.05} = 1.66 \times 10^{-6} \text{ m}^2/\text{s}$$

Example 1-10. Assuming that experimental data are absent, calculate approximately the dynamic viscosity of nitrobenzene ($\text{C}_6\text{H}_5\text{NO}_2$) at 20°C .

Solution. We shall use the empirical equation (1-11). The molar mass of nitrobenzene is 123 kg/kmol, and its density is 1200 kg/m³ (Table A-3).

We calculate the sum of the atomic constants $\sum A_n$ (Table A-9):
 $\sum A_n = 6 \times 50.2 + 5 \times 2.7 + 1 \times 37 + 2 \times 29.7 = 411.1$

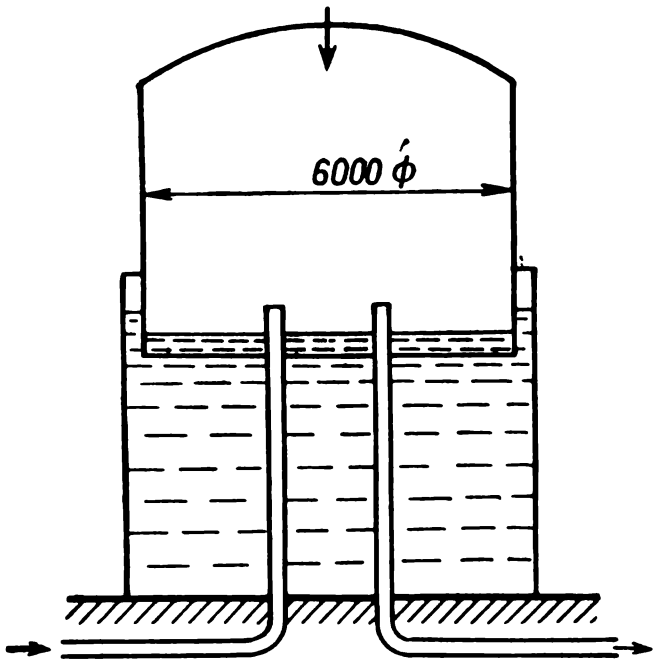


Fig. 1-11. To Example 1-8 (dimensions in figures are in millimetres unless otherwise indicated)

We calculate the sum of the corrections for structure $\sum B$ according to items 1, 3, 4, and 16 of Table A-10:

$$\sum B = 3(-15.5) + 1(-21.0) + 1(-17.0) + 1(-16.4) = -100.9$$

Hence by Eq. (1-11)

$$\begin{aligned} \log(\log \mu) &= (\sum A_n + \sum B) \frac{\rho}{10^3 M} - 2.9 = \\ &= (411.1 - 100.9) \frac{1200}{10^3 \times 123} - 2.9 = 0.126 \end{aligned}$$

According to the nomogram in Fig. A-1, we find $\mu = 2.15 \text{ cP} = 2.15 \times 10^{-3} \text{ Pa}\cdot\text{s}$, which coincides with experimental data (Fig. A-5).

Example 1-11. Determine the dynamic viscosity of flue gases having the following composition: CO₂—16%, O₂—5%, N₂—79% (by volume). The temperature of the gases is 400°C and their pressure is $p_{\text{abs}} = 1 \text{ at}$.

Solution. We find the dynamic viscosities for the components of the mixture from the nomogram in Fig. A-6. For 400°C, we get $\mu = 0.035 \text{ cP}$ (CO₂), $\mu = 0.039 \text{ cP}$ (O₂), and $\mu = 0.0335 \text{ cP}$ (N₂).

We calculate the dynamic viscosity of the mixture by Eq. (1-12):

$$\frac{M_{\text{mix}}}{\mu_{\text{mix}}} = \frac{0.16 \times 44}{0.035} + \frac{0.05 \times 32}{0.039} + \frac{0.79 \times 28}{0.0335} = 902$$

The molar mass of the mixture is:

$$M_{\text{mix}} = 0.16 \times 44 + 0.05 \times 32 + 0.79 \times 28 = 30.8 \text{ kg/kmol}$$

The dynamic viscosity of the mixture is:

$$\mu_{\text{mix}} = \frac{30.8}{902} = 0.034 \text{ cP} = 0.034 \times 10^{-3} \text{ Pa} \cdot \text{s}$$

Example 1-12. Determine the kinematic viscosity of a liquid having the composition: 70 mol.% of oxygen and 30 mol.% of nitrogen at 84 K and $p_{\text{abs}} = 1 \text{ at}$.

Solution. Considering that liquid oxygen and nitrogen belong to the class of Newtonian liquids, we calculate the dynamic viscosity of the mixture using Eq. (1-15).

At 84 K for liquid oxygen, $\mu_1 = 22.6 \times 10^{-5} \text{ Pa} \cdot \text{s}$, and for liquid nitrogen $\mu_2 = 11.8 \times 10^{-5} \text{ Pa} \cdot \text{s}$. Hence

$$\log \mu_{\text{mix}} = x_1 \log \mu_1 + x_2 \log \mu_2 = 0.7 \log (22.6 \times 10^{-5}) + 0.3 \log (11.8 \times 10^{-5}) = 6.2694$$

whence $\mu_{\text{mix}} = 18.6 \times 10^{-5} \text{ Pa} \cdot \text{s}$.

The mass fractions of the components in the mixture are

$$\bar{x}_1 = \frac{0.7 \times 32}{0.7 \times 32 + 0.3 \times 28} = 0.727 \text{ and } \bar{x}_2 = \frac{0.3 \times 28}{0.7 \times 32 + 0.3 \times 28} = 0.273$$

The densities of liquid oxygen and nitrogen are $\rho_1 = 1180 \text{ kg/m}^3$ and $\rho_2 = 780 \text{ kg/m}^3$, respectively.

The density of the mixture by Eq. (1-3) is:

$$\rho_{\text{mix}} = \frac{1}{\frac{0.727}{1180} + \frac{0.273}{780}} = 1030 \text{ kg/m}^3$$

The kinematic viscosity of the mixture is:

$$\nu_{\text{mix}} = \frac{\mu_{\text{mix}}}{\rho_{\text{mix}}} = \frac{18.6 \times 10^{-5}}{1030} = 0.18 \times 10^{-6} \text{ m}^2/\text{s}$$

Example 1-13. Calculate the dynamic viscosity of a suspension of benzidine in water if one ton of benzidine is charged into a tank per 10 m^3 of water. The temperature of the suspension is 20°C , and the relative density of the solid phase is 1.2.

Solution. The volume of the solid phase is $V = \frac{G}{\rho} = \frac{1000}{1200} = 0.833 \text{ m}^3$.

The volume concentration of the solid phase in the suspension is $\varphi = \frac{0.833}{10 + 0.833} = 0.077 \text{ m}^3/\text{m}^3$.

The dynamic viscosity of water at 20°C is 1 cP (Table A-6). The dynamic viscosity of the suspension by Eq. (1-17) is

$\mu_{\text{susp}} = \mu_{\text{liq}}(1 + 2.5\varphi) = 1(1 + 2.5 \times 0.077) = 1.19 \text{ cP} = 1.19 \times 10^{-3} \text{ Pa} \cdot \text{s}$ and by Eq. (1-18)

$$\mu_{\text{susp}} = \mu_{\text{liq}} \frac{0.59}{(0.77 - \varphi)^2} = \frac{1 \times 0.59}{(0.77 - 0.077)^2} = 1.23 \text{ cP} = 1.23 \times 10^{-3} \text{ Pa} \cdot \text{s}$$

Example 1-14. It is known that the dynamic viscosity of chlorobenzene at 20 °C is 0.9 cP, and at 50 °C is 0.6 cP. Using the rule of linearity, find the dynamic viscosity of chlorobenzene at 70 °C.

Solution. We take water as the standard liquid. We find the temperatures of the standard liquid at which its dynamic viscosity is 0.9 and 0.6 cP.

For water, the dynamic viscosity of 0.9 cP corresponds to a temperature of 25 °C, and of 0.6 cP to a temperature of 45 °C (Table A-6).

Therefore, the constant determined by Eq. (1-19) is:

$$K = \frac{t_{\mu_1} - t_{\mu_2}}{\theta_{\mu_1} - \theta_{\mu_2}} = \frac{50 - 20}{45 - 25} = 1.5$$

Next we find the temperature of water at which its dynamic viscosity equals that of chlorobenzene at 70 °C from the equation

$$1.5 = \frac{70 - 20}{\theta_{\mu_3} - 25}$$

whence $\theta_{\mu_3} = 58.4$ °C.

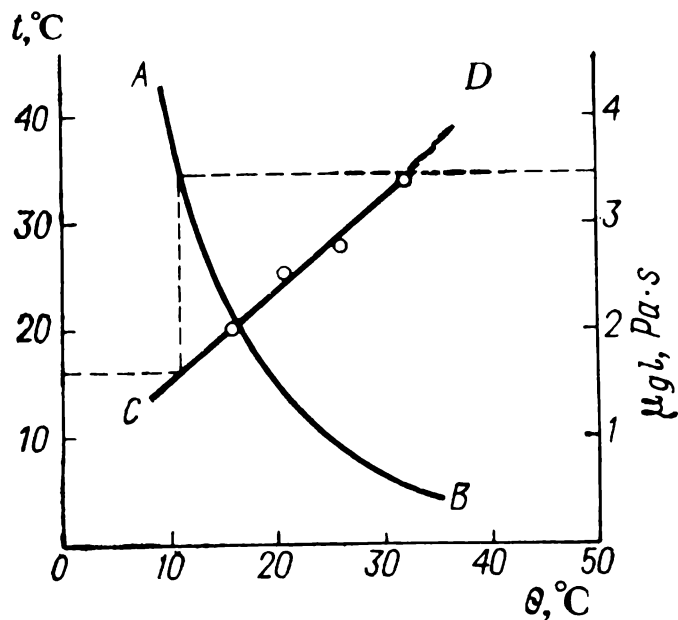


Fig. 1-12. To Example 1-15

At this temperature, we find for water that $\mu_3 = 0.48$ cP.

Consequently, the dynamic viscosity of chlorobenzene at 70 °C equals 0.48 cP.

If for purposes of comparison we revert directly to the nomogram in Fig. A-5, we find $\mu = 0.46$ cP = 0.46×10^{-3} Pa·s.

Example 1-15. The dynamic viscosity of a certain liquid is:

$t, \text{ }^\circ\text{C}$	34	28	25	20
$\mu, \text{ Pa}\cdot\text{s}$	0.554	0.934	1.42	2.09

Find the dynamic viscosity of this liquid at $t = 16$ °C.

Solution. We use the rule of linearity and take glycerine as the standard liquid. The temperature dependence of the dynamic viscosity of glycerine $\mu_{gl} = f(\theta)$ is given in Fig. 1-12 (curve AB). We use this curve to find the temperatures of glycerine θ :

$\mu_{gl}, \text{Pa}\cdot\text{s}$	0.554	0.934	1.42	2.09
$\theta, ^\circ\text{C}$	32	26	21	16

By plotting the dependence of the temperature of the liquid t on the temperature of glycerine θ at identical values of the viscosity, we get straight line CD . Next we find, as shown by the dash lines, that the required dynamic viscosity of the liquid at $t = 16^\circ\text{C}$, equal to the dynamic viscosity of glycerine at $\theta = 11^\circ\text{C}$, is $3.46 \text{ Pa}\cdot\text{s}$.

Example 1-16. A heat exchanger is made of steel tubes with a diameter of $76 \times 3 \text{ mm}$ *. A gas at atmospheric pressure flows along the tubes. Find the required diameter of the tubes for operation with the same gas, but under a pressure of $p_{\text{gauge}} = 5 \text{ at}$ if the velocity of the gas, its mass rate of flow, and the number of tubes are to remain unchanged.

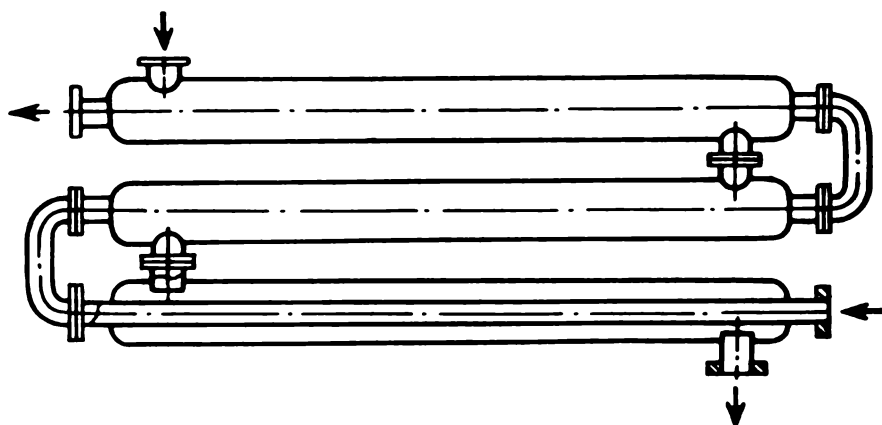


Fig. 1-13. To Example 1-17

Solution. At the pressure of $p_{\text{gauge}} = 5 \text{ at}$, the density of the gas, in accordance with Eq. (1-5), will be six times greater than at atmospheric pressure.

Since the mass rate of flow of the gas $Q_m = Q_v \rho = v A \rho$ must be retained unchanged, then

$$v_1 n_1 \times 0.785 d_1^2 \rho_1 = v_2 n_2 \times 0.785 d_2^2 \rho_2$$

Introducing $v_2 = v_1$, $n_2 = n_1$, $\rho_2 = 6\rho_1$, and $d_1 = 0.07 \text{ m}$, we get $0.07^2 = 6d_2^2$, whence

$$d_2 = \sqrt{\frac{0.07^2}{6}} = 0.0286 \text{ m} \approx 29 \text{ mm}$$

Example 1-17. Determine the conditions of flow of a liquid in the intertubular space of a heat exchanger of the "tube-in-tube" type (Fig. 1-13) if the inner tube has a diameter of $25 \times 2 \text{ mm}$, the outer tube one of $51 \times 2.5 \text{ mm}$, the mass rate of flow of the liquid is 3730 kg/h , the density of the liquid is 1150 kg/m^3 , and its dynamic viscosity is 1.2 cP .

* The first number denotes the external diameter of the tube, the second the thickness of its wall.

Solution. The velocity of the liquid from the equation for the rate of flow is

$$v = \frac{Q_v}{A} = \frac{3730}{1150 \times 3600 \times 0.785 (0.046^2 - 0.025^2)} = 0.77 \text{ m/s}$$

The equivalent diameter of an annular section by Eq. (1-30) is:

$$d_{eq} = \frac{4A}{P} = \frac{4\pi (D^2 - d^2)}{4\pi (D + d)} = D - d = 0.046 - 0.025 = 0.021 \text{ m}$$

where D = internal diameter of outer tube, m

d = external diameter of inner tube, m.

The dynamic viscosity of the liquid is $\mu = 1.2 \times 10^{-3} \text{ Pa}\cdot\text{s}$.

The Reynolds number is:

$$Re = \frac{vd\rho}{\mu} = \frac{0.77 \times 0.021 \times 1150}{1.2 \times 10^{-3}} = 15\,500$$

Consequently, the flow conditions are turbulent.

Example 1-18. Find the critical velocity in a straight pipe having a diameter of $51 \times 2.5 \text{ mm}$: (a) for air at 20°C and $p_{abs} = 1 \text{ at}$, and (b) for petroleum oil for which $\mu = 35 \text{ cP}$ and the relative density $\Delta = 0.963$.

Solution. We have the critical velocity when $Re_{cr} = 2300$. Hence from Eq. (1-27), we get $v_{cr} = \frac{2300\mu}{d\rho}$.

(a) For air we have

$$v_{cr} = \frac{2300 \times 0.018 \times 10^{-3}}{0.046 \times 1.2} = 0.75 \text{ m/s}$$

where 0.018 = dynamic viscosity of air at 20°C (according to Fig. A-6), cP

1.2 = density of air at 20°C and $p_{abs} = 1 \text{ at}$ according to Eq. (1-5), kg/m^3 .

(b) For petroleum oil we have

$$v_{cr} = \frac{2300 \times 35 \times 10^{-3}}{0.046 \times 963} = 1.8 \text{ m/s}$$

Example 1-19. A pipe with an internal diameter of 200 mm has a smooth transition to a diameter of 100 mm (Fig. 1-14). The pipe is used to deliver 1700 m^3 of methane (at standard conditions) an hour at 30°C . A U-shaped water manometer opening to the atmosphere is installed on the wide portion of the pipe before the constriction. It shows a gauge pressure in the pipe equal to $40 \text{ mm H}_2\text{O}$. What is the reading of an identical manometer installed on the narrow portion of the pipe? Disregard the resistances. The atmospheric pressure is 760 mm Hg .

Solution. Considering approximately that the density of the methane on the portion of the pipe between the points of con-

section of the manometers is constant (which we shall check at the end of our calculations), we write the Bernoulli equation for an incompressible liquid:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

whence

$$p_1 - p_2 = \frac{v_2^2 - v_1^2}{2} \rho$$

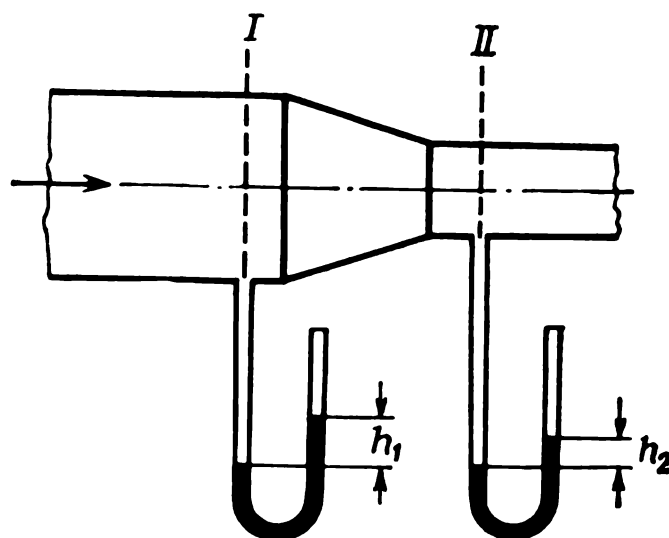


Fig. 1-14. To Example 1-19

We determine the velocities of the methane at sections I and II assuming that the pressure in the pipe approximately equals atmospheric pressure:

$$v_1 = \frac{1700 \times 303}{3600 \times 273 \times 0.785 \times 0.2^2} = 16.7 \text{ m/s}$$

According to Eq. (1-26), we have

$$v_2 = v_1 \frac{A_1}{A_2} = 16.7 \left(\frac{200}{100} \right)^2 = 66.8 \text{ m/s}$$

We calculate the density of the methane:

$$\rho = \frac{MT^\circ}{22.4T} = \frac{16 \times 273}{22.4 \times 303} = 0.645 \text{ kg/m}^3$$

We find the difference between the pressures:

$$\begin{aligned} p_1 - p_2 &= \frac{(v_2^2 - v_1^2) \rho}{2} = \frac{(66.8^2 - 16.7^2) 0.645}{2} = \\ &= 1354 \text{ Pa} = \frac{1354}{9.81} = 138 \text{ mm H}_2\text{O} \end{aligned}$$

Hence

$$p_2 = p_1 - 138 = 40 - 138 = -98 \text{ mm H}_2\text{O}$$

i.e. the pressure in section *II* will be less than in *I* by 138 mm H₂O, and the manometer in section *II* will show a vacuum of 98 mm H₂O.

The absolute pressure in section *I* is:

$$10\,330 + 40 = 10\,370 \text{ mm H}_2\text{O} = 1.037 \text{ at} = 0.1017 \text{ MPa}$$

and in section *II*:

$$10\,330 - 98 = 10\,232 \text{ mm H}_2\text{O} = 1.023 \text{ at} = 0.1003 \text{ MPa}$$

Assuming a constant pressure in the pipe when calculating the velocities and densities of methane, we tolerated an error not exceeding

$$\frac{1.037 - 1.023}{1.023} \times 100 \approx 1.4\%$$

Example 1-20. Figure 1-15 shows the so-called Mariotte vessel. It is a closed reservoir from which the liquid can flow out through

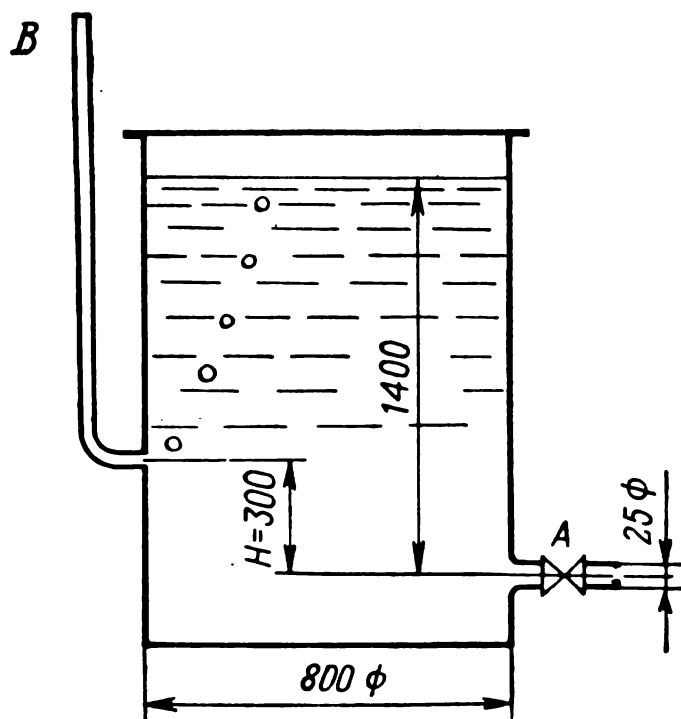


Fig. 1-15. To Example 1-20

pipe *A*. The upper end of pipe *B* is open. When liquid flows out of the reservoir via pipe *A*, a vacuum is formed in the upper part of the reservoir, and atmospheric air is sucked into it through pipe *B*. As a result, the pressure in the reservoir at the level *H* above pipe *A* will always equal atmospheric pressure, regardless of the amount of liquid in the reservoir. The liquid will flow out under a constant pressure until its level drops below *H*. According to the dimensions (in mm) shown in Fig. 1-15, determine the discharge velocity of the water and the time during which its level will lower from the initial one (1400 mm) to *H* = 300 mm. The velocity coefficient $\varphi = 0.82$. The coefficient of contraction $\kappa = 1$.

Solution. By Eq. (1-35) with $p_0 = p$, we have

$$v = \varphi \sqrt{2gH} = 0.82 \sqrt{2 \times 9.81 \times 0.3} = 1.98 \text{ m/s}$$

The volume of water that flows out of the reservoir when the level in it drops from 1400 to 300 mm is

$$V = 0.785 \times 0.8^2 (1.4 - 0.3) = 0.553 \text{ m}^3$$

The duration of the discharge is:

$$\tau = \frac{0.553}{0.785 \times 0.025^2 \times 1.98} = 569 \text{ s} \approx 9.5 \text{ min}$$

Example 1-21. A cylindrical tank with a diameter of 1 m is filled with water to a height of 2 m. The orifice in its bottom for discharge has a diameter of 3 cm. Determine the time needed for emptying the tank.

Solution. We use Eq. (1-38):

$$\tau = \frac{2A \sqrt{H}}{C_d A_o \sqrt{2g}}$$

where A = cross-sectional area of the tank, m^2

A_o = area of the orifice, m^2

H = initial height of the level of the liquid in the tank, m

C_d = discharge coefficient (for an orifice with unrounded edges we can assume that $C_d = 0.61$).

Introducing the given values into the formula, we find

$$\tau = \frac{2 \times 0.785 \times 1^2 \sqrt{2}}{0.61 \times 0.785 \times 0.03^2 \sqrt{2 \times 9.81}} = 1180 \text{ s} \approx 20 \text{ min}$$

Example 1-22. Water at a temperature of 20°C flows with an average velocity of 1.3 m/s through a horizontal hydraulically smooth pipe with an internal diameter of 152 mm. A normal measuring orifice plate having an orifice diameter of 83.5 mm is installed in the pipe. Determine the reading of the mercury differential manometer of the orifice plate (see Fig. 1-3).

Solution. The rate of flow of the water is:

$$Q_v = vA = 1.3 \times 0.785 \times 0.152^2 = 0.0236 \text{ m}^3/\text{s}$$

The Reynolds number is:

$$Re = \frac{vd}{\nu} = \frac{1.3 \times 0.152}{1.01 \times 10^{-6}} = 195\,000$$

where ν is the kinematic viscosity of water at 20°C (Table A-39), m^2/s .

We determine the discharge coefficient of the orifice plate C_d which depends on the value of the Reynolds number Re and of the quantity $(d_o/d)^2 = (83.5/152)^2 \approx 0.3$.

In Table A-15 we find $C_d \approx 0.635$.

From Eq. (1-39) and assuming that $k=1$, we get:

$$\begin{aligned} \sqrt{H} &= \frac{Q_v}{C_d A_o \sqrt{2g \frac{\rho_{\text{man}} - \rho}{\rho}}} = \\ &= \frac{0.0236}{0.635 \times 0.785 \times 0.0835^2 \sqrt{2 \times 9.81 \frac{13\,600 - 1000}{1000}}} = 0.432 \end{aligned}$$

whence $H = 0.188 \text{ m} = 188 \text{ mm}$.

Example 1-23. The reading of the water differential manometer of a Pitot-Prandtl tube (see Fig. 1-4) installed along the axis of a horizontal air pipe is 13 mm. Determine the rate of flow of the air if its temperature is 40°C , the pipe diameter is $159 \times 6 \text{ mm}$, and there is a straight section 7 m long before the tube. The air is at atmospheric pressure.

Solution. The density of the air at 40°C is:

$$\rho = 1.293 \times \frac{273}{313} = 1.13 \text{ kg/m}^3$$

The maximum (axial) velocity of the air is determined from Eq. (1-42):

$$v_{\text{max}} = \sqrt{\frac{2\Delta p_{\text{vel}}}{\rho}} = \sqrt{\frac{2 \times 13 \times 9.81}{1.13}} = 15.07 \text{ m/s}$$

The following value of the Reynolds number corresponds to this velocity:

$$Re = \frac{v_{\text{max}} d \rho}{\mu} = \frac{15.07 \times 0.147 \times 1.13}{0.019 \times 10^{-3}} = 132\,000$$

The length of the straight section of flow stabilization before the Pitot tube should be at least 40 diameters: $40 \times 0.147 = 5.9 \text{ m}$. This condition is observed because there is a straight pipe section 7 m long. According to Fig. 1-2 for $Re = 132\,000$, the ratio of the average velocity to the maximum one $v/v_{\text{max}} \approx 0.85$.

Hence, the average velocity is $v = 0.85 \times 15.07 = 12.8 \text{ m/s}$.

The rate of flow of the air is:

$$Q_v = 0.785 \times 0.147^2 \times 12.8 = 0.217 \text{ m}^3/\text{s} = 780 \text{ m}^3/\text{h}$$

Example 1-24. The flue gases from a furnace installation in Leningrad are discharged to the atmosphere through a chimney 19 m high. The composition of the gases is CO_2 —12.7%, O_2 —4.9%, N_2 —77.5%, H_2O —4.9% (by volume). The cross-sectional area of the chimney equals that of the horizontal flue. The average temperature of the gases in the flue and the chimney is 250°C . Find the velocity of the gases if the sum of the resistance coefficients

of the flue and the chimney is

$$\frac{\lambda L_n}{d_{eq}} + \frac{\lambda H}{d} + \sum \zeta = 27.3$$

where λ = pipe resistance coefficient

L_n = length of the flue

d_{eq} = equivalent diameter of the flue

d = internal diameter of the chimney

H = height of the chimney

$\sum \zeta$ = sum of the local resistance coefficients.

Solution. The density of the gas in standard conditions is:

$$\rho^\circ = \frac{M}{22.4} = \frac{1}{22.4} (44 \times 0.127 + 32 \times 0.049 + 28 \times 0.775 + 18 \times 0.049) = 1.328 \text{ kg/m}^3$$

The density of the gas at 250 °C is:

$$\rho_n = \rho^\circ \frac{T^\circ}{T} = 1.328 \times \frac{273}{273 + 250} = 0.693 \text{ kg/m}^3$$

In summer the average temperature of the air in Leningrad is 17.5 °C (Table A-40).

The density of air at this temperature is

$$\rho_{air} = 1.293 \times \frac{273}{273 + 17.5} = 1.216 \text{ kg/m}^3$$

According to the Bernoulli equation (with the reference plane at the level of the flue)

$$p_{bar'} + \frac{\rho v^2}{2} = p_{bar''} + \frac{\rho v^2}{2} + \rho_n g H + \Delta p_l$$

where $p_{bar'}$ = barometric pressure at the level of the horizontal flue

$p_{bar''}$ = barometric pressure at the top edge of the chimney

v = velocity of the gas

H = height of the chimney

Δp_l = total pressure losses (total resistance of the flue and the chimney).

The left-hand side of the Bernoulli equation considers a section drawn through the beginning of the flue, and the right-hand side—through the top edge of the chimney.

The barometric pressure at the level of the horizontal flue is:

$$p_{bar'} = p_{bar''} + \rho_{air} g H$$

Inserting this value in the Bernoulli equation, we get

$$p_{bar''} + \rho_{air} g H + \frac{\rho v^2}{2} = p_{bar''} + \frac{\rho v^2}{2} + \rho_n g H + \Delta p_l$$

whence

$$\Delta p_1 = (\rho_{\text{air}} - \rho_n) gH \quad (\text{a})$$

The resistance of the flue and the chimney, Δp_1 , can be calculated by the equation:

$$\Delta p_1 = \left(\frac{\lambda L}{d_{\text{eq}}} + \frac{\lambda H}{d} + \sum \xi \right) \frac{\rho_n v^2}{2}$$

or, with a view to the initial conditions of the example:

$$\Delta p_1 = 27.3 \frac{\rho_n v^2}{2} \quad (\text{b})$$

From Eqs. (a) and (b), we get:

$$27.3 \frac{\rho_n v^2}{2} = (\rho_{\text{air}} - \rho_n) gH$$

Introduction of numerical values yields:

$$27.3 \times \frac{0.693v^2}{2} = (1.216 - 0.693) 9.81 \times 19$$

This gives us the velocity of the gas in the flue and the chimney in summer:

$$v = 3.2 \text{ m/s}$$

We can calculate the velocity of the gas in winter in a similar way (for Leningrad the average winter temperature is -7.7°C). Hence, in winter $v = 3.48 \text{ m/s}$.

Example 1-25. Determine the loss of pressure for overcoming the friction in a coil (Fig. 1-16) through which water flows with

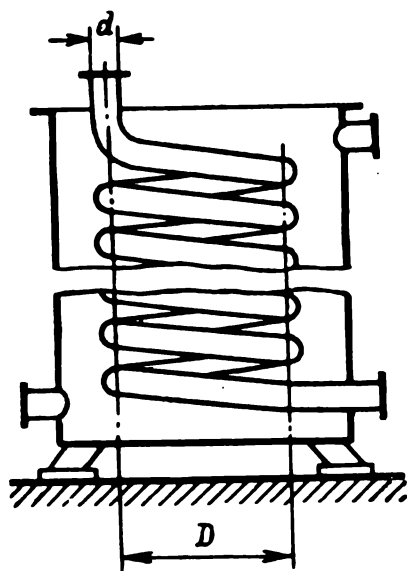


Fig. 1-16. To Example 1-25

a velocity of 1 m/s . The coil is made of a used steel pipe with a diameter of $43 \times 2.5 \text{ mm}$. The diameter of a turn of the coil is 1 m . The number of turns is 10. The average temperature of the water is 30°C .

Solution. We find the loss of pressure for overcoming the friction by Eq. (1-43) for a straight pipe, and then introduce a cor-

rection factor for the coil according to Eq. (1-54). We first determine the flow conditions. The dynamic viscosity of water at 30 °C is 0.8 cP (Table A-6). The Reynolds number is:

$$Re = \frac{vd\rho}{\mu} = \frac{1 \times 0.038 \times 1000}{0.8 \times 10^{-3}} = 47\,500$$

For a seamless steel pipe with insignificant corrosion, $e = 0.2$ mm (Table A-12). Hence, the ratio $d_{eq}/e = 38/0.2 = 190$.

According to the graph in Fig. 1-5 at $Re = 47\,500$ and $d_{eq}/e = 190$, we find $\lambda = 0.0316$.

The length of the coil is approximately

$$L = \pi Dn = 3.14 \times 1 \times 10 = 31.4 \text{ m}$$

The loss of head for overcoming the friction in a straight pipe is

$$\Delta p_{str} = \lambda \frac{L}{d} \frac{\rho v^2}{2} = 0.0316 \times \frac{31.4}{0.038} \frac{1000 \times 1^2}{2} = 13\,100 \text{ Pa}$$

We find the correction factor by Eq. (1-54):

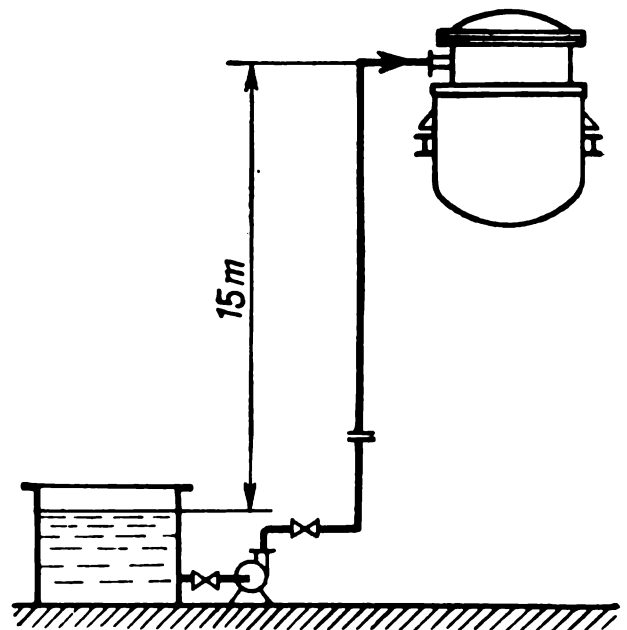
$$\psi = 1 + 3.54 \frac{d}{D} = 1 + 3.54 \times \frac{0.038}{1} = 1.134$$

Consequently, for the coil we have:

$$\Delta p_{coil} = \Delta p_{str} \psi = 13\,100 \times 1.134 = 14\,800 \text{ Pa} = 1510 \text{ mm H}_2\text{O}$$

Example 1-26. A pump feeds 30 tons of nitrobenzene an hour at a temperature of 20 °C from a tank at atmospheric pressure

Fig. 1-17. To Example 1-26



into a reactor where a gauge pressure of 0.1 at is maintained (Fig. 1-17). The pipeline is made of steel pipes with a diameter of 89×4 mm having insignificant corrosion. The length of the entire pipeline including local obstacles is 45 m. The pipeline is provided with an orifice plate ($d_o = 51.3$ mm), two gate valves, and four branch pipes at an angle of 90° with a bending radius

of 160 mm. The liquid is lifted to a height of 15 m. Find the power consumed by the pump, assuming that its overall efficiency equals 0.65.

Solution. The mass rate of flow of the nitrobenzene is $Q_m = 30\,000/3600 = 8.34$ kg/s.

The volumetric rate of flow is $Q_v = Q_m/\rho = 8.34/1200 = 0.006\,95$ m³/s, where $\rho = 1200$ kg/m³ is the density of nitrobenzene (Table A-3).

The velocity of the nitrobenzene is:

$$v = \frac{Q_v}{A} = \frac{0.006\,95}{0.785 \times 0.081^2} = 1.35 \text{ m/s}$$

The Reynolds number is:

$$Re = \frac{v d \rho}{\mu} = \frac{1.35 \times 0.081 \times 1200}{2.1 \times 10^{-3}} = 62\,500$$

where μ is the dynamic viscosity of nitrobenzene (Fig. A-5).

We determine the pipe resistance coefficient. According to Table A-12, the roughness of steel pipes with insignificant corrosion is $e = 0.2$ mm. In Fig. 1-5 for $d/e = 0.081/0.0002 = 405$ and $Re = 62\,500$, we find $\lambda = 0.0257$.

We summarize the local resistance coefficients taken from Table A-13 in the following table:

Kind of obstacle	ζ	Num-ber	$\sum \zeta$
Inlet of liquid from tank to pipe (pipe with sharp edges)	0.5	1	0.5
Orifice plate $(d_o/D)^2 = (51.3/81)^2 = 0.4$	8.25	1	8.25
Gate valve	0.5	2	1.0
Pipe bend (at $\varphi = 90^\circ$ and $R_0/d = 160/81 \approx 2$): $\zeta = AB = 1 \times 0.15 = 0.15$	0.15	4	0.6
Total			10.35

The total hydraulic resistance of the pipeline (network) by Eq. (1-57) is:

$$\begin{aligned} \Delta p &= \frac{\rho v^2}{2} \left(1 + \frac{\lambda L}{d} + \sum \zeta \right) + \rho g h_{\text{lift}} + \Delta p_{\text{add}} = \\ &= \frac{1200 \times 1.35^2}{2} \left(1 + \frac{0.0257 \times 45}{0.081} + 10.35 \right) + \\ &+ 1200 \times 9.81 \times 15 + 0.1 \times 9.81 \times 10^4 = 215\,000 \text{ Pa} \end{aligned}$$

The power consumed by the pump, according to Eq. (1-40), is

$$P = \frac{Q_v \Delta p}{1000 \eta} = \frac{0.006\,95 \times 215\,000}{1000 \times 0.65} = 2.31 \text{ kW}$$

Example 1.27. Air at a temperature of 50°C is fed by a fan to an installation having a gauge pressure of $35\text{ mm H}_2\text{O}$. The pipeline is made of steel pipes with a diameter of $102\times 6\text{ mm}$ with insignificant corrosion. The length of the entire pipeline including local obstacles is 70 m . The pipeline is provided with an orifice plate ($d_o=49.3\text{ mm}$), two gate valves, and four pipe bends at an angle of 90° with a bending radius of 300 mm . The fan motor has a power rating of 1.35 kW and an efficiency of $\eta_m=0.95$. The reading of the water differential manometer connected to the orifice plate is 400 mm . Determine the efficiency of the fan.

Solution. The density of air at 50°C is:

$$\rho = \rho^\circ \frac{T^\circ}{T} = 1.29 \times \frac{273}{328} = 1.09\text{ kg/m}^3$$

We determine the discharge coefficient of the orifice plate by Table A-15. With $(d_o/d)^2 = (49.3/90)^2 = 0.3$, we take the average value of $C_d \approx 0.64$.

The rate of air flow by Eq. (1-39) with $k=1$ (Table A-16) is:

$$\begin{aligned} Q_v &= C_d A_o \sqrt{2gH \frac{\rho_{\text{man}} - \rho}{\rho}} = \\ &= 0.64 \times 0.785 \times 0.0493^2 \sqrt{2 \times 9.81 \times 0.4 \times \frac{988 - 1.09}{1.09}} = \\ &= 0.103\text{ m}^3/\text{s} \end{aligned}$$

The velocity of the air in the pipeline is:

$$v = \frac{Q_v}{0.785d^2} = \frac{0.103}{0.785 \times 0.09^2} = 16.2\text{ m/s}.$$

The Reynolds number determined according to the diameter of the pipeline is:

$$Re = \frac{vd\rho}{\mu} = \frac{16.2 \times 0.09 \times 1.09}{0.0196 \times 10^{-3}} = 81\,000$$

The dynamic viscosity of the air $\mu = 0.0196 \times 10^{-3}\text{ Pa}\cdot\text{s}$ has been determined by Fig. A-6.

Let us check the adopted value of C_d . From Table A-15 for $(d_o/d)^2 = 0.3$ and $Re = 81\,000$, we find $C_d = 0.637$, which virtually coincides with the adopted value.

We determine the pipe resistance coefficient. According to Table A-12, the average roughness of steel pipes with insignificant corrosion is $e = 0.2\text{ mm}$. In Fig. 1-5 for $d/e = 90/0.2 = 450$ and $Re = 81\,000$, we find $\lambda \approx 0.025$.

We summarize the local resistance coefficients taken from Table A-13 in the following table:

Kind of obstacle	ζ	Num-ber	$\sum \zeta$
Inlet of air to pipeline (pipe with round edges)	0.2	1	0.2
Orifice plate, $(d_o/D)^2=0.3$	18.2	1	18.2
Gate valve	0.5	2	1.0
Pipe bend (at $\varphi=90^\circ$ and $R_o/d=300/90=3.3$): $\zeta=AB=1\times0.13=0.13$	0.13	4	0.52
Total			19.92 \approx 20

The total hydraulic resistance of the pipeline (network) is:

$$\Delta p = \frac{\rho v^2}{2} \left(1 + \frac{\lambda L}{d} + \sum \zeta \right) + \Delta p_{\text{add}} =$$
$$= \frac{1.09 \times 16.2^2}{2} \left(1 + \frac{0.025 \times 70}{0.09} + 20 \right) + 35 \times 9.81 = 6030 \text{ Pa}$$

The efficiency of the fan is:

$$\eta_{\text{fan}} = \frac{Q_v \Delta p}{1000 \eta_m P} = \frac{0.103 \times 6030}{1000 \times 0.95 \times 1.35} \approx 0.48$$

Example 1-28. A liquid having a density of 1200 kg/m³ and a dynamic viscosity of 2 cP flows by gravity from tank 1 hav-

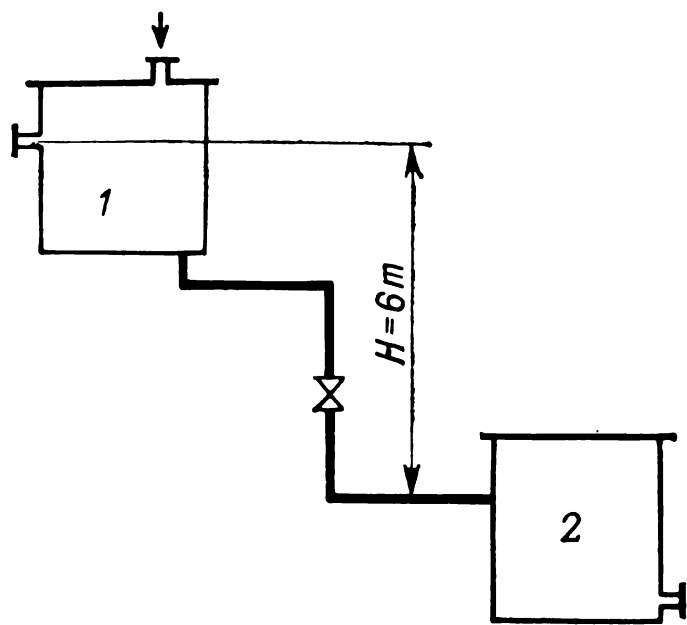


Fig. 1-18. To Example 1-28

ing a constant level into reactor 2 (Fig. 1-18). Determine the maximum amount of liquid (with the cock completely open) that can flow from the tank into the reactor. The level of the liquid in the tank is six metres higher than the inlet of the liquid to the reactor. Aluminium pipes with an internal diameter of 50 mm are used. The total length of the pipeline including local obstacles is 16.4 m. The pipeline is provided with three elbows and a cock. The pressure in the tank and the reactor is atmospheric.

Solution. We write the Bernoulli equation (1-34) for a steady flow of the liquid, taking the first section at the level of the

liquid in the tank and the second one at the end of the pipeline, i.e. at its outlet into the reactor:

$$z_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} - h_1 = z_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

Since $p_1 = p_2$ and $v_1 \ll v_2$, then

$$z_1 - z_2 = \frac{v_2^2}{2g} + h_1 = \frac{v_2^2}{2g} + h_{fr} + h_{lr}$$

or

$$H\rho g = \frac{v^2\rho}{2} \left(1 + \frac{\lambda L}{d} + \sum \zeta \right) \tag{a}$$

The head $H\rho g$ is used to overcome all the hydraulic resistances of the pipeline.

Equation (a) contains two unknown quantities— v and λ . It can be solved by consecutive approximations.

According to Table A-12 for aluminium technically smooth pipes, we find that their roughness is $e = 0.06$ mm. Hence $d/e = 50/0.06 = 840$. Let us assume (with subsequent checking) that the value of λ is 0.022.

We summarize the local resistance coefficients taken from Table A-13 in the following table:

Kind of obstacle	ζ	Num-ber	$\sum \zeta$
Inlet of liquid to pipeline	0.5	1	0.5
Cock	2.0	1	2.0
Elbow	1.1	3	3.3
Total			5.8

Introducing numerical values into equation (a), we have:

$$6 \times 1200 \times 9.81 = \frac{1200v^2}{2} \left(1 + \frac{0.022 \times 16.4}{0.05} + 5.8 \right)$$

whence $v = 2.9$ m/s.

The rate of flow of the liquid (with the cock completely open) is:

$$Q_v = vA = 2.9 \times 0.785 \times 0.05^2 = 0.0057 \text{ m}^3/\text{s} = 20.5 \text{ m}^3/\text{h}$$

We check the value of λ . We have:

$$Re = \frac{vd\rho}{\mu} = \frac{2.9 \times 0.05 \times 1200}{2 \times 10^{-3}} = 87\,000$$

In Fig. 1-5 for $Re = 87\,000$ and $d/e = 840$, we find $\lambda = 0.022$. This coincides with the value we adopted.

Example 1-29. Mineral oil in an amount of 40 000 dm³/h is pumped through a pipeline with a diameter of 108×4 mm into

a tank at a height of 20 m. The length of the horizontal section of the pipeline is 430 m.

Calculate the required power of the pump if the oil is pumped at (a) 15 °C, and (b) 50 °C. At these temperatures, the relative density of the oil is 0.96 and 0.89, respectively, and the dynamic viscosity is 3.43 and 0.187 Pa·s, respectively.

Is it economical to heat the oil to 50 °C before pumping it if 1 kWh of electric power costs 4 kopecks, and one ton of heating (waste) steam ($p_{\text{abs}} = 1$ at) costs 2 roubles (1 rouble = 100 kopecks), and the overall efficiency of the pump plant is 0.5?

Solution. The velocity of the oil is

$$v = \frac{40}{0.785 \times 0.1^2 \times 3600} = 1.414 \text{ m/s}$$

The Reynolds number for 15 and 50 °C, respectively, is:

$$Re_{15} = \frac{0.1 \times 1.414 \times 960}{3.43} = 39 \text{ and } Re_{50} = \frac{0.1 \times 1.414 \times 890}{0.187} = 670$$

Consequently, the flow is laminar in both cases.

We determine the loss of pressure due to friction (the pipe resistance) by Eq. (1-47) at 15 °C:

$$\Delta p_{\text{fr}} = \frac{32 (430 + 20) 1.414 \times 3.43}{0.1^2} = 7\,010\,000 \text{ Pa} = 71.5 \text{ at}$$

and at 50 °C:

$$\Delta p_{\text{fr}} = 7\,010\,000 \times \frac{0.187}{3.43} = 383\,000 \text{ Pa} = 3.9 \text{ at}$$

The pressure needed to lift the liquid is determined by the formula $\Delta p_{\text{lift}} = \rho g h_{\text{lift}}$ [see the explanations to Eq. (1-41)]:
at 15 °C:

$$\Delta p_{\text{lift}} = 20 \times 960 \times 9.81 = 188\,400 \text{ Pa} = 1.92 \text{ at}$$

at 50 °C:

$$\Delta p_{\text{lift}} = 20 \times 890 \times 9.81 = 174\,600 \text{ Pa} = 1.78 \text{ at}$$

The pressure needed to create a velocity of flow, by Eq. (1-42), is:

$$\Delta p_{\text{vel}} = \frac{v^2 \rho}{2} = \frac{960 \times 1.414^2}{2} = 960 \text{ Pa}$$

In the given case, we disregard this quantity, like the loss of pressure for surmounting the local obstacles.

Hence, the required power by Eq. (1-40) is:

at 15 °C:

$$P = \frac{40 (7\,010\,000 + 188\,400)}{3600 \times 1000 \times 0.5} = 160 \text{ kW}$$

at 50 °C:

$$P = \frac{40 (383\,000 + 174\,600)}{3600 \times 1000 \times 0.5} = 12.3 \text{ kW}$$

Consequently, if cold oil is pumped, the additional power needed is $160 - 12.3 = 147.7$ kW.

For the conditions of our example, the cost of the additional electric power for pumping the cold oil will be $147.7 \times 0.04 = 5.9$ roubles per hour.

The consumption of heat for heating the oil from 15 to 50 °C at the specific heat capacity of the oil $c = 1.68 \times 10^3$ J/kg·K is:

$$Q = \frac{40 \times 960 \times 1.68 \times 10^3 (50 - 15)}{3600} = 625\,000 \text{ W}$$

The specific heat of condensation of waste steam ($p_{\text{abs}} = 1$ at) is $L = 2260 \times 10^3$ J/kg.

The consumption of steam for heating the oil is:

$$Q_{\text{st}} = \frac{Q}{L} = \frac{625\,000}{2260 \times 10^3} = 0.276 \text{ kg/s} \approx 1000 \text{ kg/h}$$

One ton of steam costs two roubles. Therefore, preliminary heating of the oil is indisputably profitable at the given cost of the steam and electric power.

Example 1-30. Derive a generalized formula for calculating the pipe resistance coefficient according to the following experimental data.

(a) When pumping oil having a relative density of 0.9 through a steel pipe with an internal diameter of 300 mm on a straight section 13 metres long, the following losses of pressure occurred depending on the velocity of the oil:

Velocity, m/s	0.85	1.21	1.54	1.91	2.32
Pressure loss, Pa	738	1300	2060	3020	4050

(b) Similar experimental data were obtained when pumping water through a straight steel pipe 24 metres long with an internal diameter of 60 mm:

Velocity, m/s	0.11	0.16	0.22	0.26
Pressure loss, Pa	105	201	238	455

Solution. We shall calculate the values of the pipe resistance coefficients λ for oil and water by Eq. (1-43):

$$\Delta p_{\text{fr}} = \lambda \frac{L}{d_{\text{eq}}} \frac{v^2 \rho}{2}$$

Since the quantities L , d_{eq} , and ρ are constant for a given pipe and fluid, we can write the above equation as follows:

$$\Delta p_{\text{fr}} = C \lambda v^2$$

where $C = \frac{L\rho}{2d_{\text{eq}}}$, whence

$$\lambda = \frac{\Delta p_{\text{fr}}}{C v^2} \quad (\text{a})$$

We preliminarily calculate the values of the constant C for oil and water, respectively:

$C_{oil} = \frac{13 \times 900}{2 \times 0.3} = 19\,500$ and $C_w = \frac{24 \times 1000}{2 \times 0.06} = 200\,000$

The further calculations by Eq. (a) are summarized in Table 1-4

TABLE 1-4

Liquid	v , m/s	v^2 , (m/s) ²	Δp_{fr} , Pa	λ	Liquid	v , m/s	v^2 , (m/s) ²	Δp_{fr} , Pa	λ
Oil	0.85	0.723	738	0.0522	Water	0.11	0.0121	105	0.0433
	1.21	1.46	1300	0.0458		0.16	0.0256	201	0.0392
	1.54	2.37	2060	0.0445		0.22	0.0484	238	0.0348
	1.91	3.65	3020	0.0424		0.26	0.0676	455	0.0337
	2.32	5.38	4050	0.0386					

It follows from Table 1-4 that the pipe resistance coefficient λ diminishes with an increasing flow velocity.

If we plot the velocity dependence of the resistance coefficient for oil and water on a graph (Fig. 1-19), we get two different

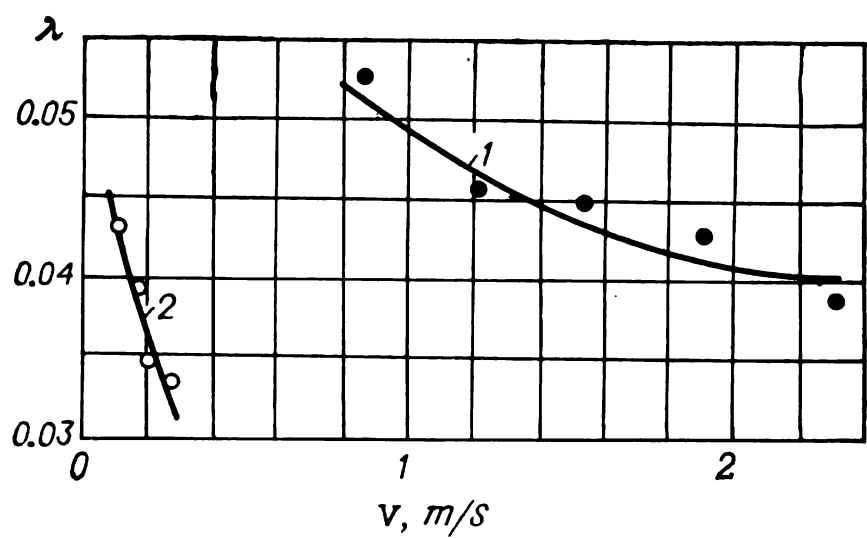


Fig. 1-19. To Example 1-30:
1 – oil, 2 – water

curves. Using the logarithmic coordinates $\log v$ against $\log \lambda$, we get two straight lines whose approximate equations are:
for oil

$\log \lambda = -1.31 - 0.25 \log v$ or $\lambda = 0.049v^{-0.25}$

for water

$\log \lambda = -1.65 - 0.3 \log v$ or $\lambda = 0.0224v^{-0.3}$

We have thus obtained two different empirical formulas each of which holds only for the conditions which it was obtained for. The latter formula, for instance, may be used to calculate the

pipe resistance coefficient only for water flowing through a pipe with an internal diameter of 60 mm with a velocity ranging from 0.1 to 0.3 m/s (and at the same temperature of the water at which the experiments were conducted).

It is possible, however, to obtain a general formula from the available experimental data for oil and water that can be applied also for other liquids at other velocities, other pipe diameters and other temperatures. Here the theory of similarity will help us.

The theory of similarity gives Eq. (1-44) in the dimensionless form for the loss of pressure for overcoming the friction resistance in pipes:

$$Eu = \varphi (Re, S_L)$$

For pipes having a given roughness of their walls in the turbulent (pre-self-similar) region, the resistance coefficient λ depends only on the value of the Reynolds number Re —see Eqs. (1-45), (1-46), (1-48), and (1-50):

$$\lambda = f (Re)$$

Consequently, to obtain a generalized formula, we must find the dependence of the resistance coefficient not on the velocity, as was done above, but on the Reynolds number Re .

The values of the kinematic viscosities during the experiments were $\nu = 0.8 \times 10^{-4}$ m²/s for oil and $\nu = 10^{-6}$ m²/s for water.

Table 1-5 compares the values of Re and λ calculated according to the above experimental data.

TABLE 1-5

Liquid	$v, \text{ m/s}$	$Re = \frac{vd}{\nu}$	λ	$\log Re$	$\log \lambda$
Oil	0.85	3 190	0.0522	3.50	$\bar{2}.72$
	1.21	4 530	0.0458	3.66	$\bar{2}.66$
	1.54	5 780	0.0445	3.76	$\bar{2}.65$
	1.91	7 160	0.0424	3.85	$\bar{2}.63$
	2.32	8 700	0.0386	3.94	$\bar{2}.59$
Water	0.11	6 600	0.0438	3.82	$\bar{2}.64$
	0.16	9 600	0.0392	3.98	$\bar{2}.59$
	0.22	13 200	0.0348	4.12	$\bar{2}.54$
	0.26	15 600	0.0337	4.19	$\bar{2}.53$

If we plot a graph of λ against Re , we get a single curve passing through all the points for oil and water.

In logarithmic coordinates, we get a straight line (Fig. 1-20) whose equation is:

$$y = -0.254x + 0.71$$

or

$$\log \lambda + 2 = -0.254 (\log Re - 3.5) + 0.71$$

$$\log \lambda = -0.254 \log Re - 0.4$$

whence

$$\lambda = \frac{0.398}{Re^{0.254}}$$

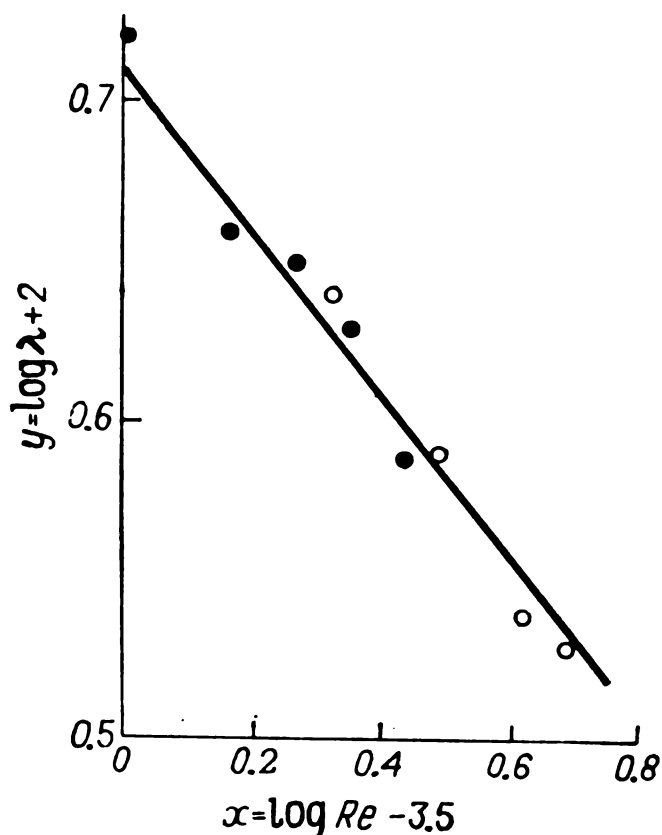


Fig. 1-20. To Example 1-30:
The black and white points are for oil and water, respectively

This formula is exactly a general relationship that can be used for calculating the pipe resistance coefficient not only for oil and water, but also for any other liquids within the range of values of the Reynolds number from 3000 to 16 000. It must be indicated, however, that this formula may be used only for pipes having the same relative roughness of their walls as those used in the experiments.

Since the experimental data are arranged along a single straight line (Fig. 1-20), we can consider that the relative roughness of the walls e/d in the experiments with oil and with water was approximately the same.

This relative roughness can be found if we plot the experimental data from Tables 1-4 and 1-5 on the graph in Fig. 1-5 or 1-6, presenting them in the form $\lambda = f(Re)$ or $Eu/(L/d_{eq}) = f(Re)$.

Example 1-31. To study a certain production process experimentally in laboratory conditions, a geometrical model of the industrial apparatus was fabricated to a scale of 1 in 10. In the industrial apparatus, the working substance is hot air (100°C, atmospheric pressure) flowing with a velocity of 3 m/s. In the

laboratory model, it is intended to use air at atmospheric pressure having a temperature of 22°C as the working substance.

Is it possible in these conditions to obtain complete hydrodynamical similarity of the industrial apparatus and the model, and what must the velocity of the air in the model be?

Solution. To observe hydrodynamical similarity, with similar boundary conditions, the Reynolds and the Froude numbers for the model and the industrial apparatus must be equal (the Euler number in this case is not a governing factor), i. e. we must have

$$Re_1 = Re_2, \text{ i. e. } \frac{v_1 L_1 \rho_1}{\mu_1} = \frac{v_2 L_2 \rho_2}{\mu_2} \quad (a)$$

$$Fr_1 = Fr_2, \text{ i. e. } \frac{v_1^2}{gL_1} = \frac{v_2^2}{gL_2} \quad (b)$$

In these equations, the subscript "1" relates to the industrial apparatus, and the subscript "2" to the model.

We have $v_1 = 3 \text{ m/s}$; $L_2 = 0.1 L_1$; $\rho_1 = 1.29 \times \frac{273}{373} = 0.945 \text{ kg/m}^3$; $\rho_2 = 1.29 \times \frac{273}{295} = 1.19 \text{ kg/m}^3$; $\mu_1 = 0.0215 \text{ cP}$; and $\mu_2 = 0.0185 \text{ cP}$.

Using these values in Eq. (a), we get

$$\frac{3 \times L_1 \times 0.945}{0.0215} = \frac{v_2 \times 0.1 \times L_1 \times 1.19}{0.0185}$$

whence

$$v_2 = \frac{3 \times 0.945 \times 0.0185}{0.1 \times 0.0215 \times 1.19} = 20.5 \text{ m/s}$$

Substitution in Eq. (b) yields:

$$\frac{3^2}{L_1} = \frac{v_2^2}{0.1 L_1}$$

whence

$$v_2 = \sqrt{0.9} = 0.946 \text{ m/s}$$

The results obtained show that the observance of complete hydrodynamical similarity, i. e. the simultaneous similarity of the resistance forces and the gravity forces, in the model and the industrial apparatus in the given conditions is impossible. We will have to limit ourselves to approximate modelling of the process, as is done in practice in the majority of cases, i. e. observe only one condition (either $Re = idem$, or $Fr = idem$) depending on which of these conditions is more important. If, for example, the friction forces are more important, i. e. the Reynolds number Re , then the velocity of the air in the model should be taken equal to 20.5 m/s.

Let us find the conditions in which it is possible to obtain simultaneous similarity of the friction forces and gravity forces in the model and the industrial apparatus.

From the condition $Re = idem$, i. e. $\frac{v_1 L_1}{\nu_1} = \frac{v_2 L_2}{\nu_2}$, where ν is the kinematic viscosity, it follows that

$$\frac{v_1}{v_2} = \frac{\nu_1 L_2}{\nu_2 L_1}$$

From the condition $Fr = idem$, i. e. $\frac{v_1^2}{gL_1} = \frac{v_2^2}{gL_2}$, we find

$$\frac{v_1}{v_2} = \left(\frac{L_1}{L_2}\right)^{1/2}$$

Both conditions ($Re = idem$ and $Fr = idem$) will be observed if

$$\frac{v_1 L_2}{\nu_2 L_1} = \left(\frac{L_1}{L_2}\right)^{1/2}, \text{ i. e. } \frac{v_1}{v_2} = \left(\frac{L_1}{L_2}\right)^{3/2}$$

The last equation expresses the required condition of the simultaneous similarity of the friction forces and gravity forces.

Example 1-32. Find the diameter of the pipeline needed for the transportation of hydrogen with its mass rate of flow equal to 120 kg/h. The length of the pipeline is 1000 m. The tolerated pressure drop is $\Delta p = 110$ mm H₂O. The density of hydrogen is 0.0825 kg/m³. The pipe resistance coefficient is $\lambda = 0.03$.

Solution. In long gas mains, the pressure is spent mainly for overcoming friction resistance, therefore we equate the given pressure loss Δp to Δp_{fr} : $\Delta p = \Delta p_{fr}$.

We use Eq. (1-43)

$$\Delta p = \lambda \frac{L}{d} \frac{v^2 \rho}{2}$$

By Eq. (1-24):

$$v = \frac{Q_v}{0.785d^2}$$

Substituting this value for v in Eq. (1-43), we get:

$$\Delta p = \frac{\lambda L}{d} \frac{Q_v^2 \rho}{2 \times 0.785^2 d^4}$$

whence

$$d = C \sqrt[5]{\frac{L Q_v^2 \rho}{\Delta p}}$$

$$\text{where } C = \sqrt[5]{\frac{\lambda}{0.785^2 \times 2}} = \sqrt[5]{\frac{0.03}{0.785^2 \times 2}} = 0.48$$

The volumetric rate of flow per second of the hydrogen is:

$$Q_v = \frac{120}{0.0825 \times 3600} = 0.405 \text{ m}^3/\text{s}$$

Since $\Delta p = 110 \times 9.81 = 1080 \text{ Pa}$, then

$$d = 0.48 \sqrt[5]{\frac{0.0825 \times 0.405^2 \times 1000}{1080}} = 0.2 \text{ m}$$

Example 1-33. Determine the initial pressure of a gas sufficient for its transportation over a pipeline 100 kilometres long if the rate of flow of the gas is 5000 kg/h, its density is 0.65 kg/m³ (in standard conditions), and the average temperature of the gas is 18°C. The diameter of the pipeline is 0.3 m, and the pipe resistance coefficient is $\lambda = 0.0253$. The pressure of the gas at the outlet from the pipeline should be $p_{\text{abs}} = 1.5 \text{ at}$.

Solution. As in the preceding example, we consider that the pressure in the pipeline goes only for overcoming the friction resistance. For long-distance gas mains, Eq. (1-43) for determining the loss of pressure for overcoming the friction resistance should be used in its differential form because the density and velocity of the gas change along the length of the main*:

$$-dp = \frac{\lambda}{d} \frac{\rho v^2}{2} dL$$

where the variable density ρ and velocity v are functions of the gas pressure, which drops as the gas flows through the pipeline. The product ρv , i. e. the mass velocity of flow, however, remains constant along the length of the pipeline.

Let us express ρ and v through the corresponding characteristics of the gas in standard conditions:

$$\rho = \rho^\circ \frac{p T^\circ}{p^\circ T}; \quad v = v^\circ \frac{p^\circ T}{p T^\circ}$$

and introduce them into the equation for the pressure loss:

$$-dp = \frac{\lambda}{2d} \frac{\rho^\circ p T^\circ}{p^\circ T} \frac{(v^\circ)^2 (p^\circ)^2 T^2}{p^2 (T^\circ)^2} dL$$

Assuming the temperature T to be constant along the length of the pipeline, we combine all constant quantities** into a single coefficient:

$$C = \frac{\lambda \rho^\circ (v^\circ)^2 p^\circ T}{2d T^\circ}$$

Hence

$$-p dp = C dL$$

* We have used a minus sign because the pressure drops with an increase in the pipeline length.

** The pipe resistance coefficient λ is constant along the length of the pipeline like the Reynolds number Re because $v\rho = v^\circ\rho^\circ = \text{const.}$

Integration of this equation within the corresponding limits yields:

$$-\int_{p_{\text{in}}}^{p_{\text{fin}}} p \, dp = C \int_0^L dL$$

we get

$$\frac{1}{2} (p_{\text{in}}^2 - p_{\text{fin}}^2) = CL$$

or

$$p_{\text{in}}^2 - p_{\text{fin}}^2 = 2CL \quad (\text{a})$$

For the conditions of our example, the velocity of the gas at 0°C and 760 mm Hg is:

$$v^0 = \frac{5000}{0.65 \times 3600 \times 0.785 \times 0.3^2} = 30 \text{ m/s}$$

Consequently,

$$C = \frac{0.0253 \times 0.65 \times 30^2 \times 101\,300 \times 291}{2 \times 0.3 \times 273} = 266 \times 10^4$$

We insert the numerical values into Eq. (a)

$$p_{\text{in}}^2 - (14.7 \times 10^4)^2 = 2 \times 266 \times 10^4 \times 10^3$$

From the latter equation we find that the initial pressure of the gas in the network must be $p_{\text{in}} = 71 \times 10^4 \text{ Pa} = 7.3 \text{ at.}$

Example 1-34. Determine the economically most advantageous diameter of a pipeline for the transportation of 6000 m³/h (at 0°C and 760 mm Hg) of methane over a distance of 4 km. The overall efficiency of the blower and its motor is 0.5. The cost of electric power is 4 kopecks per kWh. The cost of depreciation of the pipeline is 2.4 roubles a year per metre of length and per metre of diameter. The cost of repairs (operation) of the pipeline is 1.8 roubles a year per metre of length and per metre of diameter.

Solution. Let us assume approximately that the pipe resistance coefficient is $\lambda = 0.03$ and the local resistance losses are 10% of the friction resistance losses.

We shall perform our calculations for a conditional temperature of 30°C, assuming (with following verification) that the pressure drop in the pipeline is not great and the average pressure can be considered approximately equal to atmospheric pressure.

The volumetric rate of flow of the methane is

$$Q_v = \frac{6000 \times 303}{3600 \times 273} = 1.85 \text{ m}^3/\text{s}$$

From Eq. (1-24), we obtain:

$$v = \frac{Q_v}{0.785d^2} = \frac{1.85}{0.785d^2} = \frac{2.36}{d^2} \text{ m/s}$$

Disregarding the expenditure of pressure for creating the velocity, by Eq (1-41) for a horizontal gas pipeline with $\Delta p_{\text{add}} = 0$ we get:

$$\Delta p = \Delta p_{\text{fr}} + \Delta p_{\text{lr}} = 1.1\Delta p_{\text{fr}}$$

i. e.

$$\Delta p = \frac{1.1\lambda Lv^2\rho}{d \times 2} = \frac{1.1 \times 0.03 \times 4000 \times 2.36^2 \times 0.64}{d \times 2d^4} = \frac{235}{d^5} \text{ Pa}$$

where the density of the methane is:

$$\rho = \frac{16 \times 273}{22.4 \times 303} = 0.64 \text{ kg/m}^3$$

In accordance with the adopted assumption, we calculate the power consumed by the blower at $\Delta p < 0.1$ at by Eq. (1-40):

$$P = \frac{Q_v \Delta p}{1000\eta} = \frac{1.85 \times 235}{1000 \times 0.5d^5} = \frac{0.87}{d^5} \text{ kW}$$

Assuming 330 workdays in a year, 1 kW·year costs $0.04 \times 24 \times 330 = 316$ roubles.

Thus, the annual cost of the electric power depending on the diameter of the pipeline can be expressed by the equation

$$E = \frac{0.87 \times 316}{d^5} = \frac{275}{d^5} \text{ roubles a year}$$

In a similar way, we shall express the annual cost of depreciation of the pipeline:

$$D = 2.4Ld = 2.4 \times 4000d = 9600d \text{ roubles a year}$$

and its repairs (operation):

$$R = 1.8Ld = 7200d \text{ roubles a year}$$

The total annular cost as a function of the diameter is:

$$E + D + R = \frac{275}{d^5} + 16800d \text{ roubles a year}$$

To find the minimum, we take a derivative of this equation with respect to the diameter and equate it to zero:

$$\frac{\partial}{\partial d}(E + D + R) = -5 \times 275d^{-6} + 16800 = 0$$

The second derivative is positive.

From the last equation, we get $d = 0.66$ m. This is the economically most advantageous diameter of the gas pipeline. We check

the value of Δp in a pipeline with a diameter of 0.66 m:

$$\Delta p = \frac{235}{d^5} = \frac{235}{0.66^5} = 1890 \text{ Pa} = 0.0193 \text{ at}$$

i. e. less than 0.1 at, as we assumed above.

Example 1-35. In the thermal calculations of a heat exchanger for heating a certain solution, a four-pass shell-and-tube exchanger (Fig. 1-21) was selected according to a catalogue. The solution in

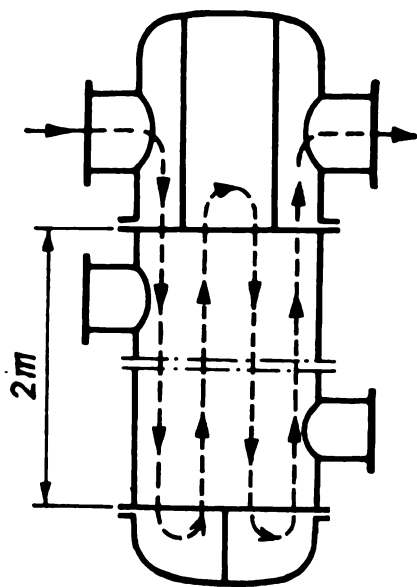


Fig. 1-21. To Example 1-35

it flows on the tube side with a velocity of 0.3 m/s. Find the hydraulic resistance of the tube space.

The exchanger has the following characteristics: the total number of tubes is 90, the tubes are of steel with a diameter of 38×2 mm with insignificant corrosion, the height of the tube side space is 2 m, the pipe connections for the solution have a diameter of 159×4.5 mm. The average temperature of the solution is 47.5°C , the dynamic viscosity is 0.83 cP, the density is 1100 kg/m^3 .

Solution. The Reynolds number for the solution is

$$Re = \frac{vd\rho}{\mu} = \frac{0.3 \times 0.034 \times 1100}{0.83 \times 10^{-3}} = 13\,500$$

Our task is to find the loss of pressure for a non-isothermal turbulent flow. But since here we have a heating flow of a dropping liquid, we shall approximately calculate the hydraulic resistance as for an isothermal flow, making an error in the direction of an increase in the pressure loss, i. e. in the direction of a margin of safety. We use Eq. (1-62) for our calculations.

We determine the pressure loss for overcoming the friction resistance in the tubes.

According to Table A-12 for steel pipes with insignificant corrosion, the roughness $e = 0.2$ mm. Hence, $d/e = 34/0.2 = 170$.

In Fig. 1-5 for $Re = 13\,500$ and $d/e = 170$, we find $\lambda = 0.035$.

The velocity pressure in the tubes is:

$$\Delta p_{vel} = \frac{\rho v^2}{2} = \frac{1100 \times 0.3^2}{2} = 49.5 \text{ Pa}$$

The loss of pressure for overcoming the friction resistance in the tubes is:

$$\Delta p_{fr} = \frac{\lambda n L}{d} \Delta p_{vel} = 0.035 \times \frac{4 \times 2}{0.034} \times 49.5 = 407 \text{ Pa}$$

where $n = 4$ is the number of passes along the tube space.

We determine the losses of pressure for overcoming the local obstacles in the heat exchanger (see Fig. 1-21) with the aid of the following table:

Kind of obstacle	ζ	Number	$\Sigma \zeta$
Inlet and outlet chambers	1.5	2	3
Tube inlet and outlet	1	8	8
180° bend from one section to another	2.5	3	7.5

The velocity of the solution in the pipe connections is

$$v_{pc} = v \frac{nd_t^2}{d_{pc}^2} = 0.3 \times \frac{(90/4) 0.034^2}{0.15^2} = 0.346 \text{ m/s}$$

The velocity pressure in the pipe connections is:

$$\Delta p'_{vel} = \frac{\rho v_{pc}^2}{2} = \frac{1100 \times 0.346^2}{2} = 66 \text{ Pa}$$

The velocity in the pipe connections is greater than that in the tubes. We therefore find the pressure losses for the inlet and outlet chambers according to the velocity in the pipe connections, and the losses at the tube inlet and outlet, as well as at the bends from one section to another, according to the velocity in the tubes:

$$\Delta p_{lr} = 3 \times 66 + (8 + 7.5) 49.5 = 966 \text{ Pa}$$

The total hydraulic resistance of the tube space of the exchanger is:

$$\Delta p = 407 + 966 = 1373 \text{ Pa} = 140 \text{ mm H}_2\text{O}$$

Example 1-36. Aniline flows on the shell side of a shell-and-tube heat exchanger (Fig. 1-22) parallel to the axes of the tubes with a velocity of 0.5 m/s. The exchanger consists of 19 steel tubes with a diameter of 26×2.5 mm and a length of 2.7 m. The internal diameter of the shell is 200 mm. The average roughness of the tubes is $e = 0.2$ mm. The aniline is cooled in the exchanger with water from 100 to 40 °C.

Determine the loss of pressure for overcoming the friction resistance for aniline: (a) considering the flow to be isothermal; and (b) for a non-isothermal flow with the average temperature of a tube wall equal to 25 °C.

Solution. (a) For an isothermal flow, we use Eq. (1-43) for an average temperature of the aniline equal to $(100 + 40)/2 = 70$ °C.

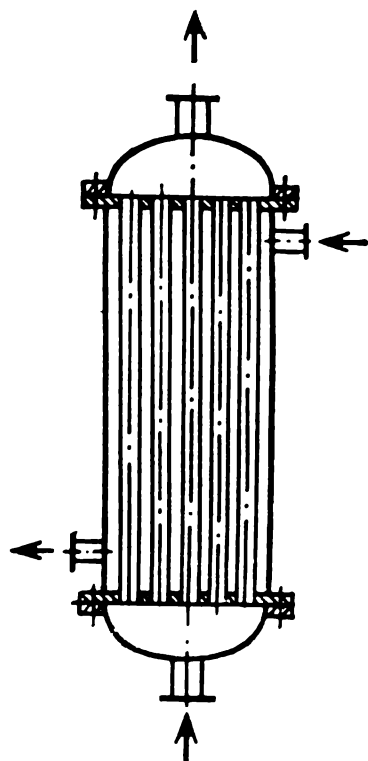


Fig. 1-22. To Example 1-36

The equivalent diameter of the shell side space of the exchanger is

$$d_{eq} = \frac{4A}{P} = \frac{D^2 - nd^2}{D + nd} = \frac{200^2 - 19 \times 26^2}{200 + 19 \times 26} = 39.2 \text{ mm}$$

where D = internal diameter of the shell, mm

d = external diameter of a tube, mm

n = number of tubes.

We find the dynamic viscosity of aniline at 70 °C from Fig. A-5: $\mu = 1.4 \times 10^{-3}$ Pa·s. The density of aniline is $\rho = 1040$ kg/m³. Now we can calculate the value of the Reynolds number:

$$Re = \frac{vd\rho}{\mu} = \frac{0.5 \times 0.0392 \times 1040}{1.4 \times 10^{-3}} = 14\,600$$

We find the pipe resistance coefficient in Fig. 1-5. For $Re = 14\,600$ and $d/e = 39.2/0.2 = 196$, we find $\lambda = 0.034$.

The loss of pressure for overcoming the friction resistance is:

$$\Delta p = \frac{\lambda L}{d} \frac{v^2 \rho}{2} = \frac{0.034 \times 2.7 \times 0.5^2 \times 1040}{0.0392 \times 2} = 304 \text{ Pa}$$

(b) If the flow is not isothermal (the temperature of the wall differs from that of the liquid), then the value of Δp_{fr} for an isothermal flow must be multiplied by a correction factor which

we determine, as for hydraulically smooth tubes, by Eq. (1-52):

$$x = \left(\frac{Pr_w}{Pr_{lq}} \right)^{1/3}$$

where Pr_w = Prandtl number for the flowing liquid at the temperature of the tube wall

Pr_{lq} = Prandtl number for the flowing liquid at its average temperature.

We find the values of the Prandtl number for aniline from the nomogram in Fig. A-13: for $t_w = 25^\circ\text{C}$ we have $Pr = 47$, and for $t_{lq} = 70^\circ\text{C}$ we have $Pr = 16.5$.

The loss of pressure for overcoming friction resistance for a non-isothermal (cooling) turbulent flow of aniline is

$$\Delta p_{fr} = 304 \left(\frac{47}{16.5} \right)^{1/3} = 430 \text{ Pa}$$

which is considerably higher than for an isothermal flow.

Example 1-37. To determine the coefficient of longitudinal mixing E_g , experiments were conducted in a packed column for a

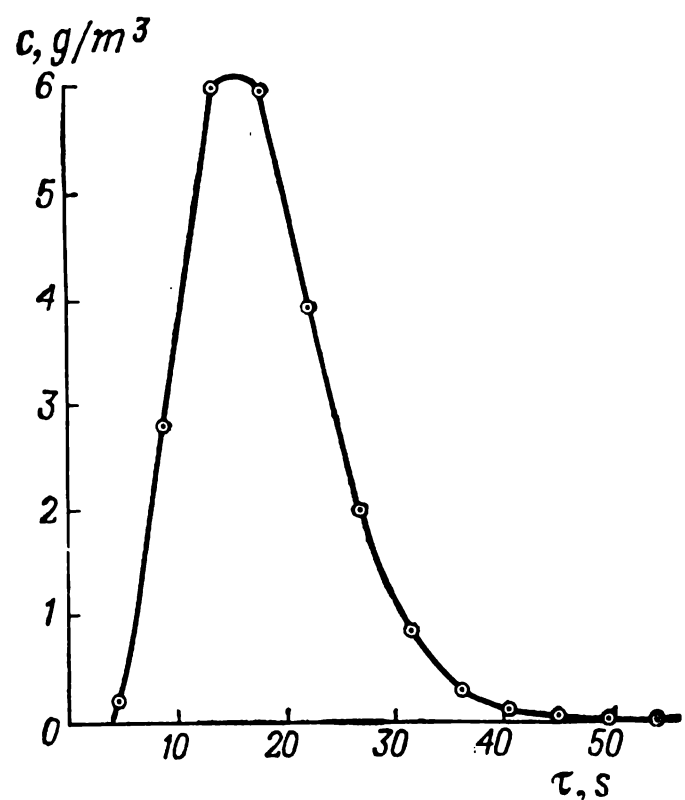


Fig. 1-23. To Example 1-37

gaseous phase to obtain the output response curve for the impulse (instantaneous) introduction of an indicator into the gas flow at the column inlet. The height of the packing layer is 6 m. The velocity of the gas in the column (fictitious) is 0.4 m/s. The results obtained are given in Table 1-6 and Fig. 1-23. Find the value of E_g .

TABLE 1-6

τ	c	τ	c
0	0.0	31.5	0.845
4.5	0.208	36.0	0.288
9.0	2.80	40.5	0.102
13.5	6.01	45.0	0.031
18.0	5.98	49.5	0.009
22.5	3.956	54.0	0.002
27.0	2.025		

τ =time from moment of introduction of indicator, s
 c =concentration of indicator at outlet, g/m³.

Solution. We find the average time spent by the particles (streams) of the gas flow in the column:

$$\tau_{av} = \frac{\int_0^\infty \tau c \, d\tau}{\int_0^\infty c \, d\tau}$$

For $\Delta\tau = \text{const}$, we approximately have

$$\begin{aligned} \int_0^\infty c \, d\tau &\approx \Delta\tau \sum_{i=1}^n \frac{c_i + c_{i-1}}{2} = \frac{\Delta\tau}{2} \left(\sum_{i=1}^n c_i + \sum_{i=1}^n c_{i-1} \right) = \\ &= \frac{4.5}{2} (22.256 + 22.254) = 100 \text{ g}\cdot\text{s}/\text{m}^3 \\ \int_0^\infty \tau c \, d\tau &\approx \frac{\Delta\tau}{2} \left(\sum_{i=1}^n \tau_i c_i + \sum_{i=1}^n \tau_{i-1} c_{i-1} \right) = \\ &= \frac{4.5}{2} (410.265 + 410.157) = 1850 \text{ g}\cdot\text{s}^2/\text{m}^3 \\ \tau_{av} &= \frac{1850}{100} = 18.5 \text{ s} \end{aligned}$$

We determine the variance:

$$\sigma^2 = \frac{\int_0^\infty \tau^2 c d\tau}{\int_0^\infty c d\tau} - \tau_{av}^2$$

$$\int_0^\infty \tau^2 c d\tau \approx \Delta\tau \sum_{i=1}^n \frac{\tau_i^2 c_i + \tau_{i-1}^2 c_{i-1}}{2} = \frac{\Delta\tau}{2} \left(\sum_{i=1}^n \tau_i^2 c_i + \sum_{i=1}^n \tau_{i-1}^2 c_{i-1} \right) =$$

$$= \frac{4.5}{2} (8352.80 + 8346.97) = 37\,590$$

$$\sigma^2 = \frac{37\,590}{100} - 342 = 33.9 \text{ s}^2$$

In dimensionless units:

$$\sigma_\theta^2 = \frac{\sigma^2}{\tau_{av}^2} = \frac{33.9}{342} = 0.0992$$

We find the value of the modified Peclet number for longitudinal mixing $Pe = \frac{vH}{E_g}$ from the equation*:

$$\sigma_\theta^2 = \frac{2}{Pe^2} (Pe - 1 + e^{-Pe})$$

Solving this equation by the method of consecutive approximations, we find: $Pe = 19.11$.

The coefficient of longitudinal mixing is:

$$E_g = \frac{vH}{Pe} = \frac{0.4 \times 6}{19.11} = 0.126 \text{ m}^2/\text{s}$$

PROBLEMS **

1-1. Find the molar mass and density of water gas at $t = 90^\circ\text{C}$ and $p_{\text{abs}} = 1.2 \text{ at}$. The composition of the gas is $\text{H}_2 - 50\%$, $\text{CO} - 40\%$, $\text{N}_2 - 5\%$, and $\text{CO}_2 - 5\%$ (by volume).

1-2. Determine the density of carbon dioxide at $t = 85^\circ\text{C}$ and a gauge pressure of $p_{\text{gauge}} = 2 \text{ at}$. The atmospheric pressure is 760 mm Hg .

1-3. The composition of the products of combustion of 1 kg of coke gas is (in kg): $\text{CO}_2 - 1.45$, $\text{N}_2 - 8.74$, and $\text{H}_2\text{O} - 1.92$. Find the composition by volume of the combustion products.

1-4. The rarefaction in the drying tower of a sulphuric acid plant is measured with a U-type draught gauge filled with sulphuric acid having a density of 1800 kg/m^3 . The reading of the draught gauge is 3 cm . What is the absolute pressure in the tower expressed in Pa if the barometric pressure is 750 mm Hg ?

1-5. A manometer on a pipe filled with a liquid shows a pressure of 0.18 kgf/cm^2 . To what height h above the point of connection of the manom-

* Van der Laan, E. T. *Chem. Eng. Sci.*, 7: 187 (1958).

** The answers to the problems are given after Chap. 11.

eter will the liquid in the pipe rise in an open piezometer if this liquid is (a) water, and (b) carbon tetrachloride (Fig. 1-24)?

1-6. The height of the level of fuel oil in a tank is 7600 mm (Fig. 1-25). The relative density of the oil is 0.96. At a height of 800 mm from the bottom,

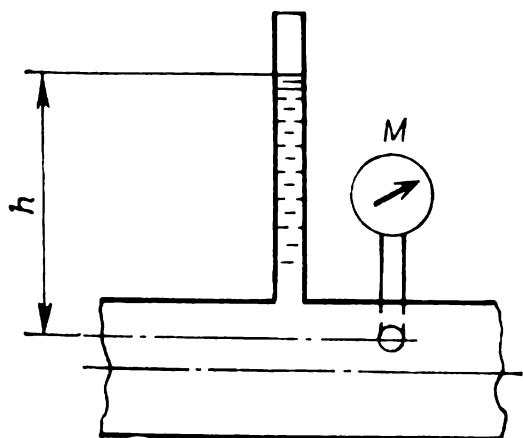


Fig. 1-24. To Problem 1-5

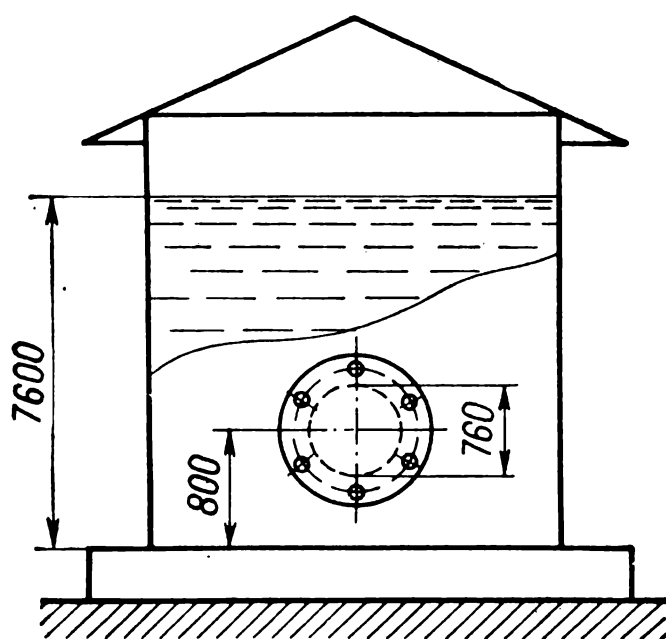


Fig. 1-25. To Problem 1-6

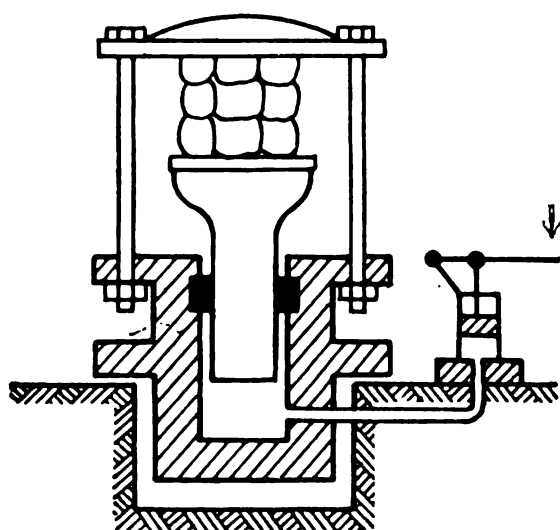


Fig. 1-26. To Problem 1-7

the tank has a round manhole 760 mm in diameter whose cover is fastened on with bolts 10 mm in diameter. Assuming that the permissible ultimate stress is 700 kgf/cm² for the bolts, determine the required number of them. Also determine the pressure of the oil on the bottom of the tank.

1-7. A force of 589 N (60 kgf) is acting on the small piston 40 mm in diameter of a hand-operated hydraulic press (Fig. 1-26). Disregarding the losses,

determine the force acting on the body being pressed if the diameter of the big piston is 300 mm.

1-8. The dynamic viscosity of a liquid at 50 °C is 30 cP. The relative density of the liquid is 0.9. Determine the kinematic viscosity.

1-9. Find the dynamic viscosity of a nitrogen and hydrogen mixture containing 75% of hydrogen and 25% of nitrogen (by volume) at 20 °C and atmospheric pressure.

1-10. The dynamic viscosity of linseed oil at 30 °C is known to equal 0.331 P, and at 50 °C to equal 0.176 P. What will the dynamic viscosity of this oil be at 90 °C? (Use the linearity rule, taking 100% glycerine as the standard liquid.)

1-11. A cooler consists of 19 tubes with a diameter of 20×2 mm (see Fig. 1-22). Water flows into the tube space of the cooler through a pipe with a diameter of 57×3.5 mm. The velocity of the water in the pipe is 1.4 m/s. The water flows upward. Determine the velocity of the water in the cooler tubes.

1-12. Nitrogen under a gauge pressure of $p_{\text{gauge}} = 3$ at flows at a rate of 6400 m³/h (counting at 0 °C and 760 mm Hg) through the tubes of a heat exchanger consisting of 379 tubes with a diameter of 16×1.5 mm. The nitrogen enters the exchanger at 120 °C and leaves it at 30 °C. Determine the velocity of the nitrogen in the exchanger tubes at the inlet and the outlet.

1-13. A cooler consists of two concentric steel pipes with a diameter of 29×2.5 mm and 54×2.5 mm. Brine having a density of 1150 kg/m³ flows through the inner pipe at a rate of 3.73 ton/h. A gas under a pressure of $p_{\text{abs}} = 3$ at an average temperature of 0 °C flows at a rate of 160 kg/h through the space between the pipes. The density of the gas at 0 °C and 760 mm Hg is 1.2 kg/m³. Find the velocities of the gas and the liquid in the cooler.

1-14. Determine the required diameter of the outer pipe in the conditions of the preceding problem if the gas flows under atmospheric pressure, but at the same velocity and with the same mass rate of flow.

1-15. Calculate the hydraulic radius in the general form with a filled cross section for an annular section, a square, a rectangle, and an equilateral triangle.

1-16. Determine the equivalent diameter of the shell side space of a shell-and-tube heat exchanger (see Fig. 1-22) consisting of 61 tubes with a diameter of 38×2.5 mm. The internal diameter of the shell is 625 mm.

1-17. Determine the conditions of flow of water in the annular space of a heat exchanger of the "tube-in-tube" type (see Fig. 1-13). The outer tube has a diameter of 96×3.5 mm, the inner one—57×3 mm, the rate of flow of the water is 3.6 m³/h, and the average temperature of the water is 20 °C.

1-18. Determine the conditions of flow of ethyl alcohol (a) in a straight pipe with a diameter of 40×2.5 mm, and (b) in a coil formed of the same pipe. The diameter of a coil turn is 570 mm. The velocity of the alcohol is 0.13 m/s, its average temperature is 52 °C.

1-19. Determine the local velocity along the axis of a pipe with a diameter of 57×3.5 mm when acetic acid flows through it at a rate of 200 litres an hour at 38 °C.

1-20. A Pitot-Prandtl tube (see Fig. 1-4) is installed at the middle of a pipe with an internal diameter of 320 mm. Its differential manometer filled with water shows a level difference of $H = 5.8$ mm. Dry air at 21 °C and atmospheric pressure flows through the pipe. Find the mass rate of flow of the air.

1-21. The amount of liquid flowing through an orifice 10 mm in diameter in the bottom of an open tank in which a constant level of the liquid is maintained at a height of 900 mm is 750 litres an hour. Determine the discharge coefficient. In how much time will the tank be emptied if the supply of liquid into it is stopped? The diameter of the tank is 800 mm.

1-22. Water flows into a head tank with a cross-sectional area of 3 m². The bottom of the tank has a drain opening. In steady flow, the discharge through the opening equals the inflow, and the level of the water is at a height of 1 m.

If the inflow of water is stopped, its level will lower and in 100 seconds the tank will be empty. Determine the inflow of water into the tank.

1-23. Mineral oil having a relative density of 0.9 flows through a horizontal pipe with an internal diameter of 200 mm. An orifice plate (see Fig. 1-3) with sharp edges (the discharge coefficient is 0.61) is installed in the pipe. The diameter of the orifice of the plate is 76 mm. A mercury differential manometer connected to the orifice plate shows a level difference of 102 mm. Determine the velocity of the oil in the pipe and the rate of its flow.

1-24. A "Venturi tube" flow meter (Fig. 1-27) with the internal diameter of its narrow part equal to 60 mm is installed on a pipe with a diameter of

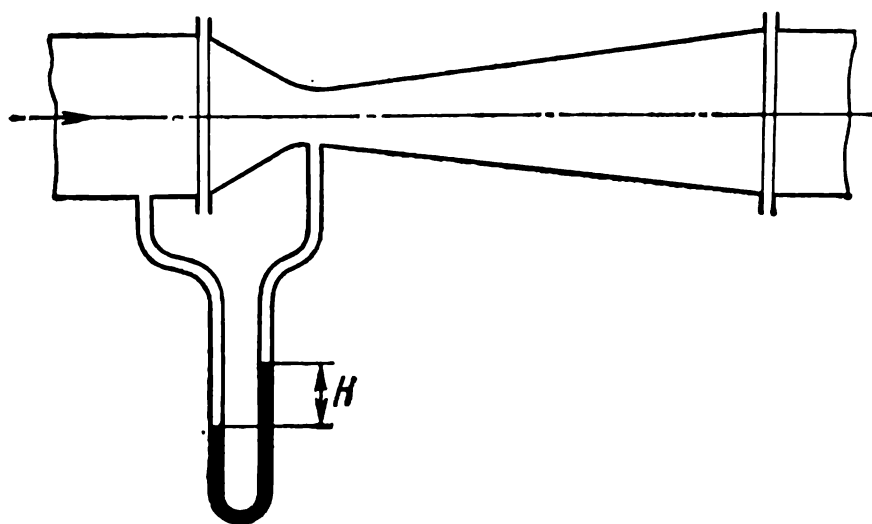


Fig. 1-27. To Problem 1-24

160×5 mm. Ethane under atmospheric pressure flows through the pipe at 25 °C. The reading of the water differential manometer of the Venturi tube is $H=32$ mm. Find the mass rate of flow of the ethane through the pipe (in kg/h), assuming the discharge coefficient to equal 0.97.

1-25. Determine the loss of pressure for overcoming the friction resistance when water flows through a brass pipe with a diameter of 19×2 mm and a length of 10 m. The velocity of the water is 2 m/s. Its temperature is 55 °C. Assume that the roughness of the pipe is $e=0.005$ mm.

1-26. Determine the loss of pressure for overcoming the friction resistance in a lead coil through which 60% sulphuric acid flows with a velocity of 0.7 m/s at an average temperature of 55 °C. Take the maximum roughness of lead pipes according to Table A-12. The internal diameter of the coil pipe is 50 mm, the diameter of a turn of the coil is 800 mm, the number of turns is 20. Determine the length of the coil approximately according to the number of turns and their diameter.

1-27. A steel pipe with an internal diameter of 200 mm and a length of 1000 m is used to transport 120 kg of hydrogen an hour. The average pressure in the network is 1530 mm Hg. The temperature of the gas is 27 °C. Determine the loss of pressure for overcoming the friction resistance in the pipe.

1-28. Find the loss of pressure for overcoming the friction resistance in a steel steam pipe 50 m long with a diameter of 108×4 mm. The pressure of the steam is $p_{\text{abs}}=6$ at, the velocity of the steam is 25 m/s.

1-29. How will the loss of pressure for overcoming the friction resistance in a gas pipe through which nitrogen flows change if with a constant mass rate of flow of the nitrogen (a) the pressure (absolute) of the nitrogen fed into the pipe is increased from 1 to 10 at at a constant temperature, and (b) the temperature of the nitrogen is increased from 0 to 80 °C at a constant pressure.

1-30. The rate of flow of water through a water main is 10 m³/h. How much

water will flow in one hour through a pipe having a double diameter at the same loss of pressure for overcoming the friction resistance? Consider the pipe resistance coefficient to be constant. The flow is turbulent.

1-31. A straight horizontal pipe 150 m long must supply 10 m^3 of a liquid an hour. The permissible loss of pressure (head) is 10 m. Determine the required diameter of the pipe assuming the pipe resistance coefficient λ to equal 0.03.

1-32. How will the loss of pressure for overcoming friction resistance change if with a constant rate of flow of the liquid the diameter of the pipeline is halved? Solve the problem in two variants: (a) considering that the flow in

Fig. 1-28. To Problem 1-33

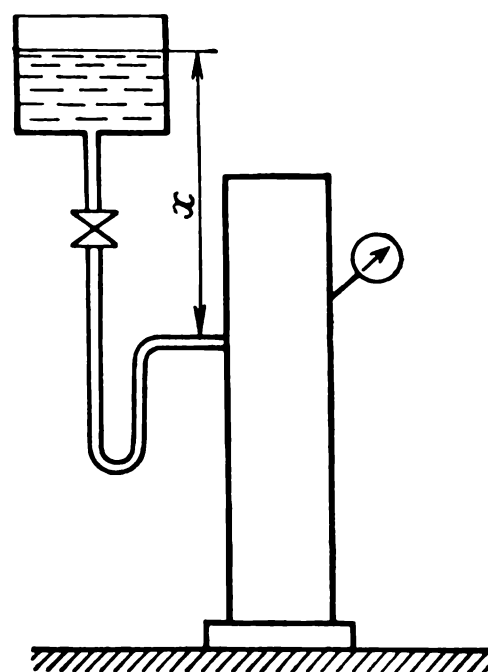
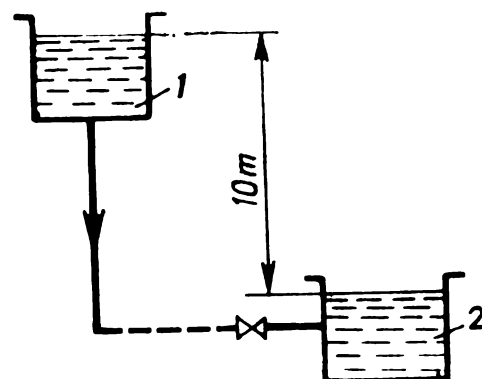


Fig. 1-29. To Problem 1-34



both cases is laminar; and (b) considering that the flow in both cases is in the self-similar (inertia) region.

1-33. A liquid whose relative density is 0.9 flows by gravity from a pressure tank in which atmospheric pressure is maintained into a rectification column (Fig. 1-28). The gauge pressure (p_{gauge}) in the column is 0.4 kgf/cm^2 . At what height x must the level of the liquid in the pressure tank be over the inlet into the column for the velocity of the liquid in the pipe to be 2 m/s ? The head lost for overcoming friction resistance and local obstacles is 2.5 m . Use the Bernoulli equation.

1-34. An 86% solution of glycerine flows down from pressure tank 1 into apparatus 2 along a pipe with a diameter of $29 \times 2 \text{ mm}$ (Fig. 1-29). The difference between the levels of the solution is 10 m . The total length of the pipeline is 110 m . Determine the rate of flow of the solution if its relative density is 1.23, and its dynamic viscosity is 97 cP . Ignore the local obstacles. Assume the flow to be laminar (with following checking of this assumption). Consider the level of the solution in the tank to be constant.

1-35. A pump delivers 20 tons of chlorobenzene an hour at 45°C from reactor 1 into pressure tank 2 (Fig. 1-30). A rarefaction of 200 mm Hg is main-

tained in the reactor over the liquid, while the pressure in the pressure tank is atmospheric. The pipeline is made of steel pipes with insignificant corrosion having a diameter of 76×4 mm and a total length of 26.6 m. The pipeline is provided with two cocks, an orifice plate ($d_o = 48$ mm) and five pipe bends at an angle of 90° ($R_o/d = 3$). The chlorobenzene is pumped to a height of $H = 15$ m.

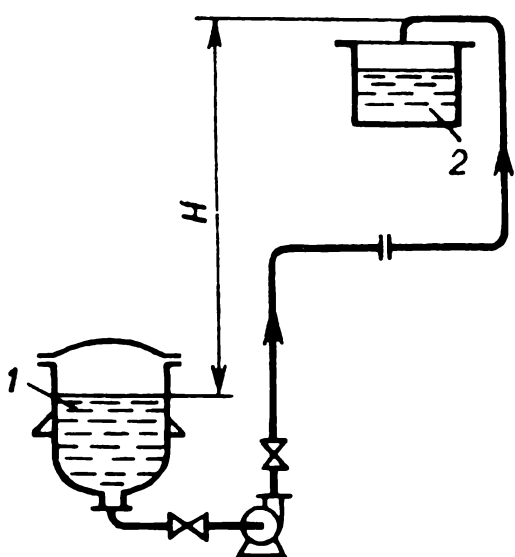


Fig. 1-30. To Problem 1-35

Find the power consumed by the pump if the overall efficiency of the pump installation is 0.7.

1-36. A shell-and-tube heat exchanger (see Fig. 1-22) consists of 187 steel tubes with insignificant corrosion ($e = 0.2$ mm) having a diameter of 18×2 mm and a length of 1.9 m. The shell is made of a tube with a diameter of 426×12 mm. Nitrogen flows through the shell side parallel to the axes of the tubes at a rate of $3000 \text{ m}^3/\text{h}$ (in standard conditions) under atmospheric pressure, the average temperature being -10°C . The diameter of the inlet and outlet pipe connections is 250 mm. Determine the hydraulic resistance of the shell side.

1-37. Toluene in an amount of 1900 kg/h is cooled from 70 to 30°C in a "tube-in-tube" type of heat exchanger (see Fig. 1-13) consisting of two concentric tubes: an internal one with a diameter of 44.5×3.5 mm and an external one with a diameter of 89×5 mm. The toluene flows through the annular space between the external and internal tubes. Cooling water that becomes heated from 14 to 21°C flows along the internal tube. Determine the loss of pressure for overcoming the friction resistance per metre of tube length for the toluene and the water, assuming that the steel tubes have insignificant corrosion. The average temperature of the wall of the internal tube is 25°C .

1-38. Bring Eq. (1-47) to the dimensionless form.

1-39. What geometrical scale must be taken for a model if the working liquid in the industrial apparatus is petroleum and in the model is water, the kinematic viscosity of water being one-fiftieth that of petroleum? What velocity must be imparted to the water in the model if the velocity of the petroleum in the industrial apparatus is 1 m/s ? The forces of friction and forces of gravity are modelled simultaneously.

1-40. Determine the power used by a pump to handle $4.6 \text{ m}^3/\text{h}$ of a cooling brine (a 25% solution of CaCl_2) from a refrigerating installation into a condenser above a rectification column. The height of lifting is 16 m, the dynamic viscosity of the brine is 9.5 cP, its density is 1200 kg/m^3 , the diameter of the pipeline is 32×2.5 mm, its total length is 80 m. The steel pipes have insignificant corrosion. The pipeline has six pipe bends at an angle of 90° ($R_o/d = 4$) and four straight-through globe valves. The overall efficiency of the pump and its motor is 0.5.

1-41. A liquid is being pumped through a horizontal pipeline. How many times will the expenditure of energy for pumping grow if a double amount

of the liquid flows through the pipeline? Consider the pipe resistance coefficient to be constant, $\Delta p_{\text{add}} = 0$.

1-42. It is necessary to pump $25 \text{ m}^3/\text{h}$ of a liquid having a density of 1200 kg/m^3 and a dynamic viscosity of 1.7 cP through a steel pipe with an internal diameter of 75 mm . The terminal point of the pipe is higher than the starting one by 24 m . The pipe is 112 m long. It is provided with two straight-through globe valves and five 90° pipe bends with a bending radius of 300 mm . The pipe has insignificant corrosion. Find the power consumed if the overall efficiency of the pump installation is 0.6 .

1-43. Water at 10°C is delivered from a river by a pump into an open reservoir (Fig. 1-31). The upper point of the pipeline is 50 m higher than the

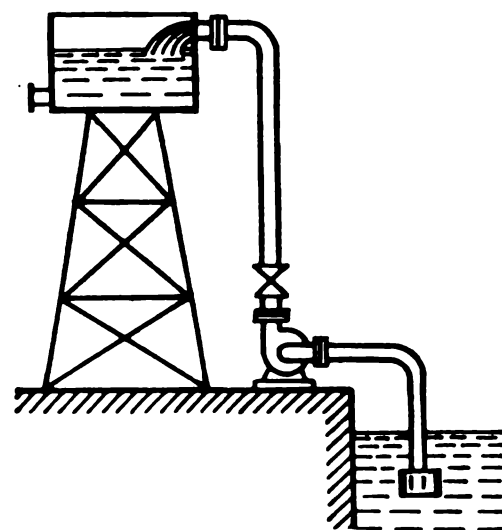


Fig. 1-31. To Problem 1-43

level of the water in the river. The pipeline is of steel with insignificant corrosion, its internal diameter is 80 mm , its total equivalent length (its actual length plus the equivalent length of local obstacles) is 165 m . The pump delivers 575 litres of water a minute. What is the power consumed by the pump if the efficiency of the pump installation is 0.55 ?

1-44. Air at 27°C and atmospheric pressure has to be delivered at a rate of $14\,400 \text{ kg/h}$ through a straight air pipeline having a rectangular cross section of $400 \times 600 \text{ mm}$ and made of roofing steel. The length of the air pipeline is 60 m . Find the required power of the motor if its efficiency is 0.95 and that of the fan is 0.4 .

1-45. Carbon dioxide at a pressure of 2 at (according to a manometer) is fed through a pipeline having an internal diameter of 100 mm at an average temperature of 75°C and with a mass velocity of $30 \text{ kg/m}^2 \cdot \text{s}$. The roughness of the pipeline is $e = 0.7 \text{ mm}$. Determine the hydraulic resistance of the horizontal pipeline if its length is 90 m and it is provided with four elbows at an angle of 90° and a gate valve. Also determine the power consumed by the gas blower for transportation of the carbon dioxide if its efficiency is 50% .

1-46. Ethyl alcohol having a concentration of 40% is allowed to flow from a tank down a pipe with a diameter of $33.5 \times 2.8 \text{ mm}$. The pipe has a cock and two elbows at an angle of 90° . The total length of the pipe is 49 m . Determine the velocity of the alcohol in the pipe (with a difference of levels of 7.2 m). Assume the coefficient of pipe resistance to approximately equal 0.025 . Find the velocity of the alcohol and check the value of the coefficient of pipe resistance. The temperature of the alcohol is 35°C .

1-47. Nitrobenzene having a temperature of 44°C flows through a pipeline with a diameter of $26.8 \times 2.5 \text{ mm}$. The starting point of the pipeline is higher than the terminal one by 200 mm . The length of the horizontal part of the pipeline is 242 m . Take into consideration only the friction resistance of the pipe. Find the mass rate of flow of the nitrobenzene and check the adopted conditions of its motion.

1-48. Water from an open basin has to be pumped through a pipeline with an internal diameter of 70 mm into an apparatus working under a pressure of $p_{\text{abs}}=2$ at. The upper point of the pipeline is higher than the level of the water in the basin by 5 m. The total equivalent length of the pipeline (its actual length plus the equivalent length of the local obstacles) is 350 m. The coefficient of pipe resistance is $\lambda=0.03$. Find the relationship between the rate of flow of the water through the pipeline and the loss of pressure for overcoming all the resistances in it (find the equation of the network characteristic).

1-49. A centrifugal pump has the following certificate characteristic:

Rate of flow of water, m ³ /h	12	18	24	30
Head created, m	38	36	32	26

How much water will be delivered by this pump if it is installed for operation in the network of Problem 1-48? (Find the working point.)

1-50. A fan supplies air sucked in from the atmosphere at a rate of 12 500 m³/h. What mass of air is supplied by the fan an hour in winter ($t=-15^{\circ}\text{C}$) and in summer ($t=30^{\circ}\text{C}$)?

1-51. Determine the pressure developed by a fan that supplies air from the atmosphere at a temperature of 18°C into a space with a gauge pressure of 43 mm H₂O. The losses of pressure in the pipe are 275 Pa, and the velocity of the air in it is 11.5 m/s.

1-52. What absolute pressure (in at) must the air fed into a monte-jus (acid elevator) (Fig. 1-32) have for lifting sulphuric acid with a relative density of 1.78 to a height of 21 m? Disregard the hydraulic losses.

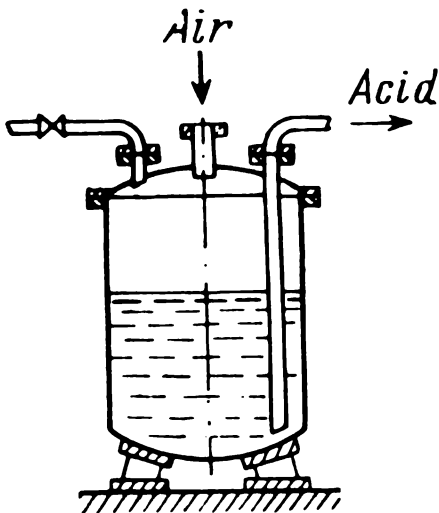


Fig. 1-32. To Problem 1-52

1-53. The velocity of the stream at the outlet from the diffuser of a horizontal water-jet pump (see Fig. 2-10) is 2.35 m/s. The water leaves the diffuser at atmospheric pressure. The diameter of the outlet opening of the diffuser is 62 mm, the diameter of the nozzle opening (section *I*) is 30 mm. Disregarding the losses, determine the theoretical height *H* to which water pumped from an open basin can be lifted.

1-54. Determine the hydraulic resistance of a layer of dry packing 3 m high consisting of ceramic rings 15×15×2 mm in size. Air is sucked through the packing at 20°C and atmospheric pressure with a velocity of 0.4 m/s (fictitious velocity).

SYMBOLS

- A area; coefficient; number of like atoms in a molecule
- a constant
- B correction for the grouping of atoms and the nature of the bond between them
- b constant; correction factor
- C constant; Sutherland's constant

- C_d discharge coefficient
 c concentration
 D diameter
 d diameter
 E coefficient of longitudinal mixing
 Eu Euler dimensionless number
 e height of slot; roughness of pipe wall
 F force
 Fr Froude dimensionless number
 G weight
 Gr Grashof dimensionless number
 g acceleration due to gravity
 H height
 h height; hydraulic resistance head
 K constant
 k relative density of froth layer; roughness coefficient
 L latent heat of transformation; length
 M molar mass
 m number of pipe rows in bank in direction of flow
 n atomic constant; dimensionless coefficient; number of passes
 P perimeter; power
 Pr Prandtl dimensionless number
 p pressure
 Q_m mass rate of flow
 Q_v volumetric rate of flow
 R molar gas constant
 Re Reynolds dimensionless number
 r radius
 s pipe spacing
 T absolute temperature
 V volume
 v velocity
 x dimensionless correction factor; mole fraction
 \bar{x} mass fraction
 z geometrical (position) head

Greek Letters

- γ unit (volume) weight
 Δ increment; relative density
 ε relative roughness
 ζ dimensionless local resistance coefficient
 η efficiency
 θ temperature
 κ coefficient of contraction
 λ dimensionless pipe resistance coefficient
 μ dynamic viscosity
 ν kinematic viscosity
 π ratio of circumference to diameter, $\pi = 3.14 +$
 ρ density
 Σ sum
 σ unit surface area
 τ time
 φ angle of attack; velocity coefficient; volume fraction
 ψ dimensionless correction factor

FUNDAMENTAL RELATIONSHIPS
AND FORMULAS FOR CALCULATIONS

1. The total head developed by a pump (Fig. 2-1) is determined by the equation

$$H = \frac{p_2 - p_1}{\rho g} + H_g + h \quad (2-1)$$

which is similar to Eq. (1-57).

Here H = total head developed by the pump in metres of column of the liquid being pumped

p_2 and p_1 = pressures in the delivery space and in the suction space, respectively, Pa

ρ = density of the liquid being pumped, kg/m³

H_g = geometrical height to which the liquid is lifted, m

h = head required for creating a velocity and for overcoming friction resistance in the pipes and local obstacles in the suction and delivery lines, m

$g = 9.81 \text{ m/s}^2$ = acceleration due to gravity.

The same total head H can also be calculated by a different equation:

$$H = \frac{p_d - p_s}{\rho g} + H_0 + \frac{v_d^2 - v_s^2}{2} \quad (2-2)$$

where p_d = pressure in the delivery pipeline at the outlet of the liquid from the pump, Pa

p_s = pressure in the suction pipeline at the inlet of the liquid into the pump, Pa

H_0 = vertical distance between the points of measuring the pressures p_d and p_s , m

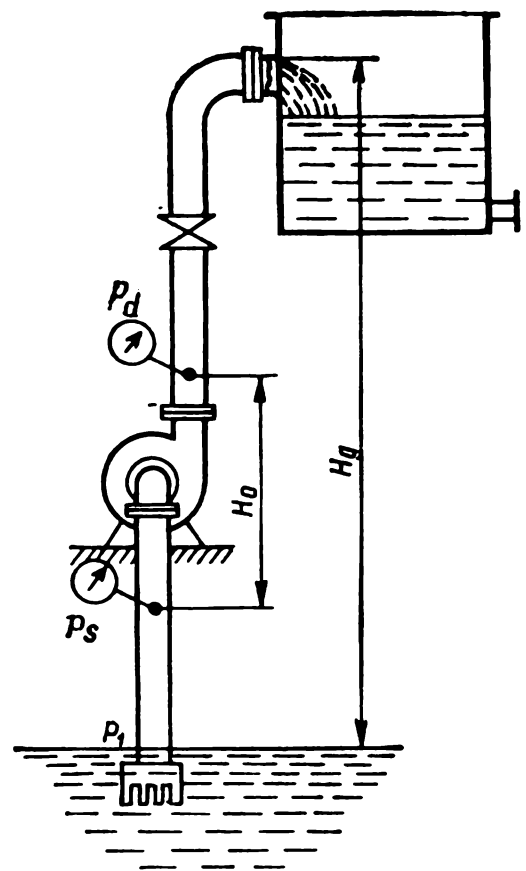
v_d = velocity of the liquid in the delivery pipeline, m/s

v_s = velocity of the liquid in the suction pipeline, m/s.

If the velocities v_d and v_s are close to each other, and the distance H_0 is small, Eq. (2-2) can be written in a simpler form:

$$H \approx \frac{p_d - p_s}{\rho g} \quad (2-3)$$

Fig. 2-1. Pump installation



Equations (2-1) and (2-2) are obtained from the Bernoulli equation compiled for the relevant cross sections of the flow. Equation (2-2) is used when testing pumps in operation, and Eq. (2-1) when designing pump installations.

2. The power P (in kW) consumed by the motor of a pump is:

$$P = \frac{Q\rho gH}{1000\eta} \quad (2-4)$$

where Q = volume output (delivery) of the pump *, m^3/s

ρ = density of the liquid being pumped, kg/m^3

$g = 9.81 \text{ m/s}^2$ = acceleration due to gravity

H = total head developed by the pump in metres of column of the liquid being pumped

η = overall efficiency of the pump installation equal to the product of the efficiencies of the pump η_p , the transmission η_{tr} , and the motor η_m :

$$\eta = \eta_p \eta_{tr} \eta_m \quad (2-5)$$

The motor installed for a pump has a somewhat greater power P_{inst} than that needed by the pump to provide a reserve for possible overloading:

$$P_{inst} = \beta P \quad (2-6)$$

* It is customary practice to denote the volumetric rate of flow of a fluid delivered by a pump (fan) by Q instead of Q_v in textbooks on pump and fan installations.

The power reserve factor β is taken depending on the value of P (Table 2-1).

TABLE 2-1

P kW	β
< 1	2-1.5
1-5	1.5-1.2
5-50	1.2-1.15
> 50	1.1

3. The theoretical suction height of a piston pump H_s (in m) is determined by the expression (Fig. 2-2):

$$H_s \leq p_{atm} - h_t - \sum h$$

(2-7)

where p_{atm} = atmospheric pressure
 h_t = pressure of the saturated vapour of the liquid being sucked in at the pumping temperature t
 $\sum h$ = losses of suction height including the expenditure of energy for imparting a velocity to the flow of liquid and overcoming the inertia of the column of liquid in the suction pipeline, and also for overcoming friction resistance and local obstacles in the suction line; $\sum h$ grows with increasing pump speed.

All the quantities in expression (2-7) are in metres of column of the liquid being pumped.

The atmospheric pressure p_{atm} depends on the altitude of the place of installation of the pump (Table A-19).

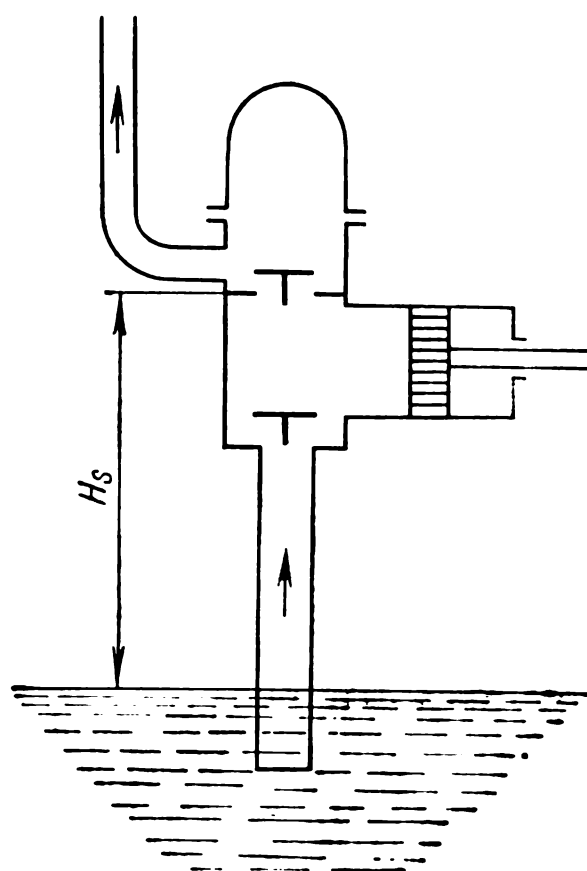
The saturated vapour pressure of the liquid being sucked in (h_t) is determined by its temperature. The temperature dependence of h_t for water is given in Table 2-2.

TABLE 2-2

Temperature, °C	5	10	20	30	40	50	60	70	80	90	100
Saturated vapour pressure h_t : mm H ₂ O kPa	0.09 0.88	0.12 1.18	0.24 2.36	0.43 4.22	0.75 7.36	1.25 12.26	2.02 19.82	3.17 31.1	4.82 47.3	7.14 70.04	10.33 101.3

In practice for determining the permissible suction height (in m) when pumping water with the aid of piston pumps, it is recommended to use the data of Table A-20.

Fig 2-2. Piston pump



4. The output of a piston pump Q (in m^3/s) is:
 (a) for single-acting and differential (trim) pumps:

$$Q = \eta_v \frac{A s n}{60} \quad (2-8)$$

- (b) for double-acting pumps:

$$Q = \eta_v \frac{(2A - A_r) s n}{60} \quad (2-9)$$

where η_v = delivery factor whose value averages from 0.8 to 0.9
 A = working area (cross-sectional area) of the piston (plunger), m^2

A_r = cross-sectional area of the rod, m^2

s = piston stroke, m

n = speed of rotation, i. e. the number of double piston strokes a minute.

5. The head and output of a centrifugal pump at a given pump speed depend on each other. If we plot on a graph showing this relationship and called the pump characteristic a curve of the network characteristic (Fig. 2-3), the intersection of the two curves gives the so-called working point determining the head and output of the pump when it is serving the given network.

The efficiency of a centrifugal pump changes with a change in the head and output.

When the speed n of a centrifugal pump changes within small limits, the changes in its output Q , head H , and power consumed

P are determined from the following relationships:

$$\frac{Q_1}{Q_2} = \frac{n_1}{n_2}; \quad \frac{H_1}{H_2} = \left(\frac{n_1}{n_2}\right)^2 \quad \text{and} \quad \frac{P_1}{P_2} = \left(\frac{n_1}{n_2}\right)^3 \quad (2-10)$$

The suction height H_s of a centrifugal pump (in metres) is calculated by the formula

$$H_s \leq p_{\text{atm}} - h_t - h_{1,s} - h_{\text{cav}} \quad (2-11)$$

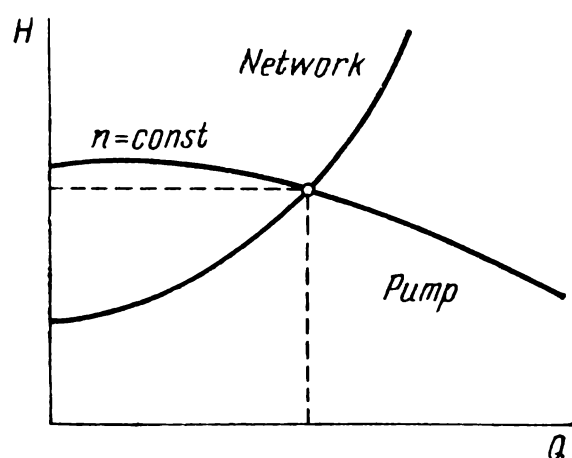


Fig. 2-3. Characteristics of centrifugal pump (at $n = \text{const}$) and network

where p_{atm} = atmospheric pressure

h_t = saturated vapour pressure of the liquid being sucked in at the pumping temperature t

$h_{1,s}$ = hydraulic resistance of the suction line including the energy needed for imparting a velocity to the flow of liquid

h_{cav} = cavitation correction (a reduction in the suction height to avoid cavitation) depending on the output of the pump Q (in m^3/s) and the speed n (in rpm).

The cavitation correction is determined by the equation

$$h_{\text{cav}} = 0.00125 (Qn^2)^{0.67} \quad (2-12)$$

The quantities p_{atm} , h_t , $h_{1,s}$, and h_{cav} are all expressed in metres of column of the liquid being pumped.

6. The pressure (more exactly the increase in pressure) created by a fan* delivering air (Fig. 2-4) is:

$$\Delta p = (p_2 - p_1) + (\Delta p_s + \Delta p_d) + \frac{v_d^2 \rho}{2} \quad (2-13)$$

or

$$\Delta p = \left(p_{\text{st}, d} + \frac{v_d^2 \rho}{2} \right) - \left(p_{\text{st}, s} + \frac{v_s^2 \rho}{2} \right) \quad (2-14)$$

where p_1 = pressure in the space from which the fan takes in air, Pa

p_2 = pressure in the space into which the fan delivers the air, Pa

* This pressure is sometimes called the total head of a fan in textbooks on fan installations and is designated by the letter H .

- Δp_s and Δp_d = losses of pressure in the suction and delivery lines, Pa
 v = velocity of the air at the outlet from the installation, m/s
 $p_{st, d}$ and $p_{st, s}$ = static pressures directly after the fan and before it, Pa
 v_d and v_s = velocities of the air in the delivery and suction pipelines, m/s
 ρ = density of the air, kg/m³.

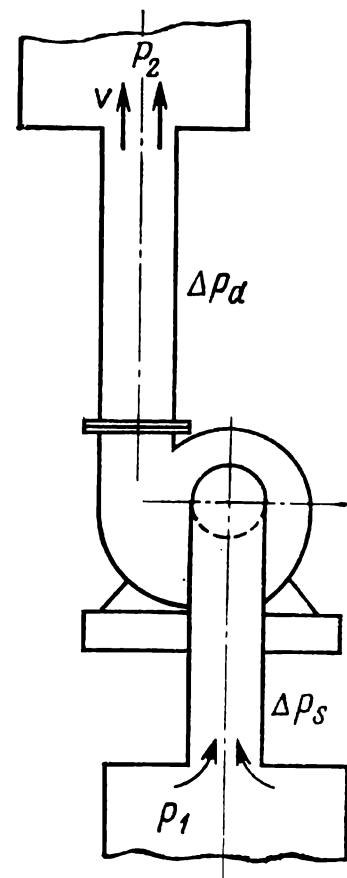


Fig. 2-4. Fan installation

Equation (2-13) is similar to Eq. (2-1) for a pump and to Eq. (1-57). Equation (2-14) is similar to Eq. (2-2) for a pump.

If instead of air a fan delivers a different gas whose density is not the same as that of the surrounding air, then in the preceding formulas ρ is the density of the gas, and it is necessary to add to the right-hand side of Eq. (2-13) the quantity

$$\Delta p_{\text{lift}} = (\rho - \rho_{\text{air}})gz \quad (2-15)$$

where z is the difference between the heights of the places of delivery and suction, m.

The power P (in kW) used by a fan installation is:

$$P = \frac{Q\Delta p}{1000\eta} \quad (2-16)$$

where Q = delivery (output) of the fan, m³/s

Δp = increase in pressure created by the fan, Pa

$\eta = \eta_f \eta_{tr} \eta_m$ = overall efficiency of the fan installation [see Eq. (2.5)].

7. The graphical characteristic of a centrifugal fan, like that of a centrifugal pump, changes its position when the speed changes. The relationship between the old and the new parameters of operation of a centrifugal fan when the speed changes within small limits is determined by Eq. (2-10).

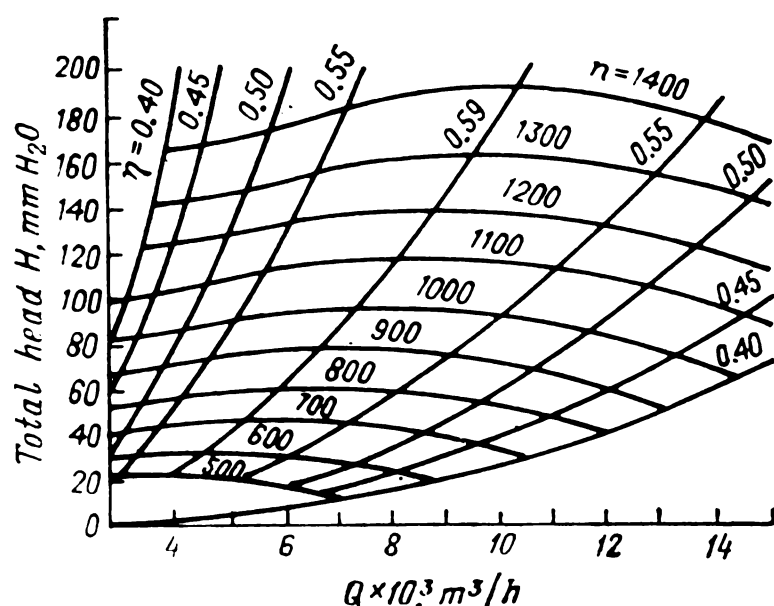


Fig. 2-5. Characteristic of centrifugal fan

Figure 2-5 shows an example of a characteristic of a centrifugal fan for various speeds n (in rpm).

8. The theoretical amount of work W_{ad} (in J/kg) done by a single-stage compressor in the adiabatic (isentropic) compression of 1 kg of a gas can be calculated by the formula:

$$W_{ad} = \frac{\gamma}{\gamma - 1} p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] = \frac{\gamma}{\gamma - 1} R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \quad (2-17)$$

or by the formula

$$W_{ad} = H_2 - H_1 \quad (2-18)$$

The temperature of the gas at the end of the process of adiabatic compression is determined by the equation

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} \quad (2-19)$$

The following notation is used in these formulas:

γ = adiabatic exponent equal to the ratio c_p/c_v (the specific heat capacities at constant pressure and constant volume)

p_1 and p_2 = initial and final pressure of the gas, Pa

v_1 = specific volume of the gas in the initial conditions, i.e. at a pressure of p_1 and a temperature of T_1 , m^3/kg

H_1 and H_2 = initial and final enthalpy of the gas, J/kg

R = gas constant equal to $8310/M$ J/kg·K

M = molar mass of the gas.

The power P (in kW) required by the motor of a single-stage compressor to compress G kg of a gas an hour from the initial pressure p_1 to the final pressure p_2 is determined by the formula:

$$P = \frac{GW_{ad}}{3600 \times 1000 \eta} \quad (2-20)$$

where η is the overall efficiency of the compressor installation.

9. The output Q (in m³/s) of a piston single-acting compressor is determined by the equation:

$$Q = \lambda \frac{Asn}{60} \quad (2-21)$$

where λ = dimensionless delivery coefficient

A = area of the piston, m²

s = piston stroke, m

n = speed, rpm.

The delivery coefficient ranges from $0.8\lambda_0$ to $0.95\lambda_0$, where λ_0 is the volumetric efficiency of a compressor, equal to

$$\lambda_0 = 1 - \varepsilon_0 \left[\left(\frac{p_2}{p_1} \right)^{\frac{1}{m}} - 1 \right] \quad (2-22)$$

where ε_0 = ratio of the volume of the dead space of the cylinder to the piston displacement

m = polytropic exponent of expansion of the compressed gas remaining in the dead space.

10. The theoretical amount of work W_{ad} (in J/kg) done by a multi-stage compressor in the adiabatic compression of 1 kg of a gas from the initial pressure p_1 to the final pressure p_{fn} is determined by the formula

$$\begin{aligned} W_{ad} &= N p_1 v_1 \frac{\gamma}{\gamma - 1} \left[\left(\frac{p_{fn}}{p_1} \right)^{\frac{\gamma - 1}{\gamma N}} - 1 \right] = \\ &= N R T_1 \frac{\gamma}{\gamma - 1} \left[\left(\frac{p_{fn}}{p_1} \right)^{\frac{\gamma - 1}{\gamma N}} - 1 \right] \end{aligned} \quad (2-23)$$

or by the formula

$$W_{ad} = \Delta H_1 + \Delta H_2 + \dots + \Delta H_N \quad (2-24)$$

where N = number of compression stages

$\Delta H_1, \Delta H_2, \dots$ = differences between the enthalpies of the gas for the first, second, etc. stages [Eq. (2-18)].

The power consumed by a multi-stage compressor is calculated by Eq. (2-20). The following equation is sometimes also used for air compressors:

$$P = \frac{1.69 G W_{is}}{3600 \times 1000} = \frac{1.69 G R T_1 \ln \frac{p_{fn}}{p_1}}{3600 \times 1000} \quad (2-25)$$

where 1.69 is a practically established factor taking into account the difference between the actual air compression process in a compressor and an isothermal process.

The output of a multi-stage piston compressor is determined by that of the first stage.

Disregarding the loss of pressure between the stages, the number of compression stages N is found approximately by the equation:

$$x^N = \frac{p_{fn}}{p_1} \quad (2-26)$$

whence

$$N = \frac{\log p_{fn} - \log p_1}{\log x} \quad (2-27)$$

where x is the compression ratio in one stage.

EXAMPLES

Example 2-1. The manometer on the delivery pipeline of a pump (see Fig. 2-1) handling 8.4 m^3 of water a minute shows a pressure of 3.8 kgf/cm^2 . The vacuum gauge on the suction pipeline shows a vacuum of 21 cm Hg . The vertical distance between the places of connection of the manometer and the vacuum gauge is 410 mm . The diameter of the suction pipeline is 350 mm , and of the delivery pipeline 300 mm . Find the head developed by the pump.

Solution. We use Eq. (2-2).

The velocity of the water in the suction pipeline is:

$$v_s = \frac{8.4}{60 \times 0.785 \times 0.35^2} = 1.45 \text{ m/s}$$

The velocity of the water in the delivery pipeline is:

$$v_d = \frac{8.4}{60 \times 0.785 \times 0.3^2} = 1.98 \text{ m/s}$$

The pressure in the delivery pipeline (assuming atmospheric pressure to equal $1.013 \times 10^5 \text{ Pa}$ or 760 mm Hg) is:

$$p_d = (3.8 + 1.03) 9.81 \times 10^4 = 474\,000 \text{ Pa}$$

The pressure in the suction pipeline is:

$$p_s = (0.76 - 0.21) 133.3 \times 1000 = 73\,300 \text{ Pa}$$

The head developed by the pump is

$$\begin{aligned} H &= \frac{474\,000 - 73\,300}{1000 \times 9.81} + 0.41 + \frac{1.98^2 - 1.45^2}{2 \times 9.81} = \\ &= 40.8 + 0.41 + 0.09 = 41.3 \text{ m H}_2\text{O} \end{aligned}$$

Example 2-2. A piston pump operating at a speed of 150 rpm is to deliver water heated to 60°C . Preliminary calculations showed

that the expenditure of energy for creating a velocity and overcoming the inertia losses and the hydraulic resistances in the suction line total 6.5 m H₂O. The average atmospheric pressure at the place of installation of the pump is 736 mm Hg. At what height above the level of the water should the pump be installed?

Solution. The quantity $p_{\text{atm}} - h_t - \sum h$ [expression (2-7)] in the given case equals:

$$\frac{0.736 \times 13\,600}{1000} - 2.02 - 6.5 = 1.48 \text{ m}$$

where $h_t = 2.02$ m has been taken from Table 2-2.

Consequently, the theoretical suction height cannot exceed 1.48 m. Actually, according to the data of Table A-20, the suction height

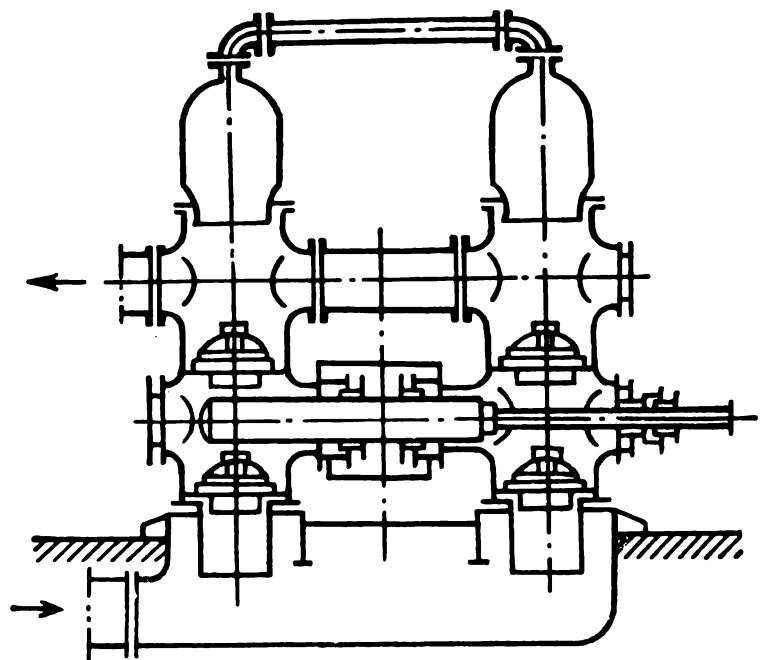


Fig. 2-6. To Example 2-3

in this case ($n = 150$ rpm) equals zero, i. e. the pump must be installed below the level of the liquid.

Example 2-3. A double-action piston pump (Fig. 2-6) delivers 22.8 m³ of liquid an hour. The pump rotates at a speed of 65 rpm, the diameter of its plunger is 125 mm, the diameter of the rod is 35 mm, the crank radius is 136 mm. Determine the delivery factor of the pump.

Solution. The volume displaced by the plunger during one revolution is

$$(2A - A_r)s = (2 \times 0.785 \times 0.125^2 - 0.785 \times 0.035^2) 0.272 = 0.006\,37 \text{ m}^3$$

where $s = 0.272$ m is the stroke of the plunger equal to the double radius of the crank.

The theoretical delivery of the pump at 65 rpm is:

$$Q_{\text{th}} = 0.006\,37 \times 65 = 0.413 \text{ m}^3/\text{min}$$

The actual delivery is:

$$Q_a = \frac{22.8}{60} = 0.38 \text{ m}^3/\text{min}$$

The delivery factor is:

$$\eta_v = \frac{Q_a}{Q_{th}} = \frac{0.38}{0.413} = 0.92$$

Example 2-4. A single-action piston pump (see Fig. 2-2) with a piston diameter of 160 mm and a piston stroke of 200 mm must deliver 430 litres a minute of a liquid having a relative density of 0.93 from a collecting tank to an apparatus in which the pressure is $p_{gauge} = 3.2$ at. The pressure in the collectin tank is atmospheric. The geometrical lifting height is 19.5 m. The total loss of head in the suction line is 1.7 m, and in the delivery line is 8.6 m. What speed must the pump have and what power must its motor have if the delivery factor of the pump is 0.85, and the efficiencies of the pump, the transmission, and the motor are 0.8, 0.95, and 0.95, respectively?

Solution. From Eq. (2-8), we find:

$$n = \frac{Q \times 60}{\eta_v A s}$$

In our case $Q = \frac{430}{1000 \times 60} = 0.00717 \text{ m}^3/\text{s}$, $A = 0.785 \times 0.16^2 = 0.0201 \text{ m}^2$, and

$$n = \frac{0.00717 \times 60}{0.85 \times 0.0201 \times 0.2} = 126 \text{ rpm}$$

We find the head developed by the pump by Eq. (2-1):

$$H = \frac{3.2 \times 9.81 \times 10^4}{930 \times 9.81} + 19.5 + 10.3 = 64.2 \text{ m}$$

We calculate the power used by the pump motor by Eq. (2-4):

$$P = \frac{0.43 \times 930 \times 9.81 \times 64.2}{60 \times 1000 \times 0.72} = 5.82 \text{ kW}$$

where 0.72 is the overall efficiency of the pump installation:

$$\eta = \eta_p \eta_{tr} \eta_m = 0.8 \times 0.95 \times 0.95 = 0.72$$

In accordance with the data of Table 2-1, it is necessary to use a motor (with a reserve power for overloading) having the power

$$5.82 \times 1.17 = 6.8 \text{ kW}$$

Example 2-5. The following data were obtained when testing a centrifugal pump running at a speed of 1200 rpm:

Q , dm ³ /s	0	10.8	21.2	29.8	40.4	51.1
H , m	23.5	25.8	25.4	22.1	17.3	11.9
P , kW	5.16	7.87	10.1	11.3	12.0	18.5

A solution was pumped having a relative density of 1.12. Determine the efficiency of the pump for each output and plot a graphical characteristic of the pump.

Solution. We find the efficiency of the pump from Eq. (2-4):

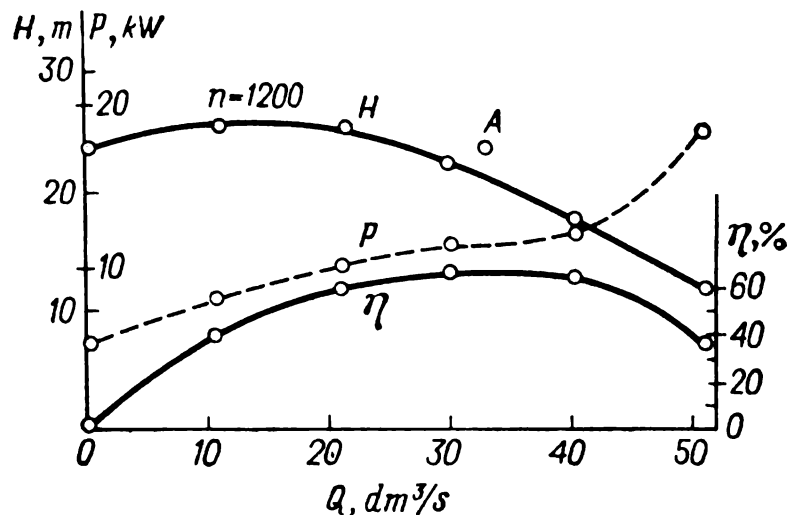
$$\eta = \frac{Q\rho gH}{1000P}$$

Calculations by this equation yield the following values of the pump efficiency:

$Q, \text{ dm}^3/\text{s}$	0	10.8	21.2	29.8	40.4	51.1
η	0	0.39	0.587	0.643	0.637	0.36

The characteristic of the pump is shown in Fig. 2-7.

Fig. 2-7. To Examples 2-5 and 2-6



Example 2-6. A solution having a relative density of 1.12 is to be fed at a rate of 115 m³/h from a tank into an apparatus at a height of 10.8 m above the liquid level in the tank. The gauge pressure in the apparatus is $p_{\text{gauge}} = 0.4$ at, the pressure in the tank is atmospheric. The pipeline has a diameter of 140×4.5 mm, its total equivalent length (the actual length plus the equivalent length of the local obstacles) is 140 m. Is it possible to use the centrifugal pump of the preceding example if the coefficient of friction resistance in the pipeline is $\lambda = 0.03$?

Solution. We find the head to be provided by the pump.

The velocity of the liquid is:

$$v = \frac{115}{3600 \times 0.785 \times 0.131^2} = 2.37 \text{ m/s}$$

The velocity head is:

$$h_{\text{vel}} = \frac{v^2}{2g} = \frac{2.37^2}{2 \times 9.81} = 0.286 \text{ m}$$

The loss of head for overcoming the pipe resistance and local obstacles is

$$h_{\text{fr} + \text{lr}} = \frac{\lambda (L + L_{\text{eq}})}{d} h_{\text{vel}} = \frac{0.03 \times 140}{0.131} \times 0.286 = 9.16 \text{ m}$$

We calculate the required total head of the pump by Eq. (2-1):

$$H = \frac{0.4 \times 10\,000 \times 9.81}{1120 \times 9.81} + 10.8 + 9.16 + 0.286 = 23.8\text{ m}$$

The required output of the pump is:

$$Q = \frac{115 \times 1000}{3600} = 32\text{ dm}^3/\text{s}$$

Reverting to Fig. 2-7, we see that point A with the coordinates $Q = 32\text{ dm}^3/\text{s}$ and $H = 23.8\text{ m}$ is above the curve of the pump characteristic and, consequently, the pump at $n_1 = 1200\text{ rpm}$ cannot ensure the required output (at $H = 23.8\text{ m}$ the pump can deliver only $26\text{ dm}^3/\text{s}$). If we increase the pump speed somewhat, however, it will be suitable. Using the relationships (2-10):

$$\frac{Q_1}{Q_2} = \frac{n_1}{n_2} \text{ and } \frac{H_1}{H_2} = \left(\frac{n_1}{n_2}\right)^2$$

we can select the new speed of the pump n_2 that is needed.

If, for instance, we take $n_2 = 1260\text{ rpm}$ and recalculate the data of Example 2-5 by means of Eqs. (2-10) for this new speed, we get the results shown in Table 2-3.

TABLE 2-3

$n_1 = 1200\text{ rpm}$	$Q_1, \text{ dm}^3/\text{s}$	21.2	29.8	40.4
	$H_1, \text{ m}$	25.4	22.1	17.3
$n_2 = 1260\text{ rpm}$	$Q_2, \text{ dm}^3/\text{s}$	22.3	31.3	42.5
	$H_2, \text{ m}$	28.0	24.4	19.1

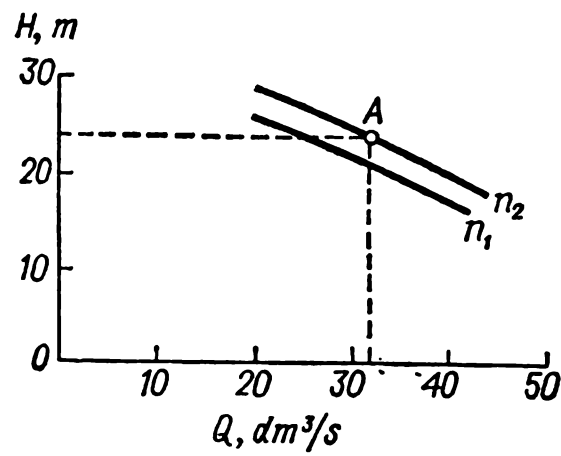


Fig. 2-8. To Example 2-6

Using the data of Table 2-3 to plot a curve of the pump characteristic at $n_2 = 1260\text{ rpm}$ (Fig. 2-8), we see that at this speed the pump can ensure the required delivery ($32\text{ dm}^3/\text{s}$) and head (23.8 m).

We determine the power used by the pump at the new speed by the equation

$$P = \frac{Q\rho gH}{1000\eta}$$

assuming approximately that the efficiency of the pump η has not changed*. We take its value according to the data of Example 2-5, in which we found that for $Q=30$ to 40 dm³/s the efficiency of the pump is $\eta \approx 0.64$.

The power used by the pump at $n_2=1260$ rpm is:

$$P = \frac{32 \times 1120 \times 9.81 \times 23.8}{1000 \times 1000 \times 0.64} = 13.1 \text{ kW}$$

Example 2-7. Determine the delivery factor of a gear pump (Fig. 2-9) having a speed of 440 rpm. The number of teeth on a

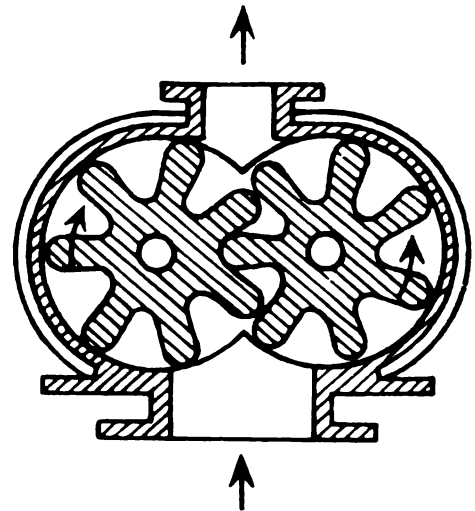


Fig. 2-9. To Example 2-7

gear wheel is 12, the width of a tooth is 42 mm, the cross-sectional area of a tooth restricted by the addendum circle of the mating gear wheel is 960 mm². The pump delivers 0.312 m³ a minute.

Solution. The output of a gear pump Q (in m³/s) is determined by the formula

$$Q = \eta_v \frac{2A_t b z n}{60}$$

where η_v = delivery factor

A_t = cross-sectional area of a tooth restricted by the addendum circle of the mating gear wheel, m²

b = width of a tooth, m

z = number of teeth on a gear wheel

n = speed of the gear wheel, rpm.

* The power can also be calculated by the equation $P_2 = P_1 \left(\frac{1260}{1200} \right)^3$, taking the value of P_1 from a graph (Fig. 2-7).

The theoretical delivery in our case is:

$$Q_{th} = \frac{2A_t b z n}{60} = \frac{2 \times 0.00096 \times 0.042 \times 12 \times 440}{60} = 0.00708 \text{ m}^3/\text{s}$$

The actual delivery is:

$$Q_a = \frac{0.312}{60} = 0.0052 \text{ m}^3/\text{s}$$

Hence the delivery factor is

$$\eta_v = \frac{Q_a}{Q_{th}} = \frac{0.0052}{0.00708} = 0.735$$

Example 2-8. Determine (ignoring the losses) the theoretical rarefaction that can be created by the working jet of water in chamber *C* of a water-jet pump (Fig. 2-10). The pressure at the

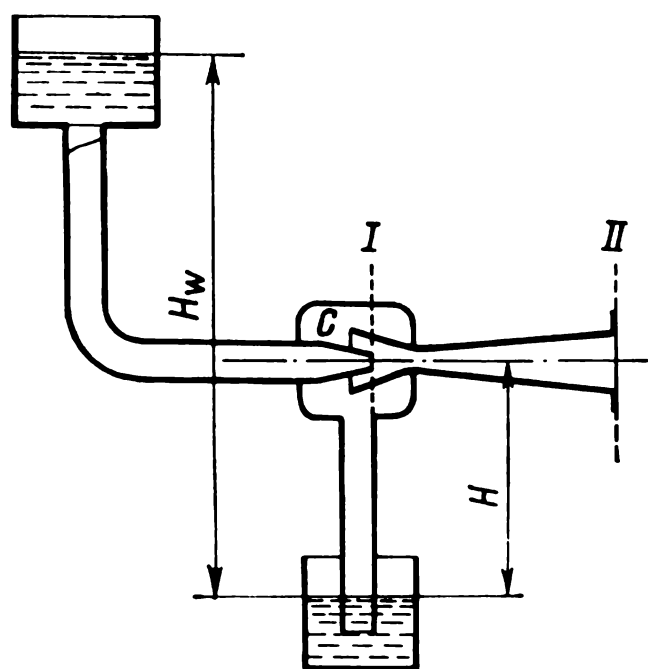


Fig. 2-10. To Examples 2-8 and 2-9

outlet from the diffuser is atmospheric (1.013×10^5 Pa or 760 mm Hg), the velocity of the jet at this place is 2.7 m/s. The diameter of the jet in section *I* is 23 mm, and in section *II* is 50 mm.

Solution. Let us write the Bernoulli equation for sections *I* and *II* of the jet, ignoring the losses:

$$z_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

With a horizontal arrangement of the pump, we have: $z_1 = z_2$. We further have:

$$v_1 = \frac{A_2}{A_1} v_2 = \left(\frac{50}{23}\right)^2 2.7 = 12.8 \text{ m/s}$$

From the Bernoulli equation, we find:

$$\begin{aligned} p_1 &= p_2 + \frac{(v_2^2 - v_1^2) \rho}{2} = 760 \times 133.3 + \frac{2.7^2 - 12.8^2}{2} \times 1000 = \\ &= 101\,300 - 78\,300 = 23\,000 \text{ Pa} \end{aligned}$$

Hence, the theoretical rarefaction is:

$$\frac{101\,300 - 23\,000}{9.81 \times 10^4} = 0.8 \text{ at}$$

Example 2-9. A water-jet pump (Fig. 2-10) lifts 7.8 m^3 of a liquid having a relative density of 1.02 to a height of $H = 4 \text{ m}$ an hour. The rate of flow of the working (pressure) water is $9.6 \text{ m}^3/\text{h}$. The head of the working water before the pump is $H_w = 22 \text{ m}$. Determine the efficiency of the pump.

Solution. The effective work (power) done by the pump is:

$$P_{\text{eff}} = \frac{7.8 \times 1020 \times 9.81 \times 4}{3600} = 86.7 \text{ W}$$

The power used by the pump is:

$$P = \frac{9.6 \times 1000 \times 9.81 (22 - 4)}{3600} = 471 \text{ W}$$

Hence, the efficiency of the water-jet pump is:

$$\eta = \frac{86.7}{471} \times 100 = 18.4 \%$$

Example 2-10. Find the pressure developed by a fan (see Fig. 2-4) delivering nitrogen ($\rho = 1.2 \text{ kg/m}^3$) from a gas holder to an apparatus. The gauge pressure in the gas holder is $60 \text{ mm H}_2\text{O}$, and in the apparatus $74 \text{ mm H}_2\text{O}$. The losses in the suction line are $19 \text{ mm H}_2\text{O}$, and in the delivery line $35 \text{ mm H}_2\text{O}$. The velocity of the nitrogen in the delivery pipeline is 11.2 m/s .

Solution. We find the pressure developed by the fan by Eq. (2-13).

The difference between the pressures at the places of delivery and suction is:

$$p_2 - p_1 = (74 - 60) 9.81 = 137 \text{ Pa} = 14 \text{ mm H}_2\text{O}$$

The total losses in the suction and delivery pipelines are:

$$\Delta p_s + \Delta p_d = (19 + 35) 9.81 = 530 \text{ Pa} = 54 \text{ mm H}_2\text{O}$$

The velocity pressure at the outlet from the pipeline is

$$\frac{v^2 \rho}{2} = \frac{11.2^2 \times 1.2}{2} = 76 \text{ Pa} = 7.7 \text{ mm H}_2\text{O}$$

The pressure developed by the fan is:

$$\Delta p = 137 + 530 + 76 = 743 \text{ Pa} = 76 \text{ mm H}_2\text{O}$$

Example 2-11. The rarefaction in the suction pipeline before a centrifugal fan is $15.8 \text{ mm H}_2\text{O}$; the manometer on the delivery pipeline after the fan shows a gauge pressure of $20.7 \text{ mm H}_2\text{O}$. A flow gauge shows that air is delivered at a rate of $3700 \text{ m}^3/\text{h}$. The suction and delivery pipes have the same diameter. The fan speed is 960 rpm . The power consumed by the fan is 0.77 kW .

Determine the pressure developed by the fan and its efficiency. How will the output of the fan change if its speed is increased to 1150 rpm, and what power will be consumed at this speed?

Solution. We find the pressure developed by the fan by Eq. (2-14). Since the suction and delivery pipelines have the same diameter, the velocity pressures in them are the same. Hence

$$\Delta p = p_{st,d} - p_{st,s} = 20.7 \times 9.81 - (-15.8 \times 9.81) = 354 \text{ Pa}$$

The amount of air delivered by the fan a second is $Q = 3700/3600 = 1.03 \text{ m}^3/\text{s}$.

The theoretical consumption of power is:

$$P_{th} = \frac{1.03 \times 354}{1000} = 0.368 \text{ kW}$$

The efficiency of the fan is:

$$\eta = \frac{P_{th}}{P} = \frac{0.368}{0.77} = 0.48$$

We determine the output of the fan at $n_2 = 1150 \text{ rpm}$ by Eq. (2-10):

$$Q_2 = Q_1 \frac{n_2}{n_1} = 3700 \times \frac{1150}{960} = 4430 \text{ m}^3/\text{h}$$

The consumption of power at the new speed is:

$$P_2 = P_1 \left(\frac{n_2}{n_1} \right)^3 = 0.77 \left(\frac{1150}{960} \right)^3 = 1.33 \text{ kW}$$

Example 2-12. The following data were obtained when testing a centrifugal fan having a speed of $n = 1440 \text{ rpm}$:

$Q, \text{ m}^3/\text{h}$	100	350	700	1000	1600	2000
$\Delta p \left\{ \begin{array}{l} \text{Pa} \\ \text{mm H}_2\text{O} \end{array} \right.$	449	424	432	427	387	316
	45.8	43.2	44.0	43.5	39.5	32.2

How much air will this fan deliver to a network (when running at the same speed as during the test) if calculations of the resistance of the network showed that when air flows through it at a rate of $1350 \text{ m}^3/\text{h}$ the pressure losses are $\Delta p_{vel} = 85 \text{ Pa} = 8.7 \text{ mm H}_2\text{O}$ and $\Delta p_{fr} + \Delta p_{lr} = 288 \text{ Pa} = 29.4 \text{ mm H}_2\text{O}$?

The difference between the pressures in the delivery and the suction spaces for the network being calculated is $\Delta p_{add} = p_2 - p_1 = 128 \text{ Pa} = 13 \text{ mm H}_2\text{O}$.

Solution. We must first find the working point at the intersection of the fan and network characteristics.

The characteristic of the network is expressed by a parabola in whose equation

$$\Delta p = aQ^2 + b$$

the first addend aQ^2 in the right-hand side equals the sum of the pressure losses $\Delta p_{vel} + \Delta p_{fr} + \Delta p_{lr}$ and changes proportionally

to the square of the rate of flow, while the second addend b does not depend on the rate of flow and is the difference between the pressures in the delivery and the suction spaces, i. e. Δp_{add} ; a is a constant quantity.

Let us use the available data to calculate several points of this parabola (Table 2-4).

TABLE 2-4

$Q, \text{ m}^3/\text{h}$	aQ^2	b		Δp	
		P_a	$\text{mm H}_2\text{O}$	P_a	$\text{mm H}_2\text{O}$
1350	38.1	128	13	501	51.4
$\frac{1350}{1.5} = 900$	$\frac{38.1}{1.5^2} = 16.9$	128	13	293	29.9
$\frac{1350}{2} = 675$	$\frac{38.1}{2^2} = 9.5$	128	13	221	22.5
$\frac{1350}{2.5} = 540$	$\frac{38.1}{2.5^2} = 6.1$	128	13	187	19.1
0	0	128	13	128	13

We plot on a common graph (Fig. 2-11) the characteristic of the fan according to the data obtained when testing it and the characteristic of the network according to the calculated points.

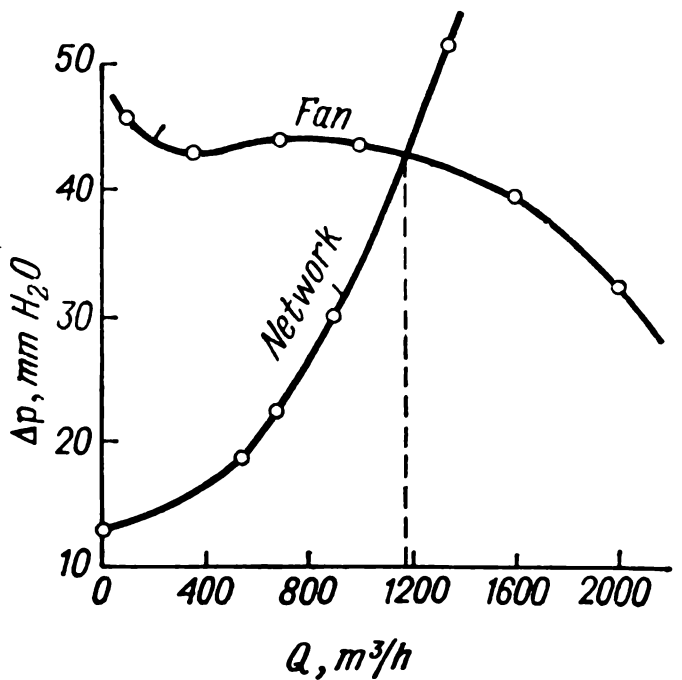


Fig. 2-11. To Example 2-12

The point of intersection of the two characteristics shows that in operation for the given network the fan will deliver 1170 m³ of air an hour.

Example 2-13. Compare the theoretical amounts of work needed to compress 1 m³ of air from $p_{abs} = 1$ at to (a) $p_{abs} = 1.1$ at, and

(b) to $p_{\text{abs}} = 5$ at. Calculate the work needed using both the thermodynamic formula for adiabatic compression and the hydraulic formula (i.e. considering air to be incompressible).

Solution.

(a) $p_1 = 9.81 \times 10^4$ Pa; $p_2 = 10.8 \times 10^4$ Pa.

By Eq. (2-17), if we relate the work of compression in the compressor to 1 m³ of gas (in the suction conditions), we get (in J/m³):

$$W_{\text{ad}} = \frac{\gamma}{\gamma-1} p_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

For air $\gamma = 1.4$ (Table A-5). Hence *

$$W_{\text{ad}} = \frac{1.4}{0.4} \times 9.81 \times 10^4 \left(1.1^{\frac{0.4}{1.4}} - 1 \right) = 9520 \text{ J/m}^3$$

Calculations according to the hydraulic formula

$$W_{\text{h}} = Q \Delta p$$

give for $\Delta p = 10.8 \times 10^4 - 9.81 \times 10^4 = 9.8 \times 10^3$ Pa:

$$W_{\text{h}} = 1 \times 9.8 \times 10^3 = 9800 \text{ J/m}^3$$

(b) $p_1 = 9.81 \times 10^4$ Pa; $p_2 = 49.1 \times 10^4$ Pa.

According to the thermodynamic formula:

$$W_{\text{ad}} = \frac{1.4}{0.4} \times 9.81 \times 10^4 \left(5^{\frac{0.4}{1.4}} - 1 \right) = 201\,000 \text{ J/m}^3$$

According to the hydraulic formula when $\Delta p = 49.1 \times 10^4 - 9.81 \times 10^4 = 39.3 \times 10^4$ Pa, we have:

$$W_{\text{h}} = 1 \times 39.3 \times 10^4 = 393\,000 \text{ J/m}^3$$

Comparing variants (a) and (b), we see that in the first case the results obtained by the thermodynamic and hydraulic formulas differ by only 3%. This case ($p_2/p_1 = 1.1$) corresponds to the limiting compression ratio of air provided by fans for which the required power is calculated, as we have seen above, according to the hydraulic formula.

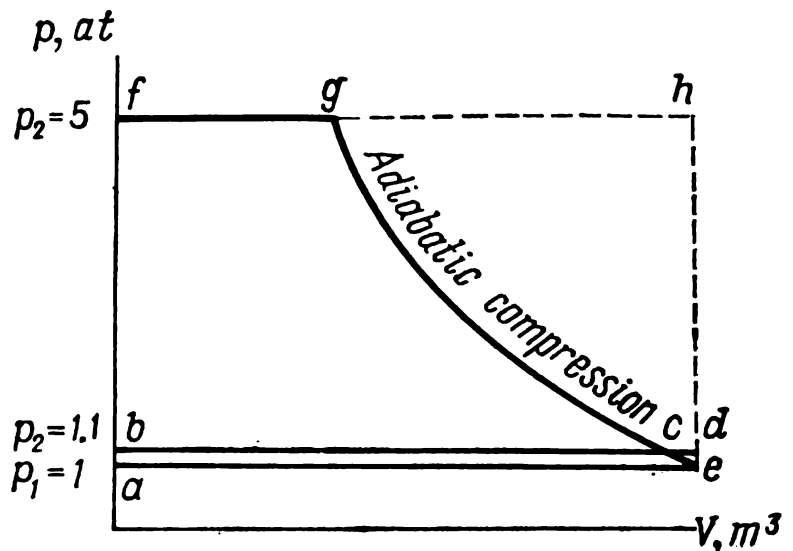
In the second case ($p_2/p_1 = 5$), which corresponds to compression of air in a compressor, the results obtained by the formula for adiabatic compression and by the hydraulic formula diverge by 100%. To calculate the power consumed by a compressor, the thermodynamic equations (2-17) and (2-18) are always used.

It is clearly seen in the theoretical indicator diagram of a piston compressor (Fig. 2-12) that area *abce* (which is the work done for adiabatic compression up to $p_2 = 1.1$ at) approximately equals area *abde*, but area *afge* (for $p_2 = 5$ at) far from equals area *afhe*.

* Since the parentheses contain a small quantity, a slide rule should not be used because its accuracy is not sufficient here.

Example 2-14. Determine the power used by a single-stage piston compressor to compress $460 \text{ m}^3/\text{h}$ in standard conditions (0°C and 760 mm Hg) of ammonia from $p_{\text{abs}} = 2.5 \text{ at}$ to $p_{\text{abs}} = 12 \text{ at}$. The initial temperature of the ammonia is -10°C , the efficiency of the compressor is 0.7. Also determine the temperature of the ammonia at the end of compression.

Fig. 2-12. To Example 2-13



Solution. We determine the theoretical work of the compressor by Eq. (2-17) for adiabatic compression.

For ammonia from Table A-5, we find $\gamma = 1.29$, $R = 488 \text{ J/kg} \cdot \text{K}$, and

$$W_{\text{ad}} = \frac{1.29}{0.29} \times 488 \times 263 \left[\left(\frac{12}{2.5} \right)^{\frac{0.29}{1.29}} - 1 \right] = 240\,000 \text{ J/kg}$$

We now calculate the same quantity by Eq. (2-18). From the T - S diagram for ammonia (Fig. A-26), we find for point 1 ($t_1 = -10^\circ\text{C}$ and $p_1 = 2.5 \text{ at}$) that $H_1 = 1440 \text{ kJ/kg}$. Drawing from this point a vertical line ($S = \text{const}$) up to its intersection with the isobar $p_2 = 12 \text{ at}$, we find point 2 for which $H_2 = 1673 \text{ kJ/kg}$. Hence by Eq. (2-18), we have:

$$W_{\text{ad}} = 1\,673\,000 - 1\,440\,000 = 233\,000 \text{ J/kg}$$

which is close to the value found above (the discrepancy is about 3%).

The mass rate of flow of the ammonia is $Q_m = 460 \times 0.76 = 350 \text{ kg/h}$. Here 0.76 is the density of ammonia in standard conditions:

$$\rho^\circ = \frac{M}{22.4} = \frac{17}{22.4} = 0.76 \text{ kg/m}^3$$

We find the power used by the compressor by Eq. (2-20):

$$P = \frac{350 \times 240\,000}{3600 \times 1000 \times 0.7} = 33.4 \text{ kW}$$

We calculate the temperature at the end of compression by Eq. (2-19):

$$T_2 = 263 \left(\frac{12}{2.5} \right)^{\frac{0.29}{1.29}} = 374 \text{ K or } t_2 = 101^\circ\text{C}$$

If we determine this temperature directly from the T - S diagram, we find at point 2 that $t_2 = 104^\circ\text{C}$.

Example 2-15. Compressed air has to be supplied at a gauge pressure of $p_{\text{gauge}} = 4.5$ at a rate of 80 kg an hour. Will it be possible to use a single-stage single-acting piston compressor having a cylinder diameter of 180 mm, a piston stroke of 200 mm, and running at a speed of 240 rpm? The dead space is 5% of the piston displacement. Assume that the polytropic exponent for expansion is 1.25.

Solution. We find the output of the compressor by Eq. (2-21). It is first necessary to find the volume efficiency of the compressor λ_0 . By Eq. (2.22) we have

$$\lambda_0 = 1 - 0.05 \left(5.5^{\frac{1}{1.25}} - 1 \right) = 0.854$$

We take the delivery coefficient equal to $\lambda = 0.85\lambda_0 = 0.85 \times 0.854 = 0.725$.

The output of the compressor is:

$$Q = \frac{0.725 \times 0.18^2 \times 3.14 \times 0.2 \times 240}{4 \times 60} = 0.0147 \text{ m}^3/\text{s} = 53 \text{ m}^3/\text{h}$$

Considering that the compressor sucks in atmospheric air having a temperature of about 20°C and a density of 1.2 kg/m^3 , we get the mass output of the compressor equal to $53 \times 1.2 = 63.6 \text{ kg/h}$.

Consequently, the compressor does not ensure the required output (80 kg/h). The required mass output can nevertheless be achieved if we increase the speed of the compressor from 240 to $\frac{80}{63.6} \times 240 = 302 \text{ rpm}$ or if we supercharge the compressor without changing its speed by installing before it a supercharger that will compress the air from atmospheric pressure to an absolute pressure equal to $80/63.6 = 1.26 \text{ at}$ and let the air be sucked into the compressor at this pressure (Fig. 2-13).

In both cases, the dynamics of the compressor must be checked by special calculations.

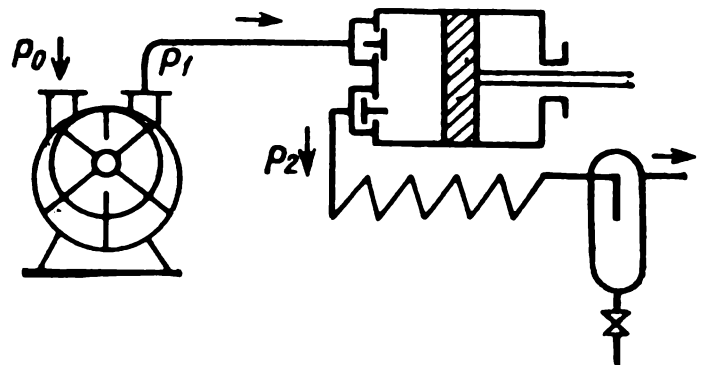
Example 2-16. In a single-stage piston compressor intended for the compression of methane, the dead space is 8.5% of the piston displacement. Considering the process of expansion of the gas from the dead space to be adiabatic, determine at what maximum delivery pressure the output of the compressor becomes equal to zero. The suction pressure is atmospheric.

Solution. The output of the compressor is equal to zero when its volume efficiency equals zero, i. e. when

$$\lambda_0 = 1 - \varepsilon_0 \left[\left(\frac{p_2}{p_1} \right)^{\frac{1}{m}} - 1 \right] = 0$$

According to the initial conditions, we consider that the gas from the dead space expands adiabatically, i. e. instead of the polytropic exponent m we take the adiabatic exponent γ equal

Fig. 2-13. To Example 2-15
 $p_0 = 1$ at; $p_1 = 1.26$ at; $p_2 = 5.5$ at



to 1.31 for methane (Table A-5). The ratio of the volume of the dead space to the piston displacement is $\varepsilon_0 = 0.085$. The suction pressure is $p_1 = 1$ at. Hence

$$1 - 0.085 \left(p_2^{\frac{1}{1.31}} - 1 \right) = 0$$

From this equation we find that $p_2^{0.763} = 12.8$, whence $p_2 \approx 28$ at.

Consequently, the output of the compressor will equal zero at a delivery pressure of $p_{abs} = 28$ at.

Example 2-17. Compare the temperatures at the end of compression, the theoretical expenditure of work, and the values of the volumetric efficiencies upon the compression of air from a pressure (absolute) of 1 to 9 at: (a) in a single-stage piston compressor, and (b) in a two-stage compressor with intermediate cooling between the stages. The initial temperature of the air and its temperature after the cooler is 20°C . The volume of the dead space is 8% of the piston displacement.

Solution. (a) Single-stage compression.

We determine the temperature at the end of compression by Eq. (2-19). For air, we have $\gamma = 1.4$ (Table A-5):

$$T_2 = 293 \times 9^{\frac{0.4}{1.4}} = 293 \times 1.88 = 551 \text{ K} = 278^\circ\text{C}$$

We find the theoretical expenditure of work by Eq. (2-17). For air from Table A-5, we find that $R = 287 \text{ J/kg} \cdot \text{K}$. Hence

$$W_{ad} = \frac{1.4}{0.4} \times 287 \times 293 (1.88 - 1) = 260\,000 \text{ J/kg}$$

We calculate the volumetric efficiency of the compressor by Eq. (2-22), assuming that the air from the dead space expands along an adiabat:

$$\lambda_0 = 1 - 0.08 \left(9^{\frac{1}{1.4}} - 1 \right) = 0.7$$

(b) Two-stage compression.

We find the compression ratio in each stage by Eq. (2-26):

$$x^2 = 9$$

whence $x = 3$.

The temperature at the end of compression in each stage is:

$$T_2 = 293 \times 3^{\frac{0.4}{1.4}} = 293 \times 1.37 = 402 \text{ K} = 129^\circ \text{C}$$

We calculate the total theoretical expenditure of work in both stages by Eq. (2-23):

$$W_{ad} = 2 \times 287 \times 293 \times \frac{1.4}{0.4} \left(1.88^{\frac{1}{2}} - 1 \right) = 218\,000 \text{ J/kg}$$

where $1.88 = (p_{fn}/p_1)^{(\gamma-1)/\gamma}$.

The volumetric efficiency is:

$$\lambda_0 = 1 - 0.08 \left(3^{\frac{1}{1.4}} - 1 \right) = 0.905$$

We compare the results obtained for single-stage and two-stage compression:

Number of compression stages	1	2
Temperature t_2 at the end of compression, $^\circ\text{C}$	278	129
Theoretical expenditure of work W_{ad} , J/kg	260 000	218 000
Volume efficiency λ_0	0.7	0.905

The above comparison clearly shows the advantages of two-stage compression. The greater the ratio p_{fn}/p_1 , the more do the advantages of multistage compression manifest themselves.

Example 2-18. A compressor must supply 210 m³/h of methane (in standard conditions—0 $^\circ\text{C}$ and 760 mm Hg) at a pressure of $p_{abs} = 55$ at. The initial pressure is atmospheric, the initial temperature is 18 $^\circ\text{C}$. Determine: (a) the number of compression stages and the distribution of pressures by stages; (b) the consumption of power, assuming the efficiency of the compressor to be 0.7; (c) the rate of flow of the water in the coolers of the compressor if it is heated by 10 K.

Solution. (a) At the tolerated compression ratio in each stage of about 4, the required number of stages, according to Eq. (2-26), is

$$N = \frac{\log 55}{\log 4} = 2.9 \approx 3$$

Ignoring the loss of pressure between the stages, we determine more precisely the compression ratio in each stage of a three-stage compressor (Fig. 2-14):

$$x = \sqrt[3]{55} = 3.8$$

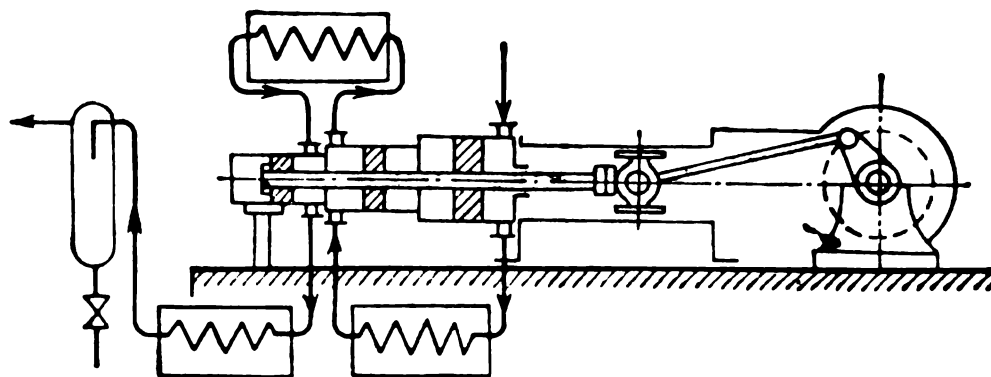


Fig. 2-14. To Example 2-18

Thus, the approximate distribution of pressure by stages is:

		p_{in}	p_{fin}
1st stage	1	3.8
2nd stage	3.8	14.45
3rd stage	14.45	55

(b) We determine the theoretical amount of work done by Eq. (2-23). For methane by Table A-5, we find $\gamma = 1.31$, $R = 519 \text{ J/kg}\cdot\text{K}$ and $\rho^0 = 0.72 \text{ kg/m}^3$ (at 0°C and 760 mm Hg).

Assuming that the methane is cooled to 30°C in the intercoolers, we have:

$$W_{ad} = 3 \times 519 \times 303 \times \frac{1.31}{0.31} \left(55^{\frac{0.31}{1.31 \times 3}} - 1 \right) = 746\,000 \text{ J/kg}$$

We calculate the power consumed by Eq. (2-20):

$$P = \frac{210 \times 0.72 \times 746\,000}{3600 \times 1000 \times 0.7} = 44.6 \text{ kW}$$

(c) To determine the consumption of the cooling water, we first find the temperatures at the end of compression in the 2nd and 3rd stages, assuming, as noted above, that in the intercoolers after the 1st and 2nd stages the methane is cooled to 303 K (Fig. 2-15). In the 1st stage cylinder of the compressor, the temperature at the end of compression will be somewhat lower because the methane is sucked into the 1st stage at 18 and not at 30°C .

By Eq. (2-19):

$$T_2 = 303 \times 3.8^{\frac{0.31}{1.31}} = 416 \text{ K} \text{ or } t_2 = 143^\circ\text{C}$$

Assuming that the heat capacity of methane at the pressures (absolute) of 3.8 , 14.45 , and 55 at is the same and approxi-

mately equal to $2.22 \text{ kJ/kg}\cdot\text{K}$, we find that in the three coolers (after the 1st, 2nd, and 3rd stages) the cooling water must always remove heat equal to

$$Q = \frac{3 \times 210 \times 0.72 \times 2.22 \times 10^3 (143 - 30)}{3600} = 31\,700 \text{ W}$$

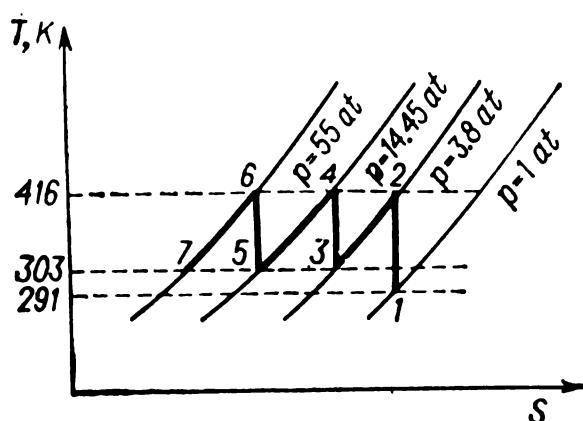


Fig. 2-15. To Example 2-18

The same quantity of heat can be calculated in a different way as the heat equivalent to the work of compression:

$$Q = W_{ad} Q_v \rho^o = \frac{745\,000 \times 210 \times 0.72}{3600} = 31\,300 \text{ W}$$

When the water is heated by 10 K , its rate of flow is

$$Q_{m, w} = \frac{Q}{c(T_2 - T_1)} = \frac{31\,700}{4.19 \times 10^3 \times 10} = 0.756 \text{ kg/s}$$

or

$$Q_{v, w} = \frac{0.756 \times 3600}{1000} = 2.72 \text{ m}^3/\text{h}$$

Here $c = 4.19 \times 10^3 \text{ J/kg}\cdot\text{K}$ is the specific heat capacity of water.

Example 2-19. A piston vacuum pump is to create a vacuum of 0.9 at in an apparatus. Considering the process of air compression in the vacuum pump to be polytropic (with a polytropic exponent of 1.25), determine the theoretical amount of work done: (a) at the moment when a vacuum of 0.1 at is reached, i.e. the residual pressure in the apparatus is equal to 0.9 at ; (b) when the pressure in the apparatus is equal to 0.3 at ; and (c) when the required vacuum is reached, i.e. the residual pressure in the apparatus is 0.1 at .

Solution. We calculate the theoretical amount of work done per cubic metre of gas sucked in (i.e. in J/m^3) by Eq. (2-17):

$$W = \frac{m}{m-1} p_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{m-1}{m}} - 1 \right]$$

where m is the compression polytropic exponent.

$$(a) \frac{p_2}{p_1} = \frac{9.81 \times 10^4}{8.83 \times 10^4} = 1.11; \quad \frac{m}{m-1} = \frac{1.25}{0.25} = 5; \quad \frac{m-1}{m} = 0.2$$

$$W = 5 \times 8.83 \times 10^4 (1.11^{0.2} - 1) = 9720 \text{ J/m}^3$$

$$(b) \frac{p_2}{p_1} = \frac{9.81 \times 10^4}{2.94 \times 10^4} = 3.33$$

$$W = 5 \times 2.94 \times 10^4 (3.33^{0.2} - 1) = 40\,000 \text{ J/m}^3$$

$$(c) \frac{p_2}{p_1} = \frac{9.81 \times 10^4}{0.981 \times 10^4} = 10$$

$$W = 5 \times 0.981 \times 10^4 (10^{0.2} - 1) = 28\,600 \text{ J/m}^3$$

We see that the work performed by the vacuum pump passes through a maximum. The power of the pump motor is calculated for this maximum.

PROBLEMS

2-1. A pump handles 30% sulphuric acid. The reading of the manometer on the delivery pipeline is 1.8 kgf/cm^2 , and the reading of the vacuum gauge (rarefaction) on the suction pipeline before the pump is 29 mm Hg . The manometer is connected 0.5 metre higher than the vacuum gauge. The suction and delivery pipelines have the same diameter. What head is developed by the pump?

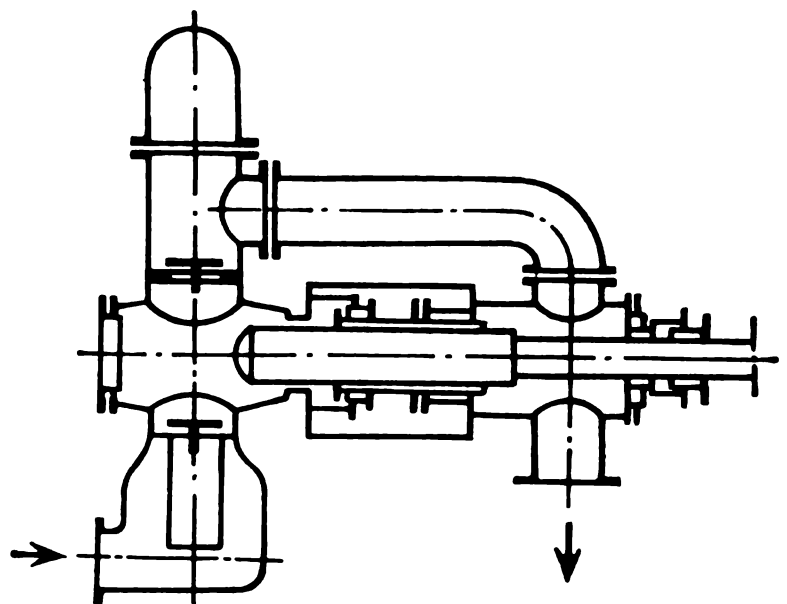
2-2. A pump feeds a liquid having a density of 960 kg/m^3 from a reservoir with atmospheric pressure into an apparatus in which the gauge pressure is $p_{\text{gauge}} = 37 \text{ at}$ (see Fig. 2-1). The lifting height is 16 m . The total resistance of the suction and delivery lines is 65.6 m . Find the total head developed by the pump.

2-3. Determine the efficiency of a pump plant. The pump delivers 380 dm^3 a minute of fuel oil having a relative density of 0.9 . The total head is 30.8 m . The power consumed by the motor is 2.5 kW .

2-4. The output of a pump is 14 dm^3 a second of a liquid having a relative density of 1.16 . The total head is 58 m . The efficiency of the pump is 0.64 , that of the transmission is 0.97 , and of the motor is 0.95 . Find the required power of the motor.

2-5. A piston pump (see Fig. 2-2) is installed at a plant at an altitude of 300 m above sea level. The total loss of suction height is $5.5 \text{ m H}_2\text{O}$. The geometrical suction height is 3.6 m . At what maximum temperature of the water is suction still possible?

Fig 2-16. To Problem 2-6



2-6. Determine the output of the differential piston of a pump (Fig. 2-16) with the greater diameter of the stepped plunger equal to 340 mm and the smaller diameter equal to 240 mm . The plunger stroke is 480 mm , the speed of

rotation is 60 rpm. The delivery factor is 0.85. Also determine the amount of liquid supplied by each side of the stepped plunger.

2-7. A double-acting piston pump (see Fig. 2-6) fills a tank 3 m in diameter and 2.6 m high during 26.5 min. The diameter of the pump plunger is 180 mm, the diameter of the rod is 50 mm, and the radius of the crank is 145 mm. The speed is 55 rpm. Find the delivery factor of the pump.

2-8. A centrifugal pump running at a speed of 1800 rpm must handle 140 m³/h of water having a temperature of 30 °C. The average atmospheric pressure at the place of installation of the pump is 745 mm Hg. The total loss of head in the suction line is 4.2 m. Determine the theoretically tolerated suction height.

2-9. A centrifugal pump when handling 280 dm³ of water a minute creates a head of $H=18$ m. Can this pump be used for delivering a liquid having a relative density of 1.06 in an amount of 15 m³/h through a pipe with a diameter of 70×2.5 mm from a collecting tank at atmospheric pressure into an apparatus with a gauge pressure of $p_{\text{gauge}}=0.3$ at? The geometrical lifting height is 8.5 m. The equivalent length of the pipeline (its actual length plus the equivalent length of local obstacles) is 124 m. The coefficient of friction resistance in the pipeline is $\lambda=0.03$. Also determine the motor power needed if the efficiency of the pump installation is 0.55.

2-10. The Manufacturer's data for a centrifugal pump handling water are as follows: $Q=56$ m³/h, $H=42$ m, $P=10.9$ kW at $n=1140$ rpm. Determine (1) the efficiency of the pump, and (2) its output, the head developed by it, and the power consumed at $n=1450$ rpm, considering that the efficiency remains unchanged.

2-11. The following data were obtained in testing a centrifugal pump:

Q , dm ³ /min	0	100	200	300	400	500
H , m	37.2	38.0	37.0	34.5	31.8	28.5

How much liquid will this pump deliver through a pipeline with a diameter of 76×4 mm and 355 m long (its actual length plus the equivalent length of the local obstacles) if the geometrical delivery height is 4.8 m? The coefficient of pipe resistance is $\lambda=0.03$, $\Delta p_{\text{add}}=0$. (Plot the characteristics of the pump and the pipeline, and find the working point.) How will the output of the pump change if the geometrical delivery height is 19 m?

2-12. Determine the output of a gear pump (see Fig. 2-9) according to the following data: speed=650 rpm, number of teeth on the gear wheel=12, width of tooth=30 mm, cross-sectional area of a tooth restricted by the addendum circle of the mating gear wheel=7.85 cm², delivery factor=0.7.

2-13. A water-jet pump (see Fig. 2-10) is to pump 215 dm³/min of a solution having a relative density of 1.06 out of a basement tank. The suction lift is 3.8 m. The gauge pressure of the water ahead of the pump is $p_{\text{gauge}}=1.9$ at. The efficiency of the pump is 0.15. How many cubic metres of water will the water-jet pump deliver an hour?

2-14. What power should a motor have that drives a fan having an output of 110 m³/min with a total head of 834 Pa (85 mm H₂O)? The efficiency of the fan is 0.47.

2-15. A centrifugal fan operating at a speed of 960 rpm delivers 3200 m³ of air an hour, using 0.8 kW. The gauge pressure created by the fan is 44 mm H₂O. What will the fan delivery, pressure and power required be at $n=1250$ rpm? Also determine the efficiency of the fan.

2-16. What amount of air will be delivered by the fan of Example 2-12 when serving a network in which at a flow rate of 1000 m³/h the sum ($\Delta p_{\text{vel}} + \Delta p_{\text{fr}} + \Delta p_{\text{lr}}$) is 265 Pa, and the difference between the pressures in the delivery and the suction spaces is 20 mm H₂O?

2-17. How much air will be delivered by the fan of Example 2-12 into a network for which at a flow rate of 1350 m³/h the sum ($\Delta p_{\text{vel}} + \Delta p_{\text{fr}} + \Delta p_{\text{lr}}$) is 167 Pa, and Δp_{add} equals 128 Pa?

2-18. What speed must be imparted to the fan of Example 2-12 if it must supply $1500 \text{ m}^3/\text{h}$ of air into a network whose total resistance at this rate of flow is 422 Pa ?

2-19. Determine analytically and according to a T - S diagram the temperature of air after its adiabatic compression from the initial pressure (absolute) of 1 at to the final pressure of 3.5 at. The initial temperature is 0°C . Also find the work needed to compress one kilogram of air.

2-20. Find the power consumed by a piston compressor having an output of $5.6 \text{ m}^3/\text{h}$ (in the conditions of suction). It compresses carbon dioxide from 20 to 70 at (absolute pressure). The initial temperature is -15°C . Assume that the efficiency of the compressor equals 0.65. Solve the problem analytically and with the aid of a T - S diagram for carbon dioxide (Fig. A-27).

2-21. Determine the volumetric efficiency of the compressor of the preceding problem if its dead space is 6% of the piston displacement, and the polytropic expansion exponent $m=1.2$.

2-22. Determine the output of a single-stage piston compressor and the power it consumes according to the following data: piston diameter = 250 mm, piston stroke = 275 mm, volume of dead space = 5.4% of piston displacement, compressor speed = 300 rpm. The atmospheric air is compressed to $p_{\text{abs}} = 4$ at in the compressor. The polytropic expansion exponent is 10% less than the adiabatic exponent. The initial temperature of the air is 25°C . The overall efficiency of the compressor is 0.72.

2-23. How will the output of and the power consumed by the compressor of the preceding problem change if a supercharger is used for feeding air into it at a gauge pressure of $p_{\text{gauge}} = 0.4$ at (see Fig. 2-13)? The final pressure (absolute) is 4 at.

2-24. At what delivery pressure will the volumetric efficiency of a single-stage piston compressor handling ethylene drop to 0.2? The suction pressure is 1 at. Consider the expansion of the gas from the dead space to be adiabatic. The volume of the dead space is 7% of the piston displacement.

2-25. On the basis of the condition that compressor lubricating oil allows a temperature in the cylinder of not over 160°C without noticeably detracting from the quality of lubrication, determine the maximum value of the delivery pressures in a single-stage piston compressor: (a) for air, and (b) for ethane. The suction pressure is atmospheric (1 at). The initial temperature is 25°C . Consider the process of compression to be adiabatic.

2-26. Using the data of Example 2-17, determine the theoretical expenditure of work by Eqs. (2-18) and (2-24) for a single- and a double-stage compressor.

2-27. A piston compressor is to compress nitrogen from 1 to 100 at (absolute pressure). How many stages should be used if the tolerated temperature at the end of compression must not exceed 140°C . Consider the compression process to be adiabatic. The initial temperature is 20°C .

2-28. Determine the theoretical expenditure of work for compressing hydrogen from 1.5 to 17 at (absolute pressure) with single- and two-stage compression. The initial temperature of the hydrogen is 20°C .

2-29. A compressor when being tested delivered atmospheric air into a cylinder having a volume of 42.4 dm^3 . In 10.5 minutes, the pressure in the cylinder increased from 0 to 52 at (gauge pressure), and the temperature of the air in it grew from 17 to 37°C . Determine the output of the compressor in m^3/h (in standard conditions).

2-30. Determine the power consumed and the rate of flow of water through the coolers of a piston compressor that compresses $625 \text{ m}^3/\text{h}$ (in standard conditions) of ethylene from a pressure (absolute) of 9.81×10^4 to $176.6 \times 10^4 \text{ Pa}$. The efficiency of the compressor is 0.75. The cooling water is heated in the coolers by 13 K. The initial temperature of the gas is 20°C .

SYMBOLS

A	area
b	width of tooth
c	specific heat capacity
G	mass of gas handled an hour
H	enthalpy; head
h	head; loss of suction lift
h_f	saturated vapour pressure at pumping temperature
M	molar mass
m	polytropic exponent
N	number of compression stages
n	speed (number of revolutions per unit of time)
p	pressure
Q	volumetric output (delivery) of pump
Q_m	mass rate of flow
Q_v	volumetric rate of flow
R	gas constant
S	entropy
s	piston stroke
T	absolute temperature
t	temperature, °C
v	velocity; specific volume
W	work
x	compression ratio
z	difference between heights of delivery and suction places; number of teeth on gear wheel

Greek Letters

β	power reserve factor
γ	adiabatic exponent (equal to ratio of heat capacities c_p/c_v)
e_0	ratio of volume of dead space of cylinder to piston displacement
η	efficiency
η_v	delivery factor
λ	dimensionless delivery coefficient
λ_0	volumetric efficiency of compressor
ρ	density

Chapter 3

HYDROMECHANICAL SEPARATION METHODS. HYDRODYNAMICS OF A FLUIDIZED BED. AGITATION IN A LIQUID MEDIUM

FUNDAMENTAL RELATIONSHIPS AND FORMULAS FOR CALCULATIONS

SETTLING

Gravity Settling (Dust Chambers, Settling Tanks)

1. The settling of a spherical particle in an unbounded medium at rest can be described in the dimensionless form by using the Archimedes Ar , Lyashchenko Ly , and Reynolds Re dimensionless numbers.

The most convenient form of a dimensionless relationship is $Ly = f(Ar)$.

2. For the so-called laminar settling conditions when the dimensionless numbers have the values of $Ar < 3.6$, $Ly < 2 \times 10^{-3}$, and $Re < 0.2$, G. Stokes theoretically obtained the following formula characterizing the settling velocity v_{set} (in m/s) of a spherical particle*:

$$v_{set} = \frac{d^2 (\rho - \rho_{fl}) g}{18\mu_{fl}} \quad (3-1)$$

Formula (3-1) is simplified as follows for the settling of particles in a gaseous medium:

$$v_{set} = \frac{d^2 \rho g}{18\mu_{fl}} \quad (3-2)$$

because in this case the quantity ρ_{fl} may be disregarded in comparison with ρ .

In Eqs. (3-1) and (3-2):

d = diameter of a spherical particle, m

ρ = density of a particle, kg/m³

ρ_{fl} = density of the fluid medium, kg/m³

μ_{fl} = dynamic viscosity of the fluid medium, Pa·s (i. e. N·s/m² or kg/m·s).

3. The settling velocity of single spherical particles in an unbounded medium at rest is determined according to the generalized method suitable for any settling conditions as follows.

* The Stokes formula may also be used in practice for higher values of the Archimedes and Lyashchenko numbers.

The Archimedes number is determined:

$$Ar = Ga \frac{\Delta\rho}{\rho_{fl}} = \frac{Re^2}{Fr} \frac{(\rho - \rho_{fl})}{\rho_{fl}} = \frac{d^3 (\rho - \rho_{fl}) \rho_{fl} g}{\mu_{fl}^2} \quad (3-3)$$

where $Ga = Re^2/Fr$ is the Galilei number.

For settling in a gaseous medium:

$$Ar = \frac{d^3 \rho \rho_{fl} g}{\mu_{fl}^2} \quad (3-4)$$

The found value of the number Ar is used to determine the Reynolds number Re or the Lyashchenko number Ly (Fig. 3-1):

$$Ly = \frac{Re^3}{Ar} = \frac{Re Fr \rho_{fl}}{\rho - \rho_{fl}} = \frac{v_{set}^3 \rho_{fl}^2}{\mu_{fl} (\rho - \rho_{fl}) g} \quad (3-5)$$

or (if the fluid is a gas):

$$Ly = \frac{v_{set}^3 \rho_{fl}^2}{\mu_{fl} g \rho} \quad (3-6)$$

Next the settling velocity is calculated:

$$v_{set} = \frac{Re \mu_{fl}}{\rho_{fl} d} \quad (3-7)$$

or

$$v_{set} = \sqrt[3]{\frac{Ly \mu_{fl} (\rho - \rho_{fl}) g}{\rho_{fl}^2}} \quad (3-8)$$

The settling velocity for a particle of an irregular shape is determined in the same way from the Lyashchenko number, but using the quantity d_{eq} instead of d in the Archimedes number.

The equivalent diameter d_{eq} of a particle having an irregular shape is calculated as the diameter of a conditional sphere whose volume V equals that of the body having an irregular shape:

$$d_{eq} = \sqrt[3]{\frac{6}{\pi} V} = 1.24 \sqrt[3]{\frac{M}{\rho}} \quad (3-9)$$

where M is the mass of the particle, kg.

4. The diameter of a settling spherical particle when the velocity is known is found in the opposite way, i. e. first the Lyashchenko number is calculated by Eq. (3-5), and the found value of Ly is used to determine the Archimedes number Ar from Fig. 3-1. Next, the value of Ar is inserted in Eq. (3-3), and the diameter of the spherical particle is calculated.

5. The equivalent diameter of a solid particle having an irregular shape is determined similarly when the settling velocity is known. First, the number Ly is found from Eq. (3-5), then the value of the number Ar is found from Fig. 3-1 for a particle of

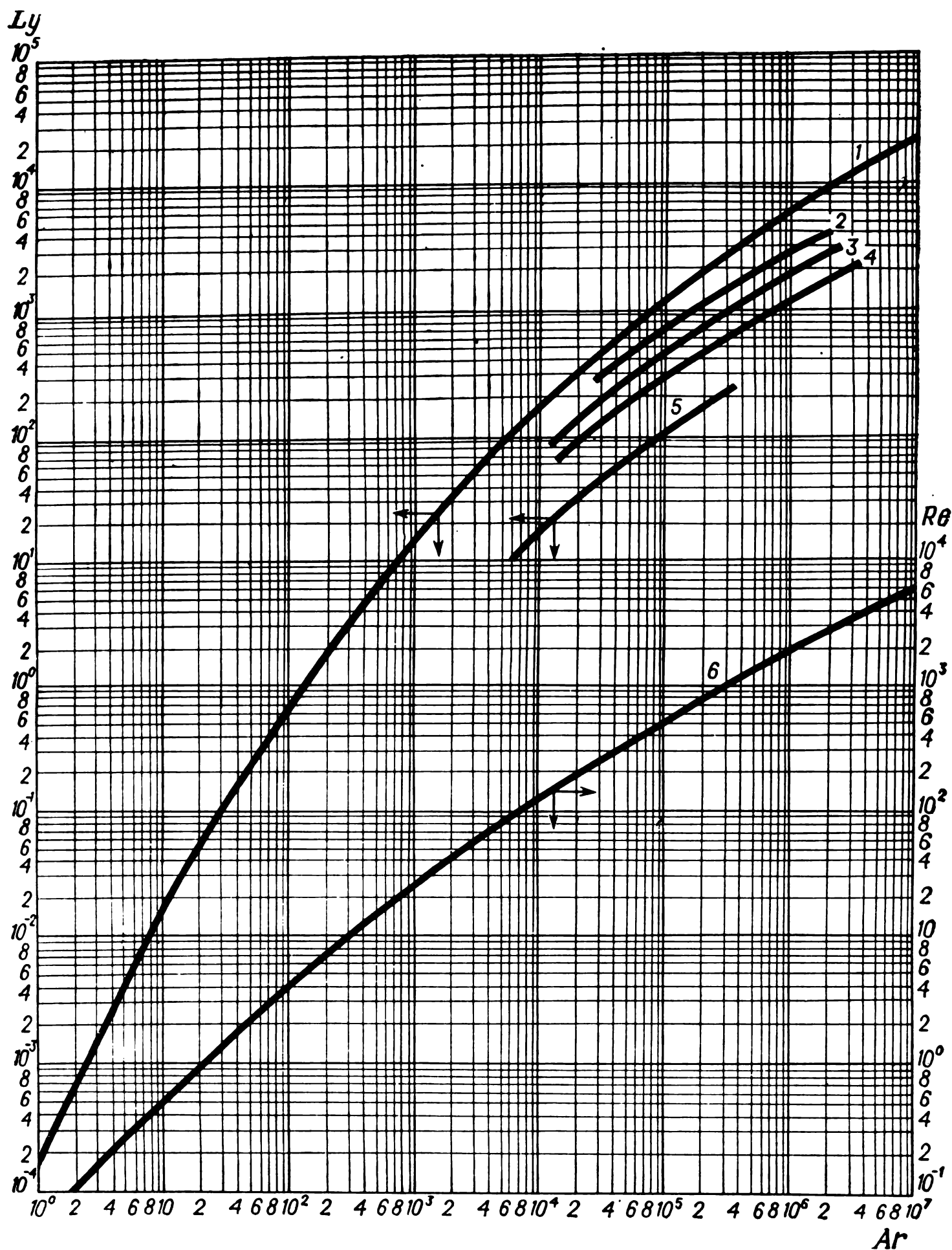


Fig. 3-1. Dependence of the numbers Re and Ly on the Archimedes number Ar for the settling of a single particle in a medium at rest:

1 and 6—spherical particles; 2—rounded; 3—sharp-edged; 4—elongated; 5—lamellar particles

the corresponding shape, and its equivalent diameter is calculated:

$$d_{eq} = \sqrt[3]{\frac{Ar\mu_{fl}^2}{(\rho - \rho_{fl})\rho_{fl}g}} \quad (3-10)$$

6. The settling area A_{set} (in m^2) of a dust chamber or a settling tank for suspensions or slurries is determined by the equation:

$$A_{set} = \frac{Q_v}{v'_{set}} \quad (3-11)$$

where Q_v = volumetric flow rate of the fluid passing through the apparatus parallel to the settling surface, m^3/s

v'_{set} = actual settling velocity of a particle, m/s .

The ratio of the actual velocity of hindered settling of particles v'_{set} to the settling velocity of a single particle v_{set} depends on the volume concentration of the suspension or slurry.

In reference calculations approximately taking into account the difference between the real settling conditions and the theoretical ones (the constricted nature of settling, the shape of the particles, the motion of the fluid), the actual settling velocity is often taken equal to half the theoretical settling velocity of a single spherical particle:

$$v'_{set} = 0.5v_{set} \quad (3-12)$$

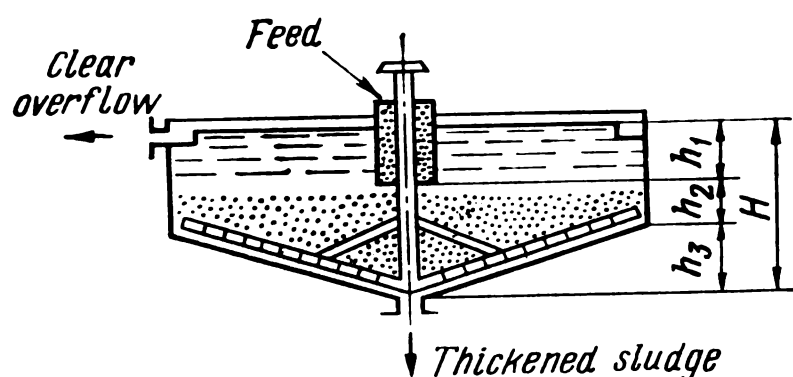


Fig. 3-2. Continuous cone-bottom settling tank

7. Equation (3-11) when applied to a continuous settling tank or thickener for the settling of suspensions (Fig. 3-2) acquires the form:

$$A_{set} = \frac{Q_{m,f} \left(1 - \frac{c_f}{c_{th}}\right)}{\rho_{cl} v'_{set}} \quad (3-13)$$

or

$$A_{set} = \frac{Q_{v,lq} (x_{th} - x_f)}{v'_{set} x_{th}} \quad (3-14)$$

where A_{set} = settling area of tank, m^2

$Q_{m,f}$ = mass flow rate of the feed—the initial (diluted) suspension, kg/s

c_f = mass concentration of the solid phase in the feed suspension, kg/kg

- c_{th} = mass concentration of the solid phase in the thickened suspension (sludge), kg/kg
 ρ_{cl} = density of clarified liquid, kg/m³
 $Q_{v, lq}$ = volumetric flow rate of the liquid phase contained in the feed suspension, m³/s
 x_f and x_{th} = concentrations (molalities) of the feed suspension and the sludge, respectively, kg of solid phase per kg of liquid phase.

Settling under the Action of a Centrifugal Force (Cyclones)

8. The basic dimensions of a cyclone (Fig. 3-3) are usually determined depending on its diameter D . For the widespread

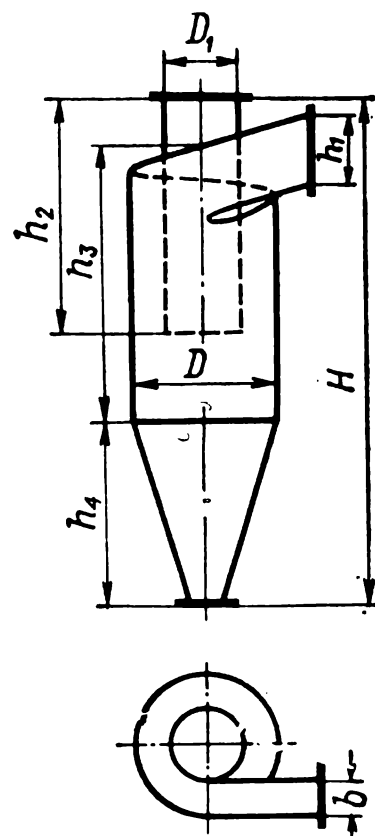


Fig. 3-3. NIIOGAZ cyclone

NIIOGAZ cyclones (the letters NIIOGAZ stand for the Research Institute for the Industrial and Sanitary Purification of Gases), these dimensions (in fractions of D) are given in Table 3-1.

A distinguishing feature of the NIIOGAZ cyclones is the inclined gas inlet pipe.

Three types of this cyclone are widely used:

1. With an angle of 24° (ЦН-24); this type ensures an increased output with the smallest hydraulic resistance. It is intended for separating large dust particles.

2. With an angle of 15° (ЦН-15); this type ensures a high efficiency of separation at a comparatively small hydraulic resistance.

3. With an angle of 11° (ЦН-11); this type ensures an increased efficiency of separation and is recommended as a unified dust separator.

TABLE 3-1

Characteristic of cyclone	Type of cyclone		
	ЦН-24	ЦН-1	ЦН-11
Diameter of gas outlet pipe D_1	0.6	0.6	0.6
Width of dust outlet pipe b	0.26	0.26	0.26
Height of gas inlet pipe h_1	1.11	0.66	0.48
Height of dust outlet pipe h_2	2.11	1.74	1.56
Height of cylindrical part h_3	2.11	2.26	2.08
Height of tapered part h_4	1.75	2.0	2.0
Total height of cyclone H	4.26	4.56	4.38
Local resistance coefficient ζ_0	60	160	250

9. The diameter D of a cyclone is determined according to the conditional velocity of the gas v_{cyc} related to the full cross section of the cylindrical part of the cyclone:

$$D = \sqrt{\frac{Q_v}{0.785 v_{\text{cyc}}}}$$

(3-15)

where Q_v is the volumetric flow rate of the gas flowing through the cyclone, m³/s.

The velocity v_{cyc} is determined from Eq. (3-16) for a given ratio $\Delta p/\rho$ (see Example 3-10). According to the data of [3-5], the values of v_{cyc} are taken equal to from 2.5 to 4 m/s.

10. The hydraulic resistance Δp (in Pa) of a cyclone is calculated by the equation:

$$\Delta p = \zeta_0 \frac{v_{\text{cyc}}^2 \rho}{2}$$

(3-16)

where ζ_0 = dimensionless resistance coefficient of a cyclone

ρ = density of the gas passing through the cyclone, kg/m³.

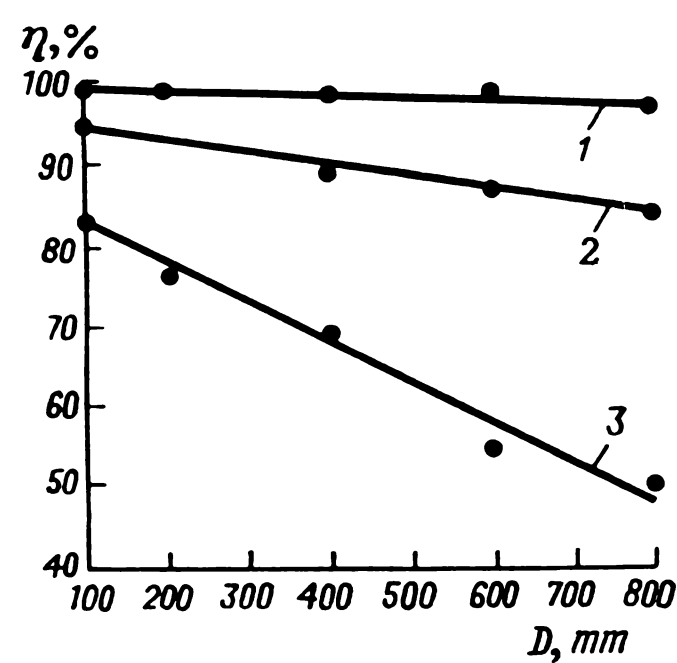


Fig. 3-4. Efficiency of separation in a type ЦН-15 cyclone:
1 - $d = 15 \mu\text{m}$; 2 - $d = 10 \mu\text{m}$; 3 - $d = 5 \mu\text{m}$

11. The efficiency of separation η in a cyclone depends, apart from the properties of the dust, on the diameter of the cyclone

and also on the velocity of the gas v_{cyc} , i. e. on the ratio $\Delta p/\rho$ in accordance with Eq. (3-16).

Figure 3-4 shows how the efficiency of dust separation η depends on the diameter of a cyclone D with different diameters of the dust particles d ; the dust particle density is 2300 kg/m^3 and with $\Delta p/\rho = 740 \text{ m}^2/\text{s}^2$.

FILTRATION

12. At $\Delta p = \text{const}$, the volume of the filtrate V passing through 1 m^2 of a filtering surface during the time τ and the duration of filtration τ are related by the equation:

$$V^2 + 2VC = K\tau \quad (3-17)$$

where C = filtration constant characterizing the hydraulic resistance of the filtering partition (cloth), m^3/m^2

K = filtration constant allowing for the conditions of the filtration process and the physicochemical properties of the sediment and the liquid [see Eq. (3-21)], m^2/s

τ = duration of filtration, s.

The constants K and C are determined experimentally.

13. The rate of filtration (in $\text{m}^3/\text{m}^2 \cdot \text{s}$) at a given moment is determined by the equation

$$\frac{dV}{d\tau} = \frac{K}{2(V + C)} \quad (3-18)$$

We can use this equation to calculate the rate of washing the sediment (filter cake) with a wash liquid if the viscosity of the latter equals that of the filtrate and if the wash liquid passes through the filter along the same path as the filtrate. In these conditions, the rate of washing equals the rate of filtration at the terminal moment.

A modified form of Eq. (3-18), i. e.

$$\frac{d\tau}{dV} = \frac{2}{K} V + \frac{2C}{K} \quad (3-19)$$

expressing a linear relationship between the quantities $d\tau/dV$ and V is used to determine the constants C and K according to experimental data. For this purpose, the measured values V_1, V_2, \dots are laid off along the axis of abscissas, and the corresponding measured values $\Delta\tau_1/\Delta V_1, \Delta\tau_2/\Delta V_2, \dots$ along the axis of ordinates. By drawing a straight line through the experimental points (Fig. 3-5), we can find K and C from the equations

$$\tan \beta = \frac{2}{K} \text{ and } a = \frac{2C}{K} \quad (3-20)$$

14. The filtration constant K (in m^2/s) related to 1 m^2 of filtering surface at $\Delta p = \text{const}$ depends on the specific resistance of

the cake according to the following equation [3-21]:

$$K = \frac{2\Delta p}{\mu cr} \quad (3-21)$$

where Δp = pressure drop in the filter, Pa

μ = dynamic viscosity of the filtrate, Pa·s

r = specific resistance of the cake (calculated per kilogram of the solid dry matter in it), m/kg

c = mass of the solid dry matter deposited on the filter when 1 m³ of the filtrate passes through the filtering surface, kg/m³.

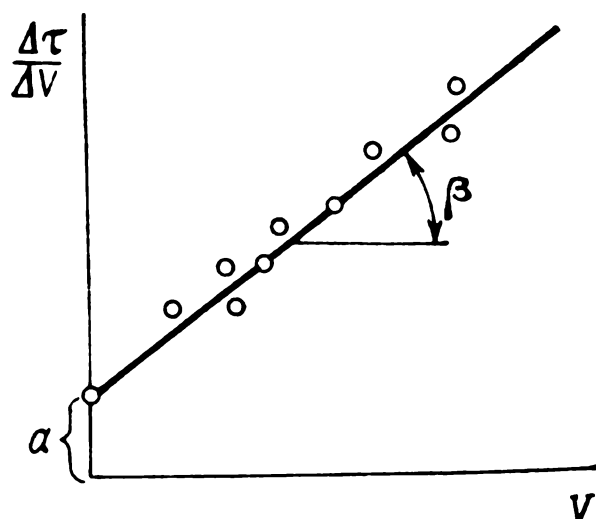


Fig. 3-5. Dependence of $\Delta\tau/\Delta V$ on V

15. The quantity c (in kg/m³) can be expressed through the concentration of the suspension being filtered x as follows (see Example 3-11):

$$c = \frac{\rho x}{1 - mx} \quad (3-22)$$

where ρ = density of the filtrate, kg/m³

x = mass concentration of the solid phase in the suspension, kg/kg

m = mass of the wet cake per kilogram of the solid dry matter contained in it, kg/kg.

When the value of c is inserted into Eq. (3-21), we get the following equation for the constant K (in m²/s):

$$K = \frac{2\Delta p (1 - mx)}{\mu r x \rho} \quad (3-23)$$

If the filtration constant K is known, the specific resistance of the cake r can be found from Eq. (3-21) solved with respect to r (in m per kg of dry matter in the cake):

$$r = \frac{2\Delta p (1 - mx)}{K \mu x \rho} \quad (3-24)$$

16. The filtration constant C (in m³/m²) characterizing the resistance of the filtering partition (cloth) and related to 1 m² of the filter surface is determined by the following expression [3-21]

when $\Delta p = \text{const}$:

$$C = \frac{r_f}{rc} \quad (3-25)$$

or in accordance with Eq. (3-22):

$$C = \frac{r_f(1 - mx)}{rx\rho} \quad (3-26)$$

where r_f = specific resistance of the filtering cloth (per m^2 of surface), m/m^2

r = specific resistance of the sediment (filter cake), m/kg .

Consequently, if the value of the filtration constant C (in m^3/m^2) is known, the specific resistance of the filter medium can be calculated by the formula:

$$r_f = \frac{Crx\rho}{1 - mx} \quad (3-27)$$

17. The concentration of the wash water at any moment of time from the beginning of the main (diffusion) period of washing* can be calculated by the equation

$$c = c_1 \exp\left(-\frac{Kv\tau}{\delta}\right) \quad (3-28)$$

where c_1 = concentration of the wash water at the beginning of the process

K = coefficient taking into account the physicochemical properties of the cake and the wash water, and also the washing conditions (it is found experimentally); it is usually called the washing velocity constant

v = unit intensity of washing or the velocity of the wash water, $\text{m}^3/\text{m}^2 \cdot \text{s}$

τ = duration of washing, s

δ = thickness of sediment layer (filter cake), m.

After taking logarithms, Eq. (3-28) becomes

$$\frac{\tau_2 - \tau_1}{\log c_1 - \log c_2} = \frac{2.3\delta}{Kv} \quad (3-29)$$

where τ_1 and τ_2 = times of beginning and terminating of observations during any period of the process of diffusion washing

c_1 and c_2 = corresponding concentrations of the wash water.

Equations (3-28) and (3-29) can be used to find the duration of washing, i. e. the time needed for the required change in the

* During the main period of washing, we have a process of diffusion of the solute into the wash water and its removal with the latter. During the initial period of washing, what occurs is only the displacement of the filtrate from the cake pores by the wash water.

concentration of the substance being washed out in the cake, and also the value of the constant K .

18. The amount of dry matter M (in kg) in the cake on a filter depends on the quantity of collected filtrate V (in m^3), the density of the filtrate ρ , the mass fraction of the solid phase in the suspension x , and the moisture content of the cake (expressed by the mass ratio m) and can be calculated by the formula

$$M = Vc = V \frac{\rho x}{1 - mx} \quad (3-30)$$

where c is the concentration in kg/m^3 .

19. The concentration of the solid phase in the suspension x , depending on the density of the suspension ρ_{susp} , is expressed by the formula:

$$x = \frac{(\rho_{\text{susp}} - \rho) \rho_{\text{sol}}}{(\rho_{\text{sol}} - \rho) \rho_{\text{susp}}} \quad (3-31)$$

The density of the suspension is:

$$\rho_{\text{susp}} = \frac{m' + 1}{\frac{1}{\rho_{\text{sol}}} + \frac{m'}{\rho}} = \frac{\rho (1 + m') \rho_{\text{sol}}}{\rho + \rho_{\text{sol}} m'} \quad (3-32)$$

In the above equations:

x = mass concentration of the solid phase in the suspension, kg/kg

ρ_{susp} = density of the suspension, kg/m^3

ρ = density of the liquid phase, kg/m^3

ρ_{sol} = density of the solid phase, kg/m^3

m' = mass of the liquid phase in the suspension per unit of mass of the solid phase (solid:liquid = 1: m').

Equation (3-32) can also be used to calculate the density of the wet cake considering it as a concentrated suspension.

20. The calculation of bag-type filters for gases consists in determining the required filtering surface A (in m^2) of the filter by the equation:

$$A = \frac{Q_v}{q_v} \quad (3-33)$$

where Q_v = volumetric rate of flow of the dust-laden gas

q_v = unit volumetric rate of flow of the dust-laden gas per square metre of cloth surface.

The value of q_v is usually taken equal to from 0.2 to 1 $\text{m}^3/\text{m}^2 \cdot \text{min}$, and when separating coarse dust up to 2.5 $\text{m}^3/\text{m}^2 \cdot \text{min}$.

CENTRIFUGAL SEPARATION

21. The centrifugal force F_c (in N) developed in centrifugal separators is determined by the equation:

$$F_c = \frac{Mv^2}{R} = M\omega^2 R \approx 40Mn^2 R = 20Mn^2 D \quad (3-34)$$

where M = mass of the cake and the liquid in the basket of the centrifuge, kg

v = linear velocity at the periphery of the basket, m/s

ω = angular velocity, rad/s

$D = 2R$ = diameter of the basket, m

n = speed of the centrifuge, rps.

The filtration pressure (in Pa) in centrifugal separation is approximately

$$\Delta p_c = \frac{F_c}{A} \quad (3-35)$$

where F_c = centrifugal force calculated by Eq. (3-34)

$A = \pi DH$ = average filtration surface area, m²

D = internal diameter of the centrifuge basket, m

H = height of the basket (in centrifuges of the batch and automatic short-cycle batch types) or the length of the filtration zone (in continuous centrifuges), m.

The filtration pressure is determined more accurately by the equation:

$$\Delta p_c = 20\rho_{\text{susp}} n^2 (R_2^2 - R_1^2) = 5\rho_{\text{susp}} n^2 (D_2^2 - D_1^2) \quad (3-36)$$

where ρ_{susp} = density of the suspension, kg/m³

$D_1 = 2R_1$ = diameter of the internal layer of liquid, m

$D_2 = 2R_2$ = internal diameter of the basket, m

n = speed of rotation of the centrifuge, rps.

22. By the separation factor in centrifuges is meant the ratio of the acceleration of the centrifugal force to the acceleration due to gravity:

$$f = \frac{F_c}{F_{\text{gr}}} = \frac{\omega^2 R}{g} \approx 20Fr_c^* \quad (3-37)$$

where R = radius of the basket, m

ω = angular velocity, rad/s

F_{gr} = force of gravity, N.

23. The rate of filtration in filtering centrifuges can be expressed in the form of the general hydraulic law

$$\frac{dV}{d\tau} = \frac{\Delta p_c}{R_c} \quad (3-38)$$

* The separation factor is a modified Froude number (centrifugal):

$$Fr_c = \frac{Dn^2}{g}$$

where Δp_c = pressure drop in centrifugal filtration
 $R_c = R_{\text{cake}} + R_f$ = total resistance in centrifugal filtration equal to the sum of the resistances of the cake and the filtering partition.

The values of the resistances R_{cake} and R_f can be calculated according to the filtration equations or determined experimentally.

24. The depth h (in m) of the funnel of liquid formed upon the rotation of the centrifuge bowl (basket) is approximately calculated by the formula

$$h = 2n^2 R^2 \quad (3-39)$$

where n = rotational speed of the bowl, rps

R = radius of the bowl, m.

This formula can also be used to approximately calculate the depth of the funnel in apparatuses with an agitator.

25. The consumption of power during the starting period for batch centrifuges is calculated by the following formulas.

(a) The power used for overcoming the inertia of the drum and its charge.

The work W_1 (in J) done to overcome the inertia of the bowl during the starting period is:

$$W_1 = \frac{v_2^2 M_b}{2} \quad (3-40)$$

where v_2 = steady peripheral velocity of the bowl (at its external surface with the radius R_2) after the latter has reached the preset rotational speed, m/s

M_b = mass of the bowl, kg.

The work W_2 (in J) done to overcome the inertia of the charge during the starting period (the volume of the charged material is taken equal to half the total volume of the bowl) is:

$$W_2 = \frac{0.75 v_1^2 \rho V}{4} \quad (3-41)$$

where v_1 = peripheral velocity at the internal radius R_1 of the bowl, m/s

ρ = density of the charged material, kg/m³

V = total volume of the centrifuge bowl, equal to $\pi R_1^2 H$, m³.

The power P_1 (in W) spent to overcome the inertia of the bowl and the charge during the starting period is:

$$P_1 = \frac{W_1 + W_2}{\tau} \quad (3-42)$$

where τ is the duration of the starting period, s.

According to practical data, τ usually ranges from 1 to 3 min.

(b) The power P_2 required to overcome friction between the shaft and its bearings is:

$$P_2 = f M v_{sh} g \quad (3-43)$$

where f = coefficient of friction*

M = mass of all the rotating parts of the centrifuge together with its charge, kg

v_{sh} = peripheral velocity of the shaft journal, m/s.

(c) The power P_3 (in W) spent on friction between the bowl wall and the air is:

$$P_3 = 2.94 \times 10^{-3} \zeta R_2^2 v_2^3 \rho_{air} \quad (3-44)$$

where ρ_{air} = density of the air, kg/m³

ζ = dimensionless resistance coefficient averaging 2.3.

(d) The total consumption of power P_{tot} (in W) for a batch centrifuge during the starting period is:

$$P_{tot} = P_1 + P_2 + P_3 \quad (3-45)$$

With a view to the efficiency of the transmission η_{tr} , the consumption of power is:

$$P = \frac{P_{tot}}{\eta_{tr}} \quad (3-46)$$

The installed capacity of motors for centrifuges should be selected with a reserve of from 10 to 20%.

26. The wall thickness of a centrifuge bowl can be calculated or the strength of the wall checked by means of the equation:

$$\sigma_u = \frac{F_{c,1} + F_{c,2}}{2A} \quad (3-47)$$

where σ_u = permissible ultimate tensile stress of the material of the bowl wall, Pa

A = cross-sectional area of the bowl wall, since the acting force is resisted by both parts of the bowl section, the total cross-sectional area equals $2A$ (Fig. 3-6)

$F_{c,1}$ = centrifugal force acting on a half-ring of the bowl wall, N

$F_{c,2}$ = centrifugal force acting on a half-ring of the charge, N.

The values of $F_{c,1}$ and $F_{c,2}$ are calculated by Eq. (3-34). Here the distance R (in m) from the centre of gravity of the rotating half-ring to the axis of rotation is determined by the equation:

$$R = \frac{4}{3\pi} \left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \right) \quad (3-48)$$

where R_1 and R_2 are the internal and external radii of the half-ring, m.

* It ranges from 0.07 to 0.1 for plain bearings with annular lubrication, and $f=0.03$ for ball bearings.

27. (a) The capacity Q_v (volumetric flow rate of the suspension fed in) of a settling centrifuge of type AOF (in m^3/s) with the sediment removed by means of an unloader knife and with lami-

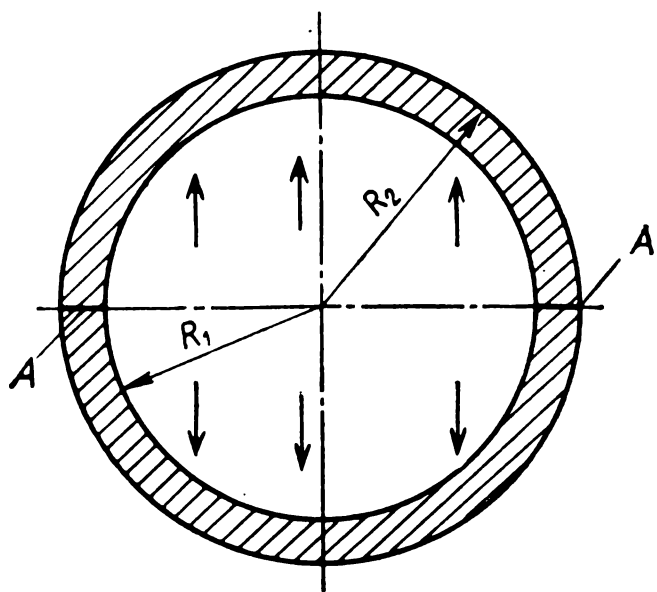


Fig. 3-6. Diagram of the forces acting in the bowl of a centrifuge

nar conditions of settling is determined by the modified equation (3-11):

$$Q_v = A' v \eta \quad (3-49)$$

where $A' = 2\pi R_0 L$ = area of the suspension surface in the bowl, m^2

R_0 = internal radius of the annular layer of the suspension, m

L = length of the bowl, m

$v = v_{\text{set}} f$ = velocity of settling of the particles under the action of the centrifugal force, m/s

v_{set} = velocity of settling of the particles under the action of the force of gravity, m/s

f = separation factor determined according to the radius R_0

η = coefficient taking into account the ratio between the actual and the theoretical capacities of the centrifuge, which in the absence of experimental data can be taken equal to from 0.4 to 0.5; it depends on the slipping of the liquid relative to the bowl, and also allows for eddies hindering settling.

Equation (3-49) can be reduced to a form convenient for calculations:

$$Q_v = 25.3 \eta L n^2 R_0^2 v_{\text{set}} k \quad (3-50)$$

where k is the ratio of the duration of feeding the suspension (the duration of centrifugal separation proper) to the total duration of centrifuge operation.

(b) The capacity Q_v (in m^3/h) relative to the feed suspension of a type HOFIII centrifuge (continuous horizontal helical-conveyor

centrifuge) is determined by the equation:

$$Q_v = 3.5 \frac{D_{dis}^2 L_{dis} (\rho - \rho_f) d^2 n^2}{\mu} \quad (3-51)$$

where D_{dis} and L_{dis} = diameter and length of the "discharge cylinder", respectively, m

ρ and ρ_f = density of the particles and fluid medium, respectively, kg/m³

d = diameter of the smallest separated particles (the "cut diameter"), m

n = speed of the rotor, rpm

μ = dynamic viscosity of the medium, Pa·s.

28. The capacity Q_v (in m³/s) of a tubular supercentrifuge (relative to the feed suspension) is determined by the expression*:

$$Q_v \leq \frac{v V_{1q}}{h} \quad (3-52)$$

where v = velocity of settling of the particles in the centrifuge, m/s

$V_{1q} = 0.785 (D^2 - D_0^2) L$ = volume of the liquid in the bowl, m³

h = depth of the flow in the bowl, m

l = length of the working part of the bowl, m

D = internal diameter of the bowl, m

D_0 = diameter of the decanting (overflow) weir, m.

A condition for good operation of a supercentrifuge (i. e. without carrying away the solid particles) having the calculated diameter is that the flow conditions in the bowl be laminar ($Re \leq 350$). At higher values of the Reynolds number, the length of the drum must be increased.

HYDRODYNAMICS OF A FLUIDIZED BED

29. For a static bed of solid particles, the porosity, i. e. the relative volume of the voids in the total volume (voids plus solid), is

$$\epsilon_0 = \frac{V_b - V}{V_b} \quad (3-53)$$

When the density of the medium (voids) between the particles may be disregarded in comparison with the density of the particles themselves, we have

$$\epsilon_0 = 1 - \frac{\rho_b}{\rho} \quad (3-54)$$

Here V and V_b = volume occupied by the particles and that of the bed, respectively, m³

ρ and ρ_b = density of the particles and that of the bed (the so-called bulk density), respectively, kg/m³.

* Sokolov V. I. *Trubchatye sverkhtsentrijugi* (Tubular Supercentrifuges). Moscow, Goskhimizdat (1949).

In practice, the porosity of a static packed (random) bed of spherical particles having the same diameter ranges from 0.38 to 0.42; the mean value of 0.40 is used in calculations.

For a fluidized bed of solid particles, the porosity is:

$$\epsilon = \frac{V_{\text{bed}} - V}{V_{\text{bed}}} \quad (3-55)$$

where V_{bed} = volume of the fluidized bed, m^3 .

30. The condition for the transition of a static bed of solid particles into the fluidized (suspended) state is the equality of the pressure force exerted by the fluid medium (which manifests itself in the existence of a pressure drop along the height of the bed) and the weight of the bed per unit of its cross-sectional area.

The basic hydrodynamic characteristic of a fluidized bed (at a constant amount of material in it) is the constancy of Δp_{bed} :

$$\Delta p_{\text{bed}} = \frac{G_{\text{bed}}}{A} = \text{const} \quad (3-56)$$

where G_{bed} = weight of the material in the bed, N

A = cross-sectional area, m^2 .

The pressure drop (in Pa) for the fluid passing through a fluidized bed of solid particles is determined by the equation:

$$\Delta p_{\text{bed}} = (\rho - \rho_{\text{fl}}) g (1 - \epsilon) h = (\rho - \rho_{\text{fl}}) g (1 - \epsilon_0) h_0 \quad (3-57)$$

where h and h_0 = heights of the fluidized and static beds, respectively, m

ρ and ρ_{fl} = densities of the solid particles and the fluid, respectively, kg/m^3 .

If the fluid is a gas, then $\rho_{\text{fl}} \ll \rho$ and we have, approximately:

$$\Delta p_{\text{bed}} = \rho g (1 - \epsilon) h = \rho g (1 - \epsilon_0) h_0 \quad (3-58)$$

The pressure drop (in Pa) in the gas-distributing grid (also called gas distributor or constriction plate) can be found by the equation:

$$\Delta p_g = \frac{0.503 v_0^2 \rho_{\text{fl}} (1 - \varphi^2)}{\zeta^2} \quad (3-59)$$

where φ = fraction of the grid area occupied by its clear openings; this quantity is often taken equal to from 0.01 to 0.05

$v_0 = v/\varphi$ = velocity of the flow in the grid openings, m/s

v = velocity of the flow related to the total cross-sectional area of the apparatus, m/s

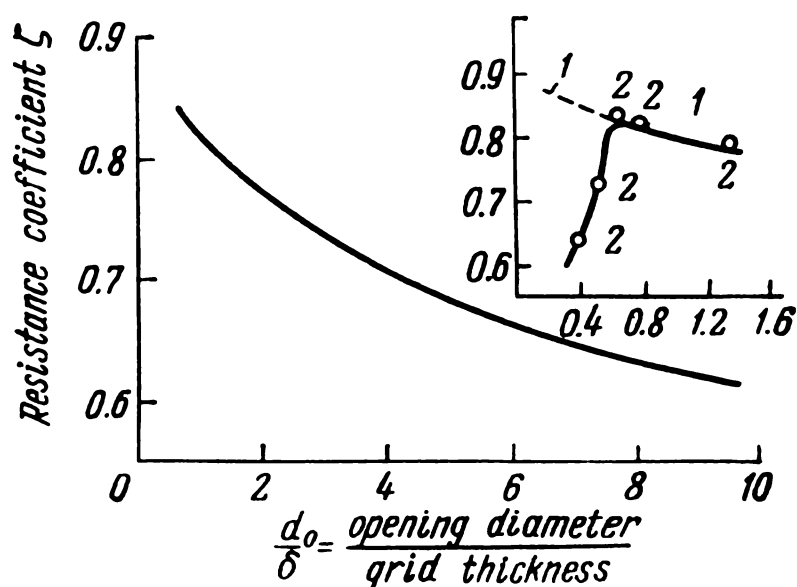
ζ = dimensionless resistance coefficient of the grid depending on the ratio d_0/δ and determined from a graph (Fig. 3-7).

d_o = diameter of a grid opening, m
 δ = thickness of the grid, m.

31. The velocity of a fluid at which the resistance of the bed becomes equal to its weight per unit of cross-sectional area and at which the particles of the static bed become fluidized is called the critical velocity or the fluidization velocity. The critical

Fig. 3-7. Grid resistance coefficient:

1—data of G. Hughmark and H. O'Connell; 2—data of D. Orochko *et al*



velocity for a bed of spherical particles having the same diameter is determined from the equation [3-1]:

$$Re_{cr} = \frac{Ar}{1400 + 5.22 \sqrt{Ar}} \quad (3-60)$$

This equation has been derived for an average porosity of a static bed of $\epsilon_0 = 0.4$ and gives an error of $\pm 20\%$. The dimensionless numbers in it are determined by the equations:

$$Re_{cr} = \frac{v_{cr} d}{\nu_{fl}} \quad (3-61)$$

$$Ar = \frac{Re^3}{Fr} \frac{\rho - \rho_{fl}}{\rho_{fl}} = \frac{d^3 (\rho - \rho_{fl}) g}{\nu_{fl}^2 \rho_{fl}} = \frac{d^3 \rho_{fl} (\rho - \rho_{fl}) g}{\mu^2} \quad (3-62)$$

where v_{cr} = critical velocity of the fluid related to the total cross-sectional area of the apparatus, m/s

d = diameter of the particles, m

ρ and ρ_{fl} = densities of the particles and the fluid, respectively, kg/m³

ν_{fl} = kinematic viscosity of the fluid, m²/s

μ = dynamic viscosity of the fluid, Pa·s.

For a gas ($\rho_{fl} \ll \rho$):

$$Ar = \frac{d^3 \rho g}{\nu_{fl}^2 \rho_{fl}} \quad (3-63)$$

For particles having an irregular shape, the critical fluid velocity can be determined with account taken of the shape factor:

$$\Phi = \sqrt{0.207 \frac{A}{V^{2/3}}} \quad (3-64)$$

where V = volume of a particle, m^3

A = surface area of a particle, m^2 .

It is assumed that the equivalent diameter d_{eq} (in m) is:

$$d_{\text{eq}} = \Phi d_{\text{sph}} \quad (3-65)$$

where d_{sph} is the diameter of a sphere (in m) whose volume equals that of a particle:

$$d_{\text{sph}} = 1.24 \sqrt[3]{V} \quad (3-66)$$

For a polydispersed bed consisting of particles having different diameters

$$d_{\text{eq}} = \frac{1}{\sum_i \frac{w_i}{d_i}} \quad (3-67)$$

where n = number of fractions

d_i = average screen size of the i th fraction (i. e. mean arithmetical value of the average size of the screen mesh through which a fraction passes and the mesh that retains the fraction)

w_i = mass content of the i th fraction in the particle mixture.

32. When the velocity of the fluid v grows, a fluidized bed expands (i. e. its height and porosity grow). The height of a fluidized bed h (in m) is related to the height of the static bed h_0 by the equation:

$$h = \frac{1 - \epsilon_0}{1 - \epsilon} h_0 \quad (3-68)$$

The porosity of the fluidized bed can be calculated by the formula

$$\epsilon = \left(\frac{18Re + 0.36Re^2}{Ar} \right)^{0.21} \quad (3-69)$$

The ratio between the working velocity of a fluid related to the total cross-sectional area of an apparatus and the critical velocity is called the fluidization number:

$$K_v = \frac{v}{v_{\text{cr}}} \quad (3-70)$$

The actual velocity of the fluid in the clear section between the particles of a bed is determined by the expression:

$$v_s = \frac{v}{\varepsilon} \quad (3-71)$$

33. The velocity of a fluid at which a single particle passes into the suspended state is called the soaring velocity. It approximately corresponds to the beginning of destruction of a mono-dispersed fluidized bed. Here $\varepsilon = 1$. The soaring velocity can be determined by the formula:

$$v_s = \frac{Re_s \mu_{fl}}{d \rho_{fl}} \quad (3-72)$$

where

$$Re_s = \frac{Ar}{18 + 0.61 \sqrt{Ar}} \quad (3-73)$$

34. Figure 3-8 gives a graph of the relationship $Ly = f(Ar, \varepsilon)$ for a fluidized bed from $\varepsilon_0 = 0.4$ to $\varepsilon = 1.0$. The graph makes it possible to determine the velocity of the fluid v needed to achieve the required porosity of a fluidized bed consisting of particles of a known diameter d , or to solve the reverse problem.

35. The average residence time τ_0 (in s) of particles of a solid material in an apparatus with a fluidized bed (a single one) is:

$$\tau_0 = \frac{M}{Q_m} \quad (3-74)$$

where M = mass of the material in the bed, kg

Q_m = rate of flow of the solid material, kg/s.

Owing to the intensive mixing of the material in a fluidized bed, the residence time of separate particles in it considerably differs from the average residence time of the solid material in the bed τ_0 . If the average residence time τ_0 is known and a certain time τ is set (for instance the duration of a process conducted in a fluidized bed), then the fraction x of the particles having a residence time in the bed not less than τ can be determined by the equation:

$$x = e^{-\tau/\tau_0} \quad (3-75)$$

where e is the base of natural logarithms, $e = 2.718$.

To obtain a more uniform distribution of the particles by their residence time, several successively arranged fluidized beds are used. The fraction x_n of the particles having a residence time of at least τ in an apparatus with n fluidized beds is

$$x_n = \left[1 + \frac{\tau}{\tau_0} + \frac{1}{2!} \left(\frac{\tau}{\tau_0} \right)^2 + \dots + \frac{1}{(n-1)!} \left(\frac{\tau}{\tau_0} \right)^{n-1} \right] e^{-\tau/\tau_0} \quad (3-76)$$

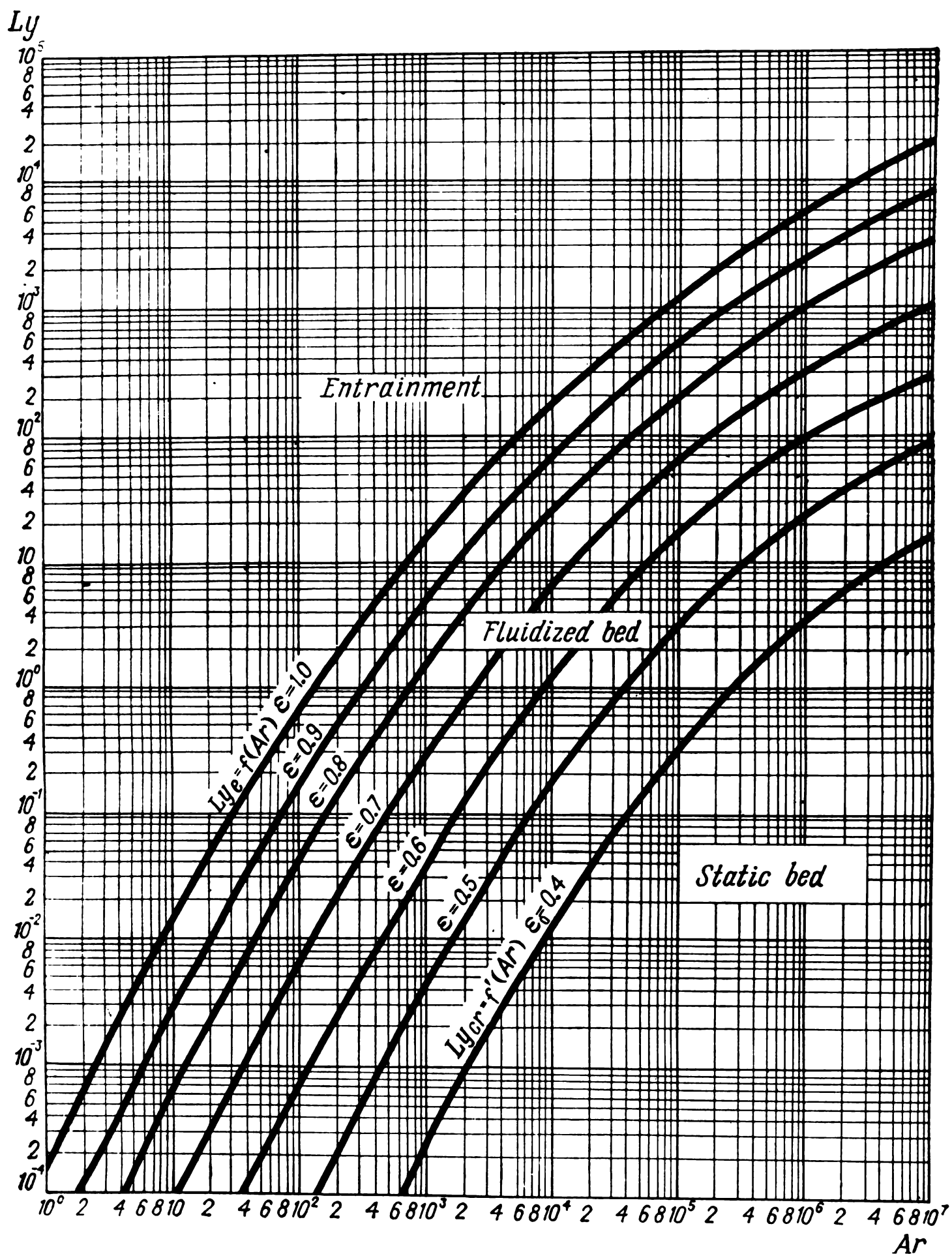


Fig. 3-8. Dependence of the number Ly on the number Ar and the porosity ϵ of a bed

AGITATION IN A LIQUID MEDIUM

36. The dimensionless numbers for the process of agitation are determined as follows.

The Reynolds number (centrifugal) is

$$Re_c = \frac{\rho n d^2}{\mu} \quad (3-77)$$

The critical value of this number is $Re_{c, cr} \approx 50$.

The power number* is:

$$N_P = \frac{P}{\rho n^3 d^5} \quad (3-78)$$

The Froude number (centrifugal) is:

$$Fr_c = \frac{n^2 d}{g} \quad (3-79)$$

In these numbers:

P = power used by the agitator, W

ρ = density of the liquid, kg/m³

μ = dynamic viscosity of the liquid, Pa·s

n = rotational speed of the agitator, rps

d = diameter of the agitator, m

$g = 9.81 \text{ m/s}^2$ = acceleration due to gravity.

37. The dimensionless equation for calculating the power used by an agitator in the general form is:

$$N_P = f(Re_c, Fr_c, S_D, S_b, S_{H_0}, \dots) \quad (3-80)$$

The forces of gravity are usually disregarded. Hence

$$N_P = \varphi(Re_c, S_D, S_b, S_{H_0}, \dots) \quad (3-81)$$

where $S_D = D/d$, $S_b = b/d$, and $S_{H_0} = H_0/d$, ... are shape factors characterizing the design of the agitator, the influence of the height of the liquid layer and other geometrical factors

d = diameter of the agitator, m

D = diameter of the tank, m

b = width of the impeller paddles, m

H_0 = height of the layer of liquid, m.

For geometrically similar apparatuses with mixers or agitators, the generalized dimensionless equation acquires the form:

$$N_P = \frac{C}{Re_c^m} \quad (3-82)$$

where C and m are constants (for a given agitator design and for definite mixing conditions).

* In publications on the subject, it is sometimes called the centrifugal Euler number and is designated Eu_c .

Data on the consumption of energy for agitation are given in various publications, for example in [3-15]. The values of the constants C and m in general equation (3-82) for various agitators are given in Table A-21; Fig. A-7 gives the relationship $N_p = f(Re_c)$ for the same agitators.

EXAMPLES

SETTLING

Example 3-1. Find the upper limit (i.e. the maximum particle diameter) of application of the Stokes formula to quartz particles having a density of 2650 kg/m^3 settling in water at 20°C .

Solution. The Stokes formula may be applied when $Ar < 3.6$. Hence the biggest quartz particles whose settling may be calculated by the Stokes formula should have a diameter equal to:

$$d = \sqrt[3]{\frac{3.6 \times 1^2 \times 10^{-6}}{(2650 - 1000) 1000 \times 9.81}} = 60 \times 10^{-6} \text{ m} = 60 \text{ } \mu\text{m}$$

Example 3-2. Find the velocity of settling in water of spherical quartz sand particles 0.9 mm in diameter if the density of the sand is 2650 kg/m^3 and the temperature of the water is 20°C .

Solution. We determine the Archimedes number Ar :

$$Ar = \frac{d^3 (\rho - \rho_n) \rho_n g}{\mu_n^2} = \frac{0.9^3 \times 10^{-9} (2650 - 1000) 1000 \times 9.81}{1^2 \times 10^{-6}} = 1.18 \times 10^4$$

where for water $\mu_n \approx 1 \text{ cP} = 1 \times 10^{-3} \text{ Pa}\cdot\text{s}$ (Table A-6).

Using the found value of $Ar = 1.18 \times 10^4$, from Fig. 3-1 we find $Re = 140$.

The velocity of settling is determined from the expression:

$$v_{\text{set}} = \frac{Re \mu_n}{d \rho_n} = \frac{140 \times 1 \times 10^{-3}}{0.0009 \times 1000} = 0.15 \text{ m/s}$$

Example 3-3. Determine the size of the greatest spherical particles of chalk that will be entrained by an ascending stream of water flowing at a velocity of 0.5 m/s . The temperature of the water is 10°C , the density of the chalk is 2710 kg/m^3 .

Solution. We determine the Lyashchenko number Ly by Eq. (3-5):

$$Ly = \frac{v_{\text{set}}^3 \rho_n^2}{\mu_n (\rho - \rho_n) g} = \frac{0.5^3 \times 1000^2 \times 10^3}{1.3 (2710 - 1000) 9.81} = 5.72 \times 10^3$$

where for water at 10°C , $\mu_n \approx 1.3 \text{ cP} = 1.3 \times 10^{-3} \text{ Pa}\cdot\text{s}$.

Using Fig. 3-1, we find $Re = 1750$ for the found value of $Ly = 5.72 \times 10^3$. Next by Eq. (3-7), we determine the maximum diameter of the chalk particles that will be entrained by the

water:

$$d = \frac{Re\mu_{fl}}{v_{set}\rho_{fl}} = \frac{1750 \times 1.3 \times 10^{-3}}{0.5 \times 1000} = 4.55 \times 10^{-3} \text{ m} = 4.55 \text{ mm}$$

Example 3-4. Find the velocity of settling of lead glance (galenite) particles having an angular shape with $d_{eq} = 1 \text{ mm}$ in water at 20°C . The density of lead glance is 7560 kg/m^3 .

Solution. We find the settling velocity of particles having an irregular shape from the Lyashchenko number Ly after first determining the number Ar by Eq. (3-62):

$$Ar = \frac{d_{eq}^3 \rho_{fl} (\rho - \rho_{fl}) g}{\mu_{fl}^2} = \frac{1^3 \times 10^{-9} \times 10^3 (7560 - 1000) 9.81}{1^2 \times 10^{-6}} = 6.44 \times 10^4$$

where $\mu_{fl} \approx 1 \text{ cP} = 1 \times 10^{-3} \text{ Pa}\cdot\text{s}$ (Table A-6).

The graph in Fig. 3-1 gives us $Ly = 3.1 \times 10^2$ for angular (sharpedged) particles.

The settling velocity by Eq. (3-8) is:

$$v_{set} = \sqrt[3]{\frac{Ly\mu_{fl}(\rho - \rho_{fl})g}{\rho_{fl}^2}} = \sqrt[3]{\frac{3.1 \times 10^2 \times 1 \times 10^{-3} (7560 - 1000) 9.81}{1000^2}} = 0.271 \text{ m/s}$$

Example 3-5. Determine the size of elongated coal particles ($\rho_1 = 1400 \text{ kg/m}^3$) and lamellar shale particles ($\rho_2 = 2200 \text{ kg/m}^3$) settling with the same velocity $v_{set} = 0.1 \text{ m/s}$ in water at 20°C .

Solution. The size of the particles d_{eq} should be calculated by Eq. (3-10):

$$d_{eq} = \sqrt[3]{\frac{Ar\mu_{fl}^2}{(\rho - \rho_{fl})\rho_{fl}g}}$$

We first determine the value of the number Ar according to that of the number Ly from the graph in Fig. 3-1 for particles of the relevant shape.

For elongated particles, we have:

$$Ly_1 = \frac{v_{set}^3 \rho_{fl}^2}{\mu_{fl} (\rho_1 - \rho_{fl}) g} = \frac{10^{-3} \times 10^6}{1 \times 10^{-3} \times 0.4 \times 10^3 \times 9.81} = 255$$

where $\mu_{fl} \approx 1 \text{ cP} = 1 \times 10^{-3} \text{ Pa}\cdot\text{s}$ (Table A-6), and $\rho_{fl} = 1000 \text{ kg/m}^3$.

For lamellar particles, we have:

$$Ly_2 = \frac{v_{set}^3 \rho_{fl}^2}{\mu_{fl} (\rho_2 - \rho_{fl}) g} = \frac{10^{-3} \times 10^6}{1 \times 10^{-3} \times 1.2 \times 10^3 \times 9.81} = 85$$

The value of $Ar_1 = 9 \times 10^4$ corresponds to $Ly_1 = 255$ for elongated particles. The value of $Ar_2 = 7 \times 10^4$ corresponds to $Ly_2 = 85$ for lamellar particles.

The equivalent diameter of the coal particles, by Eq. (3-10), is

$$d_{eq,1} = \sqrt[3]{\frac{Ar_1\mu_g^2}{(\rho_1 - \rho_g)\rho_g g}} = \sqrt[3]{\frac{9 \times 10^4 \times 1^2 \times 10^{-6}}{9.81 \times 0.4 \times 10^3 \times 10^3}} = 2.82 \times 10^{-3} \text{ m} = 2.82 \text{ mm}$$

The equivalent diameter of the shale particles is:

$$d_{eq,2} = \sqrt[3]{\frac{Ar_2\mu_g^2}{(\rho_2 - \rho_g)\rho_g g}} = \sqrt[3]{\frac{7 \times 10^4 \times 1^2 \times 10^{-6}}{9.81 \times 1.2 \times 10^3 \times 10^3}} = 1.81 \times 10^{-3} \text{ m} = 1.81 \text{ mm}$$

Example 3-6. What should the height of the layer of gas between the racks of a dust-settling chamber (Fig. 3-9) be for particles of pyrite dust $8 \mu\text{m}$ in diameter to settle if the rate of

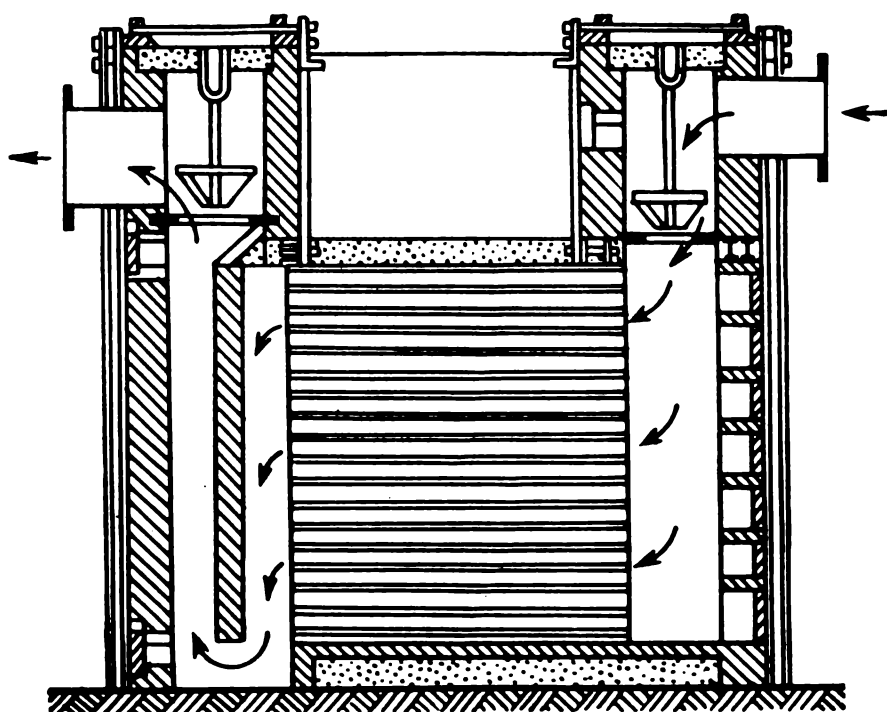


Fig. 3-9. To Example 3-6

flow of the furnace gas is $0.6 \text{ m}^3/\text{s}$ (in standard conditions)? The length of the chamber is 4.1 m , its width is 2.8 m , its total height is 4.2 m . The average temperature of the gas in the chamber is 427°C . The viscosity of the gas at this temperature is $0.034 \times 10^{-3} \text{ Pa}\cdot\text{s}$, the density of the dust is $4000 \text{ kg}/\text{m}^3$, the density of the gas is $0.5 \text{ kg}/\text{m}^3$.

Solution. We determine the rate of flow of the gas in the given conditions:

$$Q_v = \frac{0.6(273 + 427)}{273} = 1.54 \text{ m}^3/\text{s}$$

The linear velocity of the gas (ignoring the thickness of the racks) is:

$$v_g = \frac{1.54}{2.8 \times 4.2} = 0.131 \text{ m/s}$$

The residence time of the gas in the chamber is:

$$\tau = \frac{L}{v_g} = \frac{4.1}{0.131} = 31.3 \text{ s}$$

The theoretical velocity of settling of spherical particles by Eq. (3-1) is (we disregard the quantity ρ_{fl})

$$v_{set} = \frac{1}{18} \frac{(8 \times 10^{-6})^2 4000 \times 9.81}{0.034 \times 10^{-3}} = 0.0041 \text{ m/s}$$

We take the actual velocity of settling equal to $0.5 \times 0.0041 = 0.002 \text{ m/s}$.

We find the distance between the racks:

$$h = v_{set} \tau = 0.002 \times 31.3 \approx 0.06 \text{ m} = 60 \text{ mm}$$

We check the correctness of using Eq. (3-1):

$$Re = \frac{v_{set} d \rho_{fl}}{\mu_{fl}} = \frac{8 \times 10^{-6} \times 0.0041 \times 0.5}{0.034 \times 10^{-3}} = 0.00048$$

Since $Re = 0.00048 < 0.2$, then the Stokes formula (3-1) may be used.

Example 3-7. Determine the size of the smallest particles that settle in a square gas duct 16 m long and 2 m high at a linear velocity of the gas of 0.5 m/s. The viscosity of the gas is $0.03 \times 10^{-3} \text{ Pa} \cdot \text{s}$, the density of the gas is 0.8 kg/m^3 , the density of the particles is 4000 kg/m^3 .

Solution. The gas travels through the duct during the time $\tau = 16/0.5 = 32 \text{ s}$. During this time, only those particles manage to settle whose actual settling velocity is at least $v_{set} = 2/32 = 0.062 \text{ m/s}$.

We determine the diameter of the spherical particles whose theoretical settling velocity is double this value, i.e. equals 0.124 m/s .

We calculate the value of the number Ly by Eq. (3-6):

$$Ly = \frac{v_{set}^3 \rho_{fl}^2}{\mu_{fl} \rho g} = \frac{0.124^3 \times 0.8^2}{0.03 \times 10^{-3} \times 4000 \times 9.81} = 1.035 \times 10^{-3}$$

From the graph in Fig. 3-1, we find $Re = 0.14$, and

$$d = \frac{Re \mu_{fl}}{v_{set} \rho_{fl}} = \frac{0.14 \times 3 \times 10^{-2} \times 10^{-3}}{0.124 \times 0.8} = 4.24 \times 10^{-5} \text{ m} = 42.4 \text{ } \mu\text{m}$$

Example 3-8. Determine the diameter of a settling tank (see Fig. 3-2) for the continuous settling of whiting in water. The capacity of the settling tank is 80 ton/h of the initial suspension containing 8% (by mass) of CaCO_3 . The diameter of the smallest particles to be settled is $35 \text{ } \mu\text{m}$. The temperature of the suspension is 15°C . The moisture content of the sludge is 70%. The density of the whiting is 2710 kg/m^3 .

Solution. To determine the diameter of the settling tank, we must calculate the required settling area by Eq. (3-13). To do this, we first find the settling velocity:

$$v_{\text{set}} = \frac{d^2 (\rho - \rho_{\text{fl}}) g}{18\mu_{\text{fl}}} = \frac{3.5^2 \times 10^{-10} (2710 - 1000) 9.81}{18 \times 1.14 \times 10^{-3}} = 0.001 \text{ m/s}$$

where $\mu_{\text{fl}} = 1.14 \text{ cP} = 1.14 \times 10^{-3} \text{ Pa}\cdot\text{s}$ (Table A-6).

We check the value of the number Re :

$$Re = \frac{v_{\text{set}} d \rho_{\text{fl}}}{\mu_{\text{fl}}} = \frac{1 \times 10^{-3} \times 3.5 \times 10^{-5} \times 10^3}{1.14 \times 10^{-3}} = 0.0307 < 0.2$$

The actual settling velocity is:

$$v'_{\text{set}} = 0.5 \times 0.001 = 0.5 \times 10^{-3} \text{ m/s}$$

The area of the settling tank by Eq. (3-13) is:

$$A_{\text{set}} = \frac{Q_{\text{m, f}} \left(1 - \frac{c_{\text{f}}}{c_{\text{th}}}\right)}{\rho_{\text{cl}} v'_{\text{set}}} = \frac{\frac{80 \times 10^3}{3600} \left(1 - \frac{8}{100 - 70}\right)}{10^3 \times 0.5 \times 10^{-3}} = 32.5 \text{ m}^2$$

The diameter of the tank is:

$$D = \sqrt{\frac{32.5}{0.785}} = 6.4 \text{ m}$$

Example 3-9. Determine the height of a settling tank (see Fig. 3-2) if it is known that 16 hours are needed for thickening the suspension in the compression zone. The relative density of the solid phase is 2.6. The average dilution in the compression zone is solid:liquid = 1:1.5. The diameter of the tank is 10 m. The daily output of the tank is 24.2 tons of the solid phase. The liquid phase is water.

Solution. We find the relative density of the suspension in the compression zone by Eq. (3-32), dividing it by ρ :

$$\Delta_{\text{susp}} = \frac{\Delta_{\text{sol}} (m' + 1)}{\Delta_{\text{sol}} m' + 1} = \frac{2.6 (1.5 + 1)}{2.6 \times 1.5 + 1} = 1.32$$

where $m' = \text{liquid:solid} = 1.5$.

The mass concentration of the suspension in the compression zone is:

$$x = \frac{1}{1 + 1.5} = 0.4 \frac{\text{kg of solid phase}}{\text{kg of suspension}}$$

Consequently, the amount of the solid phase in 1 m³ of thickened suspension is $1320 \times 0.4 = 530 \text{ kg}$.

According to the initial conditions, during a day the amount of solid phase that settles on 1 m² is $24.2 / 0.785 \times 10^2 = 0.308 \text{ ton/m}^2 \cdot \text{day}$. Hence, during 16 hours $0.308 \times 16 / 24 = 0.205 \text{ ton}$ of the solid phase will settle per square metre of settling area.

It was found above that the suspension in the compression zone contains 0.530 ton of solid phase per cubic metre; therefore the height of this zone is:

$$h_2 = \frac{0.205}{0.530} = 0.387 \text{ m}$$

The height of the feed zone is taken from 0.45 to 0.75 m. For a dilute suspension (solid:liquid=1:10), we can take it equal to $h_1 = 0.6 \text{ m}$.

The height of the settling tank zone in which the blades rotate depends on the inclination of the blades relative to the tank bottom. We shall take it equal to 0.146 m per metre of tank radius. Hence $h_3 = 0.146 \times 5 = 0.73 \text{ m}$.

The total height of the settling tank is:

$$H = h_1 + h_2 + h_3 = 0.6 + 0.387 + 0.73 \approx 1.72 \text{ m}$$

Example 3-10. Calculate the size and the hydraulic resistance of the cyclone needed for separating particles of a dry material from the air leaving a spray drier according to the following data: the smallest size of the particles is $80 \mu\text{m}$, the rate of flow of the air is 2000 kg/h , and the temperature is 100°C .

Solution. We choose a type QH-15 cyclone (see Table 3-1) for the purpose.

Assuming that $\Delta p/\rho_g = 740$, we can find the diameter of the cyclone by Eq. (3-15):

$$D = \sqrt{\frac{Q_v}{0.785v_{\text{cyc}}}}$$

We must first determine the conditional velocity of the gas in the cylindrical part of the cyclone v_{cyc} from Eq. (3-16):

$$v_{\text{cyc}} = \sqrt{\frac{2}{\zeta_0} \frac{\Delta p}{\rho_g}}$$

From Table 3-1, we have $\zeta_0 = 160$, and

$$v_{\text{cyc}} = \sqrt{\frac{2}{160} \times 740} = 3.04 \text{ m/s}$$

The density of the air is:

$$\rho_g = 1.29 \times \frac{273}{373} = 0.95 \text{ kg/m}^3$$

$$D = \sqrt{\frac{2000}{0.95 \times 3600 \times 0.785 \times 3.04}} = 0.496 \text{ m}$$

We take the diameter of the cyclone equal to 0.5 m .

The hydraulic resistance of the cyclone, by Eq. (3-16), is

$$\Delta p_{\text{cyc}} = \zeta_0 \frac{v_{\text{cyc}}^2 \rho_g}{2} = 160 \times \frac{3.04^2 \times 0.95}{2} = 703 \text{ Pa} = 72 \text{ mm H}_2\text{O}$$

FILTRATION

Example 3-11. Derive Eq. (3-22) relating the mass of the dry cake per cubic metre of filtrate, the density of the filtrate, the concentration of the suspension, and the moisture content of the cake.

Solution. The equation of the material balance of the filtration process is:

$$M_{\text{susp}} = M_{\text{f}} + M_{\text{w, cake}}$$

where M_{susp} , M_{f} , and $M_{\text{w, cake}}$ are the masses of the suspension, the filtrate, and the wet cake, respectively.

Let us divide this equation by M_{sol} —the mass of solids contained in the suspension:

$$\frac{M_{\text{susp}}}{M_{\text{sol}}} = \frac{M_{\text{f}}}{M_{\text{sol}}} + \frac{M_{\text{w, cake}}}{M_{\text{sol}}} \quad (3-83)$$

We shall substitute the product of the volume of the filtrate and its density for its mass:

$$M_{\text{f}} = V_{\text{f}} \rho$$

We shall introduce the notations:

$$m = \frac{M_{\text{w, cake}}}{M_{\text{sol}}} \quad \frac{\text{kg of moist sediment}}{\text{kg of dry matter}}$$

$$x = \frac{M_{\text{sol}}}{M_{\text{susp}}} \quad \frac{\text{kg of dry matter}}{\text{kg of suspension}}$$

$$c = \frac{M_{\text{sol}}}{V_{\text{f}}} \quad \frac{\text{kg of dry matter}}{\text{m}^3 \text{ of filtrate}}$$

Hence, Eq. (3-83) can be written in the form:

$$\frac{1}{x} = \frac{\rho}{c} + m$$

whence

$$c = \frac{\rho x}{1 - mx}$$

Example 3-12. Determine the duration of filtering 10 dm³ of a liquid through 1 m² of a filter if in preliminary tests 1 dm³ of the filtrate was collected from 1 m² of the filter in 2.25 min and 3 dm³ in 14.5 min.

Solution. We use the experimental data to find the filtration constants in Eq. (3-17):

$$V^2 + 2VC = K\tau$$

For this purpose, we compile two equations with two unknowns:

$$1^2 + 2 \times 1C = K \times 2.25$$

$$3^2 + 2 \times 3C = K \times 14.5$$

whence $K = 0.77 \text{ dm}^6/\text{m}^4 \cdot \text{min}$ and $C = 0.37 \text{ dm}^3/\text{m}^2$.

To determine the required duration of filtering, we introduce the obtained values of the constants and the given volume of the filtrate into Eq. (3-17):

$$10^2 + 2 \times 10 \times 0.37 = 0.77\tau$$

whence $\tau = 140 \text{ min} = 2 \text{ h } 20 \text{ min}$.

Example 3-13. Determine the duration of washing the cake in the conditions of the preceding example if the quantity of wash water is $2.4 \text{ dm}^3/\text{m}^2$ and the wash water follows the same path as that of the filtrate.

Solution. Disregarding the difference between the dynamic viscosities of the filtrate and the wash water, we shall consider the rate of washing to equal the rate of filtration at the final moment.

We determine the rate of filtration at the final moment by Eq. (3-18) using the data of the preceding example:

$$\frac{dV}{d\tau} = \frac{K}{2(V+C)} = \frac{0.77}{2(10+0.37)} = 0.037 \text{ dm}^3/\text{m}^2 \cdot \text{min}$$

The duration of washing is:

$$\tau_{\text{wash}} = \frac{V_{\text{wash}}}{\left(\frac{dV}{d\tau}\right)_{\text{wash}}} = \frac{2.4}{0.037} = 65 \text{ min}$$

Example 3-14. The data given in Table 3-2 were obtained during the trial filtration of an aqueous suspension containing 13.9% of calcium carbonate at 20°C in a laboratory filter press with $A = 0.1 \text{ m}^2$ and a thickness of the cake of 50 mm. Determine the filtration constants K (in m^2/h) and C (in m^3/m^2).

TABLE 3-2

Gauge pressure		Filtrate collected, dm ³	During time from beginning of trial, s
Pa	at		
3.43×10^4	0.35	2.92	146
		7.80	888
10.3×10^4	1.05	2.45	50
		9.80	660

Solution. We find the numerical values of the filtration constants from Eq. (3-17):

$$V^2 + 2VC = K\tau$$

At a gauge pressure of $3.43 \times 10^4 \text{ Pa}$ (0.35 at), the results of the trials give:

$$V_1 = \frac{2.92}{1000 \times 0.1} = 2.92 \times 10^{-2} \text{ m}^3/\text{m}^2, \quad \tau_1 = \frac{146}{3600} = 0.0405 \text{ h}$$

$$V_2 = \frac{7.8}{1000 \times 0.1} = 7.8 \times 10^{-2} \text{ m}^3/\text{m}^2, \quad \tau_2 = \frac{888}{3600} = 0.246 \text{ h}$$

We insert the above values into Eq. (3-17) and solve the system of equations:

$$(2.92 \times 10^{-2})^2 + 2 \times 2.92 \times 10^{-2} C = K \times 0.0405$$

$$(7.8 \times 10^{-2})^2 + 2 \times 7.8 \times 10^{-2} C = K \times 0.246$$

whence

$$K = 278 \times 10^{-4} \text{ m}^2/\text{h} \text{ and } C = 4.7 \times 10^{-3} \text{ m}^3/\text{m}^2$$

In a similar way, we calculate the constants K and C for a gauge pressure of 10.3×10^4 Pa (1.05 at). We get the following values of the filtration constants:

$$K = 560 \times 10^{-4} \text{ m}^2/\text{h} \text{ and } C = 3.78 \times 10^{-3} \text{ m}^3/\text{m}^2$$

Example 3-15. Calculate the specific resistance of the calcium carbonate sediment for the conditions of the preceding example. It is additionally known that the moisture content of the cake at $p_1 = 3.43 \times 10^4$ Pa (0.35 at) was 37%, and at $p_2 = 10.3 \times 10^4$ Pa (1.05 at) it was 32% of the mass of the wet cake.

Solution. We calculate the specific resistance of the cake by Eq. (3-24).

$$r = \frac{2\Delta p (1 - mx)}{K\mu x\rho}$$

The filtration pressure $\Delta p = 3.43 \times 10^4$ Pa (0.35 at).

The density of the filtrate $\rho = 1000$ kg/m³.

The dynamic viscosity of the filtrate at 20°C is:

$$\mu = \frac{1}{3600} \times 10^{-3} = 27.8 \times 10^{-8} \text{ kg/m} \cdot \text{h}$$

The filtration constant (at a pressure $p_1 = 3.43 \times 10^4$ Pa) is $K = 278 \times 10^{-4} \text{ m}^2/\text{h}$.

The mass fraction of the solid phase in the suspension $x = 0.139$.

The mass ratio of the wet cake to the dry one is $m = 1/(1 - 0.37) = 1.59$.

The quantity of filtrate per kg of suspension is $1 - mx = 1 - 1.59 \times 0.139 = 0.779$.

We introduce the found quantities into Eq. (3-24):

$$r = \frac{2 \times 3.43 \times 10^4 \times 0.779}{278 \times 10^{-4} \times 27.8 \times 10^{-8} \times 0.139 \times 1000} = 5 \times 10^{10} \text{ m/kg of dry basis}$$

At a filtering pressure of $p_2 = 10.3 \times 10^4$ Pa (1.05 at), we have $K = 560 \times 10^{-4} \text{ m}^2/\text{h}$, $m = 1/(1 - 0.32) = 1.47$, and $1 - mx = 1 - 1.47 \times 0.139 = 0.795$. The remaining quantities have the same values as at $p_1 = 3.43 \times 10^4$ Pa (0.35 at).

We can find the new value of the specific resistance of the sediment by introducing the numerical values of these quantities

into Eq. (3-24) or from the ratio

$$\frac{r_2}{r_1} = \frac{\Delta p_2 K_1 (1 - mx)_2}{\Delta p_1 K_2 (1 - mx)_1} = \frac{10.3 \times 10^4 \times 278 \times 10^{-4} \times 0.795}{3.43 \times 10^4 \times 560 \times 10^{-4} \times 0.779} = 1.52$$

whence

$$r_2 = 5 \times 10^{10} \times 1.52 = 7.6 \times 10^{10} \text{ m/kg of dry matter}$$

Hence, when the filtering pressure is increased three-fold, the specific resistance of the calcium carbonate sediment increases one and a half times.

Example 3-16. What is the specific resistance of a cloth when filtering a suspension of calcium carbonate in the conditions of Examples 3-14 and 3-15?

Solution. We use Eq. (3-27):

$$r_f = \frac{C r_{xp}}{1 - mx}$$

When $p_1 = 3.43 \times 10^4 \text{ Pa} = 0.35 \text{ at}$, we have

$$r_f = \frac{4.7 \times 10^{-3} \times 5 \times 10^{10} \times 0.139 \times 1000}{0.779} = 4.2 \times 10^{10} \text{ m/m}^2$$

When $p_2 = 10.3 \times 10^4 \text{ Pa} = 1.05 \text{ at}$:

$$r_f = \frac{3.78 \times 10^{-3} \times 7.6 \times 10^{10} \times 0.139 \times 1000}{0.795} = 5 \times 10^{10} \text{ m/m}^2$$

A three-fold change in the pressure causes the specific resistance of the cloth to change by only 19%.

Example 3-17. It is necessary to filter a suspension in a plate-and-frame filter press and obtain 6 m^3 of the filtrate in three hours. Trial filtration of the suspension in a laboratory filter press at the same pressure and with the same thickness of the cake showed that the filtration constants related to 1 m^2 of the filter area have the following values: $K = 20.7 \times 10^{-4} \text{ m}^2/\text{h}$ and $C = 1.45 \times 10^{-3} \text{ m}^3/\text{m}^2$. Determine the required dimensions of the filter press.

Solution. We find the throughput of 1 m^2 of the filter press from Eq. (3-17):

$$V^2 + 2 \times 0.145 \times 10^{-2} V = 20.7 \times 10^{-4} \times 3$$

whence

$$\begin{aligned} V &= -0.145 \times 10^{-2} + \sqrt{(0.145 \times 10^{-2})^2 + 62.1 \times 10^{-4}} = \\ &= -0.145 \times 10^{-2} + 7.88 \times 10^{-2} = 7.73 \times 10^{-2} \text{ m}^3/\text{m}^2 \end{aligned}$$

during a filtration cycle, i.e. during 3 h.

Consequently, the filtration area needed for the given throughput is $A = 6/0.0773 = 77.5 \text{ m}^2$.

We select from a catalogue the filter press with the nearest greater area $A = 83 \text{ m}^2$ having 42 frames $1000 \times 1000 \text{ mm}$ in size.

Note. The filtration surface can also be determined directly from Eq. (3-17) compiled for an entire filter having the surface area A and the throughput $V_A \text{ m}^3$ of filtrate during a filtration cycle:

$$V_A^2 + 2V_A AC = KA^2\tau$$

$$6^2 + 2 \times 6 \times 1.45 \times 10^{-3} A = 20.7 \times 10^{-4} \times 3A^2$$

whence $A = 77.4 \text{ m}^2$.

Example 3-18. An aqueous suspension containing 17.6% of the solid phase is fed at a rate of $8.5 \text{ m}^3/\text{h}$ into a continuous rotary vacuum filter (Fig. 3-10). It is desired to have a final moisture

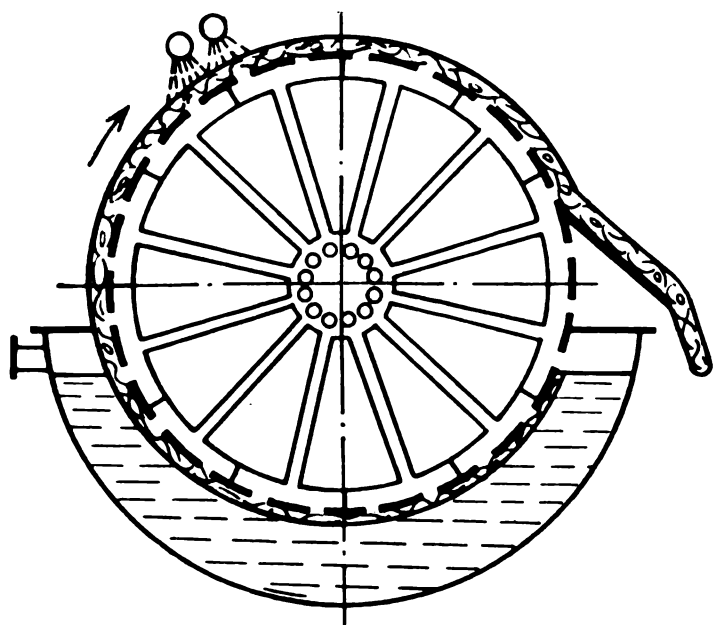


Fig. 3-10. To Example 3-18

content of the cake of 34%. The presumed vacuum at the user's plant is 600 mm Hg. Laboratory filtration conducted at a vacuum of 510 mm Hg established that the required moisture content of the cake is reached during 32 s of operation of the filtration zone. The filtration constants related to 1 m^2 were found to be $K = 11.2 \text{ dm}^6/\text{m}^4 \cdot \text{s}$ and $C = 6 \text{ dm}^3/\text{m}^2$. The density of the suspension is $1120 \text{ kg}/\text{m}^3$. Determine the required surface area of the filter and its speed of rotation.

Solution. We recalculate the filtration constant K for a vacuum of 600 mm Hg, assuming approximately that K is proportional to Δp^* :

$$\frac{K}{K'} = \frac{600}{510} = 1.17$$

whence $K = 11.2 \times 1.17 = 13.1 \text{ dm}^6/\text{m}^4 \cdot \text{s}$.

We determine the unit throughput of the filtration zone from Eq. (3-17), taking the duration of filtering τ equal to 32 s (ac-

*Actually this is not completely true—see Example 3-14.

according to the initial data):

$$V^2 + 2VC = K\tau$$

$$V^2 + 2V \times 6 = 13.1 \times 32$$

whence

$$V = -6 + \sqrt{36 + 420} = -6 + 21.4 = 15.4 \text{ dm}^3/\text{m}^2 \text{ during } 32 \text{ s}$$

Hence, the unit throughput of the filtration zone per second is $15.4/32 = 0.482 \text{ dm}^3/\text{m}^2 \cdot \text{s}$.

We reduce the given throughput with respect to the suspension to the throughput with respect to the filtrate.

With a moisture content of the cake of 34%, the mass ratio of the wet and dry cake is $m = 1/(1 - 0.34) = 1.52$.

The mass fraction of the solid phase in the suspension is $x = 0.176$; the mass rate of flow of the suspension is $M_{\text{susp}} = V_{\text{susp}} \rho_{\text{susp}} = 8.5 \times 1120 = 9500 \text{ kg/h}$; the mass of the wet cake is $M_{\text{cake}} = M_{\text{susp}} x m = 9500 \times 0.176 \times 1.52 = 2540 \text{ kg/h}$; the mass of the filtrate is $M_f = M_{\text{susp}} - M_{\text{cake}} = 9500 - 2540 = 6960 \text{ kg/h}$. At a filtrate density of $\rho = 1000 \text{ kg/m}^3$, the volumetric rate of flow of the filtrate is $6960 \text{ dm}^3/\text{h}$.

Thus, the given throughput with respect to the filtrate is $6960/3600 = 1.93 \text{ dm}^3/\text{s}$.

Therefore, the required surface area in the filtration zone is $A_f = 1.93/0.482 = 4.0 \text{ m}^2$.

Since the surface of the filtration zone in rotary vacuum filters is usually about 35% of the total surface, then the total surface area of the filter needed is $A = 4.0/0.35 = 11.43 \text{ m}^2$.

From a catalogue, we choose the rotary vacuum filter with the closest area of $A = 12 \text{ m}^2$. The diameter of the filter is $D = 2.1 \text{ m}$, its length is $L = 1.8 \text{ m}$.

The number of revolutions of the filter a minute needed to ensure the given duration of filtering $\tau = 32 \text{ s}$ is found from the proportion $32:0.35 = 60:n$, whence

$$n = \frac{60 \times 0.35}{32} = 0.655 \text{ rpm}$$

Example 3-19. How long is it necessary to wash the cake in a filter press of NaCl to reach a concentration of 5 g/dm^3 permitted in the wash water? Pure water is used for washing. The intensity of washing is $0.33 \text{ m}^3/\text{m}^2 \cdot \text{h}$. The thickness of the cake is 35 mm . The washing constant is $K = 520 \text{ cm}^3/\text{dm}^3$. At the initial moment of washing, the concentration of the NaCl in the wash water is 143 g/dm^3 .

Solution. We use Eq. (3-29), introducing into it the given quantities in the corresponding units: $\delta = 0.035 \text{ m}$, $v = 0.33 \text{ m}^3/\text{m}^2 \cdot \text{h}$,

and $K = 520 \times 0.001 \text{ m}^3/\text{m}^3$. Hence,

$$\tau = \frac{2.3 \times 0.035}{520 \times 0.001 \times 0.33} (\log 143 - \log 5) = 0.683 \text{ h} = 41 \text{ min}$$

Example 3-20. Calculate the concentration of NaCl in the wash water in 50 min after the beginning of the main washing period for the conditions of the preceding example.

Solution. We find the concentration of the salt in the wash water by Eq. (3-28):

$$c_2 = 143 \exp \left(- \frac{520 \times 0.001 \times 0.33 \times 50}{0.035 \times 60} \right) = 143e^{-4.08}$$

Taking logarithms of this expression, we get

$$\log c_2 = \log 143 - \frac{4.08}{2.3} = 2.156 - 1.75 = 0.406$$

Consequently, $c_2 = 2.53 \text{ g/dm}^3$.

CENTRIFUGAL FILTRATION

Example 3-21. Determine the technological type and plan the design of a centrifuge for the separation of potash from the mother liquor on the basis of the following data: the concentration of the solid matter in the suspension is 75%, a final cake moisture content of 3% is needed (crystalline), washing is performed with water, bleaching with steam, both the mother liquor and the cake have an alkaline reaction.

The separation process should be mechanized because the centrifuge must ensure a high output.

Solution. Centrifuges of the technological type 1-a are suitable for separating a liquid-solid system with coarse suspensions.

Since the process must be mechanized, automatic short-cycle batch centrifuges (class B) and continuous centrifuges (class C) should be used. With a view to the necessity of washing the sediment with water and bleaching with steam, we choose automatic short-cycle batch centrifuges. Mechanization of the process is ensured by automation of control. Class B includes two design groups of centrifuges — horizontal and vertical. Horizontal automatic short-cycle batch centrifuges are in the greatest favour, therefore we choose a centrifuge of the horizontal type.

Example 3-22. A batch centrifuge has a basket with an internal diameter of 1200 mm, a height of 550 mm, a wall thickness of 10 mm and a mass of 120 kg. The number of perforations in the basket wall in a vertical direction is 12, their diameter is 5 mm. Three steel hoops each with a section of $15 \times 30 \text{ mm}^2$ are fitted onto the basket. The material of the basket is steel with an ultimate tensile strength of 4500 kgf/cm^2 . The mass of a batch is 400 kg,

the thickness of the cake is 200 mm. Find the maximum permissible speed of rotation of the centrifuge if the safety factor should be at least five.

Solution. For the given grade of steel, the permissible ultimate tensile stress is

$$\sigma_u = \frac{4500 \times 9.81 \times 10^4}{5} = 8.83 \times 10^7 \text{ Pa} = 900 \text{ kgf/cm}^2$$

The cross-sectional area of the basket wall and hoops minus the perforations is:

$$A = 55 \times 1 - 12 \times 1 \times 0.5 + 3 \times 3 \times 1.5 = 62.5 \text{ cm}^2 = 6.25 \times 10^{-3} \text{ m}^2$$

It follows from Eq. (3-47) that the maximum permissible centrifugal force is

$$F_{c,1} + F_{c,2} = 8.83 \times 10^7 \times 2 \times 6.25 \times 10^{-3} = 1.1 \times 10^6 \text{ N}$$

The distance from the centre of gravity of the half-ring of the basket wall to the axis of rotation is calculated by Eq. (3-48)*:

$$R_b = \frac{4}{3 \times 3.14} \left(\frac{0.61^3 - 0.6^3}{0.61^2 - 0.6^2} \right) = 0.387 \text{ m}$$

The centrifugal force developed by half of the basket, according to Eq. (3-34), is

$$F_{c,1} = 0.011 \times 60 \times 0.387 n^2 = 0.253 n^2 \text{ N}$$

The distance from the centre of gravity of the charging half-ring to the axis of rotation is:

$$R_{ch} = \frac{4}{3 \times 3.14} \left(\frac{0.6^3 - 0.4^3}{0.6^2 - 0.4^2} \right) = 0.32 \text{ m}$$

The centrifugal force developed by the charging half-ring is:

$$F_{c,2} = 0.011 \times 200 \times 0.32 n^2 = 0.697 n^2 \text{ N}$$

The total centrifugal force is:

$$F_{c,1} + F_{c,2} = 0.253 n^2 + 0.697 n^2 = 0.950 n^2 \text{ N}$$

It was found above that the centrifugal force must not exceed $1.1 \times 10^6 \text{ N}$. Therefore, the maximum speed of the centrifuge is:

$$n = \sqrt{\frac{1.1 \times 10^6}{0.950}} = 1070 \text{ rpm} = 17.8 \text{ rps}$$

Example 3-23. Determine the hourly capacity (relative to the suspension fed in) of an automatic bowl centrifuge AOF-800 when separating an aqueous suspension of magnesium hydroxide. The density of the particles is $\rho = 2525 \text{ kg/m}^3$. Their minimum diameter is $3 \mu\text{m}$. The temperature of the suspension is 30°C . The centrifuge has the following parameters: bowl diameter = 800 mm,

* A slide rule should not be used for calculations here.

bowl length = 400 mm, rim diameter = 570 mm, speed = 1200 rpm. The centrifuge operating cycle is 20 minutes, during 18 minutes the suspension is fed in and during 2 minutes the cake is unloaded.

Solution. We determine the capacity by Eq. (3-50):

$$Q_v = 25.3 \eta L n^2 R_0^2 v_{\text{set}} k$$

We find the rate of settling of the particles by the Stokes formula (3-1):

$$v_{\text{set}} = \frac{d^2 (\rho - \rho_{\text{fl}}) g}{18 \mu_{\text{fl}}} = \frac{3^2 (2525 - 1000) 9.81}{10^{12} \times 18 \times 0.8 \times 10^{-3}} = 0.935 \times 10^{-5} \text{ m/s}$$

The dynamic viscosity of water at 30 °C is $\mu_{\text{fl}} = 0.8 \times 10^{-3} \text{ Pa} \cdot \text{s}$.

We determine the velocity of settling under the action of the centrifugal force:

$$v = v_{\text{set}} \frac{R_0 n^2}{900} = 0.935 \times 10^{-5} \times \frac{0.285 \times 1200^2}{900} = 4.26 \times 10^{-3} \text{ m/s}$$

We check the settling conditions:

$$Re = \frac{v d \rho_{\text{fl}}}{\mu_{\text{fl}}} = \frac{4.26 \times 10^{-3} \times 3 \times 10^{-6} \times 10^3}{0.8 \times 10^{-3}} = 1.6 \times 10^{-2}$$

i.e. the conditions are laminar.

We further find $k = 18/20 = 0.9$.

The capacity of the centrifuge, assuming that the coefficient $\eta = 0.45$, is

$$Q_v = 25.3 \times 0.45 \times 0.4 \times 1200^2 \times 0.285^2 \times 0.935 \times 10^{-5} \times 0.9 = 4.46 \text{ m}^3/\text{h}$$

Example 3-24. Determine the capacity that can be ensured by a tubular supercentrifuge CFO-150 with a three-blade impeller used to clarify mineral oil. The density of the oil is $\rho = 900 \text{ kg/m}^3$. The dynamic viscosity of the oil at the centrifuging temperature is $3 \times 10^{-3} \text{ Pa} \cdot \text{s}$. The density of the solid particles is $\rho = 1400 \text{ kg/m}^3$. Their diameter is $1 \text{ } \mu\text{m}$. The technical parameters of the centrifuge are: internal diameter of the bowl = 150 mm, diameter of the decanting (overflow) weir = 50 mm, length of the bowl = 750 mm, speed of rotation = 13 000 rpm.

Solution. We determine the capacity by Eq. (3-52).

Since the particles are very small, their settling conditions will probably be laminar. We use the Stokes formula (3-1) with subsequent checking of the settling conditions:

$$v_{\text{set}} = \frac{d^2 (\rho - \rho_{\text{fl}}) g}{18 \mu_{\text{fl}}} = \frac{1^2 (1400 - 900) 9.81}{10^{12} \times 18 \times 3 \times 10^{-3}} = 9.06 \times 10^{-8} \text{ m/s}$$

The velocity of settling of the particles under the action of the centrifugal force is

$$v = v_{\text{set}} f = 9.06 \times 10^{-8} \times 4700 = 4.26 \times 10^{-4} \text{ m/s}$$

Here

$$f = \frac{n^2 R_0}{900} = \frac{13^2 \times 10^6 \times 0.025}{900} = 4700$$

where $R_0 = 0.025$ m is the radius of the decanting weir.

We check the settling conditions:

$$Re = \frac{v d \rho_{\Pi}}{\mu_{\Pi}} = \frac{4.26 \times 1 \times 900}{10^4 \times 10^6 \times 3 \times 10^{-3}} = \frac{1.42}{10^4}$$

We find the useful volume of the centrifuge bowl:

$$V_{1q} = AL = 0.0157 \times 0.75 = 0.0118 \text{ m}^3$$

$$\text{where } A = \frac{\pi}{4} (D^2 - D_0^2) = 0.785 (0.15^2 - 0.05^2) = 0.0157 \text{ m}^2$$

The depth of the flow in the bowl is:

$$h = \frac{D - D_0}{2} = \frac{0.15 - 0.05}{2} = 0.05 \text{ m}$$

Hence,

$$\begin{aligned} Q_v &\leq \frac{v V_{1q}}{h} \times 3600 = \frac{4.26 \times 0.0118}{10^4 \times 0.05} \times 3600 = 0.368 \text{ m}^3/\text{h} = \\ &= \frac{0.368}{3600} = 1.05 \times 10^{-4} \text{ m}^3/\text{s} \end{aligned}$$

We check the flow conditions in the centrifuge bowl:

$$Re_{\text{flow}} = \frac{v_{\text{flow}} d_{\text{eq}} \rho}{\mu}$$

We find the values of the quantities in the above equation:

$$v_{\text{flow}} = \frac{Q_v}{A} = \frac{1.05 \times 10^{-4}}{0.0157} = 0.0067 \text{ m/s}$$

For a centrifuge with a three-blade impeller:

$$d_{\text{eq}} = \frac{4A}{P} = \frac{4\pi (D^2 - D_0^2)}{4(\pi D + 6h)} = \frac{\pi (D^2 - D_0^2)}{\pi D + 6h} = \frac{3.14 (0.15^2 - 0.05^2)}{3.14 \times 0.15 + 6 \times 0.05} = 0.0815 \text{ m}$$

Thus,

$$Re_{\text{flow}} = \frac{0.0067 \times 0.0815 \times 900}{3 \times 10^{-3}} = 164 < 350$$

i.e. the flow is laminar.

Example 3-25. Select the type of filtering apparatus for silicic acid gel and determine the number of apparatuses needed on the basis of the following data:

1. The initial concentration of the solid phase in the suspension is 4.7%, the relative density of the suspension is 1.1.
2. The daily assignment is 9 tons of wet cake.
3. The cake must have the lowest possible moisture content because it is subsequently dried (in a vacuum drier).

4. The cake is amorphous and filters poorly.

Preliminary trials involving the filtration of the gel in a vacuum (nutsch) filter and in a centrifuge produced the results given in Table 3-5.

TABLE 3-5

Parameter	Vacuum filter	Centrifuge
Filtration area, m ²	1	0.3
Cake thickness, mm	60	30
Duration of:		
filtration	1 h	45 min
washing with alkali and water	2 h	33 min
total	3 h	1.3 h
Suspension filtered, dm ³	170	108
Residual moisture content of cake, %	85	78
Vacuum, mm Hg	400	—

The parameters of the centrifuge basket are $D = 400$ mm, $H = 250$ mm, and $n = 800$ rpm.

Solution. 1. We determine the filtration pressure.

Vacuum filter

$$\Delta p = \frac{400}{760} = 0.525 \text{ at} = 5.15 \times 10^4 \text{ Pa}$$

Centrifuge. With a centrifuge basket filling factor in operation of $\varphi = 0.5$, we have a volume of the suspension (the working volume of the drum) equal to:

$$V_{\text{susp}} = 0.785 D^2 H \varphi = 0.785 \times 0.4^2 \times 0.25 \times 0.5 = 0.0157 \text{ m}^3$$

We determine the filtration pressure in centrifuging by Eq. (3-35):

$$\Delta p = \frac{\pi^2 M R n^2}{900 A} = \frac{3.14^2 \times 0.0157 \times 1100 \times 0.2 \times 800^2}{900 \times 0.3} = 8.1 \times 10^4 \text{ Pa} = 0.83 \text{ at}$$

Consequently, the pressure in centrifuging is 1.5 times greater than in filtration through a vacuum filter.

2. Determination of the unit capacity.

Vacuum filter. The amount of suspension filtered is 170 dm³ with an initial concentration of 4.7%, i.e. the amount of gel collected reduced to the dry matter is:

$$M = 0.170 \times 1100 \times 0.047 = 8.8 \text{ kg}$$

The operating cycle is 3 hours (the unloading time is excluded), consequently the unit capacity is:

$$\frac{M}{A \tau} = \frac{8.8}{3 \times 1} = 2.93 \text{ kg of dry gel/m}^2 \cdot \text{h}$$

Calculating for the moist gel (at a moisture content of 85%), we have $2.93/0.15 = 19.5$ kg of moist gel/m²·h.

Centrifuge. The amount of suspension filtered is 108 dm³, therefore the mass of the cake reduced to dry matter is:

$$M = 0.108 \times 1100 \times 0.047 = 5.54 \text{ kg}$$

The unit capacity is

$$\frac{M}{A\tau} = \frac{5.54}{0.3 \times 1.3} = 14.3 \text{ kg of dry gel/m}^2 \cdot \text{h.}$$

Calculating for the moist gel (at a moisture content of 78%), we have $14.3/0.22 = 65$ kg of moist gel/m²·h.

3. We shall compare how much more water has to be removed in drying the moist gel from the vacuum filter than in drying that from the centrifuge.

The moisture content of the cake in the vacuum filter is 85%, and in the centrifuge 78%, therefore the centrifuge removes more water than the vacuum filter. This amount is

$$\frac{1000}{0.15} - \frac{1000}{0.22} = 2120 \frac{\text{kg of water}}{\text{ton of dry gel}}$$

Upon comparing the experimental and calculated data with respect to the filtration of the gel, we can state the following.

The centrifuge, operating with a filtration pressure $0.83/0.525 = 1.6$ times greater than that of the vacuum filter and with a cake thickness half that of the vacuum filter, has a capacity $14.3/2.94 \approx 5$ times greater than that of the vacuum filter (calculating relative to the dry gel; when reduced to the wet cake the capacity of the centrifuge is $65/19.5 = 3.3$ times greater).

The centrifuge in comparison with the vacuum filter ensures a lower residual moisture content and thus facilitates operation of the drier, relieving it of the need to evaporate 2.12 tons of water (counting per ton of dry gel).

With a daily output of 9 tons of gel having a moisture content of 78% and a specific consumption of steam for drying equal to 1.5 kg per kg of evaporated moisture, the saving in steam a day will be $M_{st} = 9 \times 0.22 \times 2.12 \times 1.5 = 6.3$ tons.

The greater unit capacity of the centrifuge, the lower residual moisture content of the cake and, therefore, the appreciable saving of steam in drying the gel result in preference being given to the centrifuge in the given case, notwithstanding its higher cost.

4. We determine approximately the number of industrial centrifuges needed to cope with the daily assignment (nine tons of moist gel).

With such a comparatively large assignment, it is good to choose an automatic short-cycle batch centrifuge with mechanical unloading of the sediment. We select a horizontal centrifuge with

the following characteristic from a catalogue: $D = 1600$ mm, $H = 700$ mm, $n = 500$ rpm, the effective volume $= 500$ dm³.

The filtration surface of this centrifuge is $A = 3.14 \times 1.6 \times 0.7 = 3.5$ m².

The filtration pressure developed, by Eq. (3-35), is:

$$\Delta p = \frac{0.011 \times 0.5 \times 1100 \times 0.8 \times 500^2}{3.5} = 34.6 \times 10^4 \text{ Pa} = 3.5 \text{ at}$$

Thus, the industrial centrifuge develops a filtering pressure $3.5/0.83 = 4$ times greater than that developed by the experimental one.

The capacity of filters is approximately proportional to $\Delta p^{1/2}$, consequently we can expect an increase in the unit capacity of the centrifuge of almost two times, i. e. up to $65 \times 2 = 130$ kg of moist gel/m²·h.

Since filtration and washing of the cake take about 75% of the total operating time of an automatic short-cycle batch centrifuge, its average hourly capacity is $130 \times 0.75 = 97$ kg/m²·h.

Therefore, the daily capacity of one centrifuge, considering that it is in operation 20 hours a day, is $97 \times 3.5 \times 20 = 6800$ kg of wet cake.

To cope with the entire assigned daily output, it is necessary to install $9000/6800 = 1.32$, i. e. two centrifuges, and with a standby unit — three.

FLUIDIZED BED

Example 3-26. An apparatus contains a fluidized bed of silica gel having the following composition by fractions (the minus sign shows oversize particles):

Fraction, mm	—2.0 + 1.5	—1.5 + 1.0	—1.0 + 0.5	—0.5 + 0.25
Content, % (by mass)	43	28	17	12

The bulk density of the silica gel is $\rho_b = 650$ kg/m³, the particle density is $\rho = 1100$ kg/m³. The temperature of the air is 150 °C. The fluidization number $K_v = 1.6$.

Determine the critical, working and actual (in the clear section between the particles) velocity of the air.

Solution. We calculate the value of the Archimedes number Ar and find the corresponding value of Ly_{cr} from the graph in Fig. 3-8.

For this purpose, we determine the equivalent diameter of the silica gel particles.

The average screen diameters of the fractions are:

$$d_1 = \frac{2.0 + 1.5}{2} = 1.75 \text{ mm}$$

$$d_2 = \frac{1.5 + 1.0}{2} = 1.25 \text{ mm}$$

$$d_3 = \frac{1.0 + 0.5}{2} = 0.75 \text{ mm}$$

$$d_4 = \frac{0.5 + 0.25}{2} = 0.37 \text{ mm}$$

Hence, the equivalent diameter by Eq. (3.67) is:

$$d_{eq} = \frac{1}{\sum \frac{x_i}{d_i}} = \frac{1}{\frac{0.43}{1.75} + \frac{0.28}{1.25} + \frac{0.17}{0.75} + \frac{0.12}{0.37}} \approx 1.0 \text{ mm}$$

The dynamic viscosity of the air at 150 °C (see Fig. A-6) is $\mu = 0.024 \text{ cP} = 0.024 \times 10^{-3} \text{ Pa} \cdot \text{s}$.

The density of the air is

$$\rho = 1.293 \times \frac{273}{273 + 150} = 0.835 \text{ kg/m}^3$$

Hence,

$$Ar = \frac{d_{eq}^3 \rho \rho_f g}{\mu_f^2} = \frac{1^3 \times 10^{-9} \times 1.1 \times 10^3 \times 0.835 \times 9.81}{2.4^2 \times 10^{-10}} = 1.565 \times 10^4$$

From Fig. 3-8 for $Ar = 1.565 \times 10^4$, we find $Ly_{cr} = 3 \times 10^{-2}$. Thus

$$v_{cr} = \sqrt[3]{\frac{Ly_{cr} \mu_f \rho g}{\rho_f^2}} = \sqrt[3]{\frac{3 \times 10^{-2} \times 0.024 \times 10^{-3} \times 1.1 \times 10^3 \times 9.81}{0.835^2}} = 0.224 \text{ m/s}$$

We determine the working velocity of the air by Eq. (3-70):

$$v = K_v v_{cr} = 1.6 \times 0.224 = 0.358 \text{ m/s}$$

We find the porosity of the fluidized bed. For $K_v = 1.6$, we have:

$$Ly = K_v^3 Ly_{cr} = 1.6^3 \times 3 \times 10^{-2} = 1.23 \times 10^{-1}$$

From Fig. 3-8 for $Ly = 1.23 \times 10^{-1}$ and $Ar = 1.565 \times 10^4$, we find $\varepsilon = 0.47$.

The actual velocity of the air in the clear section of the bed, by Eq. (3.71), is:

$$v_a = \frac{v}{\varepsilon} = \frac{0.358}{0.47} = 0.762 \text{ m/s}$$

Example 3-27. Determine the dimensions and the hydraulic resistance of the apparatus according to the data of the preceding example. The output is 2.5 tons of silica gel an hour with an average residence time of it in the apparatus of $\tau_0 = 10 \text{ min}$. The rate of flow of the air in operating conditions is 4300 m³/h. The

clear section of the grid is 0.015 of the total grid area, the diameter of a grid opening is 0.8 mm, the grid thickness is 2 mm.

Solution. We determine the diameter of the apparatus. The rate of flow of the air per second is $Q_v = 4300/3600 = 1.195 \text{ m}^3/\text{s}$. The cross-sectional area of the apparatus is $A = Q_v/v = 1.195/0.358 = 3.34 \text{ m}^2$. Hence, the diameter of the apparatus is

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 3.34}{3.14}} = 2.06 \text{ m}$$

We find the height of the bed in the apparatus. The mass of the silica gel in it, from Eq. (3-74), is:

$$M = Q_m \tau_0 = \frac{2500 \times 10}{60} = 417 \text{ kg}$$

The volume of the static bed of silica gel is:

$$V_{\text{st}} = \frac{M}{\rho_b} = \frac{417}{650} = 0.642 \text{ m}^3$$

The height of the static bed is:

$$h_0 = \frac{V_{\text{st}}}{A} = \frac{0.642}{3.34} = 0.192 \text{ m}$$

The porosity of the static bed by Eq. (3-54) is:

$$\epsilon_0 = 1 - \frac{\rho_b}{\rho} = 1 - \frac{650}{1100} = 0.41$$

The height of the fluidized bed at $K_v = 1.6$, by Eq. (3-68), is:

$$h = \frac{1 - \epsilon_0}{1 - \epsilon} h_0 = \frac{1 - 0.41}{1 - 0.47} \times 0.192 = 0.214 \text{ m}$$

We calculate the hydraulic resistance of the apparatus. The resistance of the bed, by Eq. (3-58), is:

$$\Delta p_{\text{bed}} = \rho g (1 - \epsilon_0) h_0 = 1100 \times 9.81 (1 - 0.41) 0.192 = 1200 \text{ Pa}$$

The velocity of the air in the grid openings when the clear section is $\varphi = 0.015$ of the total grid section is:

$$v_0 = \frac{v}{\varphi} = \frac{0.358}{0.015} = 23.9 \text{ m/s}$$

The diameter of a grid opening is $d_o = 0.0008 \text{ m}$, the thickness of the grid is $\delta = 0.002 \text{ m}$. At $d_o/\delta = 0.4$, by Fig. 3-7 we find $\zeta = 0.63$.

The resistance (pressure drop) of the grid, by Eq. (3-59), is:

$$\Delta p_{\text{gr}} = \frac{0.503 \times 23.9^2 \times 0.835 (1 - 0.015^2)}{0.63^2} = 605 \text{ Pa}$$

The resistance of the apparatus is:

$$\Delta p = \Delta p_{\text{bed}} + \Delta p_{\text{gr}} = 1200 + 605 = 1805 \text{ Pa} = 184 \text{ mm H}_2\text{O}$$

Example 3-28. Determine the diameter of spherical quartz sand particles having a density of 2640 kg/m^3 that begin to pass into the fluidized state at a velocity of the air of 1 m/s and a temperature of 20°C .

Solution. We determine the diameter of the particles from the number Ar after first finding the value of the number Ly_{cr} according to Eq. (3-6):

$$Ly_{cr} = \frac{v_{cr}^3 \rho_{fl}^2}{\mu_{fl} g \rho} = \frac{1^3 \times 1.205^2}{0.018 \times 9.81 \times 2640 \times 10^{-3}} = 3.14$$

where $\rho_{fl} = 1.29 \times \frac{273}{293} = 1.205 \text{ kg/m}^3$ and $\mu_{fl} = 0.018 \times 10^{-3} \text{ Pa}\cdot\text{s}$.

For $Ly_{cr} = 3.14$, from Fig. 3-8, we have $Ar = 9 \times 10^5$.

From Eq. (3-4), the required diameter of the sand particles is

$$d = \sqrt[3]{\frac{Ar \mu_{fl}^2}{\rho \rho_{fl} g}} = \sqrt[3]{\frac{9 \times 10^5 \times 1.8^2 \times 10^{-4} \times 10^{-6}}{2640 \times 1.205 \times 9.81}} = 0.0021 \text{ m} = 2.1 \text{ mm}$$

Example 3-29. A fluidized bed contains 1000 kg of solid material. The rate of flow of the material through the bed (the rate of feeding and discharging the particles) is 4000 kg/h .

Determine: (1) what fraction of the particles will be in the bed during a time greater than the average residence time of the material in the bed; (2) how many of such fluidized beds must be connected sequentially for the fraction of the particles in the apparatus during a time less than the average residence time of the material in one bed not to exceed 10% .

Solution. We calculate the average residence time of the material in one bed. At $M = 1000 \text{ kg}$ and $Q_m = 4000 \text{ kg/h}$, by Eq. (3-74) we get

$$\tau_0 = \frac{M}{Q_m} = \frac{1000}{4000} = 0.25 \text{ h} = 900 \text{ s}$$

We find the fraction of the material in one fluidized bed during the time $\tau \geq \tau_0$ by Eq. (3-75):

$$x_1 = e^{-\tau/\tau_0} = e^{-\frac{900}{900}} = e^{-1} = 0.368$$

Thus, only 37% of the material is in the bed more than 15 min , and, consequently, the fraction of the material in the bed less than 15 min is

$$1 - x_1 = 0.63, \text{ i. e. } 63\%$$

To find the number of fluidized beds that must be connected successively, we use Eq. (3-76) to determine the fraction of the material whose residence time in the beds is less than τ_0 for an apparatus with two, three, etc. beds.

(a) For a two-bed apparatus:

$$x_2 = \left(1 + \frac{\tau}{\tau_0}\right) e^{-\tau/\tau_0} = \left(1 + \frac{900}{900}\right) e^{-\frac{900}{900}} = 2e^{-1} = \frac{2}{2.718} = 0.736$$

$$1 - x_2 = 0.264$$

Consequently, in an apparatus with two fluidized beds connected in succession, 26% of the material will be in the beds less than 15 min.

(b) For a three-bed apparatus:

$$x_3 = \left[1 + \frac{\tau}{\tau_0} + \frac{1}{2!} \left(\frac{\tau}{\tau_0}\right)^2\right] e^{-\tau/\tau_0} =$$

$$= \left[1 + \frac{900}{900} + \frac{1}{2!} \left(\frac{900}{900}\right)^2\right] e^{-\frac{900}{900}} = \frac{2.5}{2.718} = 0.92$$

$$1 - x_3 = 0.08$$

Thus, in an apparatus with three beds, only 8% of the material will be in the beds less than 15 min. Therefore, a three-bed apparatus meets the requirements indicated above. The average residence time of the material in a three-bed apparatus is

$$\tau_{av} = \frac{3M}{Q_m} = \frac{3000}{4000} = 0.75 \text{ h} = 45 \text{ min}$$

Let us compare this three-bed apparatus with a single-bed one in which the quantity of material in the bed is the same as in all the beds of the three-bed one (3000 kg). The average residence time of the solid material in such a single-bed apparatus will also be

$$\tau_{av} = \frac{3000}{4000} = 0.75 \text{ h} = 45 \text{ min}$$

But the fraction of the material that is in a bed less than 15 min will now be greater:

$$1 - e^{-\tau/\tau_0} = 1 - e^{-\frac{15}{45}} = 0.285, \text{ i. e. } 28.5\%$$

Hence, we can see an advantage of a sectional (three-bed) apparatus.

AGITATION IN A LIQUID MEDIUM

Example 3-30. A mixture of acids (with a density of 1600 kg/m^3 and a dynamic viscosity of $2 \times 10^{-2} \text{ Pa}\cdot\text{s}$) is prepared in an apparatus without baffles (diameter = 1200 mm, height = 1500 mm) filled up to 0.75 of its volume. The initial acids are mixed using a propeller agitator with a speed of 3.5 rps (Fig. 3-11). Determine the installed capacity of the motor needed.

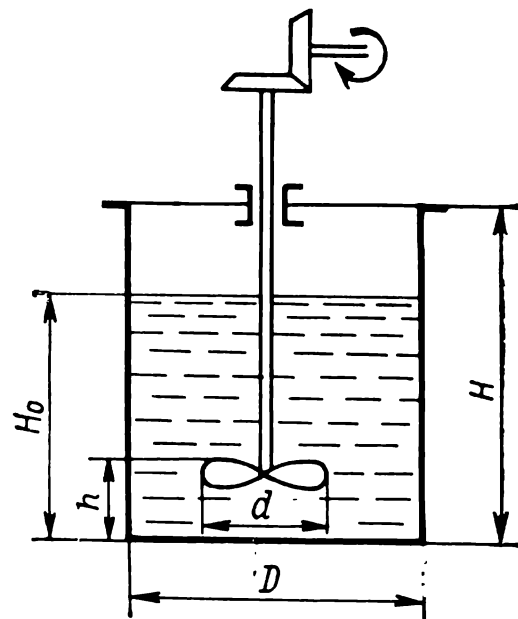
Solution. We find the diameter of an agitator in a standard catalogue:

$$d = \frac{D}{3} = \frac{1.2}{3} = 0.4 \text{ m}$$

We determine the conditions of agitation by Eq. (3-77):

$$Re_c = \frac{\rho n d^2}{\mu} = \frac{1600 \times 3.5 \times 0.4^2}{20 \times 10^{-3}} = 44\,800$$

Fig. 3-11. To Example 3-30



The conditions are turbulent.

We find the value of the power number from the graph in Fig. A-7: $N_p = 0.27$. We calculate the power used by the agitator in steady-state conditions by Eq. (3-78):

$$P_{\text{steady}} = N_p \rho n^3 d^5 = 0.27 \times 1600 \times 3.5^3 \times 0.4^5 = 200 \text{ W} = 0.2 \text{ kW}$$

The power at the moment of starting generally exceeds the operating power two or three times:

$$P_{\text{start}} = 2P_{\text{steady}} = 0.4 \text{ kW}$$

We determine the installed capacity, assuming the efficiency of the motor with the transmission to be 0.95 and taking a reserve of power of 20%:

$$P_{\text{inst}} = \frac{0.4 \times 1.2}{0.95} \approx 0.5 \text{ kW}$$

Example 3-31. A 16.5-kW motor with a transmission reducing the speed to 240 rpm drives an open turbine agitator with six blades; the agitator intensively mixes the reaction mass ($\rho = 1200 \text{ kg/m}^3$ and $\mu = 1.6 \text{ Pa}\cdot\text{s}$) in a vessel with a diameter of 1630 mm with baffles. What should the diameter of the agitator be?

Solution. Since intensive agitation is indicated, we may consider the conditions to be developed turbulent ones. By Eq. (3-77),

we have:

$$Re_c = \frac{\rho n d_{ag}^2}{\mu} = \frac{1200 \times \frac{240}{60} \times 1.63^2}{1.6} = 7980$$

In Fig. A-7, we find $N_p = 6.9$. By Eq. (3-78):

$$d = \sqrt[5]{\frac{P}{N_p \rho n^3}} = \sqrt[5]{\frac{16.5 \times 10^3}{6.9 \times 1.2 \times 10^3 \times 4^3}} = 0.5 \text{ m}$$

We check the ratio D/d (the shape factor S_D):

$$\frac{D}{d} = \frac{1.63}{0.5} = 3.26$$

Example 3-32. A theoretical analysis and the results of experiments show that the power (P) used for agitating a liquid depends on the dynamic viscosity (μ) and the density (ρ) of the liquid, the acceleration due to gravity (g), the speed of rotation of the agitator (n), its diameter (d_{ag}) and other geometrical characteristics (the diameter of the apparatus, the height of the liquid in it, the height of the agitator from the bottom of the vessel). Find the general form of the dimensionless relationship between the above variables.

Solution. For geometrically similar systems, the functional relationship between the variables are written as follows:

$$P = f(\mu, \rho, g, n, d_{ag}) \quad (a)$$

In accordance with the pi-theorem, the required dimensionless relationship should have the form

$$\varphi(\pi_1, \pi_2, \pi_3) = 0$$

or

$$\pi_1 = f(\pi_2, \pi_3)$$

where π_1, π_2, π_3 are dimensionless expressions (dimensionless numbers) that are meanwhile unknown.

Assuming as usual that the relationship between the variables within a certain range of their change can be described with sufficient accuracy by an equation of the power monomial type, we shall rewrite the initial relationship (a) as follows:

$$[P] = C [\mu]^a [\rho]^b [g]^c [n]^e [d_{ag}]^f$$

Further, introducing the units of the relevant quantities, we get an equation of the relationship between the basic units:

$$\left[\frac{\text{kg} \cdot \text{m}^2}{\text{s}^3} \right] = C \left[\frac{\text{kg}}{\text{m} \cdot \text{s}} \right]^a \left[\frac{\text{kg}}{\text{m}^3} \right]^b \left[\frac{\text{m}}{\text{s}^2} \right]^c \left[\frac{1}{\text{s}} \right]^e [\text{m}]^f \quad (c)$$

By next comparing the exponents of identical units, we arrive at a system of three equations containing five unknowns:

$$\left. \begin{aligned} a + b &= 1 \\ a + 2c + e &= 3 \\ a + 3b - c - f &= -2 \end{aligned} \right\} \quad (d)$$

The indeterminate system of equations (d) obtained can be solved relative to any three quantities assuming the other two quantities to be fixed. The total number of variants of the solutions is determined by the number of combinations of all the unknowns in the system taken the number of selected fixed quantities at a time, i.e. in the given case the number of combinations of five quantities taken two at a time:

$${}_5C_2 = \frac{5 \times 4}{1 \times 2} = 10$$

These combinations are ab , ac , ae , af , bc , be , bf , ce , cf , and ef .

An elementary analysis of the system of equations (d) permits us to establish that when a and b are selected as the fixed quantities, it is impossible to solve the system of equations*, and the number of variants of solutions diminishes to nine.

Let us solve the simultaneous equations (d), considering the quantities a and c (the exponents of μ and g) to be fixed:

$$\begin{aligned} b &= 1 - a \\ e &= 3 - a - 2c \\ f &= 5 - 2a - c \end{aligned}$$

We rewrite Eq. (b) using the found values of b , e , and f :

$$P = C\mu^a \rho^{1-a} g^c n^{3-a-2c} d_{ag}^{5-2a-c}$$

Grouping the quantities with identical letter exponents, we get:

$$\frac{P}{\rho n^3 d_{ag}^5} = C \left(\frac{\rho n d_{ag}^2}{\mu} \right)^{-a} \left(\frac{n^2 d_{ag}}{g} \right)^{-c}$$

or

$$N_p = C Re_c^{-a} Fr_c^{-c}$$

i.e. we get a system of dimensionless variables in which the physical characteristics μ and g are contained only once and each only in one dimensionless number.

* It should be stressed that the impossibility of solving the system (d) when a and b are selected as the fixed quantities is not accidental, but reflects a physical feature of real liquids—the viscosity and the density are properties that cannot be changed independently of each other.

Similarly, assuming that the fixed quantities are:

$$b \text{ and } c, \text{ we get } N_p Re_c = C Re_c^b Fr_c^{-c}$$

$$a \text{ and } e, \text{ we get } N_p^2 Fr_c^3 = C^2 Ga^{-a} Fr_c^e$$

$$b \text{ and } e, \text{ we get } N_p Re_c Fr_c = C Ga^{b/2} Fr_c^{e/2}$$

$$c \text{ and } e, \text{ we get } N_p Re_c^3 = C Ga^c Re_c^e$$

$$a \text{ and } f, \text{ we get } N_p Fr_c^5 = C \left(\frac{Fr_c^2}{Re_c} \right)^a Fr_c^f$$

$$b \text{ and } f, \text{ we get } N_p Re_c Fr_c^3 = C \left(\frac{Re_c}{Fr_c^2} \right)^b Fr_c^f$$

$$c \text{ and } f, \text{ we get } N_p^2 Re_c^5 = C^2 \left(\frac{Re_c}{Fr_c^2} \right)^c Re_c^f$$

$$e \text{ and } f, \text{ we get } N_p^3 Re_c^7 Fr_c = C^3 \left(\frac{Fr_c^2}{Re_c} \right)^e Ga^f$$

Thus, to describe the process of agitation of a liquid, we can use nine dimensionless equations that are absolutely equivalent in principle, but differ in their form.

The difference between the equations is due to the physical quantities characterizing the phenomenon being studied (in the given case two quantities) that are contained in the defining dimensionless numbers only once, and each of them is contained only in one of the numbers.

All the equations can easily be transformed into one another because the numerical values of the exponents are determined by coupling equations.

The selection of the dimensionless equation to be used is generally based either on the tendency to employ the traditional forms of generalized variables (in the given case the Reynolds and Froude numbers), or on the convenience of processing the experimental data. The numerical values of the coefficient C and the exponents can be determined only experimentally.

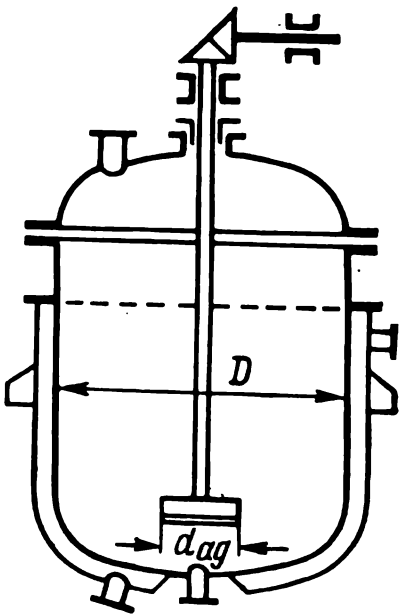
Example 3-33. It is necessary to ensure the uniform distribution of solid catalyst particles having a maximum size of 1.3 mm and a density of 2450 kg/m³ in a reactor (Fig. 3-12) having a diameter of 1000 mm filled to a height of 1000 mm with the reaction mixture. The mixture contains 1 part of solid matter to 4 parts of liquid, and at the temperature of the process its viscosity $\mu = 150 \times 10^{-3}$ Pa·s and the density of the liquid $\rho_l = 1200$ kg/m³.

What agitator is it better to use—a three-blade propeller with a square pitch (Fig. 3-13a) or a shrouded turbine impeller with eight blades (Fig. 3-13b)?

Solution. 1. We determine the diameter of an agitator in a standard catalogue:

$d_{ag} = (0.25 \text{ to } 0.30) D = (0.25 \text{ to } 0.30) \times 1 = 0.3 \text{ m}$

Fig. 3-12. To Example 3-33



2. To find the required values of the Reynolds number Re_c and the quantity determining the speed of rotation, we use the equation*:

$Re_c = CGa^k \Phi_\rho^l S_{dp}^m S_D^n$

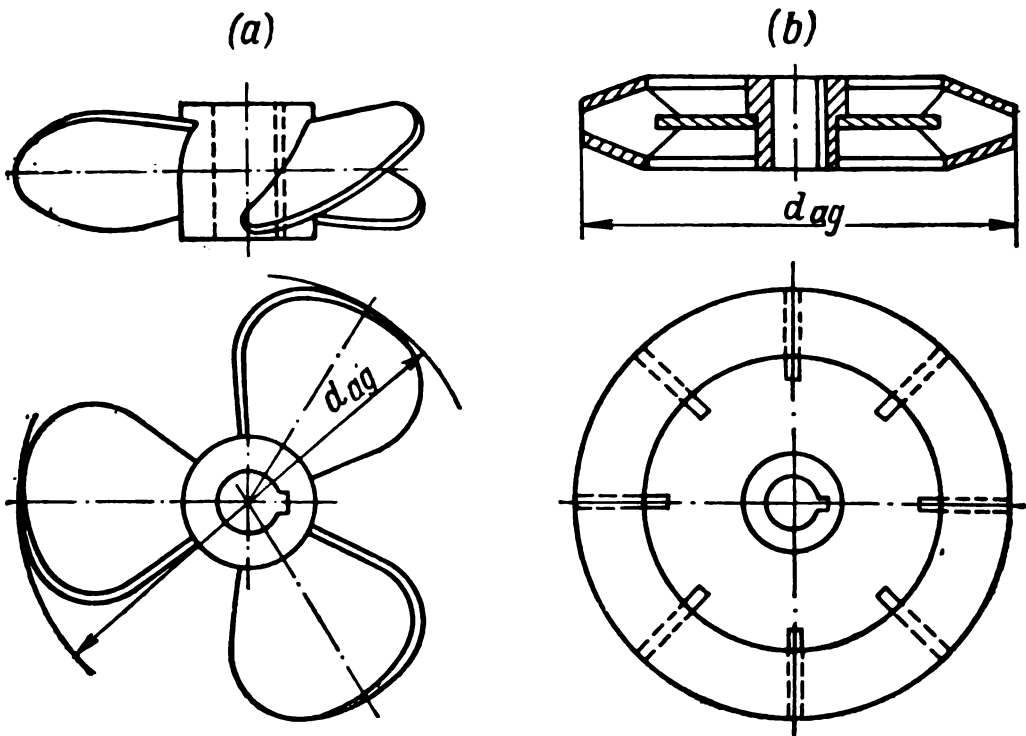


Fig. 3-13. To Examples 3-33 and 3-34

The values of the quantities C, k, l, m and n in this equation are:

	C	k	l	m	n
For a propeller	0.105	0.6	0.8	0.4	1.9
For a turbine impeller . .	0.25	0.57	0.37	0.33	1.15

* Pavlushenko, I.S., Kostin, N. M., and Matveev, S. F. *Zh. prikl. khimii.* 30: 1160 (1957).

We calculate the values of the dimensionless numbers and the shape factors:

$$Ga = \frac{d_{ag}^3 \rho_{fl}^2 g}{\mu^2} = \frac{3^3 \times 10^{-3} \times 1.2^2 \times 10^6 \times 9.81}{1.5^2 \times 10^{-2}} = 1.7 \times 10^3$$

$$\Phi_p = \frac{\rho_p}{\rho_{fl}} = \frac{2.45 \times 10^3}{1.2 \times 10^3} = 2.04$$

$$S_{d_p} = \frac{d_p}{d_{ag}} = \frac{1.3 \times 10^{-3}}{3 \times 10^{-1}} = 4.33 \times 10^{-3}$$

$$S_D = \frac{D}{d_{ag}} = \frac{1.0}{3 \times 10^{-1}} = 3.33$$

The found values are within the limits of applicability of the equation.

We find the value of the number Re_c and the determining speed of rotation for a propeller agitator:

$$Re_c = 0.105 Ga^{0.6} \Phi_p^{0.8} S_{d_p}^{0.4} S_D^{1.9} = 1.05 \times 10^{-1} \times 1.7^{0.6} \times 10^{4.2} \times 2.04^{0.8} \times 4.33^{0.4} \times 10^{-1.2} \times 3.33^{1.9} = 4.51 \times 10^3$$

From Eq. (3-77), we have

$$n_0 = \frac{Re_c \mu}{\rho_{fl} d_{ag}^2} = \frac{4.51 \times 10^3 \times 1.5 \times 10^{-1}}{1.2 \times 10^3 \times 3^2 \times 10^{-2}} = 6.26 \text{ rps} = 376 \text{ rpm}$$

We find the value of the number Re_c and the defining speed of rotation for a turbine impeller:

$$Re_c = 0.25 Ga^{0.57} \Phi_p^{0.37} S_{d_p}^{0.33} S_D^{1.15} = 2.5 \times 10^{-1} \times 1.7^{0.57} \times 10^{3.99} \times 2.04^{0.37} \times 4.33^{0.33} \times 10^{-0.99} \times 3.33^{1.15} = 2.85 \times 10^3$$

From Eq. (3-77), we have

$$n_0 = \frac{Re_c \mu}{\rho_{fl} d_{ag}^2} = \frac{2.85 \times 10^3 \times 1.5 \times 10^{-1}}{1.2 \times 10^3 \times 3^2 \times 10^{-2}} = 3.96 \text{ rps} = 238 \text{ rpm}$$

3. We find the power used directly for agitation.

From a graph of $N_p = f(Re)$ [0-46], we find $N_p = 0.32$ for a propeller agitator and $N_p = 1.3$ for a turbine impeller.

We calculate the constant factor:

$$\rho_{fl} d_{ag}^5 = 1.2 \times 10^3 \times 3^5 \times 10^{-5} = 2.91$$

The power consumed by the propeller agitator is:

$$P = N_p n^3 \rho_{fl} d_{ag}^5 = 3.2 \times 10^{-1} \times 6.26^3 \times 2.91 = 227 \text{ W} \approx 0.23 \text{ kW}$$

The power consumed by the turbine impeller is:

$$P = N_p n^3 \rho_{fl} d_{ag}^5 = 1.3 \times 3.96^3 \times 2.91 = 234 \text{ W} \approx 0.23 \text{ kW}$$

Since in the given case the power used for agitation is identical, we select the turbine impeller which operates at a lower speed.

Example 3-34. Experiments involving the oxidation of ammonium sulphide to thiosulphate by oxygen conducted in a model apparatus with a diameter of 500 mm having four baffles showed that upon agitation using a shrouded turbine impeller 125 mm in diameter and having a speed of 6 rps with a unit rate of flow of the oxygen of $Q_{v, \text{mod}} = 3.5 \times 10^{-3} \text{ m}^3 \text{ of O}_2/\text{m}^3 \cdot \text{s}$, 1.04 dm³ of oxygen is absorbed per cubic metre of reaction medium per second; the unit throughput is $q_{v, \text{mod}} = 1.04 \times 10^{-3} \text{ m}^3 \text{ of O}_2/\text{m}^3 \cdot \text{s}$.

The results of the experiments are used to design an industrial apparatus 2 m in diameter having an impeller 0.5 m in diameter geometrically similar to the model one ($S_D = D/d = 4$). It is necessary to calculate the speed of the industrial agitator at which the same unit throughput will be ensured as in model conditions. The rate of flow of oxygen in the industrial apparatus is $Q_{v, \text{ind}} = 3.5 \times 10^{-3} \text{ m}^3 \text{ of O}_2/\text{m}^3 \cdot \text{s}$. The physical properties of the reaction medium in the model and the industrial apparatuses are close to those of water. The reaction temperature is 80°C.

Solution. The following equation* can be used in designing industrial apparatuses in which $Re_c \geq 1.8 \times 10^5$:

$$Di = C Re_c^{0.09} Fr_c^{0.165} K_{Q_v}^{0.75} S_D^{-0.167} \quad (a)$$

where $Di = q_v/n$ = Dyakonov number

$K_{Q_v} = Q_v/n$ = distribution number.

If the properties of the reacting phases do not change upon transition from the model to the industrial apparatus, Eq. (a) acquires the form:

$$q_v = C_1 \frac{n^{0.67} d_{\text{ag}}^{1.015} Q_v^{0.75}}{D^{0.67}}$$

We calculate the value of the Reynolds number (centrifugal) for the model apparatus:

$$Re_c = \frac{\rho n d_{\text{ag}}^2}{\mu} = \frac{972 \times 6 \times 0.125^2}{3.56 \times 10^{-4}} = 2.55 \times 10^5 > 1.8 \times 10^5$$

Thus, the results of the experiments can be used for modelling.

We determine the coefficient C_1 for the process being considered according to the results of the experiments on the model:

$$C_1 = \frac{q_{v, \text{mod}} D_{\text{mod}}^{0.67}}{n_{\text{mod}}^{0.67} d_{\text{ag, mod}}^{1.015} Q_{v, \text{mod}}^{0.75}} = \frac{1.04 \times 10^{-3} \times 0.5^{0.67}}{6^{0.67} \times 0.125^{1.015} (3.5 \times 10^{-3})^{0.75}} = 1.15 \times 10^{-4}$$

* Pavlushenko, I.S., Braginsky, L.N., and Brylov, V.N. *Zh. prikl. khimii*, 34: 805 (1961).

Using the found values in the equation of the characteristic of the industrial apparatus, we calculate the speed of the agitator:

$$n_{\text{ind}} = \left(\frac{q_{v, \text{ind}} D_{\text{ind}}^{0.67}}{C_1 d_{\text{ag, ind}}^{1.015} Q_{v, \text{ind}}^{0.75}} \right)^{\frac{1}{0.67}} =$$

$$= \left(\frac{1.04 \times 10^{-3} \times 20^{0.67}}{1.15 \times 10^{-1} \times 0.5^{1.015} (3.5 \times 10^{-3})^{0.75}} \right)^{\frac{1}{0.67}} = 2.85 \text{ rps}$$

We take the speed of the agitator equal to $n_{\text{ind}} = 3$ rps.

We check the value of the Reynolds number for the industrial apparatus:

$$Re_c = \frac{972 \times 3 \times 0.5^2}{3.56 \times 10^{-4}} = 2.04 \times 10^6$$

Since the value of the Reynolds number for the industrial apparatus is also within the range of applicability of Eq. (a), the selected speed ensures the required throughput.

PROBLEMS

3-1. Find the ratio of the diameters of particles of lead glance ($\rho = 7800 \text{ kg/m}^3$) and quartz ($\rho = 2600 \text{ kg/m}^3$) settling with the same velocity (a) in air, and (b) in water, considering that settling occurs at $Re < 0.2$.

3-2. With what velocity will spherical particles of quartz ($\rho = 2600 \text{ kg/m}^3$) having a diameter of $10 \text{ }\mu\text{m}$ settle (a) in water at 15°C , and (b) in air at 15 and 500°C ?

3-3. What should the velocity of the air in the vertical tube of a pneumatic drier be to ensure the movement of crystals having a density of 2000 kg/m^3 with a maximum diameter of 3 mm ? The temperature of the air is 60°C . The velocity of the air should be 1.25 times greater than the soaring velocity of the particles.

3-4. Calculate the velocity of the ascending stream of air in an air separator needed to separate fine ($d < 1 \text{ mm}$) particles of apatite from coarser ones. The temperature of the air is 20°C . The density of the apatite is 3230 kg/m^3 .

3-5. What should the distance between the racks of a dust-settling chamber (see Fig. 3-9) be for pyrite dust particles with a diameter exceeding $15 \text{ }\mu\text{m}$ to settle in it? The remaining conditions are the same as in Example 3-6 (p. 130).

3-6. A dust-laden gas with a density of 1.6 kg/m^3 passes at a rate of $2000 \text{ m}^3/\text{h}$ through a dust-settling chamber (see Fig. 3-9) with the racks spaced 100 mm apart (the density and the rate of flow are given for standard conditions). The temperature of the gas is 400°C . The dynamic viscosity of the gas at this temperature is $0.03 \times 10^{-3} \text{ Pa}\cdot\text{s}$. The density of the dust is 3700 kg/m^3 . The length of the chamber is 4.55 m , its width is 1.71 m , and its height is 4 m . What size of dust particles will be separated in the chamber if it is considered that the actual settling velocity is half of the theoretical value?

3-7. Prove that Eqs. (3-13) and (3-14) are identical for calculating the area of a continuous settling tank.

3-8. Determine the diameter of the settling tank needed for the continuous compacting of an aqueous suspension of chalk having a temperature of 35°C . The remaining conditions are the same as in Example 3-8.

3-9. How will the capacity of a settling tank change if the temperature of the aqueous suspension is increased from 15 to 50 °C? In both cases, $Re < 0.2$.

3-10. Select a cyclone of the NIIOGAZ type (see Fig. 3-3 and Table 3-1) according to the following data: rate of flow of dust-laden air = 5100 m³/h (0 °C and 760 mm Hg), temperature of the air = 50 °C. The density of the dust is 1200 kg/m³. The minimum diameter of the dust particles is 15 μm. Also find the hydraulic resistance of the cyclone.

3-11. Derive Eq. (3-31) assuming that the volume of the suspension equals the sum of the volumes of the liquid and solid phases.

3-12. Calculate the density of an aqueous suspension containing 10% (by mass) of the solid phase. The relative density of the solid phase is 3.

3-13. Determine the settling velocity in water at 25 °C of elongated particles of coal ($\rho = 1400$ kg/m³) and lamellar particles of shale ($\rho = 2200$ kg/m³) having an equivalent diameter of 2 mm.

3-14. Determine the diameter of lead glance particles having a sharp-edged shape settling with a velocity of 0.25 m/s in water at a temperature of 15 °C. The density of the lead glance is 7500 kg/m³.

3-15. What amount of wet cake will be gathered on a filter as a result of the filtration of 10 m³ of a suspension having a relative specific gravity of 1.12 and containing 20% (by mass) of the solid phase? The moisture content of the sediment is 25%.

3-16. As a result of the filtration of an aqueous suspension containing 20% (by mass) of the solid phase, 15 m³ of the filtrate was collected. The moisture content of the cake is 30%. How much of the cake, dry basis, was obtained?

3-17. A filter press has 26 frames 62 × 62 cm in size. The thickness of a frame is 25 mm. The duration of filtering up to filling the frames is two hours. Water in an amount of 10% of the volume of the filtrate is used for washing. The pressure during filtration and washing is the same and constant. How much time is needed for washing? The cake is homogeneous and incompressible, its volume is 5% of that of the filtrate. Use Eq. (3-17) for the calculations assuming that $C = 0$.

3-18. The duration of filtering 20 m³ of a solution in a plate-and-frame filter press is 2.5 hours. Find the approximate duration of washing the cake with 2 m³ of water assuming that the washing velocity is about one-fourth of the filtering velocity at the final moment. Ignore the resistance of the cloth. The dynamic viscosities of the filtrate and the wash water are identical.

3-19. How will the duration of washing the cake in the conditions of the preceding problem change if the viscosity μ of the filtrate and the wash water is 1.5×10^{-3} and 1×10^{-3} Pa·s, respectively?

3-20. Find the theoretical duration of washing the cake on a filter in the following conditions: the intensity of washing is 6 dm³/m²·min, the thickness of the cake is 30 mm, the initial concentration of the salt being washed out in the wash water filtrate is 120 g/dm³, and the final concentration is 2 g/dm³. The coefficient K taking into account the washing conditions, etc. (the washing velocity constant), is equal to 350 cm³/dm³ according to experimental data.

3-21. Determine the washing velocity constant K for the following conditions: washing intensity = 10 dm³/m²·min, cake thickness = 25 mm, initial concentration of the salt in the wash water filtrate = 40 g/dm³, final concentration = 5 g/dm³, washing duration = 1 h 40 min.

3-22. How will the capacity of a filter change if:

1. The filtering surface is doubled?
2. The pressure is doubled (with a homogeneous incompressible cake)?
3. The concentration of the solid matter in the suspension being filtered is doubled?
4. The viscosity of the filtrate is halved (by elevating the temperature)?

5. The duration of a complete cycle of the filter is doubled (i.e. the thickness of the filter cake is increased)?

3-23. Show approximately how a change in the speed of a continuous rotary vacuum filter (see Fig. 3-10) will affect its capacity (for example, when the speed is increased 1.5 times). Use Eq. (3-17) assuming that $C=0$.

3-24. Determine the technological type and plan the design of a centrifuge for separating *p*-nitroaniline from a solution after recrystallization, taking the following data into account:

- (a) the concentration of the solid matter in the suspension is 35%;
- (b) the solvent is water;
- (c) the sediment is crystalline;
- (d) the required residual moisture content is 5%;
- (e) crystallization proceeds intermittently.

3-25. Cotton-seed oil is to be separated from the wash water (soap stock) after alkaline refining. Determine the technological type and plan the design of the centrifuge to be used with a view to the following data:

- (a) the nature of the mixture is an emulsion;
- (b) the relative density of the oil is 0.9;
- (c) the relative density of the aqueous solution of the salt (added for destruction of the emulsion) is 1.05.

3-26. Determine the pressure on the walls of a centrifuge bowl if the thickness of the liquid layer is 10 cm, the internal diameter of the bowl is 1 m, and its speed is 500 rpm. The density of the liquid is 1100 kg/m³.

3-27. Find the speed of a centrifuge if it is known that the height of the bowl $H=0.5$ m. The pressure at the bowl walls must be 5 at. The centrifuge is loaded with 400 kg of suspension.

3-28. Show approximately that when the permissible ultimate tensile stress of steel is $\sigma_u=88.3 \times 10^6$ Pa = 900 kgf/cm², the peripheral velocity of a centrifuge bowl must not exceed 60 m/s. Use this condition to determine the maximum permissible diameter of the bowl (a) for a filtering centrifuge running at 1100 rpm, and (b) for a tubular supercentrifuge running at 14 000 rpm.

3-29. Derive a formula for calculating the speed of centrifugal separation of solid spherical particles on the basis of the Stokes law. The rotational speed is measured in rps.

3-30. How many times faster will identical particles settle in a centrifuge than in a settling tank if the basket of the centrifuge has $D=1$ m and a speed of $n=600$ rpm? The settling conditions in both cases are laminar.

3-31. Determine approximately the duration of accelerating a centrifuge loaded with 300 kg of a moist salt, disregarding the friction of the shaft in its bearings and the friction between the bowl wall and the air. The internal diameter of the bowl is 1 m, its mass is 200 kg. The operating speed is 800 rpm. The power of the motor is 6 kW, the total efficiency of the unit is 0.8. The height of the bowl is 780 mm, the bowl filling factor is 0.5.

3-32. Find the required power of the motor (with account taken of friction) for the conditions of the preceding problem if the acceleration time of the centrifuge is 2.5 min. The diameter of the shaft is 70 mm, the shaft is mounted on ball bearings, the thickness of the bowl wall is 10 mm.

3-33. Determine the required number of batch centrifuges with bowl dimensions of $D=1200$ mm and $H=500$ mm for filtering 50 tons of suspension a day. The suspension contains 40% (by mass) of the solid phase. The relative density of the liquid phase is 1.1, and of the solid phase 1.8. The duration of one operation is 25 min. The number of working hours a day is 20. The bowl filling factor is 0.5.

3-34. How will the capacity of a filtering centrifuge change if its speed is doubled? The cake is homogeneous and incompressible. Ignore the resistance of the filtering cloth.

3-35. A type AOF-1800 automatic settling horizontal centrifuge is to operate

with an aqueous suspension of chalk. Determine the capacity of the centrifuge relative to feeding if the temperature of the suspension is 40°C . The size of the smallest chalk particles is $2\text{ }\mu\text{m}$. The specifications of the centrifuge are: bowl diameter = 1800 mm , bowl length = 700 mm , weir diameter = 1300 mm , the speed is $n = 735\text{ rpm}$; assume that the efficiency equals 0.45 .

3-36. How many times is the capacity of an industrial filtering centrifuge of type АГ greater than that of a laboratory model geometrically similar to it? The dimensions of the industrial centrifuge are three times greater than those of the laboratory model. Both centrifuges operate with an identical suspension, at the same speed and with the same duration of filling the bowl with the sediment.

3-37. Determine the capacity of a type HОГШ-600 centrifuge (continuous horizontal helical-conveyor separator) operating with an aqueous suspension of gypsum at a temperature of 50°C . The smallest gypsum particles in the suspension have a diameter of $2\text{ }\mu\text{m}$. The specifications of the centrifuge are: diameter of the discharge cylinder = 480 mm , length of the settling zone = 350 mm , bowl speed $n = 1400\text{ rpm}$.

3-38. Particles of what diameter can be settled by a type HОГШ-230 centrifuge if $3\text{ m}^3/\text{h}$ of an aqueous suspension of kaolin are fed in for separation at 35°C ? The specifications of the centrifuge are: diameter of the discharge cylinder = 180 mm , length of the cylinder = 164 mm , bowl speed = 1600 rpm .

3-39. Determine the velocity of air needed for the beginning of the formation of a fluidized bed of granulated aluminosilica gel in the following conditions: temperature of the air = 100°C , density of the gel (apparent) $\rho = 968\text{ kg/m}^3$, diameter of the particles = 1.2 mm . What will the hydraulic resistance be if the height of the static bed is 400 mm ?

3-40. Determine the porosity and the height of the fluidized bed for the conditions of the preceding problem if the velocity of the air is 1.7 times greater than the critical value.

3-41. Determine the maximum diameter of granulated coal particles that begin to become suspended in air when its velocity in the apparatus is 0.2 m/s . The temperature is 180°C .

Also determine the volume concentration of the particles if the velocity of the air grows to 0.4 m/s . The density of the coal (apparent) is 660 kg/m^3 .

3-42. A tank with a diameter of 900 mm and a height of 1100 mm provided with an agitator is filled to three-fourths of its volume with cylinder oil ($\rho = 930\text{ kg/m}^3$, $\mu = 18\text{ Pa}\cdot\text{s}$). A motor of what power should be installed for a three-blade propeller agitator with a speed of 180 rpm ?

3-43. A paddle agitator is used to obtain a dilute solution of a mineral salt by intensive agitation at 64°C . What is the speed of the agitator if its diameter is 0.5 m and the power used by its motor is 0.8 kW ? Assume that the dilute solution has the same physical characteristics as water.

3-44. A paddle agitator with a size of $d_1 = D/3$ is replaced with a smaller one having a size of $d_2 = D/4$. Agitation in both cases is conducted in laminar conditions. How will the speed of the agitator change with the motor power remaining unchanged?

3-45. Find the diameter of a propeller agitator for the intensive agitation of technical glycerine ($\rho = 1200\text{ kg/m}^3$, $\mu = 1.6\text{ Pa}\cdot\text{s}$) in a tank having a diameter of 1750 mm at a speed of $n = 500\text{ rpm}$ and with a power consumption of 17 kW .

EXAMPLE OF CALCULATING AND SELECTING A MULTI-TUBE CYCLONE SEPARATOR [3-5]

A multi-tube cyclone separator (Fig. 3-14) is to remove dust from $7800\text{ m}^3/\text{h}$ of a gas at a temperature of 310°C . The density of the gas (at 0°C and 760 mm Hg) is 1.3 kg/m^3 . The barometric pressure is 99300 Pa (745 mm Hg).

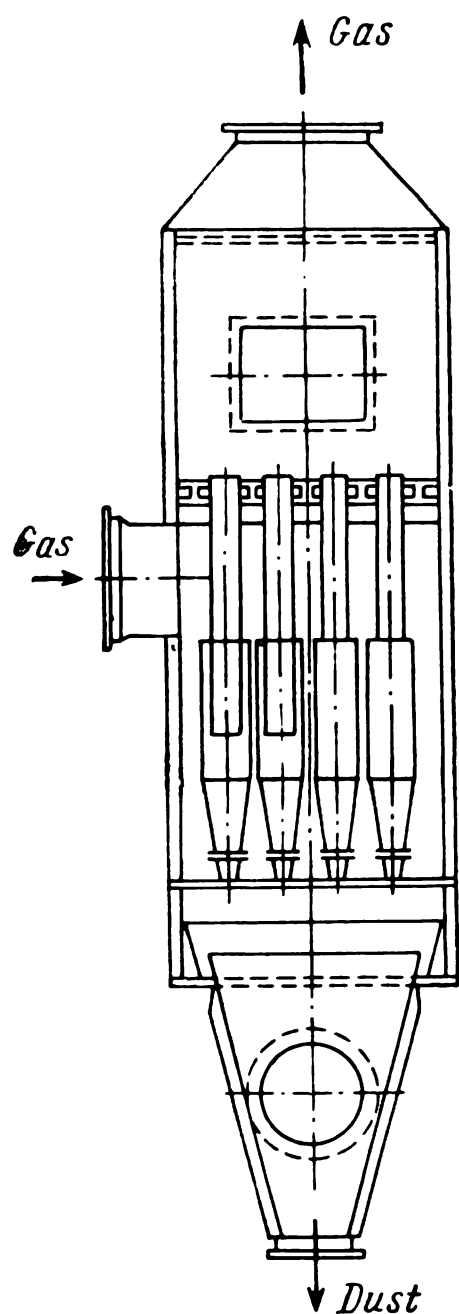


Fig. 3-14. Multi-tube cyclone separator

At the inlet to the separator, the gas is under a rarefaction of 294 Pa (30 mm H₂O). The hydraulic resistance of the separator should not exceed 392 Pa (40 mm H₂O). The density of the dust is 2450 kg/m³. The dust content of the gas is 32 g/m³ (at 0 °C and 760 mm Hg). The dust is weakly agglutinative.

Solution. The characteristics of type БЦ multi-tube cyclone separator elements with a socket-type guide rig when separating weakly agglutinative dust having a density of 2300 kg/m³ at $\Delta p/\rho=736 \text{ m}^2/\text{s}^2$ (or $\Delta p/\gamma=75 \text{ m}$) are given in Table 3-6.

TABLE 3-6

Diameter of element, mm	Maximum permissible dust content of gas, g/m ³ (at 0 °C and 760 mm Hg)	Efficiency of dust separation (in %) at particle diameter of			Hydraulic resistance coefficient ζ_0 at angle of blade inclination of	
		5 μm	10 μm	15 μm	25°	30°
250	75	72	84	93	90	65
150	35	78	88	95		
100	15	82	91	96		

Using the data of this table, we select separator elements (tubes) having a diameter of 150 mm (the permissible dust content of the gas is up to 35 g/m³).

We determine the density of the gas in operating conditions:

$$\rho = 1.3 \times \frac{273 \left(745 - \frac{30}{13.6} \right)}{(273 + 310) 760} = 0.595 \text{ kg/m}^3$$

According to the initial conditions, the pressure loss Δp must not exceed 392 Pa (40 mm H₂O). The ratio $\Delta p/\rho = 392/0.595 = 660 \text{ m}^2/\text{s}^2$ (or $\Delta p/\gamma = 40/0.595 = 67.2 \text{ m}$) is within the recommended limits of from 540 to 736 m²/s² (or from 55 to 75 m).

For a socket-type guide rig with an angle of blade inclination to the horizon of 25°, the hydraulic resistance coefficient is $\zeta_0 = 90$.

The velocity of the gas in the cylindrical part of a cyclone separator v_{cyl} , according to Eq. (3-16), is

$$v_{\text{cyl}} = \sqrt{\frac{2\Delta p}{\zeta_0 \rho}} = \sqrt{\frac{392 \times 2}{90 \times 0.595}} = 3.84 \text{ m/s}$$

The rate of flow of the gas per element of the separator is:

$$Q_{v,1} = 0.785 D^2 \times 3600 v_{\text{cyl}} = 0.785 \times 0.150^2 \times 3600 \times 3.84 = 244 \text{ m}^3/\text{h}$$

The required number of elements is:

$$n = \frac{7800}{244} = 32$$

We arrange them in four rows in the direction of flow of the gas (eight elements in each row).

EXAMPLE OF CALCULATING AND SELECTING A FOAM GAS WASHER FOR PURIFYING A GAS OF DUST [3-12]*

Determine the principal dimensions of a foam gas washer for purifying of dust 50 000 m³ of gas an hour at 80 °C. The dust content at the inlet to the apparatus is $c_{\text{in}} = 0.01 \text{ kg/m}^3$ (in standard conditions), the degree of purification is to be 0.99.

Solution. Since the velocity of the gas in the full section of the apparatus is the main factor on which good foam generation depends and, therefore, the effectiveness of purification, it is important to select the velocity properly.

The upper limit of the permissible gas velocity is such a value of it when the entrainment of the water in the form of a spray sharply grows. According to experimental data for gas washers (scrubbers) having a foam layer from 30 to 100 mm high, the jet break-through of the gas causing destruction of the foam and great spray entrainment begins at gas velocities in the full section of the apparatus (under the grid) from 2.7 to 3.5 m/s.

The higher the foam layer on the grid and the larger the clear section of the grid, the higher is the velocity of the gas that is possible without spray entrainment. A reduction in the diameter of the openings (while retaining a constant clear section of the grid) also facilitates a reduction in spray entrainment. Usually the upper limit is a gas velocity under the grid of about 3 m/s.

The lower limit of the gas velocity for a foam apparatus is the velocity at which foam generation greatly diminishes.

* Recently new improved designs of foam gas washers have appeared: with turbogrids (Valdberg, A. Yu., Tarat, E. Ya., and Zaitsev, M. M. *Vodosnabzhenie i sanitarnaya tekhnika*, No. 6, 1969) and with honeycomb foam stabilizers.

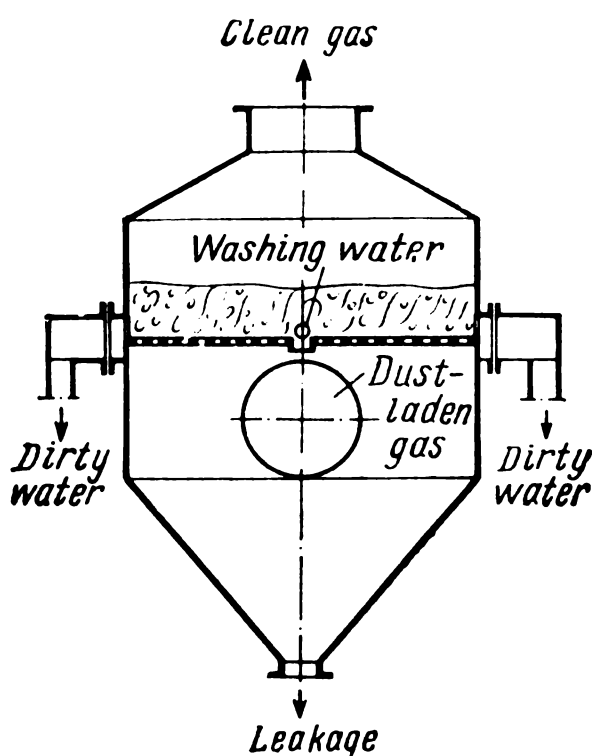


Fig. 3-15. Foam gas washer

For foam gas washers with a great clear section of the grid and a large diameter of the openings, the lower limit is such a gas velocity at which the majority of the liquid flows out through the openings, as a result of which the height of the foam layer becomes negligibly small. For conventional conditions, 1 m/s can be taken as the lower limit of the design velocity.

We take the average velocity of the gas v equal to 2.3 m/s. We determine the cross-sectional area of the apparatus:

$$A = \frac{50\,000}{3600 \times 2.3} = 6 \text{ m}^2$$

A gas washer may have a round or a square section. A round apparatus ensures a more uniform flow of the gas, and a rectangular one—a better distribution of the liquid.

We adopt an apparatus having a rectangular section 3×2 m in size with feeding in of the water at the middle (Fig. 3-15). The gas is fed in through a diffuser to improve its distribution over the area of the apparatus.

The quantity of washing water is calculated in different ways depending on the temperature of the gas fed in. Hydrodynamic factors have the greatest influence on the flow rate of the water when the gas is cold, while the heat balance determines the flow rate of the water when the gas is hot.

When gases having a temperature below 100 °C are cleaned of dust, the quantity of water supplied is calculated on the basis of the hydrodynamics of the process and the material balance of gas purification. In ordinary conditions to retain a sufficient uniformity of foam generation over the entire grid, it is essential that not over 50% of the water fed in flow through the openings because too great leakage results in the height of the layer of water on the grid lacking uniformity.

The rate of flow of the water in a gas washer consists of the rate of flow of the water leaking out and that of the water discharged from the grid.

The evaporation of the water at a given temperature of the gas may be disregarded.

The amount of water flowing through the openings of the grid is determined by the weight of the separated dust and the fixed composition of the suspension, and next a grid is chosen having such a clear section, an opening diameter and other parameters as to ensure the required leakage.

At a preset efficiency of purification η , the concentration of the dust in the cleaned gas c_{out} is determined by the equation*:

$$c_{\text{out}} = c_{\text{in}} (1 - \eta) = 0.01 (1 - 0.99) = 0.0001 \text{ kg/m}^3$$

The amount of separated dust is:

$$Q_{\text{m, sep}} = Q_{\text{v, 0}} (c_{\text{in}} - c_{\text{out}}) = 50\,000 \times \frac{273}{273 + 80} (0.01 - 0.0001) = 383 \text{ kg/h}$$

If we know the concentration of the suspension $c = \text{solid:liquid}$ (in kg/kg), then the leakage $Q_{\text{v, leak}}$, i. e. the volume of the water needed for formation of the suspension (in m^3/h), is determined by the equation

$$Q_{\text{v, leak}} = \frac{K Q_{\text{m, sep}}}{1000c}$$

where K is a coefficient showing the distribution of the dust between the leakage and discharged water, expressed by the ratio of the amount of dust getting into the leakage to the total amount of separated dust; usually K ranges from 0.6 to 0.8.

The concentration of the suspension, as a rule, is within the limits of the ratio $c = \text{solid:liquid} = (1:5)$ to $(1:10)$. The obtaining of a suspension with $c > 1:5$ may cause clogging of the grid openings (especially small ones). The obtaining of a suspension with $c < 1:10$ is not rational owing to its too great volumes.

We adopt $c = 1:8 = 0.125 \text{ kg/kg}$ and $K = 0.7$. Hence,

$$Q_{\text{v, leak}} = \frac{0.7 \times 383}{1000 \times 0.125} = 2.14 \text{ m}^3/\text{h}$$

for the entire grid or $2.14/6 = 0.36 \text{ m}^3/\text{m}^2 \cdot \text{h}$ for 1 m^2 of the grid.

Owing to the difficulty in determining the grid parameters according to a preset leakage, and also with a view to the partial evaporation of the water after it flows through the grid, we take a safety factor of about 1.5, i. e. we assume that $Q_{\text{v, leak}} = 1.5 \times 2.14 \approx 3.3 \text{ m}^3/\text{h}$, or $0.55 \text{ m}^3/\text{m}^2 \cdot \text{h}$.

The amount of discharged water is found by the equation:

$$Q_{\text{v, dis}} = ib$$

where i = intensity of the flow discharged from the grid, $\text{m}^3/\text{m} \cdot \text{h}$

b = width of the grid before the weir, equal to the length of the decanting weir, m.

Assuming that $i = 1 \text{ m}^3/\text{m} \cdot \text{h}$, we find for the selected type of apparatus (discharge to both sides):

$$Q_{\text{v, dis}} = 1 \times 2 \times 2 = 4 \text{ m}^3/\text{h}$$

The total rate of flow of the water is:

$$Q_{\text{v}} = 3.3 + 4 = 7.3 \text{ m}^3/\text{h}$$

The unit rate of flow of the water is:

$$q_{\text{v}} = \frac{7300}{50\,000} = 0.146 \text{ dm}^3/\text{m}^3 \text{ of gas}$$

The fraction of the total rate of flow of the water formed by the leakage is:

$$Q_{\text{v, leak}} = \frac{3.3}{7.3} \times 100 = 45\%$$

which is acceptable (the condition that $Q_{\text{v}} \geq 2Q_{\text{v, leak}}$ must be complied with).

* The dust concentration c_{out} is related to the volume of the dust-laden gas $Q_{\text{v, 0}}$ reduced to standard conditions. It differs insignificantly from the dust content (in kg/m^3) in the cleaned gas because the amount of the gas at the outlet increases by 1 or 2% owing to evaporation of the water in the gas washer.

The basic characteristics of the grid (the diameter and pitch of the openings) are selected with a view to the required leakage.

It has been established that the leakage of water grows with an increase in the diameter of the openings d_0 and the height of the initial layer * of liquid on the grid h_0 .

The leakage greatly increases when the velocity of the gas in the openings is lowered to below 4 to 6 m/s (depending on d_0 and h_0) and sharply diminishes when the velocity of the gas is increased to above 13 to 15 m/s, which may cause clogging of the grid with dust. In addition, an increase in the gas velocity in the openings with a small layer of water (foam) on the grid, which is characteristic of gas washers, results in the jet breaking through of the gas and great spray formation.

To ensure normal functioning of the gas washer, the velocity of the gas v_0 in the large openings of the grid should be selected within the limits from 8 to 13 m/s, and for grid with smaller openings within the limits from 7 to 10 m/s depending on the initial dust content of the gas, the possible fluctuations in the gas load, and other conditions.

With a view to the considerable concentration of the dust in the gas (10 g/m³ in standard conditions) and the relatively greater easiness of manufacturing grids with large openings (a smaller number of openings together with it being simpler to drill them), we install a grid with large openings for which design velocities of the gas of from 8 to 13 m/s are recommended. Considering that the fluctuations in the load of the apparatus with respect to the gas will mainly occur in the direction of lowering (the usual conditions), we choose a gas velocity of $v_0 = 12$ m/s.

Hence, the ratio of the area of the clear section of the grid A_0 to the cross-sectional area of the apparatus A will be

$$\frac{A_0}{A} = \frac{v}{v_0 z} = \frac{2.3}{12 \times 0.95} = 0.2$$

where $z = 0.95$ is a factor taking into account that 5% of the area of the clear section is occupied by the supports of the grid, overflow walls, etc.

When the grid openings are arranged along a hexagon with the pitch t , the hatched area in Fig. 3-16 is:

$$a = tx = t \times 2 \sqrt{t^2 - \frac{t^2}{4}} = 1.73t^2$$

This area contains two openings with a diameter of d_0 . The area of the openings is:

$$a_0 = 2 \times 0.785d_0^2 = 1.57d_0^2$$

It was shown above that the ratio a_0/a should be 0.2:

$$\frac{1.57d_0^2}{1.73t^2} = 0.2$$

whence

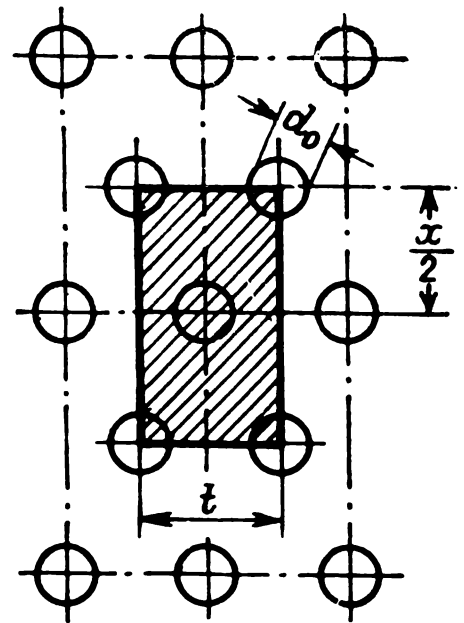
$$t = \sqrt{\frac{1.57d_0^2}{1.73 \times 0.2}}$$

With an opening diameter of $d_0 = 5$ mm, we have:

$$t = \sqrt{\frac{0.91 \times 25}{0.2}} = 10.7 \approx 11 \text{ mm}$$

* By the height of the initial layer is meant the height of the layer of unfoamed liquid used for the formation of a layer of foam of the preset height.

Fig. 3-16. Arrangement of openings on grid



The height of the weir at the discharge from the grid is established with a view to creating a layer of the foam before the weir having a height of from 60 to 100 mm (depending on the preset efficiency of purification).

Let us calculate what the height of the foam layer on the grid should be to ensure the preset efficiency of purification $\eta = 0.99$.

The coefficient of the velocity of dust separation is:

$$K_{\text{sep}} = \frac{2\eta v}{2 - \eta} = \frac{2 \times 0.99 \times 2.3}{2 - 0.99} = 4.5 \text{ m/s}$$

The relationship between the coefficient K_{sep} and the height of the foam layer H in separating hydrophilic dust with an average particle size of 15 to 20 μm is expressed by the empirical equation:

$$H = K_{\text{sep}} - 1.95v + 0.09 = 4.5 - 1.95 \times 2.3 + 0.09 = 0.1 \text{ m}$$

On the other hand, for dust separators (extractors), we have:

$$H = 0.806v^{0.5}h_0^{0.6}$$

where h_0 is the height of the initial layer of water on the grid, m. Hence,

$$h_0 = \left(\frac{H}{0.806v^{0.5}} \right)^{\frac{1}{0.6}} = \left(\frac{0.1}{0.806 \times 2.3^{0.5}} \right)^{\frac{1}{0.6}} = 0.013 \text{ m}$$

The height of the initial layer of liquid h_0 is related to the intensity of the flow at the discharge weir i and the height of the weir h_w by the empirical equation

$$h_0 = \varphi \sqrt[3]{i^2} + \psi h_w$$

where φ = coefficient characterizing a weir; for industrial calculations it can be assumed with sufficient accuracy that $\varphi = 3$

ψ = degree of water surface ascent due to the weir, which may vary quite considerably depending on the conditions of foam formation; for the operating conditions of gas washers $\psi \approx 0.4$.

Thus, the height of the weir (in mm) can be calculated by the equation:

$$h_w = 2.5h_0 - 7.5 \sqrt[3]{i^2}$$

In our case, we have $v = 2.3 \text{ m/s}$, and $i = 1 \text{ m}^3/\text{m} \cdot \text{h}$.

Consequently, the height of the weir is:

$$h_w = 2.5 \times 13 - 7.5 \sqrt[3]{1^2} = 25 \text{ mm}$$

To ensure proper functioning of the apparatus with fluctuations in its operating conditions, we adopt a weir height of 30 mm.

The total height of the gas washer consists of the heights of its separate parts: the part above the grid h_1 , the part under the grid h_2 , and the height of the bunker h_b . These heights are determined on the basis of structural considerations: h_1 depending on the formation of spray and the dimensions of the spray trap, h_2 depending on the design of the gas feeding means, and h_b depending on the properties of the suspension.

EXAMPLE OF CALCULATING AND SELECTING A ROTARY VACUUM FILTER

Design a rotary vacuum filter (see Fig. 3-10) with a capacity of 2.8 ton/day of dry cake of nickelous hydroxide according to the following data: (1) the vacuum is 53.3×10^3 Pa (400 mm Hg); (2) the average specific resistance of the cake is $r = 43.21 \times 10^{10}$ m/kg of dry matter; (3) the specific resistance of the filtering cloth is $r_f = 11.43 \times 10^{10}$ m/m²; (4) the mass of the solid matter deposited on the filter in obtaining 1 m³ of the filtrate is $c = 207.5$ kg/m³; (5) the preset cake thickness is $\delta = 5$ mm*; (6) the volume of the wet cake obtained when 1 m³ of the filtrate passes through the filter is 0.686 m³/m³; (7) the density of the wet cake is 1220 kg/m³ (with a moisture content of 75.2%), the density of the filtrate is 1110 kg/m³; (8) the total number of filter sections is $z = 24$ (similar to the rotary filters in use); (9) the dynamic viscosity of the filtrate (at a filtering temperature of 50°C) is $\mu = 1.51 \times 10^{-3}$ Pa·s; (10) the duration of drying the cake on the filter $\tau_{\text{dry}} = 1.5$ min; and (11) the concentration of the initial suspension is 10.67%.

Solution. Designing of a vacuum filter consists in determining the required filtration surface and in selecting the appropriate filter according to a catalogue.

The filtration surface A (in m²) can be determined by the expression:

$$A = \frac{Q_{v, \text{tot}}}{Q'_v}$$

where $Q_{v, \text{tot}}$ = capacity of the filter relative to the filtrate, m³/h
 Q'_v = capacity of 1 m² of the filter relative to the filtrate, equal to $q_v n$, m³/m²·h
 q_v = capacity of 1 m² of the filter during one revolution, m³/m²
 n = speed of filter, rph (revolutions per hour).

We determine the output of the filter relative to the filtrate.

The output of the filter relative to the dry cake must be 2.8 ton/day, or 117 kg/h. When recalculated for the moist cake (a moisture content of 75.2%), the corresponding figure will be $117 \times 100 / (100 - 75.2) = 472$ kg/h.

The amount of suspension fed in for filtering with a concentration of 10.67% will be $117 \times 100 / 10.67 = 1096$ kg/h. Hence, the rate of discharge of the filtrate will be $1096 - 472 = 624$ kg/h or, at a filtrate density of 1110 kg/m³, it will be $624 / 1110 = 0.56$ m³/h = 13.4 m³/day.

Thus $Q_{v, \text{tot}} = 0.56$ m³/h.

To determine Q'_v , we must know the capacity of 1 m² of the filter during one revolution, i. e. during the time of passing through the filtration zone τ , and the speed of the filter in revolutions per hour.

We know that the volume of the wet cake deposited on the filter when 1 m³ of the filtrate passes through it is 0.686 m³/m³. With the preset cake thickness

* Such a thickness of the layer is explained by the amorphous structure of the cake; the thickness is usually greater (from 15 to 20 mm) for crystalline cakes.

equal to 5 mm, the required surface of the filtration zone per 1 m³ of filtrate is $0.686/0.005 = 137.5$ m²/m³. The volume of the filtrate passing through 1 m² of the filtration zone surface will obviously be

$$q_v = \frac{1}{137.5} = 0.00728 \text{ m}^3/\text{m}^2$$

To determine the speed of rotation of the filter, we must know the duration of filtering τ (the time during which a cake 5 mm thick is formed). For this purpose, we shall use the basic equation of filtering (3-17):

$$V^2 + 2VC = K\tau$$

where $V = 7.28 \times 10^{-3}$ m³/m².

We find the constant K by Eq. (3-21):

$$K = \frac{2\Delta p}{\mu cr} = \frac{2 \times 53300}{1.51 \times 10^{-3} \times 207.5 \times 43.21 \times 10^{10}} = 0.79 \times 10^{-6} \text{ m}^2/\text{s}$$

where $\Delta p = 53300$ Pa = 400 mm Hg, $\mu = 1.51 \times 10^{-3}$ Pa·s, $c = 207.5$ kg/m³, and $r = 43.21 \times 10^{10}$ m/kg.

We determine the constant C by Eq. (3-25):

$$C = \frac{r_f}{rc} = \frac{11.43 \times 10^{10}}{43.21 \times 10^{10} \times 207.5} = 1.28 \times 10^{-3} \text{ m}^3/\text{m}^2$$

Hence,

$$\tau = \frac{7.28^2 \times 10^{-6} + 2 \times 7.28 \times 10^{-3} \times 1.28 \times 10^{-3}}{0.79 \times 10^{-6}} = 91 \text{ s} = 1.52 \text{ min}$$

To determine the bowl speed, we must find the angular velocity of the filter by the formula

$$\omega = \frac{360 - \varphi'}{\tau + \tau_{\text{dry}}}$$

where $\tau = 1.52$ min = duration of filtering

$\tau_{\text{dry}} = 1.5$ min = duration of drying the cake

φ' = angle occupied by the cake removal zone and the dead zone; we assume that it equals 1.23 rad or 70° (on the basis of previous experience).

Therefore,

$$\omega = \frac{2\pi - 1.23}{3.02} = 1.67 \text{ rad/min} \left(\text{or } \frac{360 - 70}{3.02} = 96.2 \text{ deg/min} \right)$$

The total duration of an operating cycle, or the duration of one revolution of the bowl, is:

$$\tau_{\text{rev}} = \frac{2\pi}{1.67} \left(\text{or } \frac{360}{96.2} \right) \approx 3.8 \text{ min}$$

The number of revolutions of the filter an hour is:

$$n = \frac{60}{3.8} = 15.8 \text{ rph}$$

The duration of drying, removal of the cake, and travelling through the dead zones is

$$\tau_{\text{dry}} = 3.8 - 1.52 = 2.28 \text{ min}$$

The number of sections simultaneously travelling through the drying zone, sediment removal zone and the dead zones is:

$$z = \frac{2.28 \times 24}{3.8} \approx 14$$

The number of sections in the filtering zone is $24 - 14 = 10$.
The required surface area of the filter is:

$$A = \frac{Q_{v \text{ tot}}}{q_v n} = \frac{0.56}{0.00728 \times 15.8} = 4.86 \text{ m}^2$$

We adopt a vacuum filter with a filtering surface of 5 m^2 .

SYMBOLS

A	area
Ar	Archimedes dimensionless number
a	area; constant
b	width of agitator blade
C	constant; filtration constant; number of combinations
c	concentration
D	diameter
Di	Dyakonov dimensionless number
d	diameter
e	base of natural logarithms
F	centrifugal force
Fr	Froude dimensionless number
f	coefficient of friction; separation factor
G	weight
Ga	Galilei dimensionless number
g	acceleration due to gravity
H	height
h	depth; height
i	intensity of flow
K	coefficient taking into account washing conditions; coefficient of dust separation velocity; distribution coefficient; filtration constant
K_v	fluidization number
k	ratio of times
L	length
Ly	Lyashchenko dimensionless number
M	mass
m	constant (exponent); unit mass
N_p	power number
n	mass of liquid phase in suspension per unit of mass of solid phase; speed (number of revolutions per time unit)
P	perimeter; power
p	hydraulic resistance; pressure
Q_m	mass rate of flow
Q_v	volumetric rate of flow
q_v	unit volumetric rate of flow
R	radius; resistance
Re	Reynolds dimensionless number
r	specific resistance of sediment
S	shape factor
t	pitch of openings in grid
V	volume
v	velocity

W work
 x concentration; fraction of particles
 z factor; number of filter sections

Greek Letters

Δ relative density
 δ thickness of cake
 ε porosity
 ζ dimensionless local resistance coefficient
 η coefficient taking into account the ratio between the actual and theoretical capacities of a centrifuge; degree of dust separation
 μ dynamic viscosity
 ν kinematic viscosity
 ρ density
 σ tensile stress
 τ time
 Φ ratio of densities
 φ centrifuge bowl filling factor; coefficient characterizing a weir; fraction of clear section of a grid
 φ' angle on filter
 ψ degree of water surface ascent due to a weir
 ω angular velocity

FUNDAMENTAL RELATIONSHIPS
AND FORMULAS FOR CALCULATIONS

HEAT CONDUCTION

1. The equation of heat conduction for a steady-state flow of heat through a single-layer flat wall is:

$$q = \frac{Q}{A} = \frac{T_h - T_c}{r} = \frac{\lambda}{\delta} (T_h - T_c) \quad (4-1)$$

where q = unit heat flux (unit heat load), W/m²

Q = heat flux (consumption of heat), W

A = surface area of the wall, m²

T_h and T_c = temperatures of hot and cold surfaces of wall, respectively, K

$r = \delta/\lambda$ = thermal resistance of the wall, m²·K/W

δ = thickness of the wall, m

λ = thermal conductivity, W/m·K

The equation of heat conduction for a steady-state flow of heat through a multilayer flat wall is:

$$q = \frac{Q}{A} = \frac{T_h - T_c}{\sum r} = \frac{T_h - T_c}{\frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} + \dots} \quad (4-2)$$

2. The mean area of a cylindrical single-layer wall is:

$$A_m = \pi d_m L = \frac{\pi (d_2 - d_1) L}{\ln \frac{d_2}{d_1}} \quad (4-3)$$

where L = length of the cylinder, m

d_1 and d_2 = internal and external diameters.

When the ratio d_2/d_1 is less than two, we can use the quantity

$$A_m = \frac{\pi (d_1 + d_2) L}{2} \quad (4-4)$$

for the mean area of a single-layer cylindrical wall instead of that calculated by Eq. (4-3).

The equation of heat conduction for a steady-state flow of heat through a single-layer cylindrical wall is:

$$Q = \frac{\lambda}{\delta} (T_h - T_c) A_m = \frac{2\pi\lambda (T_h - T_c) L}{\ln \frac{d_2}{d_1}} \quad (4-5)$$

The equation of heat conduction for a steady-state flow of heat through a multilayer cylindrical wall is:

$$Q = \frac{2\pi L (T_h - T_c)}{\sum \frac{1}{\lambda} \ln \frac{d_{ex}}{d_{in}}} = \frac{2\pi L (T_h - T_c)}{\frac{1}{\lambda_1} \ln \frac{d_2}{d_1} + \frac{1}{\lambda_2} \ln \frac{d_3}{d_2} + \dots} \quad (4-6)$$

Here d_{in} and d_{ex} are the internal and external diameters of each cylindrical layer.

3. In the absence of experimental data, the thermal conductivity of a liquid λ (in W/m·K) at a temperature of about 30 °C can be calculated by the formula:

$$\lambda_{30} = C_a c \rho \sqrt[3]{\frac{\rho}{M}} \quad (4-7)$$

where c = specific heat capacity of the liquid, J/kg·K

ρ = density of the liquid, kg/m³

M = molar mass of the liquid, kg/kmol

C_a = coefficient depending on the degree of association of the liquid.

For associated liquids (for example, water), $C_a = 3.58 \times 10^{-8}$, for unassociated ones (for example, benzene), $C_a = 4.22 \times 10^{-8}$.

The thermal conductivity of a liquid at the temperature t is determined by the formula

$$\lambda_t = \lambda_{30} [1 - \varepsilon (t - 30)] \quad (4-8)$$

where ε is a temperature coefficient. The values of $\varepsilon \times 10^3$ for selected liquids are given below:

Acetic acid	1.2	Chlorobenzene	1.5	Hexane	2.0
Acetone	2.2	Chloroform	1.8	Methyl alcohol	1.2
Aniline	1.4	Ethyl acetate	2.1	Nitrobenzene	1.0
Benzene	1.8	Ethyl alcohol	1.4	Propyl alcohol	1.4

The thermal conductivity of an aqueous solution at the temperature t is determined by the formula:

$$\lambda_{s, t} = \lambda_{s, 30} \frac{\lambda_{w, t}}{\lambda_{w, 30}} \quad (4-9)$$

where λ_s and λ_w are the thermal conductivities of the solution and water, respectively.

4. The thermal conductivity of a gas (in W/m·K) at moderate (not high) pressures can be calculated by the formula

$$\lambda = B c_v \mu \quad (4-10)$$

where c_v = specific heat capacity of the gas at constant volume, J/kg·K

μ = dynamic viscosity of the gas, Pa·s

$B = \frac{9\gamma - 5}{4}$ ($\gamma = c_p/c_v$ is the adiabatic exponent).

Since for gases having a given atomicity (valency), the ratio $c_p/c_v = \gamma$ is an approximately constant quantity, then for monatomic gases $B = 2.5$, for diatomic ones $B = 1.9$, and for triatomic gases $B = 1.72$.

The rule of additivity cannot be used in the general case to calculate the thermal conductivity of a mixture of gases. For an approximate calculation of λ for a mixture of gases, see Example 4-6.

HEAT TRANSFER

5. Table 4-1 gives a list of the main cases of heat transfer and the corresponding equations to be used for calculations.

TABLE 4-1

Kind of heat transfer	No. of equation
A. Convection, heat transfer not attended by a change in the state of aggregation	
I. Forced Convection	
(1) flow in tubes, pipes, and channels:	
(a) developed turbulent flow	(4-16)-(4-21)
(b) transition region	(4-22)
(c) laminar flow	(4-23), (4-24)
(2) flow perpendicular to tube banks:	
(a) smooth tubes	(4-25)-(4-28)
(b) finned tubes	(4-29), (4-30)
(3) flow along a flat wall	(4-31)-(4-33)
(4) flow of liquid film down a vertical surface	(4-34)-(4-38)
(5) mixing of liquids with agitators	(4-39)
II. Natural or Free Convection	(4-40)-(4-42)
B. Heat transfer upon a change in the state of aggregation	
(1) film condensation of a vapour	(4-43)-(4-52)
(2) boiling of liquids	(4-53)-(4-58)
C. Heat transfer in the thermal radiation of solids	(4-59)-(4-64)

6. The main dimensionless numbers in the dimensionless formulas for convective heat transfer are as follows:

The Nusselt number characterizing the intensity of heat transfer at a flow-wall boundary is:

$$Nu = \frac{\alpha_i}{\lambda}$$

(4-11)

The Prandtl number characterizing the ratio of the viscosity and thermal diffusivity properties of the heat-transfer agent is:

$$Pr = \frac{c_p \mu}{\lambda} = \frac{\nu}{a}$$

(4-12)

The Reynolds number characterizing the ratio of the forces of inertia and friction in a flow is:

$$Re = \frac{vl\rho}{\mu} = \frac{vl}{\nu}$$
 (4-13)

The Galilei number characterizing the ratio of the forces of gravity, inertia, and friction in a flow is

$$Ga = \frac{Re^2}{Fr} = \frac{gl^3\rho^2}{\mu^2} = \frac{gl^3}{\nu^2}$$
 (4-14)

The Grashof number characterizing the ratio of the forces of friction, inertia, and buoyancy due to the difference in densities at separate points of a non-isothermal flow is

$$Gr = Ga\beta\Delta t = \frac{gl^3}{\nu^2}\beta\Delta t$$
 (4-15)

The quantities in the expressions for the dimensionless numbers and their units are listed in Table 4-2.

TABLE 4-2

Quantity	Unit
α =coefficient of heat transfer	W/m ² ·K
β =coefficient of volume expansion of fluid	K ⁻¹
ρ =density of fluid	kg/m ³
λ =thermal conductivity	W/m·K
Δt =temperature drop between wall and fluid (or vice versa)	K
μ =dynamic viscosity of fluid	Pa·s
ν =kinematic viscosity of fluid	m ² /s
$a=\lambda/c\rho$ =thermal diffusivity	m ² /s
c =specific heat capacity (at constant pressure)	J/kg·K
g =acceleration due to gravity	m/s ²
l =defining geometrical dimension (indicated for each formula)	m
v =velocity	m/s
L =heat of vaporization	J/kg

The physicochemical constants of a fluid to be used in the dimensionless equations for convection heat transfer must be taken from reference data corresponding to the so-called defining temperature. The temperature adopted as the defining one is indicated for each particular case of heat transfer.

7. Many dimensionless equations for convection include the factor $(Pr/Pr_w)^{0.25}$. The latter takes into account the direction of the heat flux and is close to unity when the temperatures of the fluid and the wall do not greatly differ. When calculating the number Pr_w , the values of the physicochemical constants for

the fluid should be taken according to the temperature of the wall.

Approximate values of the number Pr for dropping liquids can be determined according to the nomogram in Fig. A-13. Its values for water are given in Table A-39.

The value of the number Pr for dropping liquids diminishes with increasing temperature—see Fig. A-13. Consequently, when such liquids are heated, we have $(Pr/Pr_w) > 1$, and when they are cooled $(Pr/Pr_w) < 1$. On these grounds, when designing heat exchangers we may assume that $(Pr/Pr_w)^{0.25} = 1$ in calculations of the heat transfer coefficients for the heating liquids. Here a slight error is allowed in the direction of lowering the heat transfer coefficient, i.e. in the direction of creating a margin of safety. For cooling liquids when $(Pr/Pr_w) \geq 0.5$, we can adopt with sufficient accuracy the mean value of $(Pr/Pr_w)^{0.25}$ equal to 0.93.

For gases, $(Pr/Pr_w) = 1$ both in heating and in cooling because for a gas of the given atomicity (at moderate pressures) the number Pr is an approximately constant quantity not depending on the temperature and pressure.

It is recommended to use the following approximate values of the number Pr for gases in calculations:

Monatomic gases	0.67
Diatomic gases	0.74
Triatomic gases	0.80
Tetratomic and polyatomic gases	1.0

8. Heat transfer in developed turbulent flow in straight pipes, tubes and channels ($Re > 10\,000$).

The formula to be used for calculations is:

$$Nu = 0.021 \epsilon_l Re^{0.8} Pr^{0.43} \left(\frac{Pr}{Pr_w} \right)^{0.25} \quad (4-16)$$

This equation has been used to construct a nomogram (Fig. A-12) recommended for calculations.

For the expressions for the numbers Nu , Re , and Pr see Eqs. (4-11) *et seq*, and also Table 4-2.

The defining temperature is the average temperature of the fluid, and the defining geometrical dimension l is the equivalent diameter d_{eq} :

$$d_{eq} = \frac{4A}{P} \quad (4-17)$$

where A = cross-sectional area of the flow

P = total perimeter of the flow section regardless of what part of the perimeter participates in heat exchange.

For round tubes (pipes), $d_{eq} = d$.

The values of the correction factor ϵ_l , allowing for the influence of the ratio of the tube length L to its diameter d on the heat transfer coefficient are given in Table 4-3.

TABLE 4-3

Value of number Re	Values of ϵ_l at ratio L/d equal to				
	10	20	30	40	50 and more
1×10^4	1.23	1.13	1.07	1.03	1
2×10^4	1.18	1.10	1.05	1.02	1
5×10^4	1.13	1.08	1.04	1.02	1
1×10^5	1.10	1.06	1.03	1.02	1
1×10^6	1.05	1.03	1.02	1.01	1

For bent tubes (coils), the value of α obtained by Eq. (4-16) is multiplied by the coefficient x taking into account the relative curvature of the coil:

$$\alpha_{coil} = x\alpha$$

(4-18)

The coefficient x is calculated by the equation:

$$x = 1 + 3.54 \frac{d}{D}$$

(4-19)

where d = internal diameter of the coil tube

D = diameter of a coil turn.

For gases, Eq. (4-16) is simplified because $Pr/Pr_w = 1$, and Pr depends only on the atomicity of a gas. Thus the equation acquires the form

$$Nu = C\epsilon_l Re^{0.8}$$

(4-20)

For example, for air we have:

$$Nu = 0.018\epsilon_l Re^{0.8}$$

(4-21)

9. Heat transfer in the transition region ($2300 < Re < 10\,000$).

No accurate relationships are available for calculations.

In practice, it is recommended to use a graph such as that shown in Fig. 4-1 or the approximate equation (for values of Re close to 10 000):

$$Nu = 0.008 Re^{0.9} Pr^{0.43}$$

(4-22)

The defining temperature is the average temperature of the fluid, and the defining dimension is the equivalent diameter calculated by Eq. (4-17).

10. Heat transfer in laminar flow through straight pipes, tubes, and channels ($Re < 2300$).

(a) In the absence of the appreciable influence of free convection, when $Gr \ll 4 Re Nu$, the formula used for calculations with

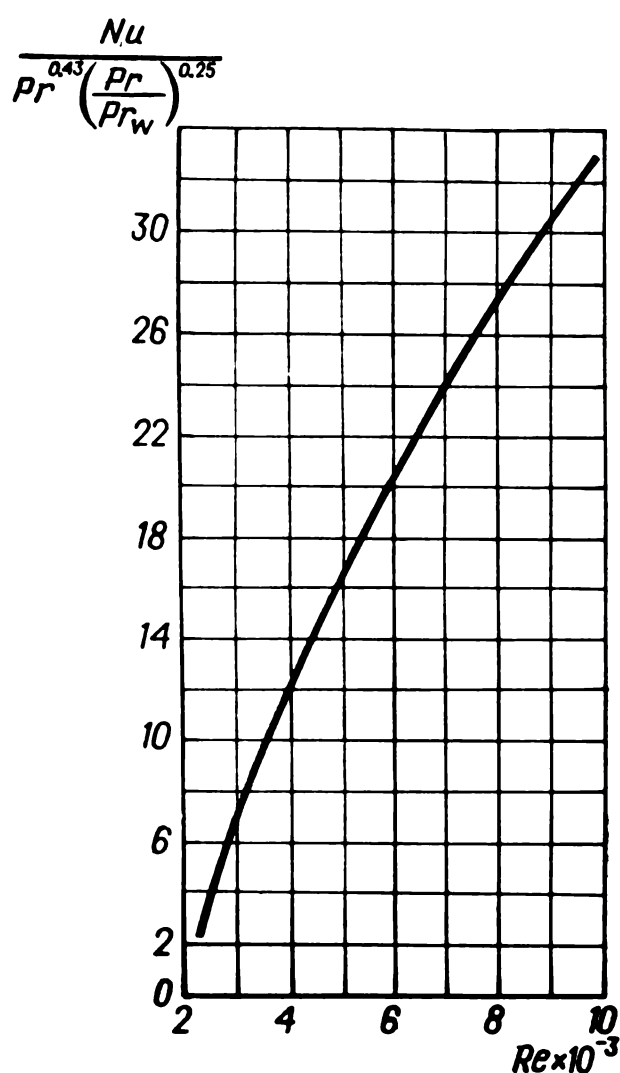


Fig. 4-1. Dependence of $\frac{Nu}{Pr^{0.43} \left(\frac{Pr}{Pr_w}\right)^{0.25}}$ on the number Re in the transition zone

$Re > 10$ and $L/d > 10$ has the form [4-18]:

$$Nu = 1.4 \left(Re \frac{d}{L} \right)^{0.4} Pr^{0.33} \left(\frac{Pr}{Pr_w} \right)^{0.25} \quad (4-23)$$

where L is the length of the tube or pipe, m.

The defining temperature is the average temperature of the fluid, the defining dimension is the diameter of the tube or pipe, or the width of the channel. Equation (4-23) holds when $Re \frac{d}{L} Pr^{5/6} > 15$. If $Re \frac{d}{L} Pr^{5/6} < 15$, it is recommended to use the approximate relationship

$$Nu = 4 \left(\frac{Pr}{Pr_w} \right)^{0.25} \quad (4-24)$$

(b) When $Gr > 4Re Nu^*$, the influence of free convection is considerable. In this case, it is recommended to determine the value of the number Nu according to a graph (Fig. 4-2) constructed on the basis of experimental data [4-19] for viscous-gravitational conditions** at values of $Gr Pr = (8 \text{ to } 25) \times 10^5$.

* The number Nu is calculated by Eq. (4-23), the number Gr —by Eq. (4-15) at $l = d$.

** According to the data of [4-16], viscous-gravitational conditions set in when $Gr Pr > 5 \times 10^5$.

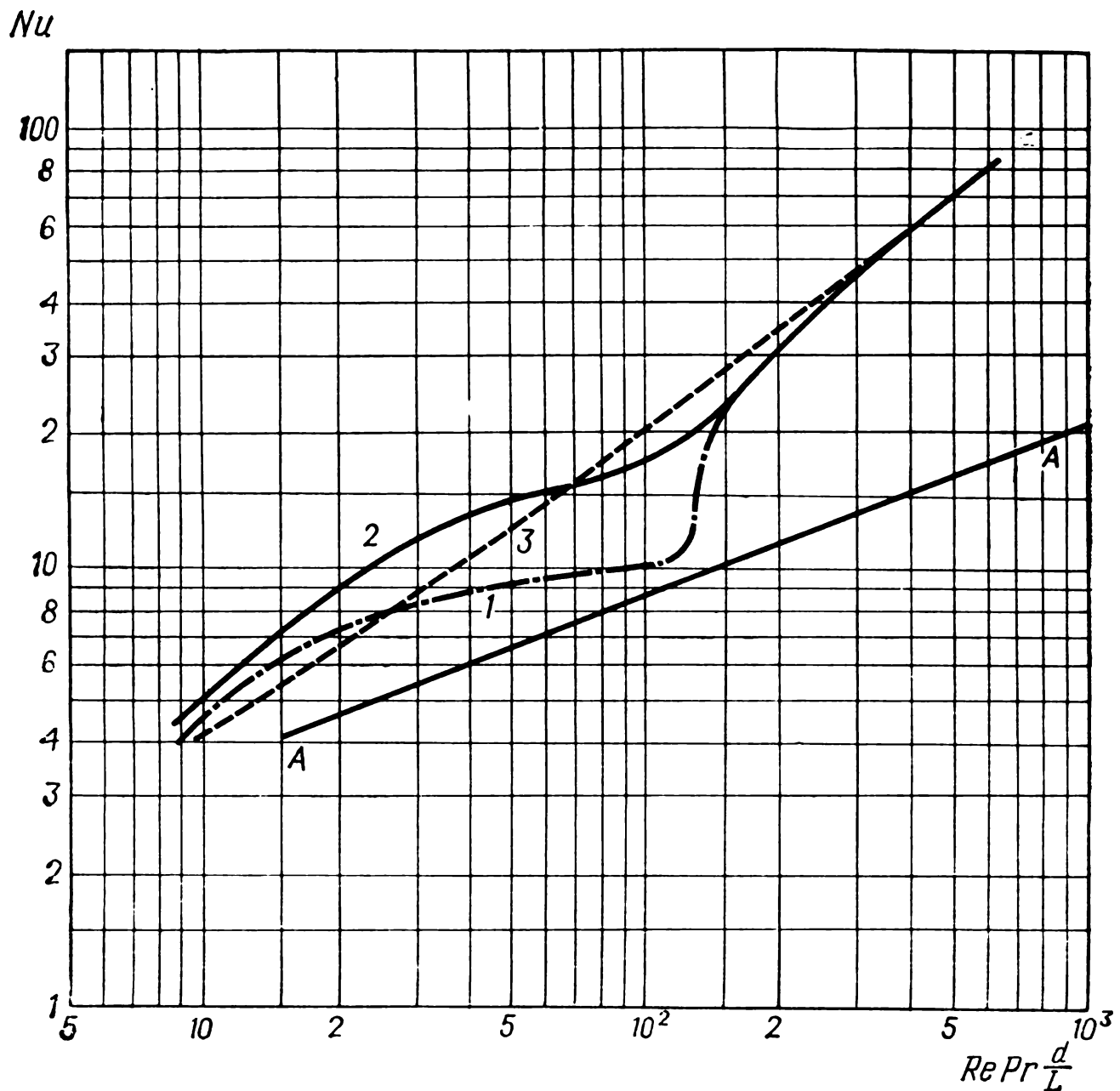


Fig. 4-2. Heat transfer in laminar flow:

1—vertical pipes (the directions of forced and free convection coincide); 2—horizontal pipes; 3—vertical pipes (the directions of forced and free convection are opposite); A-A—in the absence of free convection

In all the dimensionless numbers in the graph (Fig. 4-2), the physicochemical constants have been taken for a temperature of the boundary layer equal to $0.5(t_{fl} + t_w)$. The calculations are facilitated by the fact that the product $Re Pr \frac{d}{L} = \frac{vd^2}{aL}$ (here a is the thermal diffusivity) within small limits of a change in temperature is virtually constant (see Example 4-II).

Line AA in Fig. 4-2 has been constructed according to the equation $Nu = 1.4 \left(Re Pr \frac{d}{L} \right)^{0.4}$, which is close to Eq. (4-23) when $(Pr/Pr_w)^{0.25} = 1$.

11. Heat transfer in flow perpendicular to banks of smooth tubes.

(a) $Re < 1000$. For square pitch and triangular pitch tube-

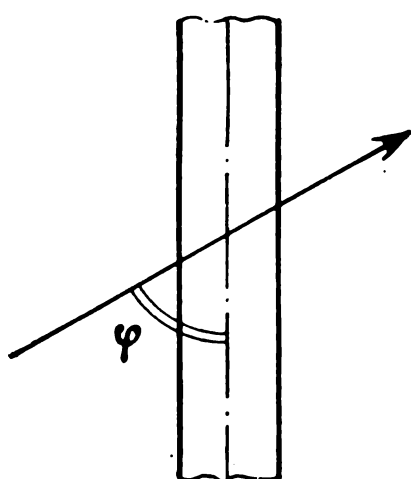


Fig. 4-3. Angle of attack

bank arrangements, we have:

$$Nu = 0.56 Re^{0.5} Pr^{0.36} \left(\frac{Pr}{Pr_w} \right)^{0.25} \epsilon_\varphi \quad (4-25)$$

(b) $Re > 1000$. For square pitch arrangements:

$$Nu = 0.22 Re^{0.65} Pr^{0.36} \left(\frac{Pr}{Pr_w} \right)^{0.25} \epsilon_\varphi \quad (4-26)$$

For triangular pitch arrangements:

$$Nu = 0.4 Re^{0.6} Pr^{0.36} \left(\frac{Pr}{Pr_w} \right)^{0.25} \epsilon_\varphi \quad (4-27)$$

The defining temperature is the average temperature of the fluid, the defining dimension is the external diameter of the tube. The velocity of the flow is determined for the narrowest section of the bank (see Example 4-II).

The coefficient ϵ_φ takes into account the influence of the angle of attack φ (Fig. 4-3). Its values are given in Table 4-4.

TABLE 4-4

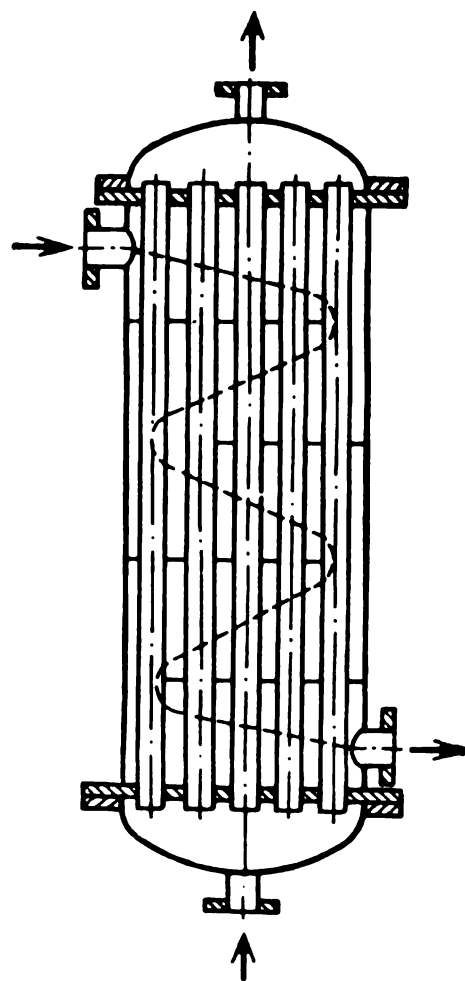
φ , deg	90	80	70	60	50	40	30	20	10
ϵ_φ	1	1	0.98	0.94	0.88	0.78	0.67	0.52	0.42

Equations (4-25) to (4-27) give the values of the heat transfer coefficients for the third and following rows of tubes in a bank. With a sufficiently great number of rows, these values can be considered approximately as average for the entire bank.

For gases, the equations become simpler because $Pr/Pr_w = 1$, and Pr depends only on the atomicity of the relevant gas. For air at $Re > 1000$ and with a triangular pitch arrangement of the tubes in the bank, we have:

$$Nu = 0.356 Re^{0.6} \epsilon_\varphi \quad (4-28)$$

Fig. 4-4. Shell-and-tube heat exchanger with segmental baffles



When Eqs. (4-25) to (4-28) are used for shell-and-tube heat exchangers with baffles (Fig. 4-4), the coefficient ε_ϕ in them is adopted equal to 0.6. This accounts for the circumstance that the shell-side fluid flows across the tubes only part of its path and with an angle of attack less than 90° ; in addition, it may leak between baffles and the shell or between baffles and tubes.

When the tubes are installed along hexagons [see Fig. A-2 (Table A-35)] to ensure their staggered arrangement (triangular pitch), which is more effective, the inlet pipe for the shell-side fluid and the edges of the segmental baffles (chords $a-a$) are arranged as shown in Fig. 4-5.

12. Heat transfer in flow around a bank of tubes with transverse fins.

The formula used for calculations has the form *:

$$Nu = C \left(\frac{d}{t} \right)^{-0.54} \left(\frac{h}{t} \right)^{-0.14} Re^n Pr^{0.4} \quad (4-29)$$

In this formula (Fig. 4-6):

d = external diameter of the tube carrying the fins, m

D = diameter of a fin, m

t = pitch of the fins, m

$h = (D - d)/2$ = height of a fin, m.

* Karasina, E. S. *Izv. VTI*, 12: 12 (1952).

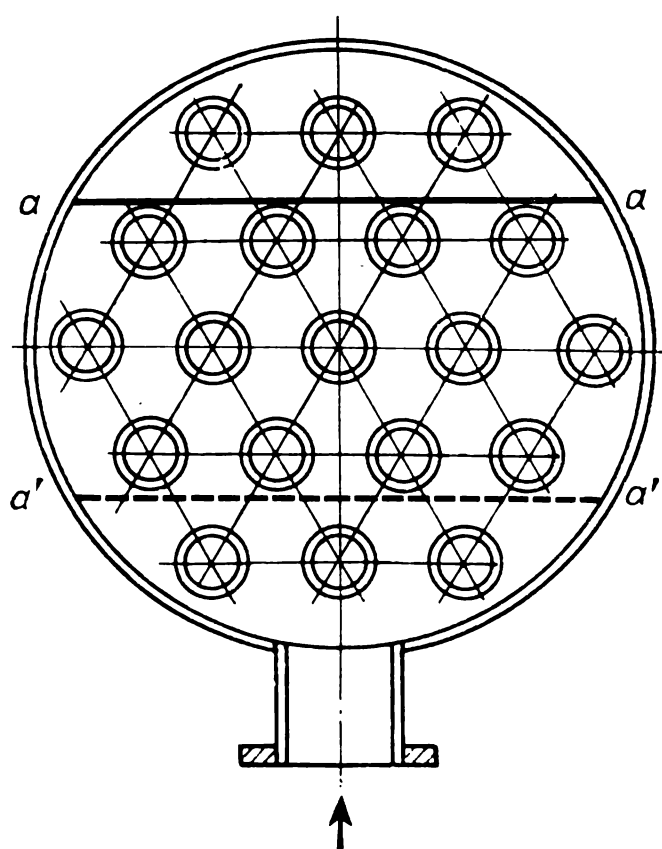


Fig. 4-5. Arrangement of inlet pipe for shell-side fluid and segmental baffles

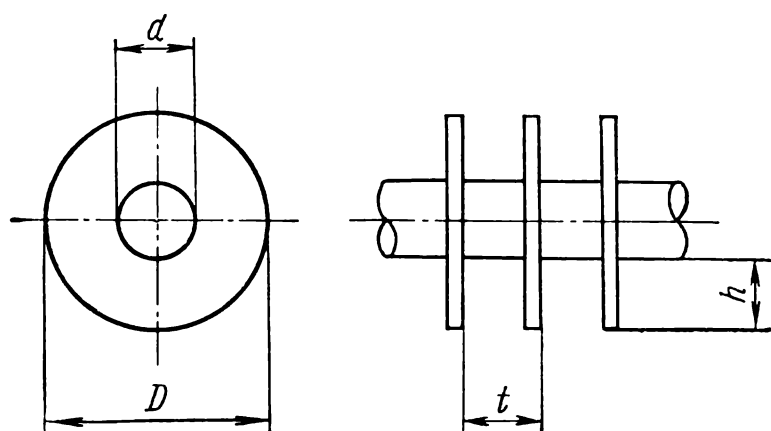


Fig. 4-6. Tube with transverse fins

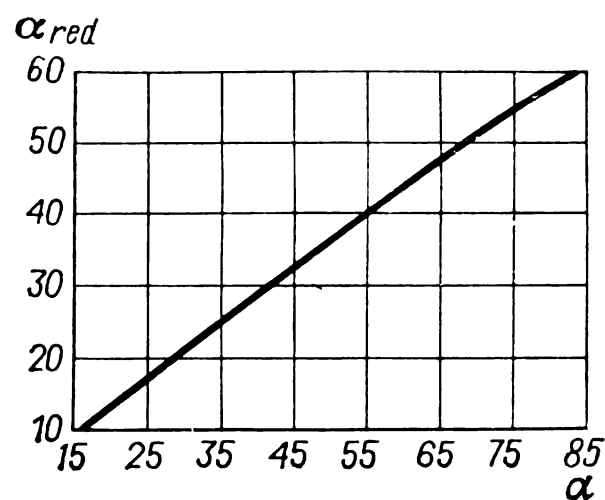


Fig. 4-7. Dependence of α_{red} on α

For square pitch banks, $C=0.116$ and $n=0.72$; for triangular pitch banks, $C=0.25$ and $n=0.65$.

The defining temperature is the average temperature of the fluid, the defining dimension is the pitch t of the fins.

Equation (4-29) may be applied for values of Re ranging from 3000 to 25 000 and values of d/t ranging from 3 to 4.8.

The heat transfer coefficient calculated by Eq. (4-29) is used to determine in a graph (Fig. 4-7) the so-called reduced heat transfer coefficient α_{red} , and the latter is inserted in the equation for the overall heat transfer coefficient K (related to the total external surface area A_{ex}):

$$K = \frac{1}{\frac{1}{\alpha_{\text{red}}} + \frac{1}{\alpha_2} \frac{A_{\text{ex}}}{A_{\text{in}}} + \sum r_w} \quad (4-30)$$

where A_{ex} = area of the total external surface of a finned tube per unit length including the surface of the fins

A_{in} = area of the internal surface of the finned tube per unit length

α_2 = heat transfer coefficient for the flow inside the tube, $\text{W/m}^2 \cdot \text{K}$

$\sum r_w$ = sum of thermal resistances of the wall and contaminants.

13. Heat transfer in flow along a flat surface.

The formulas used for calculations are:

(a) When $Re < 5 \times 10^5$:

$$Nu = 0.66 Re^{0.5} Pr^{0.33} \left(\frac{Pr}{Pr_w} \right)^{0.25} \quad (4-31)$$

(b) When $Re > 5 \times 10^5$:

$$Nu = 0.037 Re^{0.8} Pr^{0.43} \left(\frac{Pr}{Pr_w} \right)^{0.25} \quad (4-32)$$

The defining temperature is the average temperature of the fluid, the defining dimension is the length of the wall in contact with the flow in the direction of its motion.

The nomogram given in Fig. A-12 can be used for calculations by Eq. (4-32), the value of the number Nu obtained being multiplied by $0.037/0.021 = 1.76$.

The above formulas become simpler for gases. For air, Eq. (4-32) acquires the form

$$Nu = 0.032 Re^{0.8} \quad (4-33)$$

14. Heat transfer when a liquid film flows down a vertical surface.

(a) For turbulent flow ($Re > 2000$):

$$Nu = 0.01 (Ga Pr Re)^{1/3} \quad (4-34)$$

(b) For laminar flow ($Re < 2000$):

$$Nu = 0.67 (Ga^2 Pr^3 Re)^{1/9} \quad (4-35)$$

The defining temperature is the average temperature of the boundary layer equal to $(t_w + t_{\text{av, lq}})/2$. The defining dimensions

in the numbers Nu , Ga , and Re are different:

$$Nu = \frac{\alpha H}{\lambda}$$

$$Ga = \frac{H^3 \rho^2 g}{\mu^2}$$

$$Re = \frac{v d_{eq} \rho}{\mu}$$

where H = height of the surface, m

$d_{eq} = 4A/P$ = equivalent diameter of the film, m

A = cross-sectional area of the film, m²

P = perimeter in contact with the film, m.

For tubular film heat exchangers in which the liquid flows in a film down the internal surface of a vertical tube, we have:

$$A = \pi (d - b) b$$

$$P = \pi d$$

$$d_{eq} = \frac{4A}{P} = \frac{4(d - b)b}{d} \quad (4-36)$$

where d = internal diameter of a tube, m

b = thickness of the film, m.

If Q_m kg of liquid a second flows through the tubes of a film heat exchanger consisting of n tubes, then from the equation of the flow rate we have:

$$v\rho = \frac{Q_m}{An} = \frac{Q_m}{\pi (d - b) bn}$$

Consequently,

$$Re = \frac{v d_{eq} \rho}{\mu} = \frac{4Q_m}{\pi d n \mu} \quad (4-37)$$

When $Re \leq 1500$, the thickness of the film b is determined by the theoretical equation

$$b = \sqrt[3]{\frac{3Q_m \mu}{P \rho^2 g}} \quad (4-38)$$

15. Heat transfer in stirring liquids with agitators.*

The coefficient of heat transfer in an apparatus with a coil or jacket and an agitator can be calculated by the equation:

$$Nu = C Re^m Pr^{0.33} \left(\frac{\mu}{\mu_w} \right)^{0.14} S^{-1} \quad (4-39)$$

where $Nu = \frac{\alpha d_{ag}}{\lambda}$; $Re = \frac{\rho n d_{ag}^2}{\mu}$; and $S = \frac{D}{d_{ag}}$

* See also [3-15] and [3-16].

- D = diameter of the vessel
 n = speed of the agitator
 d_{ag} = diameter of the circle covered by the agitator
 μ_w = dynamic viscosity of the liquid at the temperature of the jacket or coil wall
 μ = dynamic viscosity of the liquid at the average temperature $(t_{av, lq} + t_w)/2$.

The values of the remaining physical constants must be taken at the average temperature of the liquid in the vessel $t_{av, lq}$.

For apparatuses with jackets, we have $C = 0.36$, and $m = 0.67$; for ones with coils, we have $C = 0.87$, and $m = 0.62$.

Equation (4-39) gives satisfactory results for turbine, propeller and paddle agitators with $S = D/d_{ag}$ ranging from 2.5 to 4 in apparatuses up to 1.5 m in diameter.

16. Natural convection.

The equations used for calculations are:

A. Heat transfer outside of horizontal pipes at $10^3 < Gr Pr < 10^9$:

$$Nu = 0.5 (Gr Pr)^{0.25} \left(\frac{Pr}{Pr_w} \right)^{0.25} \quad (4-40)$$

B. For vertical flat and cylindrical surfaces:

(a) at $10^3 < Gr Pr < 10^9$, we have:

$$Nu = 0.76 (Gr Pr)^{0.25} \left(\frac{Pr}{Pr_w} \right)^{0.25} \quad (4-41)$$

(b) at $Gr Pr > 10^9$, we have

$$Nu = 0.15 (Gr Pr)^{0.33} \left(\frac{Pr}{Pr_w} \right)^{0.25} \quad (4-42)$$

The defining temperature is the ambient one, the defining dimension is the diameter for horizontal pipes and the height for vertical surfaces.

17. Heat transfer in the film condensation of a saturated vapour.

(a) Condensation on a flat or cylindrical vertical surface having a height of H metres with laminar flow of the condensate film.

The theoretical equation proposed by W. Nusselt with a correction for wave formation in the film (with $\rho_{lq} \gg \rho_v$) is:

$$\alpha = 1.15 \sqrt[4]{\frac{\lambda^3 \rho^2 L g}{\mu \Delta t H}} \quad (4-43)$$

where $\Delta t = t_{cond} - t_w$.

For the remaining symbols see Table 4-2. The values of the physicochemical constants of the liquid (condensate) λ , ρ , and μ in Eq. (4-43) are related to the average temperature of the condensate film $t_f = (t_{cond} + t_w)/2$, and the value of L to the condensation temperature t_{cond} . When the temperature drop $(t_{cond} - t_w)$ does not exceed 30 to 40 kelvins, we may take the values of the

constants according to the condensation temperature with sufficient accuracy in practical calculations—see Example 4-III.

(b) Condensation on the external surface of a single horizontal pipe with a diameter of d m.

The theoretical formula is:

$$\alpha = 0.72 \sqrt[4]{\frac{\lambda^3 \rho^2 L g}{\mu \Delta t d}} \quad (4-44)$$

The quantities here are the same as in Eq. (4-43).

For a more accurate calculation of the coefficients of heat transfer in the condensation of steam on vertical and horizontal pipes and tubes see [4-18]. In practical calculations, when the second heat transfer coefficient is considerably lower than for pure steam, we can approximately assume for condensing steam that α ranges from 10 000 to 12 000 W/m²·K.

(c) Condensation of steam on the external surface of a bank of horizontal tubes.

In a bank of horizontal tubes, the layer of condensate on the lower tubes grows at the expense of the condensate dropping from the higher ones. This results in reduction of the heat transfer coefficient for the lower rows.

The average heat transfer coefficient for the entire bank can be calculated by the equation:

$$\alpha_{av} = \varepsilon \alpha \quad (4-45)$$

where α = heat transfer coefficient for a single horizontal tube calculated by Eq. (4-44)

ε = coefficient averaged for the entire bank that depends on the arrangement of the tubes in the bank and the number of tubes n_v in each vertical row; the values of ε are taken from a graph (Fig. 4-8).

(d) Equations (4-43) and (4-44) can be represented in a different form that is often more convenient for technical calculations. Introducing into these equations the value of $\Delta t = q/\alpha$ (where q is the unit heat load, W/m²), we get:
for a vertical surface

$$\alpha = 1.21 \lambda \left(\frac{\rho^2 L g}{\mu H} \right)^{1/3} q^{-1/3} \quad (4-46)$$

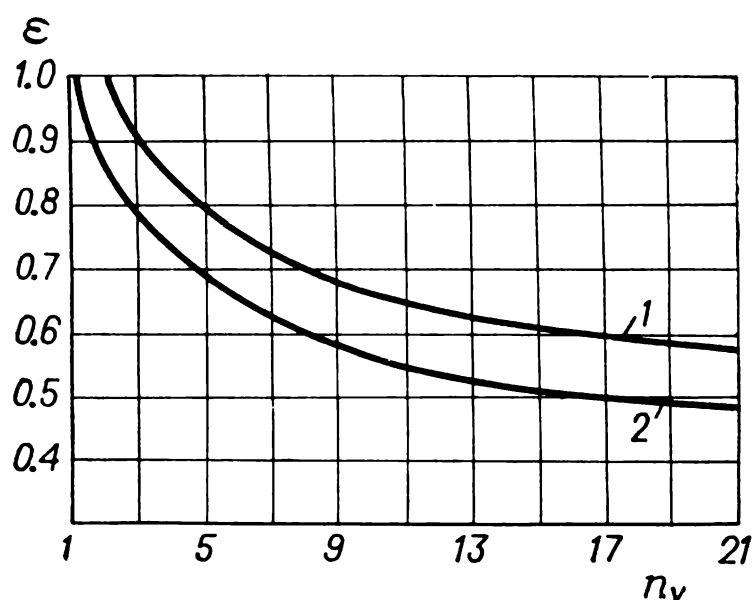
for a single horizontal pipe

$$\alpha = 0.645 \lambda \left(\frac{\rho^2 L g}{\mu d} \right)^{1/3} q^{-1/3} \quad (4-47)$$

We can obtain the dependence of the coefficient of heat transfer in a tubular condenser on the rate of flow of the condensing

Fig. 4-8. Dependence of coefficient ε on the number of tubes n_v in a vertical row

1—triangular pitch; 2—square pitch



steam. From the equations:

$$LQ_m = \alpha \Delta t A \quad (4-48)$$

$$A = \pi d n l \quad (4-49)$$

where L = specific heat of condensation of steam

Q_m = mass rate of flow of the condensing steam

A = surface area of the tubular condenser consisting of n tubes with an external diameter of d

l = length of a tube (for vertical tubes the height H should be inserted instead of l),

we find

$$H \Delta t = \frac{LQ_m}{\alpha \pi d n} \quad \text{and} \quad d \Delta t = \frac{LQ_m}{\alpha \pi n l}$$

Using these values in Eqs. (4-43) and (4-44), respectively, we get:

for vertical tubes

$$\alpha = 3.78 \lambda \sqrt[3]{\frac{\rho^2 d n}{\mu Q_m}} \quad (4-50)$$

for a bank of horizontal tubes

$$\alpha_{av} = 2.02 \varepsilon \lambda \sqrt[3]{\frac{\rho^2 n l}{\mu Q_m}} \quad (4-51)$$

where ε is the same coefficient as in Eq. (4-45).

(e) Condensation of steam inside horizontal tubes and coils.

The general dimensionless relationship for the condensation of steam is reduced to the form [5-3]:

$$\alpha = 1.36 C q^{0.5} l^{0.35} d^{-0.25} \quad (4-52)$$

where C = coefficient combining the physicochemical constants of water and steam (its values depending on the temperature of condensation are given in Fig. 4-9)

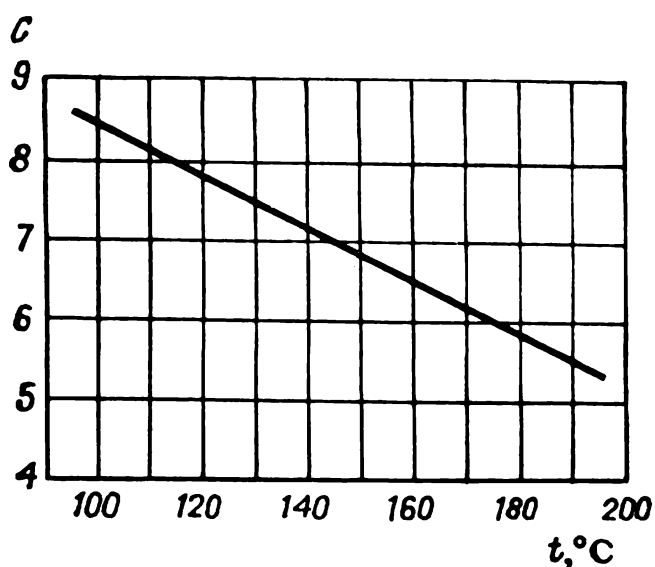


Fig. 4-9. Values of coefficient C in Eq. (4-52)

q = unit heat load, W/m^2

l = length of a tube, m

d = internal diameter of a tube, m.

The length of the coil for condensation of steam should not be very great because condensate accumulates in the lower part of long coils and this detracts from heat transfer. In addition, the vapour pressure diminishes which results in lowering of the effective temperature drop.

According to practical data, the initial velocity of the steam in steam coils should not be greater than about 30 m/s. At a mean temperature drop of Δt_m ranging from 30 to 40 K, the maximum ratio of the coil length to the tube diameter l/d depending on the vapour pressure p is:

p_{abs}							
kPa
at
$(l/d)_{\text{max}}$
	490.5	294.3	147.1	78.5			
	5	3	1.5	0.8			
	275	225	175	125			

For other values of Δt_m , the values of l/d given above for steam coils should be multiplied by the factor $6\Delta t_m^{-1/2}$.

(f) Condensation of steam containing a non-condensing gas (e. g. air).

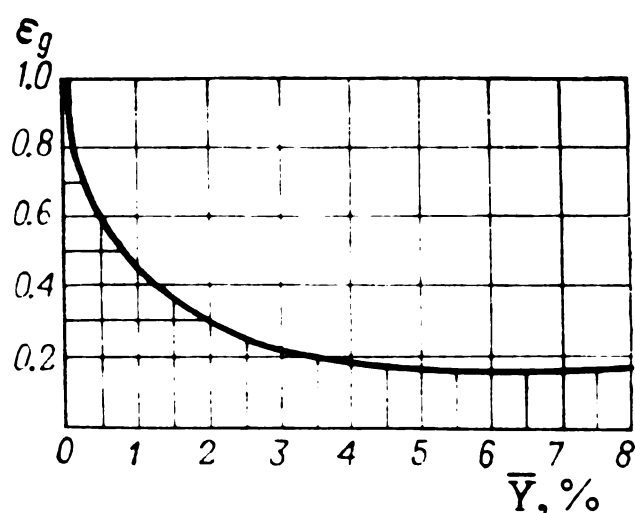
If steam contains air or another non-condensing gas, then heat transfer becomes much poorer in condensation. Figure 4-10 gives experimentally obtained values of the ratio $\epsilon_g = \alpha_{\text{air}}/\alpha$ depending on the concentration \bar{Y} of the air in the steam.

Here α = coefficient of heat transfer upon the condensation of pure steam calculated by one of the equations (4-43)-(4-52)

α_{air} = coefficient of heat transfer when the steam contains air

\bar{Y} = mass ratio of the air in the steam, kg of air per kg of steam.

Fig. 4-10. Dependence of correction factor ϵ_g on the concentration \bar{Y} of air in steam



Knowing the values of ϵ_g and α , we can calculate α_{air} by an equation similar to Eq. (4-45):

$$\alpha_{\text{air}} = \epsilon_g \alpha$$

18. Heat transfer in the boiling of liquids.

In developed nucleate boiling when the unit heat load q is less than the critical value ($q_{\text{cr},1}$), the heat transfer coefficient for a boiling liquid can be calculated [4-18] by the equation:

$$\alpha = b \left(\frac{\lambda^2}{v\sigma T_b} \right)^{1/3} q^{2/3} \quad (4-53)$$

or

$$\alpha = b^3 \left(\frac{\lambda^2}{v\sigma T_b} \right) \Delta t^2 \quad (4-54)$$

Here b = dimensionless coefficient depending only on the ratio of the densities of the liquid and its vapour (its values are given in Fig. 4-11):

$$b = 0.075 \left[1 + 10 \left(\frac{\rho_{\text{liq}}}{\rho_v} - 1 \right)^{-2/3} \right] \quad (4-55)$$

σ = surface tension, N/m

T_b = boiling point, K

q = unit heat load, W/m²

$\Delta t = t_w - t_b$.

For the remaining symbols see Table 4-2. All the physicochemical constants should be taken for the boiling point.

Equation (4-53) can be used to calculate the coefficients of heat transfer for the boiling of a liquid in a large space and in tubes. The deviations of experimental data from those calculated by Eq. (4-53) are within the limits of $\pm 35\%$. The transfer of heat in boiling is considerably affected by the material, state and cleanliness of the heating surface.

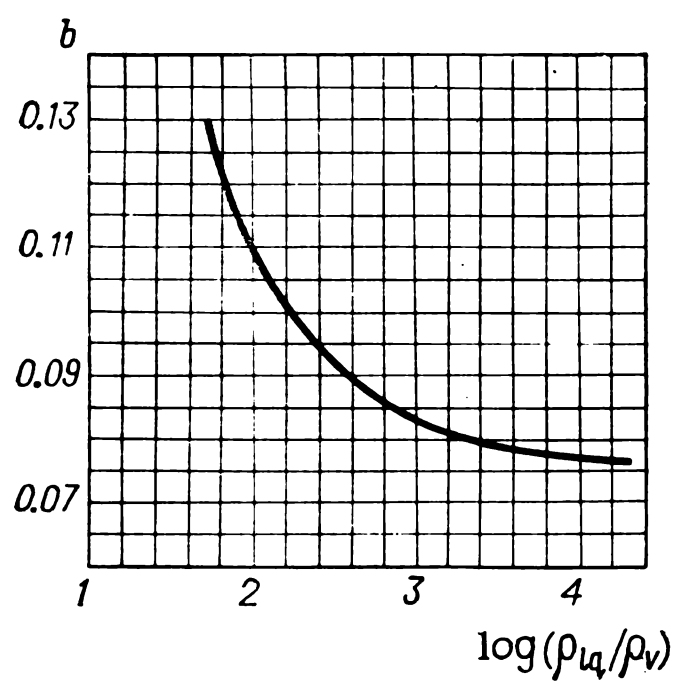


Fig. 4-11. Values of coefficient b

For boiling water, Eq. (4-3) becomes:

$$\alpha = \frac{3.4p^{0.18}}{1 - 0.0045p} q^{2/3}$$

(4-56)

where p is the pressure, at.
The value of the critical heat load $q_{cr,1}$ for the boiling of a liquid on horizontal tubes in a large space is determined by the equation (when $\rho_{lq} \gg \rho_v$):

$$q_{cr,1} = 0.14L \sqrt{\rho_v} \sqrt[4]{\sigma g \rho_{lq}}$$

(4-57)

where L is the heat of vaporization, J/kg.
The deviations of experimental data from those calculated by Eq. (4-57) are also within the limits of $\pm 35\%$.

The following equation can be used for approximate calculations of the heat transfer coefficient for nucleate boiling of a liquid in a large space on the external surface of banks of tubes and in vertical evaporators in the region of moderate heat loads (up to $0.4q_{cr}$) and pressures of p_{abs} ranging from 0.2 to 10 at [4-5]:

$$\alpha = 2.72\varphi p_{abs}^{0.4} q^{0.7}$$

(4-58)

where φ is a factor accounting for the physical properties of the liquid; p_{abs} is in at.

The experimentally found values of the factor φ (for boiling on the surface of tubes made of non-ferrous metals) are given below:

Benzene	0.31	Water	1
Ethyl alcohol	0.45	26% aqueous solution of glycerine	0.83
Gasoline	0.27	9% aqueous solution of NaCl	0.86
Heptane	0.46	24% aqueous solution of NaCl	0.62
Kerosene	0.31-0.56	10% aqueous solution of Na ₂ SO ₄	0.91
Methyl alcohol	0.36	25% aqueous solution of sugar	0.57

19. Heat transfer in the thermal radiation of solids.

The quantity of heat passing from a hotter body to a colder one by radiation is determined by the equation:

$$Q_{\text{rad}} = W_{1-2} \varphi A \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right] \quad (4-59)$$

where Q_{rad} = quantity of heat transmitted by radiation in a unit time, W

A = area of radiating surface, m^2

W_{1-2} = radiating power, $\text{W}/\text{m}^2 \cdot \text{K}^4$

T_1 = temperature of the surface of the hotter body, K

T_2 = temperature of the surface of the colder body, K

φ = dimensionless angular coefficient.

The radiating power W_{1-2} depends on the mutual arrangement and the emissivity ε of the radiating surfaces having the temperatures T_1 and T_2 .

(a) If a body whose radiating surface area is A_1 is within a hollow body with a radiating surface area A_2 , then $A = A_1$, the angular coefficient $\varphi = 1$, and

$$W_{1-2} = \frac{1}{\frac{1}{W_1} + \frac{A_1}{A_2} \left(\frac{1}{W_2} - \frac{1}{W_{\text{bl}}} \right)} \quad (4-60)$$

Here $W_1 = \varepsilon_1 W_{\text{bl}}$ = radiating power of the smaller body

$W_2 = \varepsilon_2 W_{\text{bl}}$ = radiating power of the larger (outer) body

ε_1 and ε_2 = emissivities of the surface of the smaller and larger bodies, respectively (the values of ε for selected materials are given in Table A-24)

W_{bl} = radiating power of a black body; $W_{\text{bl}} = 5.7 \text{ W}/\text{m}^2 \text{K}^4$.

(b) If the area A_2 is very great in comparison with the area A_1 (for example, an apparatus in a shop), i. e. the ratio A_1/A_2 is close to zero, then the radiating power $W_{1-2} = W_1$.

(c) If $A_1 = A_2$ (two parallel infinitely great surfaces), we have:

$$W_{1-2} = \frac{1}{\frac{1}{W_1} + \frac{1}{W_2} - \frac{1}{W_{\text{bl}}}} \quad (4-61)$$

The summary coefficient of heat transfer by radiation and convection is

$$\alpha = \alpha_{\text{rad}} + \alpha_{\text{con}} \quad (4-62)$$

where

$$\alpha_{\text{rad}} = \frac{Q_{\text{rad}}}{(T_1 - T_2) A} = \frac{W_{1-2} \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right]}{T_1 - T_2} \quad (4-63)$$

α_{con} = coefficient of heat transfer by convection determined by the relevant equations for free or forced convection.

The following approximate formula can be used to calculate the heat losses of apparatuses in closed premises when the temperature of the surface of an apparatus does not exceed 150°C:

$$\alpha = 9.74 + 0.07\Delta t \quad (4-64)$$

where α = summary coefficient of heat transfer by radiation and convection, W/m²·K

Δt = difference between the temperatures of the apparatus surface and the surrounding air, K.

OVERALL HEAT TRANSFER IN SURFACE HEAT EXCHANGERS

Surface heat exchangers in which the heat from a hot fluid is transferred to a cold fluid through the wall separating them form the main kind of heat exchange apparatuses used in industry. Contact heat exchangers in which the heat is transferred upon the direct contact of the hot and cold fluids form another kind of heat exchange equipment.

20. The equation of overall heat transfer is:

$$Q = KA\Delta t_m \quad (4-65)$$

where Q = heat flux (rate of flow of the heat being transferred), W

K = overall coefficient of heat transfer, W/m²·K

A = area of the heat transfer surface, m²

Δt_m = mean difference between the temperatures of the hot and the cold fluids, K.

The unit heat load (unit heat flux) is:

$$q = \frac{Q}{A} = K\Delta t_m \quad (4-66)$$

For a flat surface, the overall heat transfer coefficient K in Eqs. (4-65) and (4-66) is:

$$K = \frac{1}{\frac{1}{\alpha_h} + \sum r_w + \frac{1}{\alpha_c}} \quad (4-67)$$

where α_h and α_c = heat transfer coefficients for the hot and the cold fluids, respectively, W/m²·K

$\sum r_w$ = sum of the thermal resistances of all the layers which the wall consists of including fouling layers, m²·K/W.

Equations (4-65) to (4-67) may also be used with sufficient accuracy for calculating the transfer of heat through a cylindrical wall if $d_{in} > 0.5d_{ex}$. The area of the heat transfer surface A in this case is determined according to the mean diameter $d_m = 0.5(d_{ex} + d_{in})$.

The heat transfer equation for a tubular heat exchanger consisting of n tubes each L m long can be written in the form:

$$Q = K_L n L \Delta t_m \quad (4-68)$$

where the overall coefficient of heat transfer per metre of length K_L (in $\text{W/m} \cdot \text{K}$) is:

$$K_L = \frac{\pi}{\frac{1}{\alpha_{\text{in}} d_{\text{in}}} + \sum \frac{1}{2\lambda} \ln \frac{d_{\text{ex}}}{d_{\text{in}}} + \frac{1}{\alpha_{\text{ex}} d_{\text{ex}}} + \sum \frac{r_{\text{foul}}}{d_{\text{foul}}}} \quad (4-69)$$

λ = thermal conductivity of the wall material, $\text{W/m} \cdot \text{K}$.

The thermal conductivity of the fouling deposits on the walls ($1/r_{\text{foul}}$) depends on the kind of fluid, its temperature and velocity, and also on the material of the wall, the temperature of the heating fluid, and the duration of operation of the apparatus without cleaning, i.e. in the long run on the kind of deposit or corrosion product. Accurate data on r_{foul} can be obtained only experimentally.

The influence of fouling deposits on the walls can be approximately taken into consideration by using the roughly guiding values of the thermal conductivity of such deposits given in Table A-31.

When an apparatus is cleaned rarely or it is greatly corroded, the value of $1/r_{\text{foul}}$ may drop to $500 \text{ W/m}^2 \cdot \text{K}$ and even lower.

21. The mean temperature drop Δt_m in the equation of overall heat transfer is determined as follows.

(a) For counterflow and parallel flow:

$$\Delta t_m = \frac{\Delta t_{\text{gr}} - \Delta t_{\text{sm}}}{2.3 \log \frac{\Delta t_{\text{gr}}}{\Delta t_{\text{sm}}}} \quad (4-70)$$

where Δt_{gr} and Δt_{sm} are the greater and smaller temperature drops at the ends of the heat exchanger.

If the ratio $\Delta t_{\text{gr}}/\Delta t_{\text{sm}} < 2$, then instead of Eq. (4-70) we can use the following equation with sufficient accuracy:

$$\Delta t_m = \frac{\Delta t_{\text{gr}} + \Delta t_{\text{sm}}}{2} \quad (4-71)$$

It can be seen to follow from Eq. (4-70) that if $\Delta t_{\text{gr}} = 0$ or $\Delta t_{\text{sm}} = 0$, then Δt_m also equals zero; if $\Delta t_{\text{gr}} = \Delta t_{\text{sm}}$, then $\Delta t_m = \Delta t_{\text{gr}} = \Delta t_{\text{sm}}$.

If the temperature of one of the fluids does not change along a surface in the process of heat transfer (the condensation of a saturated vapour, the boiling of a liquid), then the mean temperature drop Δt_m is also determined by Eq. (4-70) or (4-71).

Equations (4-70) and (4-71) may be applied provided that in the heat exchanger the value of the overall heat transfer coefficient

K and the product of the mass flow rate and the specific heat capacity $Q_m c$ for each of the fluids can be considered constant along the entire surface of heat exchange.

When the value of the overall heat transfer coefficient K (or the value of the product $Q_m c$) changes considerably along a heat exchange surface, the use of the logarithmic mean temperature drop (LMTD) according to Eq. (4-70) becomes impossible.

In such cases, the differential equation of heat transfer is solved by the method of graphical integration—see Example 4-21.

(b) For a combined flow in multipass heat exchangers and for cross flow, we have:

$$\Delta t_m = \varepsilon_{\Delta t} \Delta t_{cf} \quad (4-72)$$

where $\varepsilon_{\Delta t}$ is a correction factor to the mean temperature drop Δt_{cf} calculated for a counterflow. The value of the factor $\varepsilon_{\Delta t}$ is taken from special graphs [0-30, 4-18]. Examples of such graphs are given in Fig. A-8.

For multipass heat exchangers with a simple combined flow (one pass on the shell side, and an even number of passes on the tube side), the mean temperature drop can be calculated [0-47] by the equation:

$$\Delta t_m = \frac{\theta}{2.3 \log \frac{\Delta t_{gr} + \Delta t_{sm} + \theta}{\Delta t_{gr} + \Delta t_{sm} - \theta}} \quad (4-73)$$

where Δt_{gr} and Δt_{sm} = greater and smaller temperature drops at the ends of the heat exchanger in counterflow with the same initial and final temperatures of the fluids

$$\theta = \sqrt{\delta T^2 + \delta t^2} \quad (4-74)$$

$\delta T = T_{in} - T_{fn}$ = change in the temperature of the hot fluid
 $\delta t = t_{fn} - t_{in}$ = change in the temperature of the cold fluid.

22. Determination of the mean temperature of fluids.

In the majority of dimensionless equations for heat transfer, the values of the physicochemical constants of a fluid are related to its mean temperature, which can be found as follows.

For the fluid whose temperature changes to a smaller extent in the heat exchanger, the mean temperature is determined as the mean arithmetical value of the initial and final temperatures:

$$t_{m,1} = \frac{t_{in,1} + t_{fn,1}}{2} \quad (4-75)$$

The mean temperature for the second fluid is found by the equation:

$$t_{m,2} = t_{m,1} \pm \Delta t_m \quad (4-76)$$

Equation (4-76) also holds when the temperature of the first fluid is constant along the heat exchange surface.

OVERALL HEAT TRANSFER IN DIRECT CONTACT OF STREAMS

23. A generalized formula for determining the overall coefficient of heat transfer from a cooling unsaturated gas to a liquid in scrubbers with packing is:

$$Ki = 0.01 Re_g^{0.7} Re_{lq}^{0.7} Pr_g^{0.33} \quad (4-77)$$

Here $Ki = \frac{K d_{eq}}{\lambda_g}$ = Kirpichev dimensionless number

$Re_g = \frac{4 v_f \rho_g}{\sigma \mu_g}$ = Reynolds number for the gas

$Re_{lq} = \frac{4S}{\sigma \mu_{lq}}$ = Reynolds number for the liquid

$Pr_g = \frac{c_p \mu_g}{\lambda_g}$ = Prandtl number for the gas

K = overall coefficient of heat transfer from the gas to the liquid, $W/m^2 \cdot K$

$d_{eq} = \frac{4V_{free}}{\sigma}$ = equivalent diameter of the packing, m

V_{free} = free volume of the packing, m^3/m^3

σ = unit surface area of the packing, m^2/m^3

v_f = fictitious velocity of the gas (related to the total cross section of the scrubber), m/s

S = density of spraying, $kg/m^2 \cdot s$

λ_g = thermal conductivity of the gas, $W/m \cdot K$

μ_g and μ_{lq} = dynamic viscosities of the gas and liquid, respectively, $Pa \cdot s$

ρ_g = density of the gas, kg/m^3 .

Equation (4-77) has been obtained from experimental data for the cooling of air from 80 to 2°C with a unit rate of spraying with water ranging from 3.5 to 10 $m^3/m^2 \cdot h$. *

24. A generalized formula for the coefficient of evaporation from the surface of a liquid in a turbulent gas stream upon its forced flow is:

$$Nu'_g = 0.027 Re_g^{0.8} (Pr'_g)^{0.33} \quad (4-78)$$

* For the overall transfer of heat from a gas to water upon their direct contact see also: Egorov, N. N. *Okhlazhdenie gaza v skrubberakh* (The Cooling of a Gas in Scrubbers). Moscow, Goskhimizdat (1954) and Komarov, I. A. *Inzh.-fiz. zhurn.*, 5 (1965).

TABLE 4-6.

APPROXIMATE VALUES OF OVERALL
HEAT TRANSFER COEFFICIENTS K
(IN $\text{W/m}^2 \cdot \text{K}$)

Kind of heat exchange	Values of K for	
	forced flow	free flow
From gas to gas (at moderate pressures)	10-40	4-12
From gas to liquid (gas coolers)	10-60	6-20
From condensing steam to gas (air heaters)	10-60	6-12
From liquid to liquid (water)	800-1700	140-340
From liquid to liquid (hydrocarbons, oils)	120-270	30-60
From condensing steam to water (condensers, heaters)	800-3500	300-1200
From condensing steam to organic liquids (heaters)	120-340	60-170
From condensing vapour of organic substances to water (condensers)	300-800	230-460
From condensing vapour to boiling liquid (evaporators)	—	300-2500

EXAMPLES

Example 4-1. An apparatus 2 m in diameter and 5 m high is covered with a layer of asbestos thermal insulation 75 mm thick. The temperature of the apparatus wall is 146°C, and that of the external surface of the insulation is 40°C. Determine the rate of heat flow (the heat flux) through the layer of insulation.

Solution. The mean area which the heat flows through is:

$$A_m = \pi \left(D_m L + 2 \frac{D^2}{4} \right) = 3.14 (2.075 \times 5 + 0.5 \times 2^2) = 38.8 \text{ m}^2$$

We find the thermal conductivity of asbestos $\lambda = 0.151 \text{ W/m} \cdot \text{K}$ in Table A-28.

The heat flux through the insulation is

$$Q = \frac{\lambda}{\delta} (t_h - t_c) A_m = \frac{0.151}{0.075} (146 - 40) 38.8 = 8280 \text{ W}$$

Example 4-2. Calculate the thermal conductivity of liquid nitrobenzene at 120°C.

Solution. The specific heat capacity of nitrobenzene (Table A-26) is $c = 1380 \text{ J/kg} \cdot \text{K}$.

The density of nitrobenzene at 30°C is $\rho \approx 1200 \text{ kg/m}^3$ (Table A-3).

The thermal conductivity of nitrobenzene at 30°C by Eq. (4-7) is

$$\lambda_{30} = C_a c \rho \sqrt[3]{\frac{\rho}{M}} = 4.22 \times 10^{-8} \times 1380 \times 1200 \sqrt[3]{\frac{1200}{123}} = 0.149 \text{ W/m} \cdot \text{K}$$

where $C_a = 4.22 \times 10^{-8}$ for unassociated liquids
 $M = 123 \text{ kg/kmol}$ = molar mass of nitrobenzene.

The thermal conductivity of nitrobenzene at 120 °C, by Eq. (4-8), is:

$$\lambda_t = \lambda_{30} [1 - \varepsilon (t - 30)] = 0.149 [1 - 1.0 \times 10^{-3} (120 - 30)] = 0.136 \text{ W/m} \cdot \text{K}$$

According to experimental data (Fig. A-10), we have: $\lambda_t = 0.137 \text{ W/m} \cdot \text{K}$.

Example 4-3. Calculate the thermal conductivity of a 25% aqueous solution of sodium chloride at 80 °C.

Solution. According to the nomogram in Fig. A-11, the specific heat capacity of a 25% solution of sodium chloride at 30 °C is $c = 3390 \text{ J/kg} \cdot \text{K}$.

The density of a 25% solution of sodium chloride according to Table A-3 is $\rho = 1189 \text{ kg/m}^3$.

The molar mass of the solution is determined by the equation:

$$M = xM_1 + (1 - x)M_2$$

where M_1 and M_2 = molecular masses of sodium chloride and water, respectively

x = mole fraction of the sodium chloride in the solution.

Since $M_1 = 58.5$, $M_2 = 18$, and $x = \frac{25/58.5}{25/58.5 + 75/18} = 0.093$, we have $M = 0.093 \times 58.5 + (1 - 0.093) 18 = 21.7 \text{ kg/kmol}$.

The thermal conductivity of a 25% solution of sodium chloride at 30 °C by Eq. (4-7) is:

$$\lambda_{30} = 3.58 \times 10^{-8} \times 3390 \times 1189 \sqrt[3]{\frac{1189}{21.7}} = 0.548 \text{ W/m} \cdot \text{K}$$

The thermal conductivity of the solution at 80 °C by Eq. (4-9) is:

$$\lambda_{80} = 0.548 \times \frac{0.674}{0.615} = 0.60 \text{ W/m} \cdot \text{K}$$

where 0.674 and 0.615 W/m · K are the thermal conductivities of water at 80 and 30 °C (Fig. A-10).

Example 4-4. Calculate the thermal conductivity for liquid methane at $t = -160.6$ °C and compare the value obtained with the experimental one.

Solution. Liquid methane relates to unassociated liquids. By Eq. (4-7), we have

$$\lambda = C_a c \rho \sqrt[3]{\frac{\rho}{M}} = 4.22 \times 10^{-8} \times 3.47 \times 10^3 \times 423 \sqrt[3]{\frac{423}{16}} = 0.184 \text{ W/m} \cdot \text{K}$$

where $C_a = 4.22 \times 10^{-8}$ for unassociated liquids

$c = 3.47 \times 10^3 \text{ J/kg} \cdot \text{K}$ = specific heat capacity of liquid methane at $T = 112.5 \text{ K}$

$\rho = 423 \text{ kg/m}^3 = \text{density of liquid methane}$
 $M = 16 \text{ kg/kmol} = \text{molar mass of methane.}$

According to a reference book [4-22], the thermal conductivity of liquid methane at $t = -160.6^\circ\text{C}$ equals $0.194 \text{ W/m}\cdot\text{K}$. The error when calculating by Eq. (4-7) is:

$$\frac{0.194 - 0.184}{0.184} \times 100 = 5.4\%$$

Example 4-5. Calculate the thermal conductivity of dry air at 300°C .

Solution. By Eq. (4-10) we have:

$$\lambda = Bc_v\mu = 1.9 \times 0.748 \times 10^3 \times 2.97 \times 10^{-5} = 0.0422 \text{ W/m}\cdot\text{K}$$

where $B = 1.9$ for diatomic gases

$c_v = \text{specific heat capacity at constant volume; it is determined from the relationship } c_p/c_v = 1.4:$

$$c_v = \frac{c_p}{1.4} = \frac{1.05 \times 10^3}{1.4} = 0.748 \times 10^3 \text{ J/kg}\cdot\text{K}$$

$c_p = 1.05 \times 10^3 \text{ J/kg}\cdot\text{K} = \text{specific heat capacity of dry air at } 300^\circ\text{C}$

$\mu = 2.97 \times 10^{-5} \text{ Pa}\cdot\text{s} = \text{dynamic viscosity of air at } 300^\circ\text{C}.$

Example 4-6. Calculate the thermal conductivity at 0°C for a gas mixture having the following composition by volume: H_2 —50%, CO —40%, N_2 —10%.

Solution. The additivity rule cannot be applied. We can approximately determine the thermal conductivity of a gas mixture by Eq. (4-10). We write out the values of the physicochemical constants for the separate components of the gas mixture:

Component	$\rho_0, \text{ kg/m}^3$	$c_v \times 10^{-3}, \text{ J/kg}\cdot\text{K}$	$\gamma = c_p/c_v$	$\mu \times 10^3, \text{ Pa}\cdot\text{s}$
H_2	0.09	10.14	1.41	0.00842
CO	1.25	0.75	1.4	0.0166
N_2	1.25	0.75	1.4	0.017

We find the mass composition of the gas mixture:

Component	m^3 (% by volume)	kg	% by mass
H_2	50	$50 \times 0.09 = 4.5$	6.7
CO	40	$40 \times 1.25 = 50.0$	74.6
N_2	10	$10 \times 1.25 = 12.5$	18.7
Total	100	67.0	100.0

We calculate c_v for the mixture of gases:

$$c_v = 0.067 \times 10.14 \times 10^3 + 0.746 \times 0.75 \times 10^3 + 0.187 \times 0.75 \times 10^3 = 1.379 \times 10^3 \text{ J/kg} \cdot \text{K}$$

We find the dynamic viscosity of the gas mixture by Eq. (1-13) and by Table A-11:

$$\mu_{\text{mix}} = \frac{(0.5 \times 8.13 \times 0.00842 + 0.4 \times 61.4 \times 0.0165 + 0.1 \times 59.5 \times 0.017) 10^{-3}}{0.5 \times 8.13 + 0.4 \times 61.4 + 0.1 \times 59.5} = 0.0156 \times 10^{-3} \text{ Pa} \cdot \text{s}$$

We calculate the coefficient B in Eq. (4-10):

$$B = 0.25(9\gamma - 5) = 0.25(9 \times 1.4 - 5) = 1.9$$

We determine the thermal conductivity of the gas mixture:

$$\lambda_{\text{mix}} = Bc_v\mu = 1.9 \times 1.379 \times 10^3 \times 0.0156 \times 10^{-3} = 0.041 \text{ W/m} \cdot \text{K}$$

If we compute the thermal conductivity of the gas mixture using the additivity rule and the data of Table A-30, we get for additivity by volume $\lambda_{\text{mix}} = 0.0926 \text{ W/m} \cdot \text{K}$ and by mass $\lambda_{\text{mix}} = 0.0317 \text{ W/m} \cdot \text{K}$

Example 4-7. A furnace wall consists of two layers—refractory brick ($\delta_1 = 500 \text{ mm}$) and ordinary building brick ($\delta_2 = 250 \text{ mm}$).

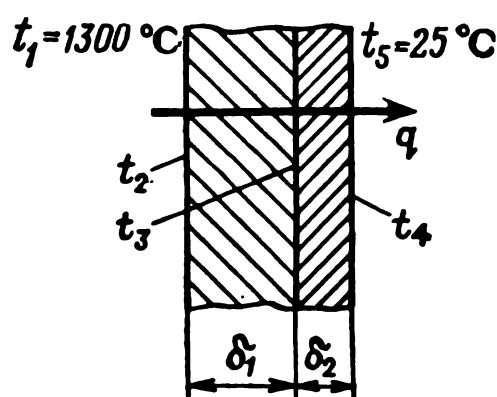


Fig. 4-12. To Example 4-7

The temperature inside the furnace t_1 is 1300°C and the ambient temperature t_5 is 25°C . Determine: (a) the heat losses from 1 m^2 of wall surface, and (b) whether the temperature t_3 at the boundary between the refractory and the building brick is low enough. The coefficient of heat transfer from the furnace gases to the wall is $\alpha_1 = 34.8 \text{ W/m}^2 \cdot \text{K}$, and that from the wall to the air is $\alpha_2 = 16.2 \text{ W/m}^2 \cdot \text{K}$. The thermal conductivity of refractory brick is $\lambda_1 = 1.16 \text{ W/m} \cdot \text{K}$ and that of building brick is $\lambda_2 = 0.58 \text{ W/m} \cdot \text{K}$.

Solution. The process of heat transfer through the furnace wall is shown schematically in Fig. 4-12.

(a) The overall heat transfer coefficient is:

$$K = \frac{1}{\frac{1}{\alpha_1} + \frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} + \frac{1}{\alpha_2}} = \frac{1}{\frac{1}{34.8} + \frac{0.5}{1.16} + \frac{0.25}{0.58} + \frac{1}{16.2}} = 1.05 \text{ W/m}^2 \cdot \text{K}$$

The heat losses from 1 m² of the wall surface are:

$$q = K (t_1 - t_5) = 1.05 (1300 - 25) = 1340 \text{ W/m}^2$$

(b) The temperature t_3 at the boundary between the refractory and the building brick can be found from the system of equations:

$$q = \alpha_1 (t_1 - t_2) = \frac{\lambda_1}{\delta_1} (t_2 - t_3)$$

whence

$$t_2 = t_1 - \frac{q}{\alpha_1} = 1300 - \frac{1340}{34.8} = 1261 \text{ }^\circ\text{C}$$

$$t_3 = t_2 - \frac{q\delta_1}{\lambda_1} = 1261 - \frac{1340 \times 0.5}{1.16} = 684 \text{ }^\circ\text{C}$$

Building brick may be used up to 800 °C. Consequently, the temperature of $t_3 = 684$ °C on the internal surface of the building brick is quite permissible.

Example 4-8. Determine the temperatures of the internal t_2 and the external t_3 surfaces of the wall of a heat exchanger, and also

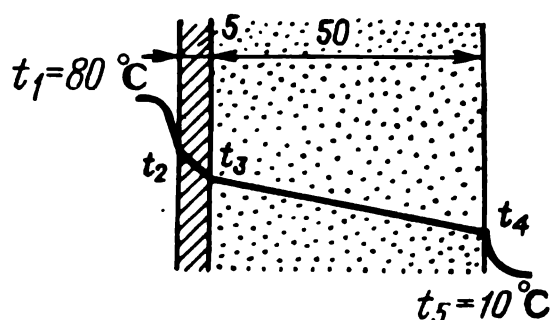


Fig. 4-13. To Example 4-8

the temperature t_4 of the external surface of the insulation covering the apparatus. The temperature of the liquid in the exchanger is $t_1 = 80$ °C, and that of the ambient air is $t_5 = 10$ °C. The exchanger is made of steel; the thickness of the steel wall is $\delta_w = 5$ mm, the thickness of the insulation is $\delta_{ins} = 50$ mm. The coefficient of heat transfer from the liquid to the steel wall is $\alpha_1 = 232 \text{ W/m}^2 \cdot \text{K}$, and that from the surface of the insulation to the air is $\alpha_2 = 10.4 \text{ W/m}^2 \cdot \text{K}$. The thermal conductivity of the insulation is $\lambda_{ins} = 0.12 \text{ W/m} \cdot \text{K}$.

Solution. Figure 4-13 gives a schematic cross section of the apparatus wall covered with insulation.

The overall heat transfer coefficient is:

$$K = \frac{1}{\frac{1}{\alpha_1} + \frac{\delta_w}{\lambda_w} + \frac{\delta_{ins}}{\lambda_{ins}} + \frac{1}{\alpha_2}} = \frac{1}{\frac{1}{232} + \frac{0.005}{46.5} + \frac{0.05}{0.12} + \frac{1}{10.4}} = 1.86 \text{ W/m}^2 \cdot \text{K}$$

where $\lambda_w = 46.5 \text{ W/m} \cdot \text{K}$ has been taken from Table A-28.

The unit heat flux is:

$$q = K (t_1 - t_5) = 1.86 (80 - 10) = 130 \text{ W/m}^2$$

The temperatures t_2 , t_3 , and t_4 are determined from the system of equations:

$$q = \alpha_1 (t_1 - t_2) = \frac{\lambda_w}{\delta_w} (t_2 - t_3) = \alpha_2 (t_4 - t_5)$$

The temperature of the internal surface of the apparatus wall is:

$$t_2 = t_1 - \frac{q}{\alpha_1} = 80 - \frac{130}{232} = 79.4^\circ\text{C}$$

The temperature of the external surface of the apparatus wall is:

$$t_3 = t_2 - q \frac{\delta_w}{\lambda_w} = 79.4 - \frac{130 \times 0.005}{46.5} \approx 79.4^\circ\text{C}$$

The temperature of the external surface of the insulation is:

$$t_4 = \frac{q}{\alpha_2} + t_5 = \frac{130}{10.4} + 10 = 22.4^\circ\text{C}$$

It can be seen that when insulation is used the thermal resistance of the steel wall may be disregarded ($t_2 \approx t_3$).

Example 4-9. Determine the mean wall temperature in a steam heater in which the heating steam ($p_{\text{abs}} = 4$ at) is used to heat (a) air at atmospheric pressure, and (b) water. The mean temperature of both the air and the water is 30°C . The wall thickness of the steel tubes is $\delta_w = 4$ mm. Take the approximate values of the heat transfer coefficients for steam, air, and water according to the mean data of Table 4-5 (turbulent flow in tubes). Take into account the presence of rust on both sides of the wall. The thermal conductivity of one layer of rust $1/r_r = 2320$ W/m²·K. For the designation and values of the temperatures see Fig. 4-14.

Solution. We find the temperatures t_2 and t_3 of the wall surfaces from the system of equations:

$$q = \alpha_1 (t_1 - t_2) = \alpha_2 (t_3 - t_4)$$

The temperature of condensation of steam at $p_{\text{abs}} = 4$ at is $142.9 \approx 143^\circ\text{C}$ (Table A-57).

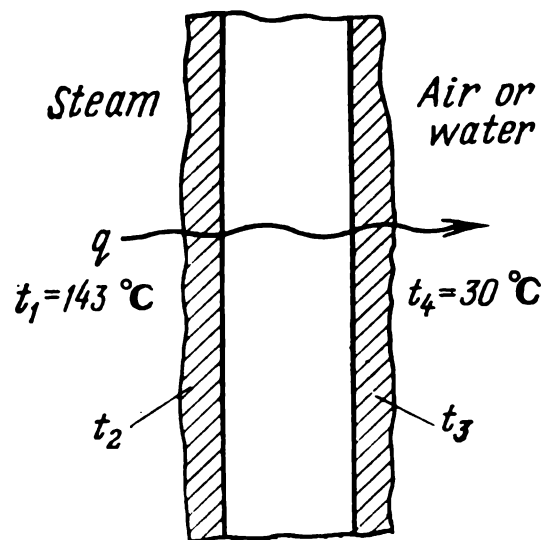
(a) The steam is used to heat air.

The overall heat transfer coefficient is:

$$\begin{aligned} K &= \frac{1}{\frac{1}{\alpha_1} + r_r + \frac{\delta_w}{\lambda_w} + r_r + \frac{1}{\alpha_2}} = \\ &= \frac{1}{\frac{1}{13\,300} + \frac{1}{2320} + \frac{0.004}{46.5} + \frac{1}{2320} + \frac{1}{46.4}} = 44.3 \text{ W/m}^2 \cdot \text{K} \end{aligned}$$

Here the heat transfer coefficient for condensing steam is $\alpha_1 = 13\,300$ W/m²·K, that for air is $\alpha_2 = 46.4$ W/m²·K, and the thermal conductivity of steel (Table A-28) is $\lambda_w = 46.5$ W/m·K.

Fig. 4-14. To Example 4-9



The unit heat flux is:

$$q = K(t_1 - t_2) = 44.3(143 - 30) = 5010 \text{ W/m}^2$$

The temperature t_2 is:

$$t_2 = t_1 - \frac{q}{\alpha_1} = 143 - \frac{5010}{13300} = 142.6 \text{ }^\circ\text{C}$$

The temperature t_3 is:

$$t_3 = t_4 + \frac{q}{\alpha_2} = 30 + \frac{5010}{46.4} = 138 \text{ }^\circ\text{C}$$

The mean temperature of the wall is

$$t_m = \frac{t_2 + t_3}{2} = \frac{142.6 + 138}{2} \approx 140 \text{ }^\circ\text{C}$$

(b) The steam is used to heat water.

The overall heat transfer coefficient is:

$$K = \frac{1}{\frac{1}{\alpha_1} + r_r + \frac{\delta_w}{\lambda_w} + r_r + \frac{1}{\alpha_2}} = \frac{1}{\frac{1}{13300} + \frac{1}{2320} + \frac{0.004}{46.5} + \frac{1}{2320} + \frac{1}{3420}} = 761 \text{ W/m}^2 \cdot \text{K}$$

Here the heat transfer coefficient for water is $\alpha_2 = 3420 \text{ W/m}^2 \cdot \text{K}$.

The unit heat load is:

$$q = K(t_1 - t_4) = 761(143 - 30) = 86000 \text{ W/m}^2$$

The temperature t_2 is:

$$t_2 = t_1 - \frac{q}{\alpha_1} = 143 - \frac{86000}{13340} = 136.5 \text{ }^\circ\text{C}$$

The temperature t_3 is:

$$t_3 = t_4 + \frac{q}{\alpha_2} = 30 + \frac{86000}{3420} = 55.2 \text{ }^\circ\text{C}$$

The mean temperature of the wall is:

$$t_m = \frac{t_2 + t_3}{2} = \frac{136.5 + 55.2}{2} \approx 96 \text{ } ^\circ\text{C}$$

Example 4-10. Ammonia at a pressure of $p_{\text{abs}} = 11.9$ at and at a temperature of $95 \text{ } ^\circ\text{C}$ is fed into a counterflow tubular condenser at a rate of 200 kg/h . The condenser is cooled with water supplied at a temperature of $15 \text{ } ^\circ\text{C}$. The liquid ammonia leaves the apparatus at the condensation temperature.

What amount of water has to be supplied to the condenser if the smallest difference allowed between the temperatures of the ammonia and the water in the condenser is 5 K ? What temperature will the water have at the outlet from the condenser?

Solution. From a T - S diagram for ammonia (Fig. A-26) or from Table A-48, we find that the condensation temperature of ammonia at a pressure of $p_{\text{abs}} = 11.9$ at equals $30 \text{ } ^\circ\text{C}$. Therefore, the ammonia fed into the condenser at a temperature of $95 \text{ } ^\circ\text{C}$ is in the state of a superheated vapour. To cool it at a constant pressure of $p_{\text{abs}} = 11.9$ at from $95 \text{ } ^\circ\text{C}$ to the beginning of condensation, i. e. to $30 \text{ } ^\circ\text{C}$, it is necessary to remove the quantity of heat

$$Q_1 = \frac{200 (1647 \times 10^3 - 1467 \times 10^3)}{3600} = 10\,000 \text{ W}$$

where $(1647 \times 10^3 - 1467 \times 10^3) \text{ J/kg}$ is the difference between the specific enthalpies of ammonia vapour at a pressure of $p_{\text{abs}} = 11.9$ at and temperatures of 95 and $30 \text{ } ^\circ\text{C}$ —see the T - S diagram in Fig. A-26.

To next condense the ammonia vapour, it is necessary to remove the quantity of heat

$$Q_2 = \frac{200 (1467 \times 10^3 - 323 \times 10^3)}{3600} = 63\,600 \text{ W}$$

where $323 \times 10^3 \text{ J/kg}$ is the specific enthalpy of liquid ammonia at $p_{\text{abs}} = 11.9$ at and $t = 30 \text{ } ^\circ\text{C}$.

Thus, the following quantity of heat must be removed by the water:

$$Q = Q_1 + Q_2 = 10\,000 + 63\,600 = 73\,600 \text{ W}$$

The change in the temperature of the ammonia in the condenser depending on the quantity of heat it gives up is shown in Fig. 4-15.

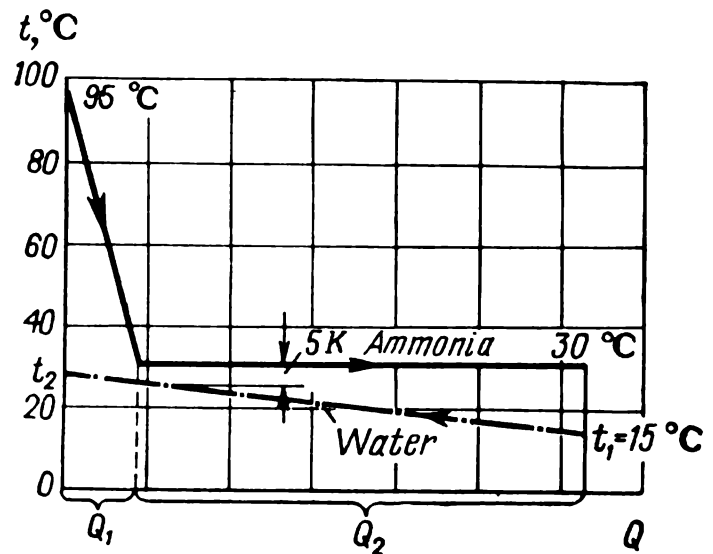
Since the difference between the temperatures of the ammonia and the water in any section of the condenser must be at least 5 K , we take the temperature of the water in the section of the condenser where condensation of the ammonia begins and there is the smallest temperature difference equal to $30 - 5 = 25 \text{ } ^\circ\text{C}$.

Hence, the required rate of flow of the water can be found from the equation:

$$63\,600 = Q_{m,w} \times 4.19 \times 10^3 (25 - 15)$$

whence $Q_{m,w} = 1.515 \text{ kg/s}$.

Fig. 4-15. To Example 4-10



We determine the temperature of the water at the outlet from the condenser t_2 by the equation:

$$73\,600 = 1.515 \times 4.19 \times 10^3 (t_2 - 15)$$

whence

$$t_2 = \frac{73\,600}{1.515 \times 4.19 \times 10^3} + 15 = 26.6^\circ\text{C}$$

Example 4-11. The heat of the residue leaving a cracking plant is used to heat the petroleum delivered for processing in the plant. Determine the mean temperature drop in the heat exchanger between the heating residue and the petroleum being heated if the residue has temperatures of $t_{in} = 300^\circ\text{C}$, $t_{fn} = 200^\circ\text{C}$, and the petroleum has temperatures of $t_{in} = 25^\circ\text{C}$ and $t_{fn} = 175^\circ\text{C}$.

Solution. We shall consider two cases.

Case 1. Parallel flow:

$$\begin{array}{ccc} 300 \rightarrow & & 200 \\ 25 \rightarrow & & 175 \\ \hline \Delta t_{gr} = 275 & & \Delta t_{sm} = 25 \end{array}$$

$$\frac{\Delta t_{gr}}{\Delta t_{sm}} = \frac{275}{25} > 2$$

Hence,

$$\Delta t_m = \frac{275 - 25}{2.3 \log \frac{275}{25}} = 104 \text{ K}$$

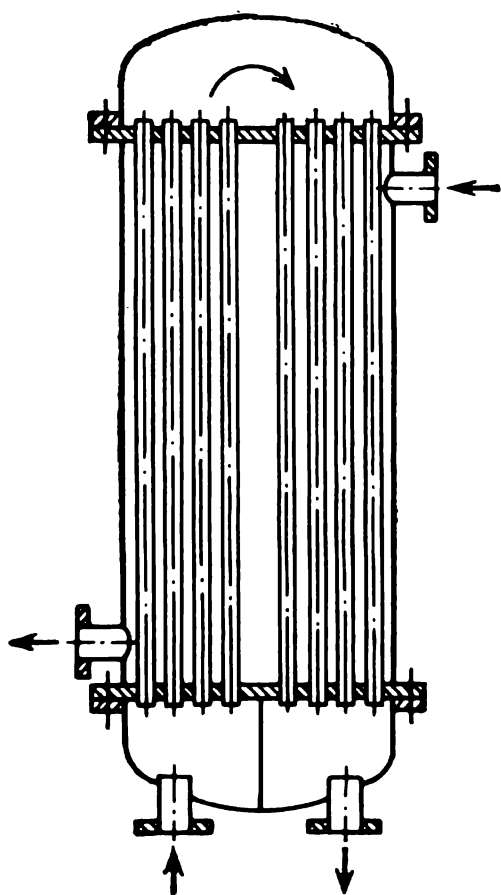


Fig. 4.16. To Example 4-12

Case 2. Counterflow:

$$\begin{array}{ccc} 300 \rightarrow & & 200 \\ 175 \leftarrow & & 25 \\ \hline \Delta t_{sm} = 125 & \Delta t_{gr} = 175 & , \quad \frac{\Delta t_{gr}}{\Delta t_{sm}} = \frac{175}{125} < 2 \end{array}$$

Hence,

$$\Delta t_m = \frac{125 + 175}{2} = 150 \text{ K}$$

If we calculate the mean temperature drop for the counterflow as the logarithmic mean drop, we get 149 K.

The above calculations show that when other conditions are equal, the mean temperature drop in counterflow is greater than in parallel flow.

It must be noted that in counterflow the liquid being heated (petroleum) can be brought up to a much higher temperature than 175 °C (for example, up to 290 °C), while the residue can be cooled to a temperature much lower than 200 °C. This is the main advantage of counterflow.

Example 4-12. Determine the mean temperature drop in a multipass heat exchanger having one pass on the shell side and two passes on the tube side (Fig. 4-16):

Initial temperature of hot fluid — $T_1 = 80 \text{ }^\circ\text{C}$
 Final temperature of hot fluid — $T_2 = 40 \text{ }^\circ\text{C}$
 Initial temperature of cold fluid — $t_1 = 20 \text{ }^\circ\text{C}$
 Final temperature of cold fluid — $t_2 = 40 \text{ }^\circ\text{C}$

Solution. We shall use Eq. (4-73) for our calculations. By Eq. (4-74), we get

$$\theta = \sqrt{\delta T^2 + \delta t^2} = \sqrt{40^2 + 20^2} = 44.7$$

The temperature diagram in counterflow is:

$$\begin{array}{ccc} 80 \rightarrow & & 40 \\ 40 \leftarrow & & 20 \\ \hline \Delta t_{\text{gr}} = 40 & & \Delta t_{\text{sm}} = 20 \end{array}$$

The logarithmic mean temperature drop in a multipass exchanger, by Eq. (4-73), is:

$$\Delta t_m = \frac{\theta}{2.3 \log \frac{\Delta t_{\text{gr}} + \Delta t_{\text{sm}} + \theta}{\Delta t_{\text{gr}} + \Delta t_{\text{sm}} - \theta}} = \frac{44.7}{2.3 \log \frac{40 + 20 + 44.7}{40 + 20 - 44.7}} = 23.2 \text{ K}$$

We use Eq. (4-72) for calculations:

$$\Delta t_m = \varepsilon_{\Delta t} \Delta t_{\text{cf}}$$

We find the mean temperature drop for counterflow:

$$\Delta t_{\text{cf}} = \frac{40 + 20}{2} = 30 \text{ K}$$

The value of $\varepsilon_{\Delta t}$ is found from Fig. A-8. For this purpose, we must first compute the values of the coefficients P and R :

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{40 - 20}{80 - 20} = 0.33 \text{ and } R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{80 - 40}{40 - 20} = 2$$

From the graph in Fig. A-8a, with $P = 0.33$ and $R = 2$, we obtain $\varepsilon_{\Delta t} = 0.78$. Hence, the mean temperature drop in the multipass exchanger by Eq. (4-72) is:

$$\Delta t_m = \varepsilon_{\Delta t} \Delta t_{\text{cf}} = 0.78 \times 30 = 23.4 \text{ K}$$

Example 4-13. Calculate the heat transfer coefficient for water being heated in a tubular heat exchanger consisting of tubes with a diameter of 40×2.5 mm. The water flows through the tubes with a velocity of 1 m/s. The mean temperature of the water is 47.5°C . The temperature of the tube wall is 95°C . The length of a tube is 2 m.

Solution. We determine the flow conditions, for which purpose we find the Reynolds number by Eq. (1-27):

$$Re = \frac{vd\rho}{\mu}$$

We find the dynamic viscosity of water at 47.5°C in Table A-6. Interpolation gives us $\mu \approx 0.57 \times 10^{-3} \text{ Pa}\cdot\text{s}$. In Table A-39, we

find the density of water at 47.5 °C. Interpolation gives us $\rho = 989 \text{ kg/m}^3$. Hence

$$Re = \frac{1 \times 0.035 \times 989}{0.57 \times 10^{-3}} = 60\,800$$

The value of Re exceeds 10 000 (developed turbulent flow). We determine the heat transfer coefficient with the aid of a nomogram (Fig. A-12) constructed according to Eq. (4-16):

$$Nu = 0.021 \varepsilon_l Re^{0.8} Pr^{0.43} \left(\frac{Pr}{Pr_w} \right)^{0.25}$$

From Table 4-3 for $L/d = 2000/35 = 57$, we have $\varepsilon_l = 1$. From Table A-39 for $t_{m, iq} = 47.5 \text{ °C}$ we find $Pr = 3.73$ and for $t_w = 95 \text{ °C}$ we find $Pr_w = 1.85$ (by interpolation). Hence $Pr/Pr_w = 3.73/1.85 = 2.02$.

The nomogram in Fig. A-12 gives us $Nu = 300$ (the example shown in the nomogram uses the above values).

From Eq. (4-11), we have

$$\alpha = \frac{Nu \lambda}{d}$$

We find the thermal conductivity of water at 47.5 °C in Table A-39— $\lambda = 0.644 \text{ W/m} \cdot \text{K}$. Hence

$$\alpha = \frac{300 \times 0.644}{0.035} = 5510 \text{ W/m}^2 \cdot \text{K}$$

Example 4-14. Benzene is heated in a tube with an internal diameter of 53 mm and a length of 3 m. The temperature of the tube wall is 70 °C. The velocity of the benzene is 0.1 m/s. The mean temperature of the benzene is 40 °C.

Find the heat transfer coefficient for benzene.

Solution. We determine the Reynolds number by Eq. (1-27):

$$Re = \frac{vd\rho}{\mu}$$

We find the dynamic viscosity of benzene at 40 °C in Fig. A-5: $\mu = 0.49 \times 10^{-3} \text{ Pa} \cdot \text{s}$. The density of benzene at 40 °C is $\rho = 858 \text{ kg/m}^3$ [4-22]. Hence

$$Re = \frac{0.1 \times 0.053 \times 858}{0.49 \times 10^{-3}} = 9300$$

The Reynolds number $Re = 9300$, hence we have a transition region. Using the graph in Fig. 4-1, for $Re = 9.3 \times 10^3$, we find:

$$\frac{Nu}{Pr^{0.43} \left(\frac{Pr}{Pr_w} \right)^{0.25}} = 30.9$$

The nomogram in Fig. A-13 gives us the values of the Prandtl number for benzene at 40 and 70 °C, namely, $Pr = 7.4$ and $Pr_w =$

=6.6, respectively. After substituting these values in the above expression, we get:

$$Nu = 30.9 \times 7.4^{0.43} \left(\frac{7.4}{6.6} \right)^{0.25} = 75.2$$

Then we find the thermal conductivity of benzene at 40 °C in Fig. A-10: $\lambda = 0.122 \times 1.163 = 0.142 \approx 0.14$ W/m·K. Hence, by Eq. (4-11),

$$\alpha = \frac{75.2 \times 0.14}{0.053} = 199 \text{ W/m}^2 \cdot \text{K}$$

We use Eq. (4-22) for calculations:

$$Nu = 0.008 Re^{0.9} Pr^{0.43} = 0.008 \times 9300^{0.9} \times 7.4^{0.43} = 70.3$$

$$\alpha = \frac{Nu \lambda}{d} = \frac{70.3 \times 0.14}{0.053} = 186 \text{ W/m}^2 \cdot \text{K}$$

Example 4-15. Aniline is pumped through the tube side of a horizontal shell-and-tube heat exchanger at an average temperature of 120 °C and with a velocity of 0.03 m/s. The internal diameter of the tubes is 20 mm, their length is 3 m. The average temperature of the internal surface of the tubes is 110 °C.

Determine the coefficient of heat transfer.

Solution. We determine the Reynolds number for the average temperature of the liquid. We find the density of aniline in Table A-4: $\rho = 933$ kg/m³, and its dynamic viscosity in Fig. A-5: $\mu = 0.52 \times 10^{-3}$ Pa·s. Hence by Eq. (1-27):

$$Re = \frac{vd\rho}{\mu} = \frac{0.03 \times 0.02 \times 933}{0.52 \times 10^{-3}} = 1075$$

The flow is laminar. To use the graph in Fig. 4-2, we calculate the numbers Re and Pr according to the average temperature of the boundary layer equal to $(110 + 120)/2 = 115$ °C. For this temperature, $\rho = 938$ kg/m³, $\mu = 0.55 \times 10^{-3}$ Pa·s, and by Eq. (1-27) we get:

$$Re = \frac{0.03 \times 0.02 \times 938}{0.55 \times 10^{-3}} = 1025$$

In Fig. A-13, we find $Pr = 6$. Consequently:

$$Re Pr \frac{d}{L} = 1025 \times 6 \times \frac{20}{3000} = 41$$

We find the approximate value of the Nusselt number (Nu') without account taken of free convection from the graph in Fig. 4-2. For $Re Pr \frac{d}{L} = 41$, on line AA we find $Nu' = 6.2$.

To determine the influence of free convection, we must calculate the values of the dimensionless number Gr and the product $4Re Nu'$.

Introducing μ^2/ρ^2 instead of ν^2 in Eq. (4-15), we get:

$$Gr = \frac{gd^3\rho^2}{\mu^2} \beta\Delta t$$

We find the value of $\beta\Delta t$ from the equation:

$$\beta\Delta t = \frac{v_2 - v_1}{v_1} = \frac{\rho_1 - \rho_2}{\rho_2}$$

In Table A-4, we find the densities of aniline at 110 and 120 °C: $\rho_1 = 943$ and $\rho_2 = 933$ kg/m³. Hence,

$$\beta\Delta t = \frac{943 - 933}{933} = 0.01075$$

and the Grashof number is:

$$Gr = \frac{9.81 \times 0.02^3 \times 938^2}{(0.55 \times 10^{-3})^2} \times 0.01075 = 24.5 \times 10^5$$

The product $4ReNu'$ is:

$$4ReNu' = 4 \times 1025 \times 6.2 = 25.4 \times 10^3$$

Since $Gr \gg 4ReNu'$, the influence of free convection is considerable. Using curve 2 for horizontal tubes in Fig. 4-2, we find the actual value of the Nusselt number $Nu = 13$.

The heat transfer coefficient is*:

$$\alpha = \frac{Nu\lambda}{d} = \frac{13 \times 0.165}{0.02} = 107 \text{ W/m}^2 \cdot \text{K}$$

Here $\lambda = 0.165$ W/m·K is the thermal conductivity of aniline at 115 °C found from Fig. A-10.

Example 4-16. In a vertical shell-and-tube heat exchanger consisting of 61 tubes with a diameter of 32×2.5 mm and a height of 1.25 m, carbon tetrachloride flows through the tubes at a rate of 13 m³/h. The mean temperature of the carbon tetrachloride is 50 °C, and the temperature of the internal surface of the tube walls is 24 °C.

Determine the coefficient of heat transfer from the carbon tetrachloride to the wall for two cases: (a) the carbon tetrachloride flows upward through the tubes completely filling their cross sections, and (b) the carbon tetrachloride flows downward in a thin film along the internal surface of the tubes.

Solution. (a) Complete filling of the tubes.

We use Eq. (1-24) to find the velocity of the carbon tetrachloride:

$$v = \frac{13}{16 \times 0.785 \times 0.027^2 \times 3600} = 0.103 \text{ m/s}$$

*This value of the heat transfer coefficient, like that of the Nusselt number Nu , can be considered as somewhat diminished because in the given example the product $GrPr > 25 \times 10^5$, which curve 2 in Fig. 4-2 corresponds to.

We calculate the Reynolds number by Eq. (1-27):

$$Re = \frac{vd\rho}{\mu}$$

The density of carbon tetrachloride at 50 °C according to Table A-4 is $\rho = 1536 \text{ kg/m}^3$, and its dynamic viscosity at the same temperature according to Fig. A-5 is $\mu = 0.7 \times 10^{-3} \text{ Pa}\cdot\text{s}$. Hence,

$$Re = \frac{0.103 \times 0.027 \times 1536}{0.7 \times 10^{-3}} = 6100$$

and $2300 < Re < 10\,000$.

From the graph in Fig. 4-1 for the transition region at $Re = 6.10 \times 10^3$, we find

$$\frac{Nu}{Pr^{0.43} \left(\frac{Pr}{Pr_w} \right)^{0.25}} = 21.0$$

The nomogram in Fig. A-13 gives us $Pr = 5.4$ for carbon tetrachloride at 50 °C and $Pr_w = 6.6$ at 24 °C. Hence,

$$Nu = 21.0 \times 5.4^{0.43} \left(\frac{5.4}{6.6} \right)^{0.25} = 41.4$$

The thermal conductivity of carbon tetrachloride at 50 °C from Fig. A-10 is $\lambda = 0.104 \text{ W/m}\cdot\text{K}$, and from Eq. (4-11) we find the coefficient of heat transfer with complete filling of the tubes:

$$\alpha = \frac{Nu\lambda}{d} = \frac{41.4 \times 0.104}{0.027} = 159 \text{ W/m}^2\cdot\text{K}$$

(b) Flowing down in a film.

The heat transfer coefficient is determined by Eq. (4-34) or (4-35) depending on the flow conditions. In both equations, the values of the physicochemical constants should be taken for the temperature of the boundary layer equal to:

$$\frac{t_{m,lq} + t_w}{2} = \frac{50 + 24}{2} = 37 \text{ }^\circ\text{C}$$

The dynamic viscosity of carbon tetrachloride at this temperature is $0.84 \times 10^{-3} \text{ Pa}\cdot\text{s}$. Its density from Table A-4 is $\rho = 1561 \text{ kg/m}^3$. Hence by Eq. (4-37)

$$Re = \frac{4Q_v}{\pi d n \mu} = \frac{4 \times 13 \times 1561}{3600 \times 3.14 \times 0.027 \times 61 \times 0.84 \times 10^{-3}} = 5200$$

Since $Re > 2000$, the flow of the film is turbulent. We use Eq. (4-34):

$$Ga = \frac{H^3 \rho^2 g}{\mu^2} = \frac{1.25^3 \times 1561^2 \times 9.81}{0.84^2 \times 10^{-6}} = 66.1 \times 10^{12}$$

According to the nomogram in Fig. A-13 for 37 °C, we find $Pr = 6$. Hence,

$$Nu = 0.01 (Ga Pr Re)^{1/3} = 0.01 (66.1 \times 10^{12} \times 6 \times 5200)^{1/3} = 12\,700$$

From Eq. (4-11), we have

$$\alpha_f = \frac{Nu\lambda}{H}$$

The thermal conductivity of carbon tetrachloride at 37 °C from Fig. A-10 is $\lambda = 0.109 \text{ W/m}\cdot\text{K}$, and

$$\alpha_f = \frac{12\,700 \times 0.109}{1.25} = 1110 \text{ W/m}^2\cdot\text{K}$$

The ratio $\alpha_f/\alpha = 1110/159 = 7$. Thus, in the conditions of the given example, the coefficient of heat transfer when the carbon tetrachloride flows down in a film is seven times greater than the coefficient with complete filling of the tubes.

Example 4-17. An air heater is made in the form of a multirow triangular pitch bank of tubes with an external diameter of 44.5 mm. The heating fluid flows inside the tubes, and the air being heated at atmospheric pressure flows in a direction perpendicular to the tubes. Find the heat transfer coefficient for the air if its average temperature is 200 °C. The velocity of the air in the narrow section of the bank is 12 m/s.

Solution. We calculate the Reynolds number by Eq. (1-27):

$$Re = \frac{vd\rho}{\mu}$$

The density of air at 200 °C (473 K) is $\rho = 1.29 \times (273/473) = 0.745 \text{ kg/m}^3$ and its dynamic viscosity is $\mu = 0.026 \times 10^{-3} \text{ Pa}\cdot\text{s}$ (Fig. A-6). Hence,

$$Re = \frac{12 \times 0.0445 \times 0.745}{0.026 \times 10^{-3}} = 15\,300$$

From Table 4-4, we find $\epsilon_\varphi = 1$, and by Eq. (4-28):

$$Nu = 0.356 Re^{0.6} \epsilon_\varphi = 0.356 \times 15\,300^{0.6} \times 1 = 115$$

We find the thermal conductivity of air at 200 °C from Table A-30: $\lambda = 0.0395 \text{ W/m}\cdot\text{K}$, and from Eq. (4-11) we have:

$$\alpha = \frac{Nu\lambda}{d} = \frac{115 \times 0.0395}{0.0445} = 102 \text{ W/m}^2\cdot\text{K}$$

If the heater is a shell-and-tube heat exchanger with baffles on the shell side, then $\epsilon_\varphi = 0.6$, and the coefficient of heat transfer will be $\alpha = 0.6 \times 102 = 61 \text{ W/m}^2\cdot\text{K}$.

Example 4-18. Isopropyl alcohol is heated in a tank in conditions of free convection by hot water supplied by a pump through a row of horizontal tubes with an external diameter of 30 mm. Determine the heat transfer coefficient for isopropyl alcohol if its average temperature is 60 °C and the average temperature of the external surface of the tubes is 70 °C.

Solution. We calculate the coefficient of heat transfer in the free flow of a liquid around horizontal tubes by Eq. (4-40):

$$Nu = 0.5 (Gr Pr)^{0.25} \left(\frac{Pr}{Pr_w} \right)^{0.25}$$

We find the values of the quantities ρ and μ in the formula for the Grashof number $Gr = \frac{d^3 \rho^2 \beta \Delta t g}{\mu^2}$ from a reference book [0-30, Vol. 1] for the defining temperature of 60 °C: $\rho = 789 \text{ kg/m}^3$ and $\mu = 0.8 \times 10^{-3} \text{ Pa}\cdot\text{s}$.

We determine the value of $\beta \Delta t$ by the equation

$$\beta \Delta t = \frac{v_2 - v_1}{v_1}$$

where v_2 and v_1 are the specific volumes of isopropyl alcohol at $t_2 = 70$ and $t_1 = 60$ °C, respectively.

The specific volumes can be computed by the equation:

$$v_t = v_0 (1 + at + bt^2 + ct^3)$$

In a reference book [0-30], we find the following values of the coefficients (approximate): $a = 1.043 \times 10^{-3}$, $b = 0.443 \times 10^{-6}$, and $c = 2.73 \times 10^{-8}$. Hence,

$$\begin{aligned} \beta \Delta t &= \frac{a(t_2 - t_1) + b(t_2^2 - t_1^2) + c(t_2^3 - t_1^3)}{1 + at_1 + bt_1^2 + ct_1^3} = \\ &= \frac{1.043 \times 10^{-3} (70 - 60) + 0.443 \times 10^{-6} (70^2 - 60^2) + 2.73 \times 10^{-8} (70^3 - 60^3)}{1 + 1.043 \times 10^{-3} \times 60 + 0.443 \times 10^{-6} \times 60^2 + 2.73 \times 10^{-8} \times 60^3} = \\ &= 0.0135 \end{aligned}$$

Now we can calculate the Grashof number:

$$Gr = \frac{d^3 \rho^2 \beta \Delta t g}{\mu^2} = \frac{0.03^3 \times 789^2 \times 0.0135 \times 9.81}{(0.8 \times 10^{-3})^2} = 3.46 \times 10^6$$

Using the nomogram in Fig. A-13, we find for isopropyl alcohol that $Pr = 19$ at 60 °C and $Pr_w = 16.5$ at 70 °C. Introducing the values obtained in Eq. (4-40), we get:

$$Nu = 0.5 (3.46 \times 10^6 \times 19)^{0.25} \left(\frac{19}{16.5} \right)^{0.25} = 46.4$$

We find the thermal conductivity of isopropyl alcohol by Eq. (4-8):

$$\lambda_t = \lambda_0 [1 - \varepsilon (t - 0)]$$

The thermal conductivity of isopropyl alcohol at 0 °C [0-30] is $\lambda_0 = 0.154 \text{ W/m}\cdot\text{K}$. We take the value of ε equal to that for propyl alcohol, i. e. $\varepsilon = 1.4 \times 10^{-3}$. Hence,

$$\lambda_t = 0.154 (1 - 1.4 \times 10^{-3} \times 60) = 0.141 \text{ W/m}\cdot\text{K}$$

According to Eq. (4-11), the heat transfer coefficient is:

$$\alpha = \frac{Nu\lambda}{d} = \frac{46.4 \times 0.141}{0.03} = 218 \text{ W/m}^2 \cdot \text{K}$$

Example 4-19. Determine the individual and overall coefficients of heat transfer in an ethyl alcohol evaporator heated with steam ($p_{\text{abs}} = 3 \text{ at}$). The alcohol boils in a large volume at atmospheric pressure. The heating steam condenses inside steel horizontal tubes with a diameter of $51 \times 3 \text{ mm}$ and a length of 1.6 m . Take into consideration the thermal resistances of the solid deposits on the wall.

Solution. The boiling point of the alcohol is 78.4°C . The temperature of condensation of the heating steam is 132.9°C (Table A-57).

The mean temperature drop is:

$$\Delta t_m = 132.9 - 78.4 = 54.5 \text{ K}$$

We find the heat transfer coefficient for steam condensing inside horizontal tubes by Eq. (4-52):

$$\alpha_{\text{st}} = 1.36 C q^{0.5} l^{0.35} d^{-0.25} = 1.36 \times 7.5 \times 1.6^{0.35} \times 0.045^{-0.25} q^{0.5} = 26.1 q^{0.5}$$

where the value of $C = 7.5$ has been taken from Fig. 4-9 for $t = 132.9^\circ\text{C}$.

We determine the heat transfer coefficient for boiling alcohol by Eq. (4-53) in which we have substituted μ/ρ for ν :

$$\alpha_{\text{alc}} = b \left(\frac{\lambda^2 \rho_{\text{iq}}}{\mu \sigma T_b} \right)^{1/3} q^{2/3}$$

The physicochemical constants for ethyl alcohol at 78.4°C are as follows:

$$\lambda = 0.167 \text{ W/m} \cdot \text{K} \text{—Fig. A-10}$$

$$\mu = 0.45 \times 10^{-3} \text{ Pa} \cdot \text{s} \text{—Fig. A-5}$$

$$\rho_{\text{iq}} = 735 \text{ kg/m}^3 \text{—Table A-4}$$

$$\rho_{\text{st}} = \frac{46 \times 273}{22.4 \times 351.5} = 1.6 \text{ kg/m}^3$$

$$\sigma = (\sigma_0 - 0.092t) 10^{-3} = (24.1 - 0.092 \times 78.4) 10^{-3} = 0.0169 \text{ N/m} \text{—Table A-22.}$$

We find the value of the coefficient $b = 0.0885$ from Fig. 4-11 for $\rho_{\text{iq}}/\rho_{\text{st}} = 735/1.6 = 460$.

The heat transfer coefficient for the alcohol is:

$$\alpha_{\text{alc}} = 0.0885 \left(\frac{0.167^2 \times 735}{0.45 \times 10^{-3} \times 0.0169 \times 351.5} \right)^{1/3} q^{0.67} = 1.74 q^{0.67}$$

The sum of the thermal resistances of the steel wall and the solid fouling deposits on it (Tables A-28 and A-31) is:

$$\Sigma r_w = \frac{\delta}{\lambda_w} + r_{\text{foul},1} + r_{\text{foul},2} = \frac{0.003}{46.5} + \frac{1}{5800} + \frac{1}{5800} = 0.00041 \text{ m}^2 \cdot \text{K/W}$$

Since for tubes having a diameter of 51×3 mm the ratio $d_{\text{in}}/d_{\text{ex}} > 0.5$, we use Eq. (4-67) to calculate the overall heat transfer coefficient K for a flat wall:

$$K = \frac{1}{\frac{1}{\alpha_{\text{st}}} + \Sigma r_w + \frac{1}{\alpha_{\text{alc}}}} = \frac{1}{\frac{1}{26.1q^{0.5}} + 0.00041 + \frac{1}{1.74q^{0.67}}}$$

Next by Eq. (4-66), we find:

$$q = K\Delta t_m = \frac{54.5}{0.0384q^{-0.5} + 41 \times 10^{-5} + 0.575q^{-0.67}}$$

whence

$$0.0384q^{0.5} + 41 \times 10^{-5}q + 0.575q^{0.33} - 54.5 = 0$$

We solve this equation graphically. To do this, we assume that its left-hand side equals y and adopt such values for q that give us values of y both greater and smaller than 0 (see Table 4-7)*.

TABLE 4-7

q	$q^{0.5}$	$q^{0.33}$	y
60 000	245	38.6	1.7
50 000	224	36.4	-4.5
55 000	235	37.5	-1.4

We use the data of Table 4-7 to plot a graph (Fig. 4-17) from which for $y=0$ we find $q=57\,300$ W/m².

The overall heat transfer coefficient is:

$$K = \frac{q}{\Delta t_m} = \frac{57\,300}{54.5} = 1050 \text{ W/m}^2 \cdot \text{K}$$

The individual heat transfer coefficients are:
for the condensing steam:

$$\alpha_{\text{st}} = 26.1q^{0.5} = 26.1 \times 57\,300^{0.5} = 6230 \text{ W/m}^2 \cdot \text{K}$$

for the boiling alcohol:

$$\alpha_{\text{alc}} = 1.74q^{0.67} = 1.74 \times 57\,300^{0.67} = 2680 \text{ W/m}^2 \cdot \text{K}$$

To check the correctness of our calculations, we find the value of K :

$$K = \frac{1}{\frac{1}{6230} + 41 \times 10^{-5} + \frac{1}{2680}} = 1060 \text{ W/m}^2 \cdot \text{K}$$

* To determine the order of magnitude of the expected value of q , we assume approximately according to the data of Table 4-6 that $K \approx 1200$ W/m²·K whence $q = K\Delta t_m \approx 65\,000$ W/m².

For purposes of comparison, we calculate the coefficient of heat transfer for boiling alcohol by the approximate formula (4-58) at $p_{\text{abs}} = 1$ at and $\varphi = 0.45$:

$$\alpha_{\text{alc}} = 2.72\varphi p^{0.4} q^{0.7} = 2.72 \times 0.45 \times 57\,300^{0.7} = 2620 \text{ W/m}^2 \cdot \text{K}$$

which is close to the value of α_{alc} found above. We check the value of the critical heat load $q_{\text{cr},1}$ by Eq. (4-57):

$$q_{\text{cr},1} = 0.14L\sqrt{\rho_{\text{st}}}\sqrt[4]{\sigma g \rho_{\text{lg}}} = 0.14 \times 849 \times 10^3 \times 1.6^{0.5} (0.0169 \times$$

$$\times 9.81 \times 735)^{0.25} = 497\,000 \text{ W/m}^2$$

where $L = 849 \times 10^3 \text{ J/kg}$ is the specific heat of vaporization of ethyl alcohol (Table A-45).

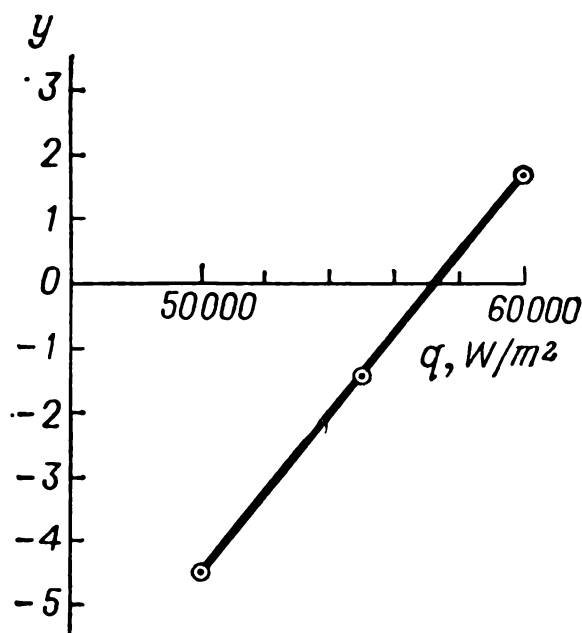


Fig. 4-17. To Example 4-19

The unit heat load in the evaporator ($q = 57\,300 \text{ W/m}^2$) is much smaller than the critical load.

Example 4-20. A 20% aqueous solution of ammonium nitrate boils in the tubes of an evaporating apparatus 4 m high under a vacuum of 0.64 at at a mean temperature of 80°C . The heating steam has a pressure of $p_{\text{abs}} = 1.1$ at. Determine the unit heat load. Take into consideration the thermal resistance of the steel wall of a tube ($\delta = 2 \text{ mm}$) and the fouling deposits.

Solution. The physicochemical constants of a 20% solution of ammonium nitrate at 80°C [5-3] are:

$$\lambda = 0.445 \times 1.16 = 0.517 \text{ W/m} \cdot \text{K}$$

$$\rho_{\text{lg}} = 1051 \text{ kg/m}^3$$

$$\mu = 41 \times 10^{-6} \times 9.81 = 0.402 \times 10^{-3} \text{ Pa} \cdot \text{s}$$

$$\sigma = 65.3 \times 10^{-3} \text{ N/m} \text{ (we assume that the change in } \sigma \text{ with the temperature is the same as for water)}$$

$$\rho_{\text{st}} = \frac{18 \times 273 \times 0.36}{22.4 \times 353} = 0.224 \text{ kg/m}^3$$

The condensation temperature of the heating steam is 101.7°C (Table A-57).

The mean temperature drop is:

$$\Delta t_m = 101.7 - 80 = 21.7 \text{ K}$$

We find the heat transfer coefficient for the condensing heating steam by Eq. (4-46):

$$\begin{aligned}\alpha_{st} &= 1.21\lambda \left(\frac{\rho^2 L g}{\mu H} \right)^{1/3} q^{-1/3} = \\ &= 1.21 \times 0.683 \left(\frac{957^2 \times 2257 \times 10^3 \times 9.81}{0.278 \times 10^{-3} \times 4} \right)^{1/3} q^{-1/3} = 2.17 \times 10^5 q^{-0.33}\end{aligned}$$

The physicochemical properties of the condensate have been taken from Table A-39, and the heat of condensation L from Table A-57.

The coefficient of heat transfer for the boiling solution, by Eq. (4-53), is:

$$\begin{aligned}\alpha_s &= b \left(\frac{\lambda^2 \rho_{lq}}{\mu \sigma T_b} \right)^{1/3} q^{2/3} = \\ &= 0.078 \left(\frac{0.517^2 \times 1051}{0.402 \times 10^{-3} \times 65.3 \times 10^{-3} \times 353} \right)^{1/3} q^{2/3} = 2.43 q^{0.67}\end{aligned}$$

where the value of the coefficient $b = 0.078$ has been determined for $\rho_{lq}/\rho_{st} = 1051/0.224 = 4700$ by Eq. (4-55):

$$b = 0.075 \left[1 + 10 \left(\frac{\rho_{lq}}{\rho_{st}} - 1 \right)^{-2/3} \right]$$

The sum of the thermal resistances of the wall and the fouling deposits (Tables A-28 and A-31) is:

$$\sum r_w = \frac{\delta}{\lambda_w} + r_{foul,1} + r_{foul,2} = \frac{0.002}{46.5} + \frac{1}{5800} + \frac{1}{5800} = 3.88 \times 10^{-4} \text{ m}^2 \cdot \text{K/W}$$

The overall heat transfer coefficient is:

$$\begin{aligned}K &= \frac{1}{\frac{1}{\alpha_{st}} + \sum r_w + \frac{1}{\alpha_s}} = \\ &= \frac{1}{\frac{1}{2.17 \times 10^5 q^{-0.33}} + 3.88 \times 10^{-4} + \frac{1}{2.43 q^{0.67}}}\end{aligned}$$

The unit heat load is:

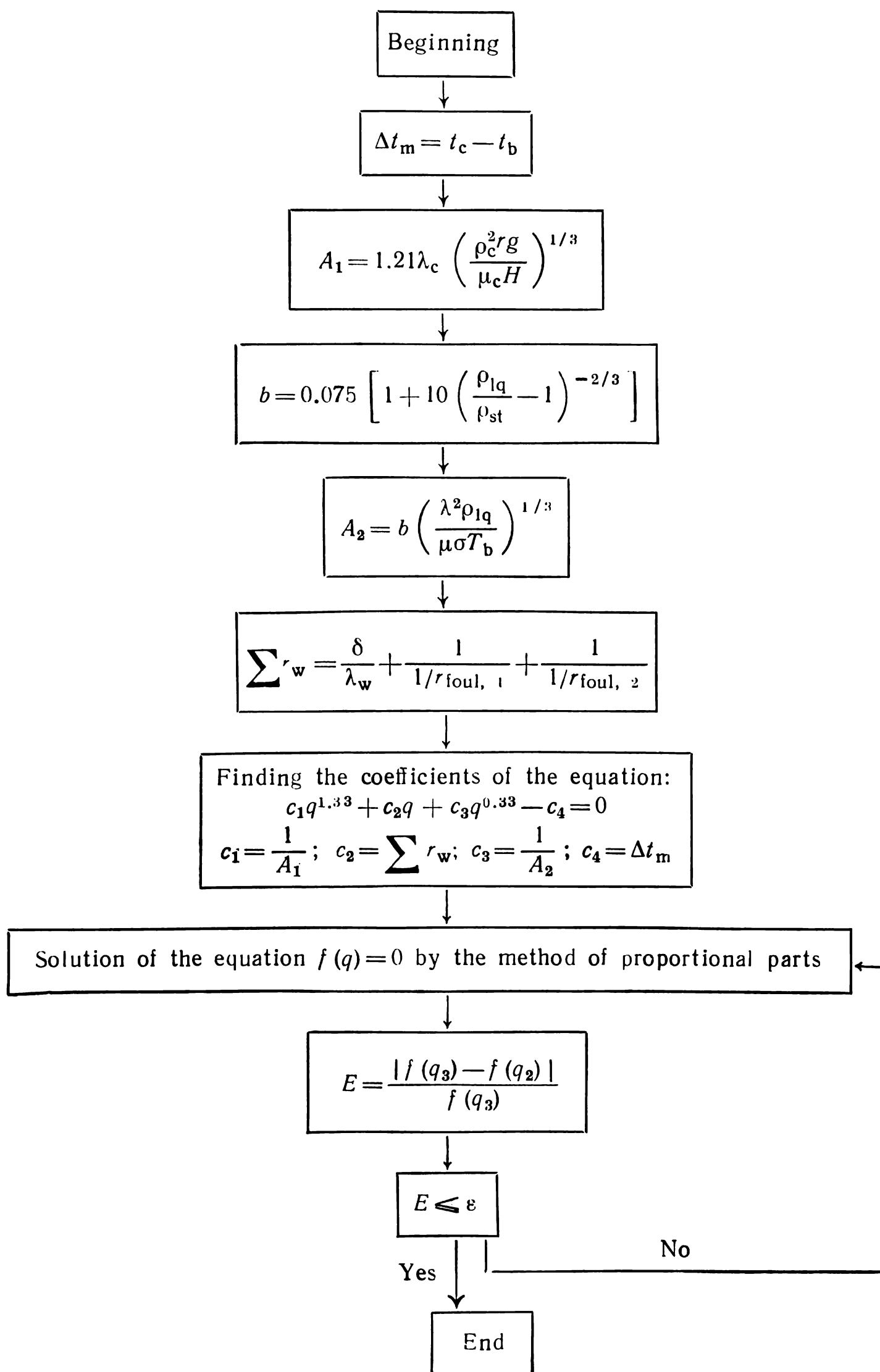
$$q = K \Delta t_m = \frac{21.7}{0.461 \times 10^{-5} q^{1.33} + 3.88 \times 10^{-4} + 0.412 q^{-0.67}}$$

whence

$$0.461 \times 10^{-5} q^{1.33} + 3.88 \times 10^{-4} q + 0.412 q^{0.33} - 21.7 = 0$$

Graphical solution of this equation (see Example 4-19) yields $q = 20\,500 \text{ W/m}^2$.

The flow chart of the algorithm for solving Example 4-20 is as follows:



The table of identifiers is given below (Table 4-8).

TABLE 4-8

Quantity	Δt_m	t_c	t_b	A_1	λ_c	ρ_c	r	μ_c	H	b	ρ_{lq}	ρ_{st}
Identifier	delta	tc	tb	A1	lambda c	rc	r	mc	H	b	rl	rs

A_2	λ	μ	σ	$\sum r_w$	δ	λ_w	$1/r_{foul, 1}$	$1/r_{foul, 2}$	c_1	c_2	c_3	c_4
A2	lambda	m	sigma	sr	dw	lw	prov 1	prov 2	c1	c2	c3	c4

Approximate values of roots of equation $f(x)=0$				Values of polynomial $p(q)=c_1q^{1.33}+c_2q+c_3q^{0.33}-c_4$ at $q=q_1, q_2, q_3$		
q_1	q_2	q_3		$f(q_1)$	$f(q_2)$	$f(q_3)$
q1	q2	q3		fq1	fq2	fq3

Determined error of calculations E	Preset (permissible) error of calculations ϵ	Parameter of cyclic calculations
E	eps	i

The ALGOL record of the programme is as follows:

```
Begin integer i;
real delta, tc, tb, A1, lambda c, rc, r, mc, H, b, rl, rs, A2, lambda, m, sigma,
sr, dw, lw, prov 1, prov 2, c1, c2, c3, c4, q1, q2, q3, fq1, fq2, fq3, E, eps,
input (tc, tb, lambda c, rc, r, mc, H, rl, rs, lambda, m, sigma, sr, dw, lw,
prov 1, prov 2, eps):
delta:=tc-tb;
A1:=1.21×lambda c×(rc↑2×r×9.81/mc/H)↑.33;
b:=.075×(1+10/(rl/rs-1)↑.67);
A2:=b×(lambda↑2×rl/m/sigma/(tb+273))↑.33;
sr:=dw/lw+1/prov 1+1/prov 2;
c1:=1/A1; c2:=sr; c3:=1/A2; c4:=delta;
q1:=0; q2:=50 000; i:=0; E:=2×eps;
M:for i:=i+1 while E > eps do
begin fq1:=c1×q1↑1.33+c2×q1+c3×q1↑.33-c4;
fq2:=c1×q2↑1.33+c2×q2+c3×q2↑.33-c4;
```

```

      q3:=q1+(-(q2-q1)×fq1/(fq2-fq1));
      fq3:=c1×q3↑1.33+c2×q3+c3×q3↑.33-c4;
      E:=abs(fq3-fq2)/fq3;
go to if fq1 < 0 then AL else AN;
AL:if fq3 < 0 then begin q1:=q3; q2:=q2 end
      else begin q1:=q1; q2:=q3 end;
go to M;
AN:if fq3 > 0 then begin q1:=q3; q2:=q2 end
      else begin q1:=q1;q2:=q3 end;
end of cycle with respect to i;
output (q3); end

```

The numerical information fed in with the aid of punched cards will have the form:

+	+	+	03	1017	+	+	+	00	517
+	+	+	02	8	+	+	-	03	402
+	+	+	00	683	+	+	-	01	653
+	+	+	03	957	+	+	-	02	2
+	+	+	07	2257	+	+	+	02	465.
+	+	-	03	278	+	+	+	04	58
+	+	+	01	4	+	+	+	04	58
+	+	+	04	1051	+	+	-	01	5
+	+	+	00	224					

Upon the tape feeding of the programme into an electronic digital computer, the numerical information is written as follows:

101.7, 80, .683, 957, 2257₁₀³, 278₁₀⁻³, 4, 1051, .224,
.517, .402₁₀⁻³, 65.3₁₀⁻³, .002, 46.5, 5800, 5800, .05

Example 4-21. Determine the surface area of a counterflow heat exchanger in which the hot fluid (absorbing oil) in an amount of 3 ton/h is cooled from 100 to 25 °C by a cold fluid heating from 20 to 40 °C. The overall heat transfer coefficient is known to change with the temperature of the oil as follows:

$T, ^\circ\text{C}$	100	80	60	40	30	25
$K, \text{W/m}^2\cdot\text{K}$	354	350	342	308	232	166

The specific heat capacity of the oil is $1.67 \times 10^3 \text{ J/kg}\cdot\text{K}$.

Solution. According to the initial condition of the example, the overall heat transfer coefficient greatly changes along the heat exchange surface, therefore the logarithmic mean temperature drop cannot be used. We employ the equation of overall heat transfer in the differential form:

$$Q_{m,h}c_h dT = -K(T-t) dA$$

whence

$$A = -Q_{m,h}c_h \int_{T_{\text{in}}}^{T_{\text{fn}}} \frac{dT}{K(T-t)} = Q_{m,h}c_h \int_{T_{\text{fn}}}^{T_{\text{in}}} \frac{dT}{K(T-t)}$$

Here T is the temperature of the hot fluid and t that of the cold one. We solve the integral

$$\int_{T_{\text{fin}}}^{T_{\text{in}}} \frac{dT}{K(T-t)}$$

graphically.
We preliminarily find the data needed to plot a graph having the ordinate $1/K(T-t)$ and the abscissa the temperature of the hot fluid T . First from the heat balance equation

$$Q_{m,h}c_h(T_{\text{in}}-T)=Q_{m,c}c_c(t_{\text{fin}}-t)$$

we determine the ratio

$$\frac{Q_{m,h}c_h}{Q_{m,c}c_c}=\frac{t_{\text{fin}}-t_{\text{in}}}{T_{\text{in}}-T_{\text{fin}}}=\frac{40-20}{100-25}=0.267$$

Consequently,

$$t=t_{\text{fin}}-\frac{Q_{m,h}c_h}{Q_{m,c}c_c}(T_{\text{in}}-T)=40-0.267(100-T)$$

Assigning different values to T , we use this equation to calculate the corresponding values of the temperature t of the cold fluid. We summate the data obtained in Table 4-9.

TABLE 4-9

T	t	$T-t$	K	$\frac{1}{K(T-t)}\times 10^4$
100	40.0	60.0	354	0.47
80	34.7	45.3	350	0.63
60	29.3	30.7	342	0.95
40	24.0	16.0	308	2.30
30	21.3	8.7	232	4.96
25	20.0	5.0	166	12.07

Using the data of Table 4-9, we plot a graph (Fig. 4-18).

We determine the area under the curve $A_{\text{curve}}=\int_{T_{\text{fin}}=25}^{T_{\text{in}}=100} \frac{dT}{K(T-t)}$

approximately using the formula for trapeziums:

$$A_{\text{curve}}=\frac{T_{\text{in}}-T_{\text{fin}}}{n}\left(\frac{y_0+y_n}{2}+y_1+y_2+\dots+y_{n-1}\right)$$

We assume that $n=10$ and compile Table 4-10, taking the values of the ordinates y from the graph.

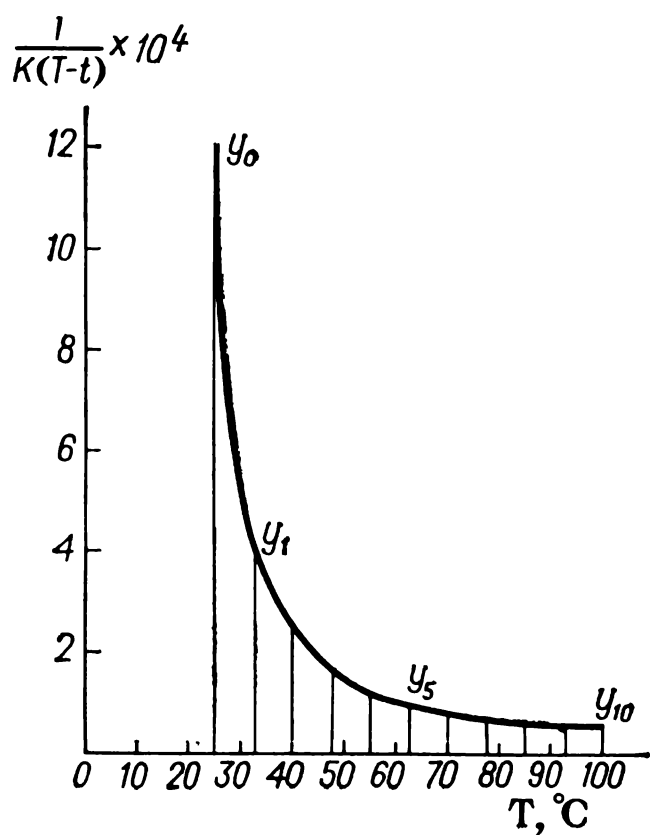


Fig. 4-18. To Example 4-21

TABLE 4-10

$T, ^\circ\text{C}$	Ordinate number	$y = \frac{1}{K(T-t)} \times 10^4$	$T, ^\circ\text{C}$	Ordinate number	$y = \frac{1}{K(T-t)} \times 10^4$
25	0	12.07	70	6	0.74
32.5	1	3.85	77.5	7	0.65
40	2	2.30	85	8	0.58
47.5	3	1.60	92.5	9	0.50
55	4	1.13	100	10	0.47
62.5	5	0.85			

From the data of Table 4-10, we find

$$\begin{aligned} A_{\text{curve}} &= \frac{T_{\text{in}} - T_{\text{fn}}}{n} \left(\frac{y_0 + y_n}{2} + \sum_1^{n-1} y \right) = \\ &= \frac{100 - 25}{10} \left(\frac{12.07 + 0.47}{2} + 12.2 \right) 10^{-4} = 138 \times 10^{-4} \text{ m}^2 \cdot \text{K/W} \end{aligned}$$

The required heat exchange surface area is:

$$A = Q_{\text{m,h}} c_{\text{h}} \int_{T_{\text{fn}}}^{T_{\text{in}}} \frac{dT}{K(T-t)} = \frac{3000}{3600} \times 1.67 \times 10^3 \times 138 \times 10^{-4} = 19.2 \text{ m}^2$$

If we determine the area under the curve using a planimeter, we get $A = 18.9 \text{ m}^2$.

For purposes of comparison, let us calculate the required heat exchange surface area if we assume the value of the overall heat exchange coefficient K to be constant (at the mean temperature)

and apply the logarithmic mean temperature drop:

$$\begin{array}{ccc} 100 \rightarrow & & 25 \\ 40 \leftarrow & & 20 \\ \hline \Delta t_{gr} = 60 & \Delta t_{sm} = 5 & \\ \Delta t_m = \frac{60-5}{2.3 \log \frac{60}{5}} = 22.2 \text{ K} & & \end{array}$$

The mean temperature of the fluid being cooled is:

$$T_m = t_m + \Delta t_m = 30 + 22.2 = 52.2^\circ\text{C}$$

At this temperature, we have $K = 329 \text{ W/m}^2 \cdot \text{K}$.

The consumption of heat transferred is:

$$Q = \frac{3000}{3600} \times 1.67 \times 10^3 (100 - 25) = 104\,500 \text{ W}$$

The area of the heat exchange surface is:

$$A = \frac{Q}{K\Delta t_m} = \frac{104\,500}{329 \times 22.2} = 14.3 \text{ m}^2$$

Example 4-22. Determine the loss of heat by radiation from the surface of a steel cylindrical apparatus in a room whose walls have an oil paint surface finish. The dimensions of the apparatus are $H = 2 \text{ m}$ and $D = 1 \text{ m}$. The dimensions of the room are: height — 4 m , length — 10 m , width — 6 m . The temperature of the apparatus wall is 70°C , the temperature of the air in the room is 20°C .

Also determine the total loss of heat from the apparatus by radiation and convection.

Solution. We calculate the loss of heat by radiation according to Eqs. (4-59) and (4-60):

$$Q_{\text{rad}} = W_{1-2} A_1 \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right]$$

$$W_{1-2} = \frac{1}{\frac{1}{W_1} + \frac{A_1}{A_2} \left(\frac{1}{W_2} - \frac{1}{W_{bl}} \right)}$$

In our case:

$$T_1 = 273 + 70 = 343 \text{ K}$$

$$T_2 = 273 + 20 = 293 \text{ K}$$

$$A_1 = \pi DH + 2 \times 0.785 D^2 = 3.14 \times 1 \times 2 + 2 \times 0.785 \times 1^2 = 7.85 \text{ m}^2$$

$$A_2 = 2(4 \times 6 + 4 \times 10 + 6 \times 10) = 248 \text{ m}^2$$

Since the area A_2 is great in comparison with the area A_1 , the radiating power $W_{1-2} \approx W_1$.

In Table A-24 for oxidized steel, we find the mean value of the emissivity $\varepsilon = 0.85$. Hence, $W_1 = 5.7 \times 0.85 = 4.84 \text{ W/m}^2 \cdot \text{K}^4$.

The loss of heat by radiation is:

$$Q_{\text{rad}} = W_1 A_1 \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right] =$$

$$= 4.84 \times 7.85 (3.43^4 - 2.93^4) = 2490 \text{ W}$$

We find the total loss of heat by radiation and convection with the aid of the equation:

$$Q = \alpha A_1 (t_w - t_{\text{air}})$$

where α is the summary coefficient of heat transfer by radiation and convection. It is determined by Eq. (4-64):

$$\alpha = 9.74 + 0.07 \Delta t = 9.74 + 0.07 (70 - 20) = 13.2 \text{ W/m}^2 \cdot \text{K}$$

The total amount of heat lost by the apparatus is

$$Q = 13.2 \times 7.85 (70 - 20) = 5200 \text{ W}$$

Example 4-23. Determine the required thickness of the layer of insulation of an apparatus inside of which the temperature is 154°C . The insulating material is Sovelite (85% of magnesia and 15% of asbestos). The temperature of the external surface of the insulation should not exceed 40°C .

Solution. We take the ambient temperature equal to $t_a = 20^\circ\text{C}$ and determine the summary coefficient of heat transfer to the surrounding air by radiation and convection with the aid of Eq. (4-64):

$$\alpha = 9.74 + 0.07 \Delta t = 9.74 + 0.07 (40 - 20) = 11.1 \text{ W/m}^2 \cdot \text{K}$$

The unit heat flux is:

$$q = \alpha (t_w - t_a) = 11.1 (40 - 20) = 222 \text{ W/m}^2$$

Assuming approximately that the entire thermal resistance is concentrated in the layer of insulation, we can write that

$$q = K (t_{\text{in}} - t_a) \approx \frac{\lambda}{\delta} (t_{\text{in}} - t_a)$$

whence the insulation thickness is:

$$\delta = \frac{\lambda}{q} (t_{\text{in}} - t_a) = \frac{0.098}{222} (154 - 20) = 0.059 \text{ m}$$

where $\lambda = 0.098 \text{ W/m} \cdot \text{K}$ is the thermal conductivity of Sovelite (Table A-28).

Example 4-24. Toluene in an amount of 1400 kg at 105°C is charged into a tank provided with a coil through which cooling water flows. The coil surface area is 3.2 m^2 . The water enters the coil at a temperature of 13°C . How much time is needed to cool the toluene to 25°C if the overall heat transfer coefficient is assumed to be constant and equal to $255 \text{ W/m}^2\text{K}$? Also determine the flow rate of the water.

Solution. A solution of the problem of an unsteady liquid cooling process is given in [0-34].

The symbols used are $t_{1,\text{in}} = 105^\circ\text{C}$, $t_{1,\text{fn}} = 25^\circ\text{C}$, $t_{2,\text{in}} = 13^\circ\text{C}$.

We assume that $t_{2,\text{fn}} = 18^\circ\text{C}$ and determine the value of the quantity B which is constant for the entire cooling process:

$$B = \frac{t_{1,\text{fn}} - t_{2,\text{in}}}{t_{1,\text{in}} - t_{2,\text{fn}}} = \frac{25 - 13}{105 - 18} = 1.715$$

From the equation

$$\frac{KA}{Z_2} = 2.3 \log B$$

where $Z_2 = G_{m,w} c_w$, we find the rate of flow of the water:

$$Q_{m,w} = \frac{KA}{2.3 \log B c_w} = \frac{255 \times 3.2}{2.3 \log 1.715 \times 4190} = 0.362 \text{ kg/s} = 1300 \text{ kg/h}$$

We find the duration of cooling τ by the equation:

$$\begin{aligned} \tau &= -\frac{Z_1}{Z_2} 2.3 \log \frac{t_{1,\text{fn}} - t_{2,\text{in}}}{t_{1,\text{in}} - t_{2,\text{in}}} \frac{B}{B-1} = \\ &= -\frac{1400 \times 1.8 \times 10^3}{0.362 \times 4190} 2.3 \log \frac{25 - 13}{105 - 18} \frac{1.715}{0.715} = 8.09 \times 10^3 \text{ s} = 2.25 \text{ h} \end{aligned}$$

Here $Z_1 = Q_{m,t} c_t = 1400 \times 1.8 \times 10^3 \text{ J/K}$

$c_t = 1.8 \times 10^3 \text{ J/kg} \cdot \text{K}$ is the specific heat capacity of toluene at the mean temperature of 65°C (Fig. A-11).

Example 4-25. Zeolite NaX is subjected to desorption—heating in a stream of hot air in a continuously functioning apparatus with a fluidized bed of zeolite. The mean temperature of the air is 190°C . The zeolite enters the apparatus having an initial temperature of 20°C . The diameter of a grain of zeolite is 4 mm, its density is 1100 kg/m^3 , its specific heat capacity is $870 \text{ J/kg} \cdot \text{K}$, and its thermal conductivity is $0.24 \text{ W/m} \cdot \text{K}$. The fluidization number [see Eq. (3-70)] is 4. Find the time needed to heat a zeolite grain from 20°C to a mean temperature of 185°C .

Solution. The heating of a zeolite grain is an unsteady process of thermal conductivity in a solid. Solution of the equation of unsteady thermal conductivity (at constant thermal and physical characteristics of the body being heated)

$$a \nabla^2 t = \frac{\partial t}{\partial \tau}$$

together with the boundary and initial conditions [4-7] leads to the equation

$$\frac{t_{\text{fn}} - t_{\text{in}}}{t_a - t_{\text{in}}} = f(Bi, Fo) \quad (a)$$

whose right-hand part is a complex function of the Biot (Bi) and Fourier (Fo) numbers.

For a grain having the shape of a sphere with the radius R , we have:

$$Bi = \frac{\alpha R}{\lambda_{sol}}$$

$$Fo = \frac{a\tau}{R^2} = \frac{\lambda_{sol}\tau}{c_{sol}\rho_{sol}R^2}$$

In the above equations:

$$a = \frac{\lambda_{sol}}{c_{sol}\rho_{sol}}$$

∇ = Laplacian operator

t_{in} and t_{fin} = initial and final temperatures of the body being heated

t_a = ambient temperature (temperature of the surrounding medium)

α = coefficient of heat transfer from the surrounding medium to the surface of the body being heated

λ_{sol} , c_{sol} , ρ_{sol} = thermal conductivity, specific heat capacity, and density of the solid body, respectively.

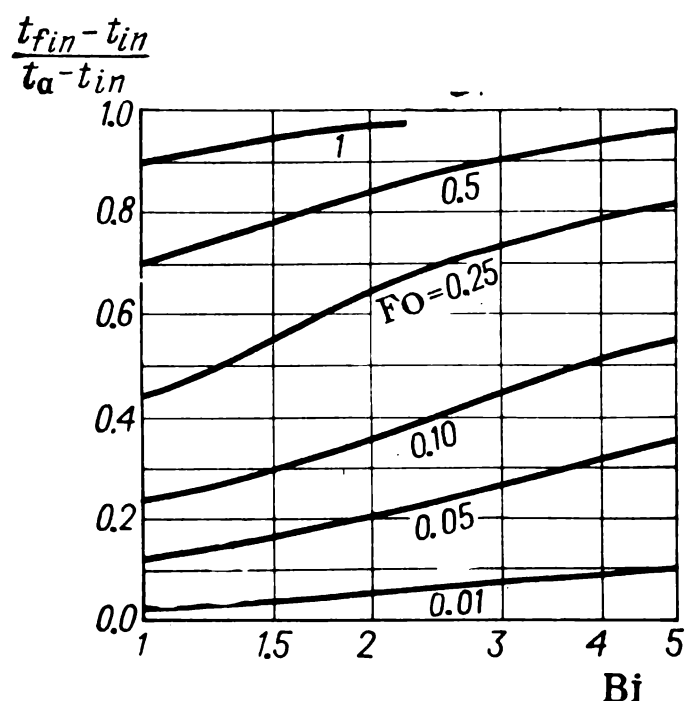


Fig. 4-19. To Example 4-25

Figure 4-19 gives a graph of Eq. (a) for a sphere [4-18, 4-16].

We determine the coefficient of heat transfer α from air to the surface of a zeolite grain in the fluidized bed. We first find the value of the Archimedes number by Eq. (3-3):

$$Ar = \frac{d^3 \rho_{sol} \rho_a g}{\mu_a^2} = \frac{0.004^3 \times 1100 \times 0.763 \times 9.81}{25.7^2 \times 10^{-12}} = 8 \times 10^5$$

where $\rho_a = 0.763 \text{ kg/m}^3$ and $\mu_a = 25.7 \times 10^{-6} \text{ Pa}\cdot\text{s}$ are the density and dynamic viscosity of the surrounding medium (air) at 190°C [4-18].

We find the Lyashchenko number from the graph in Fig. 3-8 at $Ar = 8 \times 10^5$ and $\varepsilon = 0.4$, namely, $Ly_{cr} = 2.75$. By Eq. (3-6) we determine the critical velocity of fluidization:

$$v_{cr} = \sqrt[3]{\frac{Ly_{cr} \mu_a g \rho_{sol}}{\rho_a^2}} = \sqrt[3]{\frac{2.75 \times 25.7 \times 10^{-6} \times 9.81 \times 1100}{0.763^2}} = 1.09 \text{ m/s}$$

The operating velocity of the air (related to the total cross section of the apparatus) is:

$$v = 4v_{cr} = 4 \times 1.09 = 4.36 \text{ m/s}$$

We use this velocity to calculate the Lyashchenko number:

$$Ly = \frac{v^3 \rho_a^3}{\mu_a \rho_{sol} g} = \frac{4.36^3 \times 0.763^2}{25.7 \times 10^{-6} \times 1100 \times 9.81} = 174$$

and in Fig. 3-8 at $Ar = 8 \times 10^5$ we find the porosity of the fluidized bed $\varepsilon = 0.67$.

The Reynolds number for the stream of air, using Eqs. (1-27) and (3-71), is:

$$Re = \frac{v d \rho_a}{\varepsilon \mu_a} = \frac{4.36 \times 0.004 \times 0.763}{0.67 \times 25.7 \times 10^{-6}} = 773$$

We calculate the Nusselt number by the equation [0-34]:

$$Nu = 0.4 Re^{0.67} Pr^{0.33} = 0.4 \times 773^{0.67} \times 0.681^{0.33} = 30.2$$

The coefficient of heat transfer from the air to the surface of a grain is [from Eq. (4-11)]:

$$\alpha = \frac{Nu \lambda_a}{d} = \frac{30.2 \times 0.0385}{0.004} = 290 \text{ W/m}^2 \cdot \text{K}$$

where $\lambda_a = 0.0385 \text{ W/m} \cdot \text{K}$ is the thermal conductivity of air at 190°C [4-18].

The Biot number is:

$$Bi = \frac{\alpha R}{\lambda_{sol}} = \frac{290 \times 0.002}{0.24} = 2.42$$

The simplex of the temperature drops is:

$$\frac{t_{fin} - t_{in}}{t_a - t_{in}} = \frac{185 - 20}{190 - 20} = 0.97$$

We use these data to find the value of the Fourier number Fo in Fig. 4-19. We get $Fo = 1$ whence the time needed for heating a grain of the zeolite is

$$\tau = \frac{Fo c_{sol} \rho_{sol} R^2}{\lambda_{sol}} = \frac{1 \times 870 \times 1100 \times 0.002^2}{0.24} \approx 16 \text{ s}$$

PROBLEMS

4-1. How many times will the thermal resistance of the wall of a steel coil formed of a tube having a diameter of 38×2.5 mm increase if it is coated with a layer of enamel 0.5 mm thick? Consider the wall to be flat. The thermal conductivity of enamel is $1.05 \text{ W/m}\cdot\text{K}$.

4-2. A steam pipe 40 m long with a diameter of 51×2.5 mm is covered with a layer of insulation 30 mm thick; the temperature of the external surface of the insulation is $t_2 = 45^\circ\text{C}$, and of the internal surface is $t_1 = 175^\circ\text{C}$. Determine the quantity of heat lost by the steam pipe an hour. The thermal conductivity of the insulation is $\lambda = 0.116 \text{ W/m}\cdot\text{K}$.

4-3. A steel pipe with a diameter of 60×3 mm is insulated with a layer of cork 30 mm thick and on top of it with another layer of Sovelite (85% of magnesia and 15% of asbestos) 40 mm thick. The temperature of the pipe wall is -110°C and that of the external surface of the insulation is 10°C . Calculate the loss of cold per hour from 1 m of pipe length.

4-4. How will the loss of cold in the conditions of the preceding problem change if the internal layer is made of Sovelite 40 mm thick, and the external one of cork 30 mm thick?

4-5. Find the temperature of the internal surface of the lining of an apparatus (Fig. 4-20) if the temperature on its external surface is 35°C . The thickness of the lining is 260 mm. A thermometer embedded in it to a depth of 50 mm shows a temperature of 70°C .

4-6. Calculate the thermal conductivities for (a) liquid chloroform at $t = 20^\circ\text{C}$, (b) sulphur dioxide at $t = 160^\circ\text{C}$ and $p_{\text{abs}} = 1$ at, and (c) a 30% aqueous solution of calcium chloride whose specific heat capacity is $2.72 \times 10^3 \text{ J/kg}\cdot\text{K}$.

4-7. It is necessary to evaporate 1600 kg/h of a liquid boiling at $t = 137^\circ\text{C}$ and fed into an evaporator at this temperature. The specific heat of vaporization of the liquid is $L = 377 \times 10^3 \text{ J/kg}$. The temperature of the heating steam should be at least 150°C . Determine the flow rate of the heating steam having the following parameters: (a) dry saturated, $p_{\text{gauge}} = 4$ at; (b) superheated to 250°C , $p_{\text{gauge}} = 4$ at; and (c) superheated to 250°C , $p_{\text{gauge}} = 3$ at. The specific heat capacity of superheated steam is $2.14 \times 10^3 \text{ J/kg}\cdot\text{K}$.

Show the processes of the change in the state of the heating steam in a T - S diagram. The condensate of the heating steam is removed at the temperature of condensation.

4-8. Up to what temperature will 2 tons of a calcium chloride solution be heated by indirect heating steam if the flow rate of the steam ($p_{\text{abs}} = 2$ at) is 200 kg during 2.5 hours, while the heat used for heating the apparatus and its losses to the surroundings average 2030 W? The initial temperature of the solution is 10°C . The specific heat capacity of the solution is $2.5 \times 10^3 \text{ J/kg}\cdot\text{K}$.

4-9. Determine the quantity of heat transferred in a counterflow condenser in which 850 kg of carbon disulphide vapour condense an hour under atmospheric pressure. The carbon disulphide vapour enters the condenser at a temperature of 90°C . The liquid carbon disulphide leaves the condenser at a tempera-

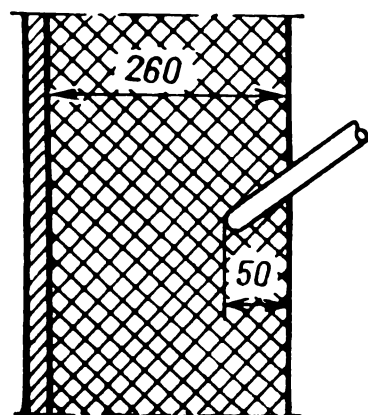
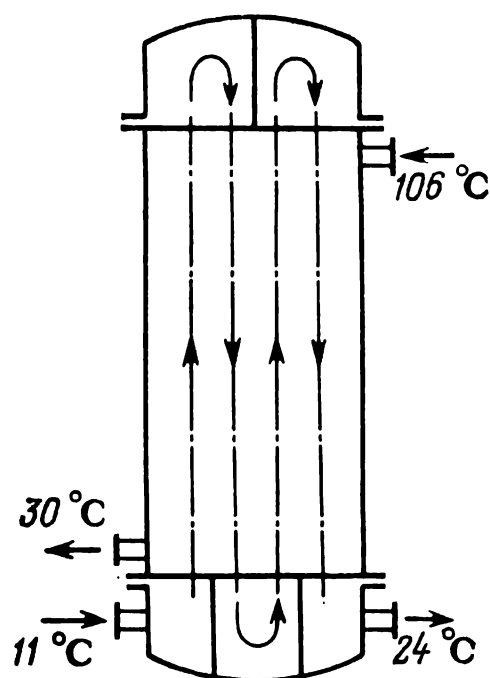


Fig. 4-20. To Problem 4-5

Fig. 4-21. To Problem 4-13



ture lower than the temperature of condensation by 8°C . The specific heat capacity of the carbon disulphide vapour is $0.67 \times 10^3 \text{ J/kg}\cdot\text{K}$.

4-10. Dry saturated carbon dioxide vapour under a pressure of $p_{\text{abs}} = 60$ at flows into a tubular condenser at a rate of 120 kg/h . Liquid carbon dioxide leaves the condenser at the same pressure and at the temperature of condensation. Assuming that the difference between the temperatures of the carbon dioxide and the water at the outlet of the latter from the condenser is 5 K , determine the required rate of flow of the water if it is fed into the condenser with a temperature of 10°C .

4-11. Butyl alcohol is to be cooled from 90 to 50°C in a counterflow heat exchanger having a surface area of 6 m^2 at a rate of 1930 kg/h . Cooling is performed by water having an initial temperature of 18°C . The overall coefficient of heat transfer in the heat exchanger is $230 \text{ W/m}^2\cdot\text{K}$; calculate Δt_m as the mean arithmetical value. How many cubic metres of water have to be passed through the heat exchanger an hour?

4-12. The hot concentrated solution leaving an evaporating apparatus with a temperature of 106°C is used to heat up to 50°C the cold dilute solution fed in for evaporation at a temperature of 15°C . The concentrated solution is cooled to 60°C . Determine the mean temperature drop for parallel flow and counterflow arrangements.

4-13. Toluene is cooled with water from 106 to 30°C in a multipass shell-and-tube heat exchanger having four passes on the tube side and one pass on the shell side (Fig. 4-21). The water while passing along the tubes is heated from 11 to 24°C . Determine the mean temperature drop in the exchanger.

4-14. A tubular heat exchanger consists of 19 brass tubes with a diameter of $18 \times 2 \text{ mm}$ and a length of 1.2 m . Is its surface sufficient for the condensation of 350 kg/h of saturated ethyl alcohol vapour if the overall heat transfer coefficient is taken equal to $700 \text{ W/m}^2\cdot\text{K}$, the initial temperature of the water is 15°C , and its final temperature is 35°C ? Condensation of the alcohol will be conducted at atmospheric pressure, the liquid alcohol is removed at the condensation temperature.

4-15. A tubular counterflow heat exchanger (Fig. 4-22) before the contact apparatus at a sulphuric acid plant has a heat exchange surface area of 360 m^2 . The purified gas from the pyrite furnaces enters the shell side of the exchanger at 300°C and leaves it at 430°C . The hot gas from the contact apparatus enters the exchanger tubes at 560°C . The flow rate of the gas is 10 ton/h , its mean specific heat capacity is $1.05 \times 10^3 \text{ J/kg}\cdot\text{K}$. The losses of heat through the shell of the exchanger are one-tenth of the amount of heat received by the gas being heated. Determine the overall coefficient of heat transfer in the exchanger.

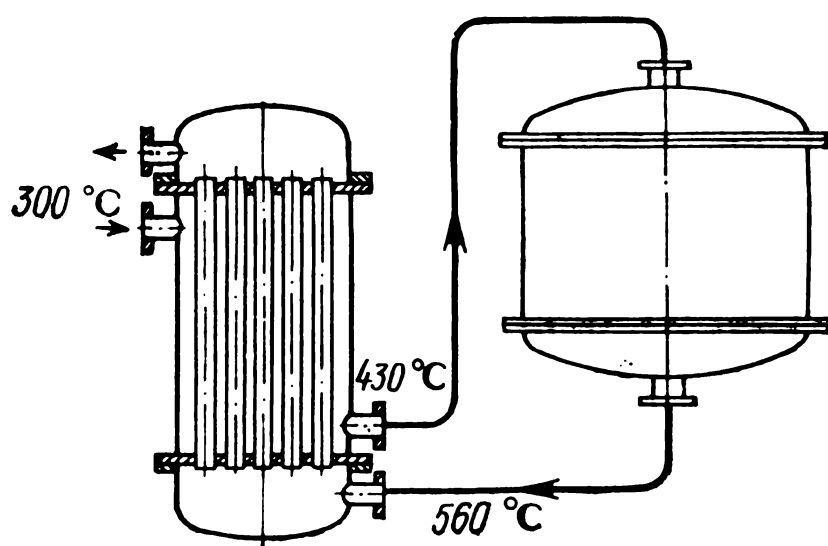


Fig. 4-22. To Problem 4-15

4-16. Determine the overall coefficient of heat transfer in a coil heat exchanger according to the following data: the heat exchange surface area is 48 m^2 , the apparatus heats 85.5 ton/h of water from 77 to 95°C , the heating fluid is saturated steam at a gauge pressure of 0.23 at .

4-17. A column for the rectification of liquid air is covered with a layer of thermal insulation made of slag wool 250 mm thick. The temperature of the liquid inside the column is -190°C , the ambient temperature is 20°C . What amount of heat can penetrate from the surrounding air into the column through 1 m^2 of its surface if we disregard the thermal resistances from the side of the liquid, the surrounding air and the metal wall of the column?

4-18. How will the overall coefficient of heat transfer in a column change if we replace the steel tubes having a diameter of $38 \times 2.5 \text{ mm}$ with copper tubes having the same size:

(a) in a steam air heater in which $\alpha_{\text{air}} = 41 \text{ W/m}^2 \cdot \text{K}$ and $\alpha_{\text{steam}} = 11\,600 \text{ W/m}^2 \cdot \text{K}$;

(b) in an evaporator in which $\alpha_{\text{boiling sol.}} = 2320 \text{ W/m}^2 \cdot \text{K}$ and $\alpha_{\text{steam}} = 11\,600 \text{ W/m}^2 \cdot \text{K}$?

Disregard fouling of the surface.

4-19. How will the value of the overall coefficient of heat transfer change in a heat exchanging apparatus made of steel tubes 3 mm thick if the surface of the tubes is covered with a scale deposit (boiling stone) 2 mm thick:

(a) in a water cooler for a gas in which $\alpha_{\text{gas}} = 58 \text{ W/m}^2 \cdot \text{K}$ and $\alpha_{\text{water}} = 580 \text{ W/m}^2 \cdot \text{K}$, and

(b) in an evaporator in which $\alpha_{\text{boiling sol.}} = 2780 \text{ W/m}^2 \cdot \text{K}$ and $\alpha_{\text{steam}} = 11\,600 \text{ W/m}^2 \cdot \text{K}$?

4-20. What can the maximum specific heat load (in W/m^2) be in a toluene evaporator if its steel tubes 4 mm thick are covered on both sides with rust? The thickness of one layer of rust is 0.6 mm . The evaporator is heated with saturated steam ($p_{\text{gauge}} = 3 \text{ at}$). The toluene boils under atmospheric pressure. Consider that the thermal resistance of the wall and the two layers of rust is considerably greater than the sum of the other thermal resistances.

4-21. Determine the required surface area of a counterflow heat exchanger when cooling $0.85 \text{ m}^3/\text{h}$ of carbon disulphide from its boiling point at atmospheric pressure to 22°C . The cooling water becomes heated from 14 to 25°C . The heat transfer coefficients for CS_2 and H_2O are 270 and $720 \text{ W/m}^2 \cdot \text{K}$, respectively. The thickness of the steel wall is 3 mm . Take into account the presence of fouling deposits—rust and scale ($\sum r_{\text{foul}} = 0.00069 \text{ m}^2 \cdot \text{K/W}$). Also determine the rate of flow of the water.

4-22. Methane under a gauge pressure of 5 at flows on the shell side of a shell-and-tube heat exchanger parallel to the tubes with a velocity of 4.6 m/s . The mean temperature of the methane is 75°C . The exchanger consists of 37 steel tubes with a diameter of $18 \times 2 \text{ mm}$ contained in a shell whose internal diameter is 190 mm . Determine the heat transfer coefficient.

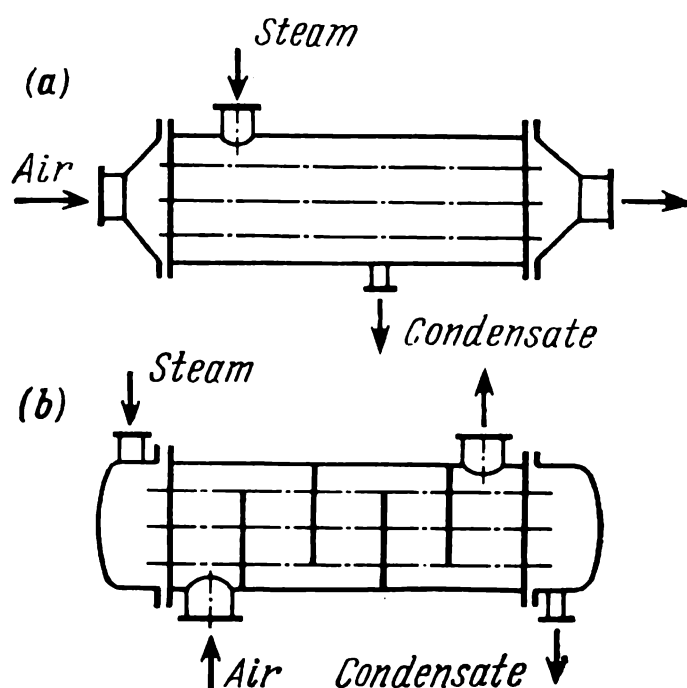


Fig. 4-23. To Problem 4-25

4-23. Determine the heat transfer coefficient for 98% sulphuric acid flowing along the annular (intertubular) space of a “tube-in-tube” heat exchanger with a velocity of 0.7 m/s. The mean temperature of the acid is 70 °C, and that of the wall is 60 °C. The external tube of the exchanger has a diameter of 54×4.5 mm, the inner one— 26×3 mm.

4-24. Methyl alcohol is heated from 10 to 50 °C at a rate of 3700 kg/h while passing through the tube side of a heat exchanger consisting of 19 tubes with a diameter of 16×2 mm. Determine the coefficient of heat transfer if the temperature of the wall is taken equal to 60 °C.

4-25. Air at atmospheric pressure is heated in a tubular steam heater made of tubes with a diameter of 26×3 mm. The average temperature of the air is 60 °C. Compare the overall heat transfer coefficients for two cases: (1) the air flows through the tubes ($L/d > 50$) and the heating steam condenses in the shell side (Fig. 4-23a), and (2) the air flows through the shell side provided with baffles and the steam condenses in the tubes (Fig. 4-23b). In both cases assume that the velocity of the air is 10 m/s and the heat transfer coefficient of the steam is 11 000 W/m²·K. The tubes have a staggered arrangement (triangular pitch).

4-26. Determine the coefficient of heat transfer for ethylene cooled under an absolute pressure of 2 at from 80 to 30 °C in the shell side of a shell-and-tube heat exchanger with baffles. The tubes with a diameter of 32×3 mm have a staggered arrangement. The velocity of the ethylene (at the narrowest section of the tube bank) is 7.5 m/s.

4-27. In heat exchange between two turbulent streams ($Re > 10\,000$) for the first stream $\alpha_1 = 230$ W/m²·K and for the second one $\alpha_2 = 400$ W/m²·K. How many times will the overall coefficient of heat transfer grow if the velocity of the first stream doubles and that of the second one triples (other conditions remaining unchanged)? Disregard the thermal resistance of the wall.

4-28. A vertical shell-and-tube heat exchanger consists of 91 tubes with a diameter of 57×3 mm and a height of 4 m. A film of water flows down the internal surface of the tubes at a rate of 52 m³/h, and it is heated from 18 to 25 °C. The average temperature of the internal surface of the tubes is 26 °C. Determine the heat transfer coefficient.

4-29. A film of 60% sulphuric acid flows down the vertical wall of a film cooler at a rate of 2.1 dm³/s per metre of wall width. The height of the cooler is 5 m. The average temperature of the wall surface is 24 °C, and that of the acid is 50 °C. Calculate the coefficient of heat transfer for the acid if its thermal conductivity is 0.43 W/m·K.

4-30. Water is heated in the conditions of free flow. The external diameter of the horizontal heating tubes is 76 mm. Determine the heat transfer coefficient if the temperature of the tube surface is 45 °C. The average temperature of the water is 25 °C.

4-31. Calculate the coefficient of heat transfer for a 30% aqueous solution of calcium chloride boiling under atmospheric pressure. The temperature drop between the heating surface and the boiling solution is 12 K.

4-32. The saturated vapour of methyl alcohol condenses under atmospheric pressure at a rate of 4 ton/h on the shell side of a vertical shell-and-tube heat exchanger consisting of 266 tubes with a diameter of 25×2 mm. Determine the coefficient of heat transfer.

4-33. Saturated steam condenses on the external surface of a horizontal ten-row triangular pitch bank of tubes. The external diameter of the tubes is 30 mm. The condensation temperature is 180 °C. Find the mean coefficient of heat transfer assuming that the temperature of the external surface of the tubes is 172 °C. The mass ratio of the air in the steam is 0.005 (0.5%).

4-34. Water flows through the tubes of a shell-and-tube heat exchanger with a velocity of 0.7 m/s. The diameter of the tubes is 46×3 mm. Determine the heat transfer coefficient if the mean temperature of the surface of the wall in contact with the water is 90 °C, and the average temperature of the water is 46 °C.

4-35. Toluene is cooled from 90 to 30 °C while flowing through a coil at a rate of 1.5 ton/h. Cooling (in counterflow) is performed by water that becomes heated from 15 to 40 °C. The steel coil tube has a diameter of 57×3.5 mm; $\alpha_{\text{water}} = 580 \text{ W/m}^2 \cdot \text{K}$. The diameter of a turn of the coil is 0.4 m. Determine the required length of the coil and the rate of flow of the water. Assume that the thermal resistance of the wall and its fouling deposits is $0.0007 \text{ m}^2 \cdot \text{K/W}$, while the ratio Pr/Pr_w for toluene equals 0.75.

4-36. A steam heater is used to heat 5200 m³/h (in standard conditions) of air from 2 to 90 °C under atmospheric pressure. The air heater is made of steel tubes having a diameter of 38×3 mm, the number of tubes is 151. The air flows through the tubes, the heating steam through the shell side. The absolute pressure of the heating steam is 2 at. Determine the required length of the tubes and the rate of flow of the steam if its moisture content is 6%.

4-37. It is necessary to condense 10 ton/h of the saturated vapour of *n*-hexane at 70 °C. The condenser can be cooled (a) with water whose temperature grows from 16 to 36 °C, and (b) with air whose temperature grows from 25 to 48 °C. Assume that the coefficient of heat transfer for the condensed hexane vapour in both cases is $1700 \text{ W/m}^2 \cdot \text{K}$. Take the coefficients of heat transfer for the water and air approximately (the mean values) from Table 4-5—for water for turbulent flow through tubes, for air for cross flow on the shell side. The liquid hexane is removed at the condensation temperature. Ignore the thermal resistances of the wall and the fouling deposits. The specific heat of condensation of the hexane is $33.3 \times 10^4 \text{ J/kg}$. Determine the rates of flow of the water and air (in m³/h) and the required heat exchange surface areas.

4-38. Methyl alcohol (100%) is heated in the tube side of a single-pass shell-and-tube heat exchanger from 15 to 40 °C. Water passes in counterflow on the shell side and is cooled from 90 to 40 °C. The exchanger consists of 121 steel tubes with a diameter of 25×2 mm. The velocity of the alcohol in the tubes is 0.75 m/s. The heat transfer coefficient for water is $840 \text{ W/m}^2 \cdot \text{K}$, the total thermal conductivity of the fouling deposits on both the inside and the outside surface of the tubes is $1700 \text{ W/m}^2 \cdot \text{K}$, the average temperature of the surface of the fouling deposit in contact with the alcohol is 38 °C. Using Table A-34, determine the smallest number of heat exchangers having the same tube length that must be installed (in series) if the spare surface area is to be at least 20%.

4-39. Air is heated in the tube side of a single-pass shell-and-tube heat exchanger from 20 to 90 °C at a mean pressure of $p_{\text{abs}} = 810 \text{ mm Hg}$. The rate of flow of the air for standard conditions is 7770 m³/h. The exchanger has

211 tubes with a diameter of 38×2 mm. Saturated steam at a pressure of $p_{\text{abs}} = 2$ at is fed into the shell side. Take the coefficient of heat transfer for water approximately, without calculations. Take the spare surface area equal to 18%. What tube length should the exchanger have? Also determine the rate of flow of the steam if its moisture content is 5%.

4-40. Determine the overall coefficient of heat transfer in a "tube-in-tube" ("double-tube") heat exchanger according to the conditions of Problem 1-37. Take into account the thermal resistance of the fouling deposits on the wall at the side of the toluene and at the side of the water. Perform the calculations (a) disregarding the ratio Pr/Pr_w , and (b) taking this ratio into account.

4-41. Hot air at atmospheric pressure having a mean temperature of 85°C flows at a velocity of 2.5 m/s in a dryer along its flat wall 6 m long. The steel wall of the dryer 5 mm thick is coated with rust and is provided outside with a layer of thermal insulation 30 mm thick. The thermal conductivity of the insulation is $0.1 \text{ W/m}\cdot\text{K}$. The ambient temperature is 18°C . Determine the amount of heat lost in one hour from 1 m^2 of the dryer wall by convection and radiation, and also the overall coefficient of heat transfer through the wall.

4-42. The vertical wall of an evaporator is covered with a layer of insulation ($\lambda = 0.12 \text{ W/m}\cdot\text{K}$) 45 mm thick. The temperature of the boiling solution is 120°C , the ambient temperature is 20°C . Determine the loss of heat by convection and radiation from 1 m^2 in one hour, assuming the temperature of the wall surface in contact with the boiling solution to equal the temperature of the latter.

4-43. Saturated steam at a pressure of $p_{\text{abs}} = 4$ at passes through a horizontal steam pipe with a diameter of 51×2.5 mm and a length of 50 m. Determine the amount of condensate formed during a day in the non-insulated pipe. The ambient temperature is 15°C .

4-44. By how many per cent will the heat loss diminish if the steam pipe considered in the preceding problem is coated with a thermal-insulating layer 40 mm thick having a thermal conductivity of $0.093 \text{ W/m}\cdot\text{K}$?

4-45. An apparatus is insulated with a layer of fireclay brick 125 mm thick ($\lambda = 0.68 \text{ W/m}\cdot\text{K}$) and a layer of an insulating compound ($\lambda = 0.12 \text{ W/m}\cdot\text{K}$). The temperature of the external surface of the metal wall of the apparatus is 500°C . Find the thickness of the layer of insulating compound sufficient for the temperature of its external surface not to exceed 50°C at an ambient temperature of 25°C .

EXAMPLES OF CALCULATING AND SELECTING HEAT EXCHANGERS

Example 4-1. Perform the thermal calculations of a heat exchanger for cooling 5 ton/h of diethyl ether from 33 to 6°C . The ether is cooled by brine supplied from a refrigerating plant at a temperature of -6°C . The concentration of the brine is 20% (mass) of sodium chloride. The ether and the brine are handled by pumps.

Solution. We adopt the final temperature of the brine equal to -2°C . The temperature scheme with counterflow is as follows:

$$\begin{array}{ccc}
 & \text{Ether} & \\
 33 & \xrightarrow{\quad} & 6 \\
 & \text{Brine} & \\
 -2 & \xleftarrow{\quad} & -6 \\
 \hline
 \Delta t_{\text{gr}} = 35 & & \Delta t_{\text{sm}} = 12
 \end{array}$$

The ratio $\Delta t_{\text{gr}}/\Delta t_{\text{sm}} = 35/12 = 2.92 > 2$, hence the mean temperature drop is, by Eq. (4-70):

$$\Delta t_{\text{m}} = \frac{35 - 12}{2.3 \log 2.92} = 21.5 \text{ K}$$

The average temperature of the brine is:

$$t_{\text{br}} = \frac{-6 + (-2)}{2} = -4 \text{ }^{\circ}\text{C}$$

The average temperature of the ether is:

$$t_{\text{eth}} = t_{\text{br}} + \Delta t_{\text{m}} = -4 + 21.5 = 17.5 \text{ }^{\circ}\text{C}$$

The heat load with account taken of the losses of cold amounting to 5% is:

$$Q = 1.05 Q_{\text{m, eth}} c_{\text{eth}} (t_{\text{in}} - t_{\text{fin}})$$

We find the mean specific heat capacity of the ether from Fig. A-11, namely, $c_{\text{eth}} = 2.15 \times 10^3 \text{ J/kg} \cdot \text{K}$, and

$$Q = 1.05 \times \frac{5000}{3600} \times 2.15 \times 10^3 (33 - 6) = 84\,600 \text{ W}$$

The specific heat capacity of the brine from Table A-52 is $c_{\text{br}} = 3.40 \times 10^3 \text{ J/kg} \cdot \text{K}$. The mass rate of flow of the brine is:

$$Q_{\text{m, br}} = \frac{Q}{c_{\text{br}} (t'_{\text{fin}} - t'_{\text{in}})} = \frac{84\,600}{3.40 \times 10^3 [-2 - (-6)]} = 6.22 \text{ kg/s}$$

We direct the brine along the tube side and the ether along the shell side. We adopt turbulent flow conditions for the brine and for preliminary calculations assume that $Re = 20\,000$.

From Eqs. (1-27) [or (4-13)] and (1-23), i. e.

$$Re = \frac{vd\rho}{\mu} \quad \text{and} \quad v = \frac{Q_{\text{m}}}{0.785d^2 n\rho}$$

we get

$$nd = \frac{Q_{\text{m}}}{0.785 Re \mu} = \frac{6.22}{0.785 \times 20\,000 \times 3.23 \times 10^{-3}} = 0.123 \text{ m}$$

where $\mu = 3.23 \times 10^{-3} \text{ Pa} \cdot \text{s}$ is the dynamic viscosity of the brine at $-4 \text{ }^{\circ}\text{C}$.

Standard shell-and-tube exchangers have tubes with a diameter of 25×2 or $38 \times 2 \text{ mm}$. Consequently, the required number of tubes for them in our case is $n_1 = 0.123/0.021 \approx 6$ or $n_2 = 0.123/0.034 \approx 4$, which in both cases is considerably lower than the minimum number of tubes permitted by the relevant standards. We adopt a design of the "tube-in-tube" type with three parallel sections. The diameter of the inner tubes is $38 \times 2 \text{ mm}$, and of the outer ones $76 \times 3 \text{ mm}$. The brine will flow through the inner tubes, and the ether through the annular intertubular space.

We determine the coefficient of heat transfer from the ether to the wall. The cross-sectional area of the flow (per section) is:

$$A = 0.785 (D^2 - d^2) = 0.785 (0.070^2 - 0.038^2) = 0.002\,71 \text{ m}^2$$

The density of the ether is $\rho = 716 \text{ kg/m}^3$, and its velocity is:

$$v = \frac{Q_{\text{m, eth}}}{n\rho A} = \frac{5000}{3 \times 3600 \times 716 \times 0.002\,71} = 0.238 \text{ m/s}$$

The equivalent diameter of the annular intertubular space is:

$$d_{\text{eq}} = D - d = 0.07 - 0.038 = 0.032 \text{ m}$$

The dynamic viscosity of the ether at 17.5 °C is $\mu = 0.249 \times 10^{-3} \text{ Pa} \cdot \text{s}$, and the Reynolds number for it at 17.5 °C is, by Eq. (1-27):

$$Re = \frac{vd_{\text{eq}}\rho}{\mu} = \frac{0.238 \times 0.032 \times 716}{0.249 \times 10^{-3}} = 22\,000$$

We find the Prandtl number for the ether from the nomogram in Fig. A-13. At $t_{\text{eth}} = 17.5 \text{ °C}$, we get $Pr = 4$.

We determine the Nusselt number Nu from the nomogram in Fig. A-12 with account of the mean value of $(Pr/Pr_w)^{0.25} = 0.93$ for the liquid being cooled:

$$Nu = 0.93 \times 110 = 102$$

whence from Eq. (4-11), we have

$$\alpha_{\text{eth}} = \frac{Nu\lambda}{d_{\text{eq}}} = \frac{102 \times 0.133}{0.032} = 425 \text{ W/m}^2 \cdot \text{K}$$

where $\lambda = 0.133 \text{ W/m} \cdot \text{K}$ is the thermal conductivity of ether (Fig. A-10).

We determine the coefficient of heat transfer from the wall to the brine by Eq. (4-11):

$$\alpha_{\text{br}} = \frac{Nu\lambda}{d}$$

We find the density of brine having a mass concentration of 20% from Table A-50 $\rho = 1150 \text{ kg/m}^3$. Hence, the velocity of the brine is:

$$v = \frac{Q_m}{n\rho \times 0.785d^2} = \frac{6.22}{3 \times 1150 \times 0.785 \times 0.034^2} = 1.98 \text{ m/s}$$

The Reynolds number for the brine at $t_{\text{br}} = -4 \text{ °C}$ is:

$$Re = \frac{vd\rho}{\mu} = \frac{1.98 \times 0.034 \times 1150}{3.23 \times 10^{-3}} = 24\,000$$

The thermal conductivity of the brine from Table A-50 at $t_{\text{br}} = -4 \text{ °C}$ by interpolation is $\lambda = 0.530 \text{ W/m} \cdot \text{K}$, and the Prandtl number for the brine at this temperature by Eq. (4-12) is:

$$Pr = \frac{c\mu}{\lambda} = \frac{3.4 \times 10^3 \times 3.23 \times 10^{-3}}{0.530} = 20.7$$

We find the Nusselt number $Nu = 245$ from the nomogram in Fig. A-12, assuming for the liquid being heated that $(Pr/Pr_w)^{0.25} = 1$, then by Eq. (4-11)

$$\alpha_{\text{br}} = \frac{Nu\lambda}{d} = \frac{245 \times 0.530}{0.034} = 3820 \text{ W/m}^2 \cdot \text{K}$$

We calculate the sum of the thermal resistances of the wall and the fouling deposits.

According to Table A-31, we assume that:

$$\frac{1}{r_{\text{foul},1}} = \frac{1}{r_{\text{foul},2}} = 5800 \text{ W/m}^2 \cdot \text{K}$$

$$\sum r_w = r_{\text{foul},1} + \frac{\delta_w}{\lambda_w} + r_{\text{foul},2} = \frac{1}{5800} + \frac{0.002}{46.5} + \frac{1}{5800} = 0.0004 \text{ m}^2 \cdot \text{K/W}$$

The overall coefficient of heat transfer (calculated for a flat wall because $d_{in}/d_{ex} = 34/38 > 0.5$), by Eq. (4-67), is:

$$K = \frac{1}{\frac{1}{\alpha_{eth}} + \sum r_w + \frac{1}{\alpha_{br}}} = \frac{1}{\frac{1}{425} + 0.0004 + \frac{1}{3820}} = 333 \text{ W/m}^2 \cdot \text{K}$$

The required surface area for heat exchange, by Eq. (4-65), is:

$$A = \frac{Q}{K\Delta t_m} = \frac{84\,600}{333 \times 21.5} = 11.8 \text{ m}^2$$

Taking a reserve area of 20%, we have $A = 1.2 \times 11.8 = 14.2 \text{ m}^2$.

The length of each of the three sections is:

$$L = \frac{A}{n\pi d_m} = \frac{14.2}{3 \times 3.14 \times 0.036} = 41.8 \text{ m}$$

We install in each section seven six-metre elements.

Example 4-II. Calculate a shell-and-tube heat exchanger for cooling 1240 m³/h (in standard conditions) of nitrogen from 76 to 31 °C with water. The pressure of the nitrogen is $p_{abs} = 1.5 \text{ at}$. The cooling water, which gives a deposit of scale, has a temperature of 16 °C.

Solution. We assume that the water in the exchanger becomes heated to 26 °C. The temperature scheme of the exchanger with counterflow is:

$$\begin{array}{ccc} 76 \rightarrow & & 31 \\ 26 \leftarrow & & 16 \\ \hline \Delta t_{gr} = 50 & \angle & t_{sm} = 15 \end{array}$$

The mean temperature drop, by Eq. (4-70), is:

$$\Delta t_m = \frac{50 - 15}{2.3 \log \frac{50}{15}} = 29.5 \text{ K}$$

The mean temperature of the water is:

$$t_{wat} = \frac{26 + 16}{2} = 21 \text{ °C}$$

The mean temperature of the nitrogen is:

$$t_n = t_{wat} + \Delta t_m = 21 + 29.5 = 50.5 \text{ °C}$$

The amount of heat transferred from the nitrogen to the water is:

$$Q = Q_{m,n} \rho_n c_n (t_{in} - t_{fn})$$

We find the density of nitrogen at 0 °C and 760 mm Hg from Table A-5: $\rho_n = 1.25 \text{ kg/m}^3$, and the mean specific heat capacity of nitrogen from Table A-27: $c = C/M = 29/28 \approx 1.05 \text{ kJ/kg} \cdot \text{K} = 1.05 \times 10^3 \text{ J/kg} \cdot \text{K}$. Hence,

$$Q = \frac{1240}{3600} \times 1.25 \times 1.05 \times 10^3 (76 - 31) = 20\,300 \text{ W}$$

The specific heat capacity of water at 21 °C from Table A-39 is $c_{wat} = 4.19 \times 10^3 \text{ J/kg} \cdot \text{K}$, and the rate of flow of the water is:

$$Q_{m,wat} = \frac{Q}{c_{wat} (t'_{fn} - t'_{in})} = \frac{20\,300}{4.19 \times 10^3 (26 - 16)} = 0.485 \text{ kg/s}$$

We determine approximately the required surface area of the shell-and-tube exchanger. Since scale may be deposited on the surface of the tubes at the side of the water being heated, and this greatly hinders the transfer of heat, it is necessary to provide for the possibility of cleaning the exchanger. For this reason, the water should flow through the tubes, whose internal surface in a shell-and-tube exchanger is easily accessible for cleaning, whereas the external surface of the tubes—the shell side—is virtually not accessible for cleaning.

To increase the coefficient of heat transfer of the nitrogen (for which it is much lower than for water), we adopt the design of the exchanger with cross flow of the nitrogen around the tubes, i.e. with baffles on the shell side (see Fig. 4-4).

We adopt approximately an overall heat transfer coefficient $K = 60 \text{ W/m}^2 \cdot \text{K}$. Hence the required heat exchange surface area, from Eq. (4-65), is:

$$A = \frac{Q}{K \Delta t_m} = \frac{20\,300}{60 \times 29.5} = 11.5 \text{ m}^2$$

According to the data of Table A-34, we adopt a single-pass shell-and-tube exchanger with a shell diameter of 400 mm, and with 121 tubes 1.5 m long. The diameter of the tubes is $25 \times 2 \text{ mm}$ and their pitch (the distance between their axes) is 32 mm.

We perform more accurate calculations of the heat exchange surface area.

We assume that $Re = 20\,000$ for the nitrogen and calculate the heat transfer coefficient by Eq. (4-28) for a triangular pitch of the tubes. We have:

$$Nu = 0.356 Re^{0.6} \epsilon_\varphi = 0.356 \times 20\,000^{0.6} \times 0.6 = 81.3$$

The thermal conductivity of nitrogen from Table A-30 for 50.5°C is $\lambda = 0.0267 \text{ W/m} \cdot \text{K}$. Hence, the heat transfer coefficient for nitrogen, by Eq. (4-11), is:

$$\alpha_n = \frac{Nu \lambda}{d} = \frac{81.3 \times 0.0267}{0.025} = 87 \text{ W/m}^2 \cdot \text{K}$$

We determine the conditions of flow of the water in the tubes.

The velocity of the water is:

$$v = \frac{Q_{m, \text{wat}}}{0.785 n d^2 \rho} = \frac{0.485}{0.785 \times 121 \times 0.021^2 \times 1000} = 0.0116 \text{ m/s}$$

The kinematic viscosity of water at 21°C from Table A-39 is $\nu = 0.986 \times 10^{-6} \text{ m}^2/\text{s}$, and the Reynolds number, by Eq. (1-27), is:

$$Re = \frac{vd}{\nu} = \frac{0.0116 \times 0.021}{0.986 \times 10^{-6}} = 247$$

We have laminar flow. To find the heat transfer coefficient for water, we must calculate the product

$$Re Pr \frac{d}{L} = \frac{vd}{\nu} \frac{\nu}{a} \frac{d}{L} = \frac{vd^2}{aL}$$

where a (in m^2/s) is the thermal diffusivity of water, which is virtually constant for a small temperature range. The value of a must be taken according to the temperature of the boundary layer $t_{b,1}$, equal to:

$$t_{b,1} = \frac{t_{\text{wat}} + t_w}{2} = t_{\text{wat}} + \frac{\Delta t_{\text{wat}}}{2}$$

Assuming that in our case $\Delta t_{\text{wat}} = t_w - t_{\text{wat}}$ is within the limits of 5 to 15°K (this will be checked later), we find that the temperature of the boundary layer of water $t_{b,1}$ equals from 23.5 to 28.5°C and $a = 1.46 \times 10^{-7} \text{ m}^2/\text{s}$ (Table A-39).

Thus,

$$Re Pr \frac{d}{L} = \frac{vd^2}{aL} = \frac{0.0116 \times 0.021^2}{1.46 \times 10^{-7} \times 1.5} = 23.4$$

In the graph in Fig. 4-2, we find for vertical tubes with account taken of free convection (curve 3) that $Nu = 7.5$.

The coefficient of heat transfer for water is:

$$\alpha_{\text{wat}} = \frac{Nu\lambda}{d} = \frac{7.5 \times 0.61}{0.021} = 281 \text{ W/m}^2 \cdot \text{K}$$

where $\lambda = 0.61 \text{ W/m} \cdot \text{K}$ is the thermal conductivity of water at a temperature of 25°C (Table A-39). Like the thermal diffusivity a , the thermal conductivity λ is virtually constant within a small temperature interval.

The total thermal resistance of the wall and the fouling deposits (Table A-31) is:

$$\sum r_w = \frac{1}{2800} + \frac{0.002}{46.5} + \frac{1}{2000} = 0.0009 \text{ m}^2 \cdot \text{K/W}$$

The overall heat transfer coefficient ($d_{\text{in}}/d_{\text{ex}} = 21/25 > 0.5$) is:

$$K = \frac{1}{\frac{1}{\alpha_n} + \sum r_w + \frac{1}{\alpha_{\text{wat}}}} = \frac{1}{\frac{1}{87} + 0.0009 + \frac{1}{218}} = 59 \text{ W/m}^2 \cdot \text{K}$$

The required heat exchange surface area is:

$$A = \frac{Q}{K\Delta t_m} = \frac{20\,300}{59 \times 29.5} = 11.7 \text{ m}^2$$

In the adopted exchanger, $A = 14 \text{ m}^2$. The reserve surface area is 20%.

We check the assumptions made in calculating the coefficient of heat transfer for water.

1. We assumed that $\Delta t_{\text{wat}} = t_w - t_{\text{wat}} = 5$ to 15 K .

The unit heat load in the exchanger is:

$$q = K\Delta t_m = 59 \times 29.5 = 1740 \text{ W/m}^2$$

Hence,

$$\Delta t_{\text{wat}} = \frac{q}{\alpha_{\text{wat}}} = \frac{1740}{218} = 8 \text{ K}$$

2. The influence of free convection manifests itself when $Gr > 4Re Nu'$ and when $Gr Pr > 5 \times 10^5$. We have:

$$4Re Nu' = 4 \times 247 \times 5 = 4940$$

where the value of the Nusselt number without account taken of free convection $Nu' = 5$ has been taken from the graph in Fig. 4-2 on line AA.

The Grashof number, by Eq. (4-15), is:

$$Gr = \frac{gd^3}{\nu^2} \beta \Delta t_{\text{wat}} = \frac{9.81 \times 0.021^3}{0.986 \times 10^{-12}} \times 25.5 \times 10^{-5} \times 8 = 1.9 \times 10^5 \gg 4Re Nu'$$

where $\beta = 25.5 \times 10^{-5} \text{ K}^{-1}$ is the coefficient of volume expansion of water (Table A-32).

$$Gr Pr = 1.9 \times 10^5 \times 6.22 > 5 \times 10^5$$

where the value of the Prandtl number $Pr = 6.22$ has been taken from Table A-39 for $t_{\text{b,l}} = 25^\circ\text{C}$.

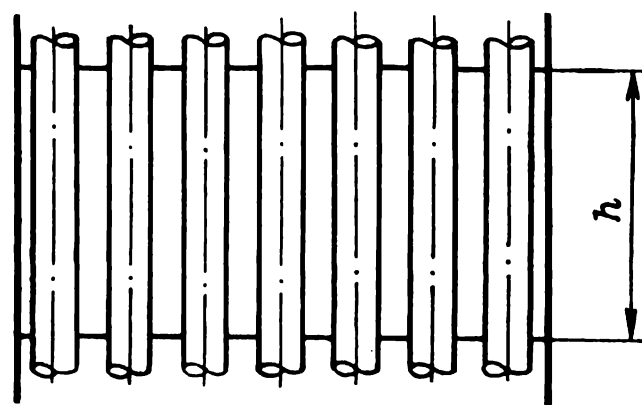


Fig. 4-24. To Example 4-II

We determine the distance between the segmental baffles on the shell side. We find the cross-sectional area for the nitrogen flow as follows (Fig. 4-24).

We adopt the rise of the segmental cut in the baffle $b = 0.5R$. Hence, the length of the chord $L = R\sqrt{3} = 1.732R$. The mean chord length is $L_m = (L + D)/2 = 1.866R$. The number of tubes on this chord is $n' = L_m/t$, where t is the tube pitch.

The cross-sectional area for the cross flow of the nitrogen is:

$$A = h(L_m - n'd) = hL_m \left(1 - \frac{d}{t}\right) \quad (a)$$

After determining A from the equation for the rate of flow, we can find h from Eq. (a).

The density of the nitrogen in the operating conditions ($t = 50.5^\circ\text{C}$ and $p_n = 1.5$ at) is:

$$\rho = 1.25 \times \frac{273 \times 1.5}{1 \times 323.5} = 1.58 \text{ kg/m}^3$$

At $Re = 20\,000$, the velocity of the nitrogen should be:

$$v = \frac{Re\mu}{d\rho} = \frac{20\,000 \times 0.019 \times 10^{-3}}{0.025 \times 1.58} = 9.6 \text{ m/s}$$

where $\mu = 0.019 \times 10^{-3} \text{ Pa}\cdot\text{s}$ is the dynamic viscosity of nitrogen at 50.5°C (Fig. A-6).

The volumetric flow rate of the nitrogen in operating conditions is:

$$Q_v = \frac{1240 \times 323.5}{3600 \times 273 \times 1.5} = 0.272 \text{ m}^3/\text{s}$$

The required cross-sectional area is:

$$A = \frac{Q_v}{v} = \frac{0.272}{9.6} = 0.0284 \text{ m}^2$$

From Eq. (a), we find

$$h = \frac{A}{L_m \left(1 - \frac{d}{t}\right)} = \frac{0.0284}{0.373 \left(1 - \frac{25}{32}\right)} \approx 0.35 \text{ m}$$

where $L_m = 1.866R = 1.866 \times 0.2 = 0.373 \text{ m}$.

Example 4-III. Calculate a vertical shell-and-tube heat exchanger for the condensation of 3100 kg/h of saturated benzene vapour under atmospheric pressure. The liquid benzene leaves the condenser at the condensation temperature. The cooling water flowing through the tubes is heated from 16 to 40 °C. According to the erection conditions, the height of the apparatus must not exceed 3.5 m.

Solution. The temperature of condensation of benzene under atmospheric pressure is 80.1 °C. The temperature scheme of the condenser is:

$$\begin{array}{ccc} 80.1 & \leftarrow & 80.1 \\ 16 & \rightarrow & 40 \\ \hline \Delta t_1 = 64.1 & & \Delta t_2 = 40.1 \end{array}$$

Since $\Delta t_1/\Delta t_2 < 2$, the mean temperature drop is:

$$\Delta t_m = \frac{64.1 + 40.1}{2} = 52.1 \text{ K}$$

The average temperature of the cooling water is:

$$t_{m, \text{wat}} = t_{\text{cond}} - \Delta t_m = 80.1 - 52.1 = 28 \text{ °C}$$

The heat load (the flow of transferred heat) is

$$Q = Q_{m, b} L = \frac{3100}{3600} \times 95.5 \times 4190 = 345\,000 \text{ W}$$

where $L = 95.5 \times 4190 \text{ J/kg}$ is the heat of condensation of benzene at 80 °C [4-22].

The rate of flow of the cooling water is:

$$Q_{m, \text{wat}} = \frac{Q}{c_{\text{wat}} (t_{\text{fin}} - t_{\text{in}})} = \frac{345\,000}{4190 (40 - 16)} = 3.44 \text{ kg/s}$$

We adopt tubes for the exchanger with a diameter of $25 \times 2 \text{ mm}$. We assume that $Re = 15\,000$ for the water (developed turbulent flow) and find the required number of tubes from the expressions:

$$Re = \frac{vd\rho}{\mu} \quad \text{and} \quad v = \frac{Q_{m, \text{wat}}}{0.785d^2n\rho}$$

whence

$$n = \frac{Q_{m, \text{wat}}}{0.785Re d\mu} = \frac{3.44}{0.785 \times 15 \times 10^3 \times 0.021 \times 0.836 \times 10^{-3}} = 16.7 \approx 17$$

where $\mu = 0.836 \times 10^{-3} \text{ Pa}\cdot\text{s}$ is the dynamic viscosity of water at 28 °C (Table A-6).

Turning to Table A-34, we see that the nearest numbers of tubes are $n = 13$ for a single-pass exchanger and $n = 90/4 = 22.5$ (two passes with 22 tubes each and two with 23 tubes each) for a four-pass exchanger.

To choose one of them, we assess approximately the required heat transfer area.

According to Table 4-6, the approximate mean value of the overall coefficient of heat transfer in condensers of vapours of organic substances is

$K = 550 \text{ W/m}^2 \cdot \text{K}$. Hence, the required surface area of the condenser from Eq. (4-66) is:

$$A = \frac{Q}{K \Delta t_m} = \frac{345\,000}{550 \times 52.1} = 12 \text{ m}^2$$

There are no single-pass exchangers with a surface area of such a magnitude (Table A-34), consequently the condenser being designed is a four-pass one.

We determine the coefficient of heat transfer for the water α_{wat} .

We find the Reynolds number Re by Eq. (4-37):

$$Re = \frac{4Q_{\text{m, wat}}}{\pi d n \mu} = \frac{4 \times 3.44}{3.14 \times 0.021 \times 22.5 \times 0.836 \times 10^{-3}} = 11\,100$$

We have developed turbulent flow. We use the nomogram in Fig. A-12 to find Nu , assuming that $(Pr/Pr_w)^{0.25} = 1$ because the water becomes heated (see "Heat Transfer", paragraph 7).

First we find the Prandtl number Pr for water at 28°C from Table A-39, namely, $Pr = 5.74$, and then in the nomogram in Fig. A-12 we find $Nu = 79$. Hence, from Eq. (4-11), we get

$$\alpha_{\text{wat}} = \frac{Nu \lambda}{d} = \frac{79 \times 0.609}{0.021} = 2300 \text{ W/m}^2 \cdot \text{K}$$

where $\lambda = 0.609 \text{ W/m} \cdot \text{K}$ is the thermal conductivity of water at 28°C .

We calculate the coefficient of heat transfer for the condensing benzene vapour α_b by Eq. (4-50):

$$\alpha_b = 3.78 \lambda \sqrt[3]{\frac{\rho^2 dn}{\mu Q_m}}$$

We take the values of the physicochemical constants for liquid benzene for the condensation temperature of 80°C :

$$\lambda = 0.14 \text{ W/m} \cdot \text{K} \text{— Fig. A-10}$$

$$\rho = 815 \text{ kg/m}^3 \text{— [4-22]}$$

$$\mu = 0.32 \times 10^{-3} \text{ Pa} \cdot \text{s} \text{— Fig. A-5}$$

The rate of flow of the benzene is $Q_m = 3100/3600 = 0.862 \text{ kg/s}$. Hence,

$$\alpha_b = 3.78 \times 0.14 \sqrt[3]{\frac{815^2 \times 0.025 \times 90}{0.32 \times 10^{-3} \times 0.862}} = 928 \text{ W/m}^2 \cdot \text{K}$$

The thermal conductivity of steel from Table A-28 is $\lambda_w = 46.5 \text{ W/m} \cdot \text{K}$, and the thermal resistance of the steel wall of a tube is:

$$\frac{\delta}{\lambda_w} = \frac{0.002}{46.5} = 0.000\,043 \text{ m}^2 \cdot \text{K/W}$$

The thermal conductivity of the fouling deposits on the benzene side from Table A-31 is:

$$\frac{1}{r_{\text{foul, b}}} = 10\,000 \times 1.163 = 11\,630 \text{ W/m}^2 \cdot \text{K}$$

The thermal conductivity of the fouling deposits on the water side is:

$$\frac{1}{r_{\text{foul wat}}} = 2000 \text{ W/m}^2 \cdot \text{K}$$

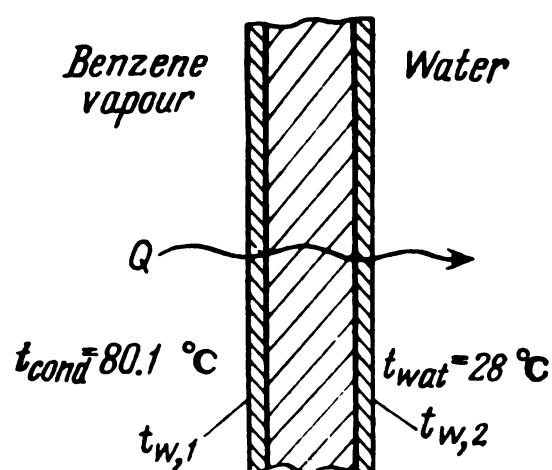


Fig. 4-25. To Example 4-III

The total thermal resistance of the wall and the fouling deposits is:

$$\sum r_w = 0.000\,043 + \frac{1}{11\,630} + \frac{1}{2000} = 6.3 \times 10^{-4} \text{ m}^2 \cdot \text{K/W}$$

We calculate the overall heat transfer coefficient K as for a flat wall because the ratio $d_{\text{in}}/d_{\text{ex}} = 21/25 = 0.84$ is greater than 0.5:

$$K = \frac{1}{\frac{1}{\alpha_b} + \sum r_w + \frac{1}{\alpha_{\text{wat}}}} = \frac{1}{\frac{1}{928} + 6.3 \times 10^{-4} + \frac{1}{2300}} = 466 \text{ W/m}^2 \cdot \text{K}$$

The required surface area of the condenser is by Eq. (4-66):

$$A = \frac{Q}{K \Delta t_m} = \frac{345\,000}{466 \times 52.1} = 14.2 \text{ m}^2$$

With a reserve area of 20%, we get $A = 14.2 \times 1.2 = 17 \text{ m}^2$.

A four-pass exchanger with a shell diameter of $D = 400 \text{ mm}$ and with a tube height of $H = 2.5 \text{ m}$ has such a surface area. This corresponds to our assignment.

For purposes of checking, we perform more precise calculations of the overall heat transfer coefficient with account taken of the temperatures of the wall surfaces.

We assume (with following checking) that $t_{w,1} = 52.8^\circ\text{C}$ (Fig. 4-25).

Hence, the temperature of the film of benzene condensate is:

$$t_f = \frac{t_{\text{cond}} + t_{w,1}}{2} = \frac{80.1 + 52.8}{2} = 66.4^\circ\text{C}$$

The physicochemical constants of liquid benzene at 66°C are:

$$\lambda = 0.144 \text{ W/m} \cdot \text{K}$$

$$\rho = 829 \text{ kg/m}^3$$

$$\mu = 0.36 \times 10^{-3} \text{ Pa} \cdot \text{s}$$

The coefficient of heat transfer for condensing benzene vapour according to Eq. (4-43) is:

$$\alpha_b = 1.15 \sqrt[4]{\frac{\lambda^3 \rho^2 L g}{\mu \Delta t H}} = 1.15 \sqrt[4]{\frac{0.144^3 \times 829^2 \times 95.5 \times 4190 \times 9.81}{0.36 \times 10^{-3} (80.1 - 52.8) 2.5}} = 870 \text{ W/m}^2 \cdot \text{K}$$

The unit heat load is:

$$q = \alpha_b (t_{\text{cond}} - t_{w,1}) = 870 (80.1 - 52.8) = 23\,800 \text{ W/m}^2$$

The temperature of the wall surface at the water side is:

$$t_{w,2} = t_{w,1} - q \sum r_w = 52.8 - 23\,800 \times 6.3 \times 10^{-4} = 37.8^\circ\text{C}$$

We calculate the coefficient of heat transfer for water with account taken of the factor $(Pr/Pr_w)^{0.25} = (5.74/4.52)^{0.25} = 1.06$, where 4.52 is the number Pr_w for water at the temperature of 38 °C (Table A-39):

$$\alpha_{\text{wat}} = 2300 \times 1.06 = 2440 \text{ W/m}^2 \cdot \text{K}$$

We check the unit heat load:

$$q = \alpha_{\text{wat}} (t_{w,2} - t_{\text{wat}}) = 2440 (37.8 - 28) = 23\,900 \text{ W/m}^2$$

Consequently, we took the correct wall temperature $t_{w,1}$.
The overall heat transfer coefficient is by Eq. (4-66):

$$K = \frac{q}{\Delta t_m} = \frac{23\,900}{52.1} = 460 \text{ W/m}^2 \cdot \text{K}$$

which virtually coincides with the value found above.

Since the average temperature of the tube walls $(52.8 + 37.8)/2 = 45.3$ °C differs from the shell temperature (80 °C) by more than 30 °C, we should use a design of an exchanger with a lens expansion piece.

Example 4-IV. Determine the main dimensions of a horizontal shell-and-tube heat exchanger for heating 9.5 ton/h of toluene from 20 to 100 °C. The heating steam has a pressure of $p_{\text{abs}} = 1.7$ at and contains 0.5% of air.

Solution. The condensation temperature of the heating steam is 114.5 °C (Table A-57). The temperature scheme is:

$$\begin{array}{ccc} 114.5 & \leftarrow & 114.5 \\ 20 & \rightarrow & 100 \\ \hline \Delta t_{\text{gr}} = 94.5 & & \Delta t_{\text{sm}} = 14.5 \end{array}$$

The mean temperature drop, by Eq. (4-70), is:

$$\Delta t_m = \frac{\Delta t_{\text{gr}} - \Delta t_{\text{sm}}}{2.3 \log \frac{\Delta t_{\text{gr}}}{\Delta t_{\text{sm}}}} = \frac{94.5 - 14.5}{2.3 \log \frac{94.5}{14.5}} = 42.8 \text{ K}$$

The average temperature of the toluene, by Eq. (4-75), is:

$$t_t = t_{\text{st}} - \Delta t_m = 114.5 - 42.8 = 71.7 \text{ °C}$$

We find the mean specific heat capacity of the toluene from Fig. A-11 — $c_t = 1800$ J/kg·K, and the amount of heat needed to heat the toluene is:

$$Q = Q_{m,t} c_t (t_{\text{fin}} - t_{\text{in}}) = \frac{9500}{3600} \times 1800 (100 - 20) = 380\,000 \text{ W}$$

The specific heat of condensation of the steam according to Table A-57 is $L = 2222 \times 10^3$ J/kg, and the rate of flow of the heating steam with account taken of a 5% moisture content and heat losses amounting to 5% is:

$$Q_{m,\text{st}} = \frac{1.05Q}{0.95L} = \frac{1.05 \times 380\,000}{0.95 \times 2222 \times 10^3} = 0.19 \text{ kg/s}$$

We determine the coefficient of heat transfer for the toluene flowing through the tubes. The dynamic viscosity of the toluene at 72 °C from Fig. A-5 is $\mu = 0.37 \times 10^{-3}$ Pa·s. We assume that $Re = 15\,000$ and find the required number of tubes with a diameter of 25×2 mm from the equation (see Example 4-I):

$$n = \frac{Q_{m,t}}{0.785 Re \mu} = \frac{9.5 \times 10^3}{3600 \times 0.785 \times 15 \times 10^3 \times 0.021 \times 0.37 \times 10^{-3}} = 29$$

According to the data of Table A-34, we adopt a six-pass exchanger with $204/6=34$ tubes in one pass and with a shell diameter of 600 mm.

We recalculate the Reynolds number:

$$Re = 15\,000 \times \frac{29}{34} = 12\,800$$

We find the Prandtl number Pr for the toluene at 72°C from Fig. A-13 — $Pr=4.5$.

We assume, with a reserve, that $(Pr/Pr_w)^{0.25}=1$ for the liquid being heated, and from the nomogram in Fig. A-12 we find the value of the Nusselt number — $Nu=72$.

The thermal conductivity of the toluene from Fig. A-10 is $\lambda=0.125\text{ W/m}\cdot\text{K}$, and from Eq. (4-11) the coefficient of heat transfer of the toluene is

$$\alpha_t = \frac{Nu\lambda}{d} = \frac{72 \times 0.125}{0.021} = 428\text{ W/m}^2\cdot\text{K}$$

To calculate the coefficient of heat transfer of the condensing heating steam by Eq. (4-51), we have to know the length of the tubes l . According to the data of Table 4-6, we assume approximately that the overall coefficient of heat transfer is $K=230\text{ W/m}^2\cdot\text{K}$. Hence, from Eq. (4-66), the heat exchange surface area is:

$$A = \frac{Q}{K\Delta t_m} = \frac{380\,000}{230 \times 42.8} = 38.6\text{ m}^2$$

For an exchanger with a surface area of 39 m^2 , Table A-34 gives us a tube length of 2.5 m.

We determine the coefficient of heat transfer for the condensing steam by Eq. (4-51) accounting for the influence of the air contained in the steam:

$$\alpha_{st} = 2.02\varepsilon\varepsilon_g\lambda \sqrt[3]{\frac{\rho^2 nl}{\mu Q_m}}$$

where ε = coefficient depending on the number of tubes in a vertical row (for a triangular pitch we assume with a reserve that $\varepsilon=0.57$ — see Fig. 4-8)
 $\varepsilon_g=0.6$ = coefficient depending on the content of air in the steam (Fig. 4-10).

From Table A-39 for water at 114°C we get $\lambda=0.685\text{ W/m}\cdot\text{K}$, $\rho=947\text{ kg/m}^3$, and $\mu=0.244 \times 10^{-3}\text{ Pa}\cdot\text{s}$. Hence

$$\alpha_{st} = 2.02 \times 0.57 \times 0.6 \times 0.685 \sqrt[3]{\frac{947^2 \times 204 \times 2.5}{0.244 \times 10^{-3} \times 0.19}} = 10\,100\text{ W/m}^2\cdot\text{K}$$

The total thermal resistance of the steel wall and the fouling deposits from Table A-31 is:

$$\sum r_w = \frac{1}{5800} + \frac{0.002}{46.5} + \frac{1}{5800} = 0.0004\text{ m}^2\cdot\text{K/W}$$

The overall coefficient of heat transfer, by Eq. (4-67), is:

$$K = \frac{1}{\frac{1}{\alpha_{st}} + \sum r_w + \frac{1}{\alpha_t}} = \frac{1}{\frac{1}{10\,100} + 0.0004 + \frac{1}{428}} = 352\text{ W/m}^2\cdot\text{K}$$

The required heat exchange surface area, by Eq. (4-66), is:

$$A = \frac{Q}{K\Delta t_m} = \frac{380\,000}{352 \times 42.8} = 25.3\text{ m}^2$$

We adopt an exchanger with $A = 31 \text{ m}^2$ (Table A-34) having a tube length of 2 m. The reserve surface area is 22%. The coefficient of heat transfer of the condensing steam calculated for $l = 2.5 \text{ m}$ does not have to be recalculated.

We determine the temperature of the wall of the tubes. From the equations

$$q = K \Delta t_m = \alpha_{st} \Delta t_{st} = \alpha_t \Delta t_t$$

we find:

$$\Delta t_{st} = t_{st} - t_{w,1} = \frac{K \Delta t_m}{\alpha_{st}} = \frac{352 \times 42.8}{10 \cdot 100} = 1.5 \text{ K}$$

$$\Delta t_t = t_{w,2} - t_t = \frac{K \Delta t_m}{\alpha_t} = \frac{352 \times 42.8}{428} = 35.2 \text{ K}$$

Hence,

$$t_{w,1} = t_{st} - \Delta t_{st} = 114.5 - 1.5 = 113 \text{ }^\circ\text{C}$$

$$t_{w,2} = t_t + \Delta t_t = 71.7 + 35.2 = 107 \text{ }^\circ\text{C}$$

The mean temperature of the wall is $(113 + 107)/2 = 110 \text{ }^\circ\text{C}$, which differs from the temperature of the shell by $4.5 \text{ }^\circ\text{C}$.

Example 4-V. Calculate a coil for the periodical heating of *m*-xylene in a tank (in conditions of free convection). The xylene in an amount of 1600 kg should be heated from 16 to $80 \text{ }^\circ\text{C}$ during one hour. Heating is performed with steam having a pressure of $p_{abs} = 2 \text{ at}$. The steel coil is made of a tube with a diameter of $53 \times 2 \text{ mm}$.

Solution. The temperature of condensation of the heating steam is $119.6 \text{ }^\circ\text{C}$ (Table A-57). Since it is constant, the mean temperature drop during the heating can be calculated by the equation:

$$\Delta t_m = \frac{\Delta t_{in} - \Delta t_{fn}}{2.3 \log \frac{\Delta t_{in}}{\Delta t_{fn}}}$$

The initial and final temperatures are:

$$\Delta t_{in} = 119.6 - 16 = 103.6 \text{ K}$$

$$\Delta t_{fn} = 119.6 - 80 = 39.6 \text{ K}$$

Hence,

$$\Delta t_m = \frac{103.6 - 39.6}{2.3 \log \frac{103.6}{39.6}} = 66.6 \text{ K}$$

The mean temperature of the xylene is:

$$t_x = t_{cond} - \Delta t_m = 119.6 - 66.6 = 53 \text{ }^\circ\text{C}$$

The mean amount of heat transferred is:

$$Q = Q_{m,x} c_x (t_{fn} - t_{in}) = \frac{1600}{3600} \times 1840 (80 - 16) = 52 \,300 \text{ W}$$

where $c_x = 1840 \text{ J/kg} \cdot \text{K}$ is the mean specific heat capacity of the xylene (Fig. A-11).

The total thermal resistance of the steel wall and the fouling deposits (Table A-31) is:

$$\sum r_w = r_{foul,1} + \frac{\delta}{\lambda_w} + r_{foul,2} = \frac{1}{5800} + \frac{0.002}{46.5} + \frac{1}{5800} = 0.0004 \text{ m}^2 \cdot \text{K/W}$$

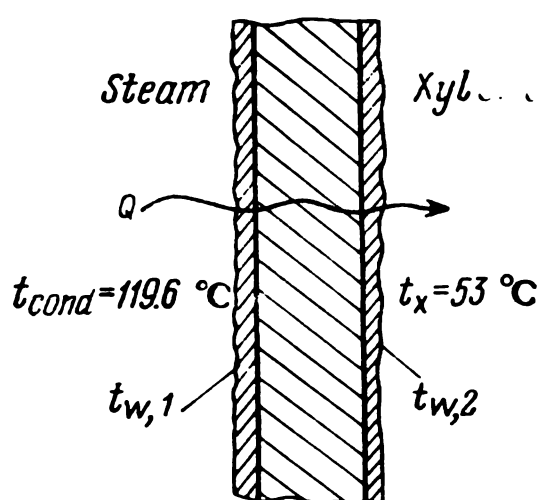


Fig. 4-26. To Example 4-V

We calculate the coefficient of heat transfer for xylene by Eq. (4-40), assuming, to create a reserve, that $(Pr/Pr_w)^{0.25} = 1$ for the liquid being heated:

$$Nu = 0.5 (Gr Pr)^{0.25} = 0.5 \left(\frac{gd^3 \rho^2 \beta}{\mu^2} Pr \right)^{0.25} \Delta t_x^{0.25}$$

The density of xylene at 53 °C, from Table A-4, is $\rho = 837 \text{ kg/m}^3$; the dynamic viscosity of the xylene [4-22] is $\mu = 0.43 \times 10^{-3} \text{ Pa}\cdot\text{s}$; the Prandtl number for xylene at 53 °C, from Fig. A-13, is $Pr = 4.5$; the coefficient of volume expansion β within the temperature interval from 53 to 119 °C is determined by the formula:

$$\beta = \frac{v_2 - v_1}{v_1 \Delta t} = \frac{\rho_1 - \rho_2}{\rho_2 \Delta t}$$

Here the subscripts 1 and 2 stand for the temperatures 53 and 119 °C, respectively. The density of xylene at 119 °C, from Table A-4, is $\rho = 778 \text{ kg/m}^3$, and

$$\beta = \frac{837 - 778}{778 (119 - 53)} = 1.15 \times 10^{-3} \text{ K}^{-1}$$

Then we get

$$Nu = 0.5 \left(\frac{9.81 \times 0.053^3 \times 837^2 \times 1.15 \times 10^{-3} \times 4.5}{0.43 \times 10^{-6}} \right)^{0.25} \Delta t_x^{0.25} = 36.6 \Delta t_x^{0.25}$$

According to Fig. 4-26, we have

$$\Delta t_x = t_{w,2} - t_x$$

The coefficient of heat transfer for xylene, from Eq. (4-11), is:

$$\alpha_x = \frac{Nu \lambda}{d} = \frac{36.6 \times 0.128}{0.053} \Delta t_x^{0.25} = 88.4 \Delta t_x^{0.25}$$

where $\lambda = 0.128 \text{ W/m}\cdot\text{K}$ is the thermal conductivity of xylene at 53 °C (Fig. A-10).

Since the coefficient of heat transfer for the xylene is much smaller than that for the condensing steam, the latter without calculations may be taken equal to about $10\,000 \text{ W/m}^2\cdot\text{K}$. Next we can write the following system of equations:

$$q = \alpha_{st} \Delta t_{st} = \frac{\Delta t_w}{\sum r_w} = \alpha_x \Delta t_x$$

$$\Delta t_m = \Delta t_{st} + \Delta t_w + \Delta t_x$$

$$\text{where } \Delta t_{st} = t_{cond} - t_{w,1}$$

$$\Delta t_w = t_{w,1} - t_{w,2}$$

Introducing numerical values, we have:

$$10\,000 \Delta t_{st} = \frac{\Delta t_w}{0.0004} = 88.4 \Delta t_x^{1.25}$$

$$66.6 = \Delta t_{st} + \Delta t_w + \Delta t_x$$

From this system of equations, we get:

$$0.0442 \Delta t_x^{1.25} + \Delta t_x - 66.6 = 0$$

Solving the latter equation (graphically), we get $\Delta t_x = 59.3$ K.
Hence,

$$q = \alpha_x \Delta t_x = 88.4 \times 59.3^{1.25} = 14\,550 \text{ W/m}^2$$

The required heat exchange surface area is:

$$A = \frac{Q}{q} = \frac{52\,300}{14\,550} = 3.62 \text{ m}^2$$

The length of the coil is:

$$l = \frac{A}{\pi d_m} = \frac{3.62}{3.14 \times 0.051} = 22.6 \text{ m}$$

With a reserve length, we have: $l = 22.6 \times 1.15 = 26 \text{ m}$.

The rate of flow of the heating steam with account taken of 5% heat losses is:

$$Q_{m, st} = \frac{1.05 Q}{Lx}$$

The heat of condensation of the steam is $L = 2208 \times 10^3 \text{ J/kg}$ (Table A-57), and the adopted dryness fraction of the steam is $x = 0.95$. Hence,

$$Q_{m, st} = \frac{1.05 \times 52\,300}{2208 \times 10^3 \times 0.95} = 0.0262 \text{ kg/s}$$

According to practical data for normal operation of a steam coil, the initial velocity of the steam must be not over 30 m/s, and the ratio l/d must be not over

$$\frac{l}{d} = C_{st} \times 6(\Delta t_m)^{-1/2}$$

where C_{st} depends on the pressure of the condensing steam and for $p_{abs} = 2 \text{ at}$ equals about 190.

We check the initial velocity of the steam:

$$v_{in} = \frac{Q_{m, st}}{\rho_{st} \times 0.785 d^2}$$

From Table A-57 we find the density of the steam at $p = 2 \text{ at}$, $\rho_{st} = 1.107 \text{ kg/m}^3$, and:

$$v_{in} = \frac{0.0262}{1.107 \times 0.785 \times 0.049^2} = 12.6 \text{ m/s}$$

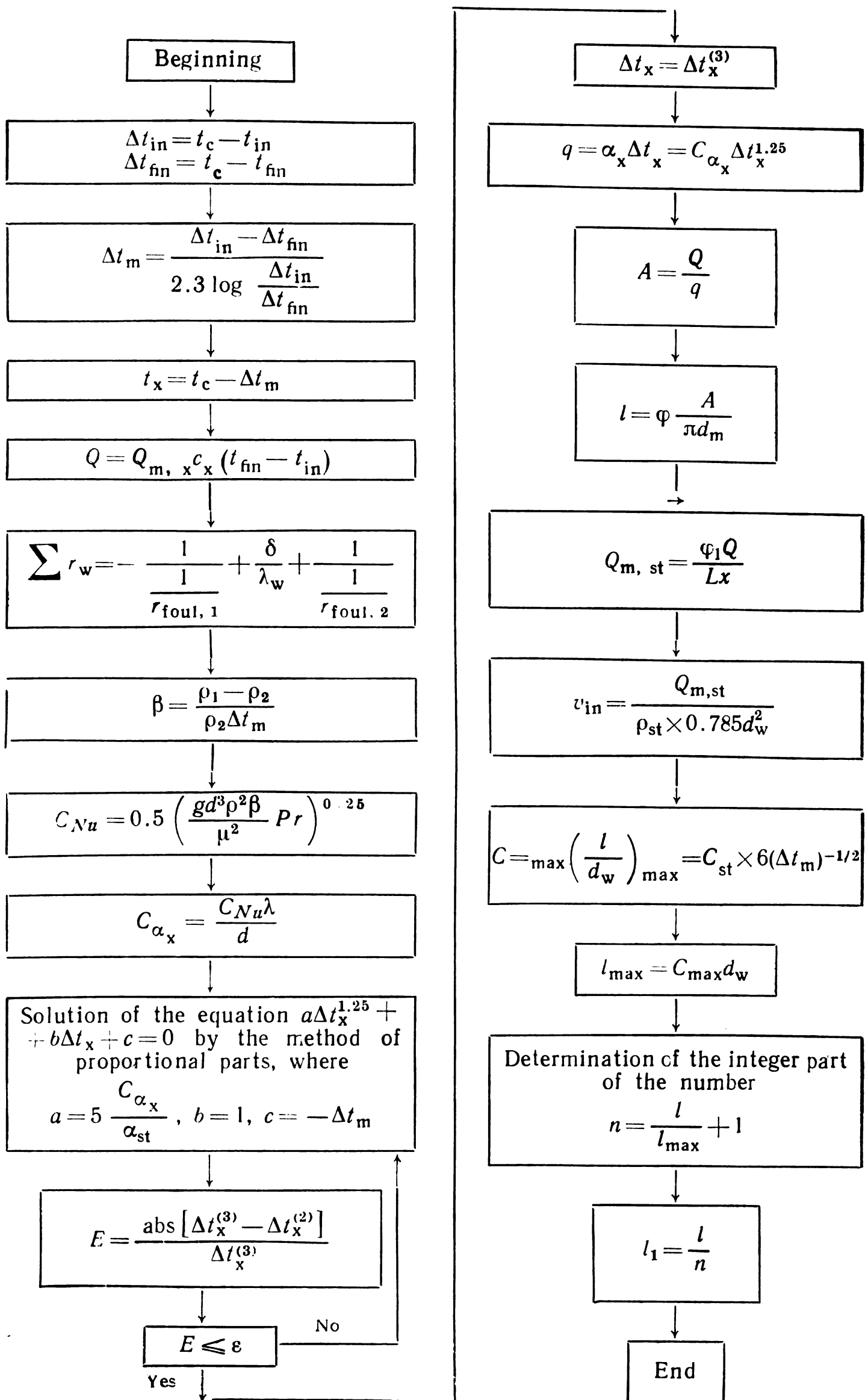
The maximum permissible ratio l/d is:

$$\left(\frac{l}{d}\right)_{\max} = 190 \times 6 \times 66.6^{-1/2} = 140$$

Therefore, the length of the coil must not exceed $l = 140 d = 140 \times 0.049 = 6.9 \text{ m}$.

We adopt the number of parallel coils equal to $n = 4$. The length of each coil is $26/4 = 6.5 \text{ m}$.

The flow chart of the algorithm for solving Example 4-V is as follows:



The identifiers are given in Table 4-11.

TABLE 4-11

Quantity	Δt_{in}	Δt_{fin}	t_{in}	t_{fin}	t_c	Δt_m	t_x	Q	$Q_{m, x}$	c_x	
Identifier	DTI	DTF	TI	TF	TC	DTM	TX	Q	QMX	CX	
$\sum r_w$	$1/r_{foul, 1}$	δ	λ_w	$1/r_{foul, 2}$	β	ρ	ρ_2	C_{Nu}	d	μ	Pr
SR	PROV1	DELTA	LW	PROV2	BETA	RO	RO2	CNU	D	M	PR
C_{α_x}	λ	α_{st}	Approximate values of Δt_x			a	b	c			
			$\Delta t_x^{(1)}$	$\Delta t_x^{(2)}$	$\Delta t_x^{(3)}$						
CALPHA	LAMBDA	ALPHA	DTX1	DTX2	DTX3	A	B	C			
Values of polynomial $P(\Delta t_x) = a\Delta t_x^{1.25} +$ $+ b\Delta t_x + c$ with $\Delta t_x = \Delta t_x^{(1)}; \Delta t_x^{(2)}; \Delta t_x^{(3)}$			Error of calcula- tion E	Permissible error of calculation ε	q	A	l	φ	$Q_{m, st}$		
F1	F2	F3	E	EPS	Q1	A1	L	FI	QMS		
φ_1	L	x	v_{in}	ρ_{st}	C_{max}	C_{st}	l_{max}	n	l_1	Parameter of cyclic calculations	
FI1	LT	X	V	R S	CMAX	CS	LMAX	N	LI	I	

The FORTRAN-IV record of the programme is as follows:
1 FORMAT ((E 10.2))
3 FORMAT ((30 X, 'INITIAL DATA',/10 X, (5E10.2, 3X))
READ 1, TI, TF, TC, QMX, CX, PROV1, DELTA, LW, PROV 2, RO,
*RO2, D, M, PR, LAMBDA, ALPHA, EPS, FI, FI 1, LT, X, ROS, C1
WRITE 3, TI, TF, TC, QMX, CX, PROV 1, DELTA, LW, PROV 2, RO,
*RO2, D, M, PR, LAMBDA, ALPHA, EPS, FI, FI 1, LT, X, ROS, C1
DTI = TC - TI
DTF = TC - TF
DTM = (DTI - DTF)/CLOG (DTI/DTF)
TX = TC - DTM


```

Q = QMX * CX * (TF - TI) / 3600
SR = 1. / PROV 1 + DELTA / LW + 1. / PROV 2
BETA = (RO - RO2) / RO2 / DTM
CNU = .5 * (9.81 * D ** 3. * RO ** 2. * BETA * PR / M ** 2.) ** .25
CALPHA = CNU * LAMBDA / D
A = 5. * CALPHA / ALPHA
B = 1.
C = -DTM
DTX 1 = 0
DTX 2 = DTM
I = 0
E = 2. * EPS
4 I = I + 1.
IF (E.LE.EPS) GO TO 5
F1 = A * DTX 1 ** 1.25 + B * DTX 1 + C
F2 = A * DTX 2 ** 1.25 + B * DTX 2 + C
DTX 3 = DTX 1 + (-(DTX 2 - DTX 1) * F1 / (F2 - F1))
F3 = A * DTX 3 ** 1.25 + B * DTX 3 + C
E = ABS (DTX 3 - DTX 2) / DTX 3
IF (F1.LT.0) GO TO 6
GO TO 7
6 IF (F3) 8, 9, 9
8 DTX1 = DTX3
GO TO 4
9 DTX2 = DTX3
GO TO 4
IF (F3. GT. 0.) DTX1 = DTX3
Q1 = CALPHA * DTX 3 ** 1.25
A1 = Q / Q1
L = FI * A1 / 3.14 / (D - DELTA)
QMS = FI1 * Q / LT / X
V = QMS / ROS / .785 / (D - 2. * DELTA) ** 2.
CMAX = CS * 6. / DTM ** .5
LMAX = CMAX * (D - 2. * DELTA)
N = INT (L / LMAX + 1.)
L1 = L / N
5 WRITE 10, DTX3, Q1, A1, L, QMS, V, N, L1
10 FORMAT (10 X, 'OUTPUT OF VALUES', /10 X, 'DTX3 =', E12.4,
* 'Q1 =', E12.4, 'A1 =', E12.4, 'L =', E12.4, /10 X,
'QMS =', E12.4,
* 'V =', E12.4, 'N =', 15, 7X, 'L1 =', E12.4)
STOP
END

```

The numerical information can be written in the form:

16., 80., 119.6, 1600., 1840., 5800., 0.002, 46.5, 5800., 837.,
 778., 0.053, 0.00043, 4.5, 0.128, 10 000., 0.05, 1.15, 1.05,
 2208000., 0.95, 1.107, 190.

The following text will be printed on the result output form:

INITIAL DATA

TI = 16. TF = 80. IC = 119.6. QMX = 1600. CX = 1840. PROV1 = 5800.
 DELTA = 0.002. LW = 46.5. PROV2 = 5800. RO = 837. RO2 = 778. D = 0.053.
 M = 0.00043. PR = 4.5. LAMBDA = 0.128. ALPHA = 10 000. FPS = 0.05.
 FI = 1 15. FI1 = 1.05. LT = 2208000. X = 0.95. ROS = 1.107 CS = 190

RESULTS OF CALCULATION

DTX = 59.3 Q1 = 14550. A1 = 3.62 L = 26. QMS = 0.0262

V = 12.6 N = 4. L1 = 6.5

SYMBOLS

<i>A</i>	area
<i>Ar</i>	Archimedes dimensionless number
<i>a</i>	constant; thermal diffusivity
<i>B</i>	coefficient; constant
<i>Bi</i>	Biot dimensionless number
<i>b</i>	constant; dimensionless coefficient; film thickness
<i>C</i>	coefficient
<i>c</i>	constant; specific heat capacity
<i>D</i>	diffusion coefficient
<i>d</i>	diameter
<i>Fo</i>	Fourier dimensionless number
<i>Fr</i>	Froude dimensionless number
<i>Ga</i>	Galilei dimensionless number
<i>Gr</i>	Grashof dimensionless number
<i>g</i>	acceleration due to gravity
<i>K</i>	overall heat transfer coefficient
<i>Ki</i>	Kirpichev dimensionless number
<i>L</i>	latent heat of transformation; length
<i>l</i>	defining geometrical dimension; length
<i>M</i>	molar mass
<i>Nu</i>	Nusselt dimensionless number
<i>Nu'</i>	Nusselt dimensionless number for mass transfer
<i>n</i>	exponent; number of tubes
<i>P</i>	coefficient; perimeter
<i>Pr</i>	Prandtl dimensionless number
<i>Pr'</i>	Prandtl dimensionless number for mass transfer
<i>p</i>	pressure
<i>Q</i>	heat flux; amount of heat
<i>Q_m</i>	mass rate of flow
<i>q</i>	unit heat flux (load)
<i>R</i>	coefficient
<i>Re</i>	Reynolds number
<i>r</i>	thermal resistance of wall
<i>S</i>	density of spraying; shape factor
<i>T</i>	absolute temperature
<i>t</i>	pitch of fins; temperature, °C
<i>V</i>	volume
<i>v</i>	specific volume; velocity
<i>W</i>	radiating power
<i>x</i>	coefficient; dryness fraction; mole fraction
<i>Y</i>	mass ratio
<i>Z</i>	product of mass rate of flow and specific heat capacity

Greek Letters

α	individual coefficient of heat transfer
β	coefficient of volume expansion; evaporation coefficient
γ	adiabatic exponent
δ	increment; thickness

ϵ	coefficient depending on the arrangement of the tubes in a bank; correction factor; emissivity; ratio; temperature coefficient
θ	coefficient
λ	thermal conductivity
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
σ	surface tension; unit surface area
φ	angle of attack; angular coefficient; factor

FUNDAMENTAL RELATIONSHIPS
AND FORMULAS FOR CALCULATIONS

1. The equations of the material balance of the evaporation process are:

$$G_{in} = G_{fn} + W \quad (5-1)$$

$$G_{in} \bar{x}_{in} = G_{fn} \bar{x}_{fn} \quad (5-2)$$

where G_{in} , G_{fn} = mass rates of flow of the initial and the final (evaporated) solutions, respectively, kg/s

\bar{x}_{in} , \bar{x}_{fn} = mass fractions of the solute in the initial and final solution, respectively

W = mass rate of flow of the evaporated water, kg/s

$$W = G_{in} \left(1 - \frac{\bar{x}_{in}}{\bar{x}_{fn}} \right) \quad (5-3)$$

2. It follows from the heat balance of an evaporator that the consumption of heat for evaporation Q (in W) includes the heat spent for heating the initial solution Q_h , for evaporation of the water Q_e , and that lost to the surroundings Q_l :

$$Q = Q_h + Q_e + Q_l \quad (5-4)$$

The heat spent for heating the initial solution is:

$$Q_h = G_{in} c_{in} (t_b - t_{in}) \quad (5-5)$$

where c_{in} = specific heat capacity of the initial solution, J/kg·K

t_b = mean temperature at which the solution boils in the evaporator, °C or K—see Eq. (5-21). Approximately,

$$t_b = t_{fn}$$

t_{in} = temperature of the initial solution fed to the evaporator, °C or K.

If a solution is fed to an evaporator in the superheated state ($t_{in} > t_b$), then Q_h has a negative sign, and the consumption of heat in the evaporator diminishes because part of the water evaporates at the expense of the heat liberated when the initial so-

lution cools from t_{in} to t_b . The quantity $G_{in}c_{in}(t_{in} - t_b)$ is called the heat of spontaneous evaporation.

The consumption of heat for the evaporation of the water is

$$Q_e = W(H - c_{wat}t_b) \quad (5-6)$$

where H = specific enthalpy of the secondary steam, J/kg

c_{wat} = specific heat capacity of water, J/kg·K.

Considering the secondary steam to be saturated and the heat of vaporization to be the same as for pure water, we can approximately assume that $H - c_{wat}t_b \approx L$, where L is the specific heat of vaporization of water (in J/kg), which when calculating evaporators is usually taken in accordance with the mean pressure in the evaporator.

The consumption of heat to compensate for losses to the surroundings Q_l is taken equal to from 3 to 5% of the sum ($Q_h + Q_e$). The value of Q_l can be calculated by the equation:

$$Q_l = \alpha A_{ex}(t_w - t_{air}) \quad (5-7)$$

where $\alpha = \alpha_{rad} + \alpha_{c,n}$ = summary coefficient of heat transfer by radiation and convection, W/m²·K

A_{ex} = area of the external surface of the heat insulated evaporator, m²

t_w = temperature of the external insulated surface, °C or K

t_{air} = ambient temperature, °C or K.

In the general case, the heat used for dehydration of the solute must also be considered in the heat balance of an evaporator. Usually this heat in comparison with the other terms of the heat balance is small and may be ignored (see Example 5-5).

The consumption of heating steam G_{st} (in kg/s) in an evaporator is determined by the equation:

$$G_{st} = \frac{Q}{(H'' - H')x} = \frac{Q}{L_{st}x} \quad (5-8)$$

where H'' = specific enthalpy of dry saturated steam, J/kg

H' = specific enthalpy of condensate at the condensation temperature, J/kg

x = steam content (dryness fraction) of the heating steam

L_{st} = specific heat of condensation of the heating steam, J/kg.

The specific rate of flow of steam for evaporation g is the ratio of the rate of flow of the heating steam G_{st} to that of the water being evaporated W :

$$g = \frac{G_{st}}{W} \quad (5-9)$$

3. The specific heat capacity of a solution can be calculated by the general formula:

$c = c_1x_1 + c_2x_2 + c_3\bar{x}_3 + \dots$ (5-10)

where c_1, c_2, c_3, \dots = specific heat capacities of the components
 x_1, x_2, x_3, \dots = mass fractions of the components.

The following approximate formula may be used to calculate the specific heat capacity of two-component (water + solute) dilute aqueous solutions ($\bar{x} < 0.2$):

$c = 4190 (1 - \bar{x})$ (5-11)

where 4190 J/kg·K is the specific heat capacity of water and \bar{x} is the mass fraction of the solute.

For concentrated two-component aqueous solutions ($\bar{x} > 0.2$), the following formula is used for calculations:

$c = 4190 (1 - \bar{x}) + c_1\bar{x}$ (5-12)

where c_1 is the specific heat capacity of the anhydrous solute, J/kg·K.

The specific heat capacity of a chemical compound in the absence of experimental data can be calculated by the equation:

$Mc = n_1C_1 + n_2C_2 + n_3C_3 + \dots$ (5-13)

where M = molecular mass of the chemical compound
 c = specific heat capacity of the compound, J/kg·K
 n_1, n_2, n_3, \dots = numbers of atoms of the respective elements in the compound
 C_1, C_2, C_3, \dots = atomic heat capacities of the elements in the compound, J/kg-atom·K.

When calculating by Eq. (5-13) the values of the atomic heat capacities given in Table 5-1 are used.

TABLE 5-1

Element	Atomic heat capacity C of element for chemical compounds, kJ/kg-atom·K		Element	Atomic heat capacity C of element for chemical compounds, kJ/kg-atom·K	
	In solid state	In liquid state		In solid state	In liquid state
C	7.5	11.7	F	20.95	29.3
H	9.6	18.0	P	22.6	31.0
B	11.3	19.7	S	22.6	31.0
Si	15.9	24.3	All others	26.0	33.5
O	16.8	25.1			

4. The boiling point of a solution or an organic liquid can be determined by means of the equation *

$$\frac{\log p_{A_1} - \log p_{A_2}}{\log p_{B_1} - \log p_{B_2}} = C \quad (5-14)$$

or the nomogram given in Fig. A-14.

Here p_{A_1} and p_{B_1} = saturated vapour pressures of two liquids at the same temperature t_1

p_{A_2} and p_{B_2} = their saturated vapour pressures at the temperature t_2

C = constant.

The boiling point of a liquid can also be found according to the rule of linearity of chemical engineering functions (discovered experimentally by Dühring):

$$\frac{t_1 - t_2}{\theta_1 - \theta_2} = K \quad (5-15)$$

where t_1 and t_2 = boiling points of a solution or an organic liquid at two different pressures p_1 and p_2

θ_1 and θ_2 = boiling points of water or some other standard liquid ** at the same pressures (Figs. A-15 and A-16)

K = constant depending on the concentration of the solution and the solute.

To find the boiling point of a liquid at a given pressure by means of Eq. (5-14) or (5-15), we must know the boiling points of this liquid at two other pressures. If we know the boiling point of a solution at one pressure, we can find its boiling point at a different pressure by using Babo's law

$$\left(\frac{p}{p_0} \right)_t = \text{const} \quad (5-16)$$

with the correction proposed by V. Stabnikov for concentrated aqueous solutions boiling under a vacuum (Table 5-2).

Here p = vapour pressure of the solution

p_0 = saturated vapour pressure of the pure solvent at the same temperature.

The application of Babo's law is illustrated in Example 5-8.

If the heat of solution is positive (heat is liberated upon solution), the correction is taken with the plus sign, and if it is negative, with the minus sign.

* Kireev, V. A. *Zh. fiz. khim.*, 2: 233 (1931).

** Hexane is usually taken as the standard liquid when determining the boiling points of organic compounds using the linearity rule. The dependence of its saturated vapour pressure on the temperature is given in Fig. A-17.

TABLE 5-2

Ratio p/p_0							Correction $\pm \Delta t$, K
0.9	0.8	0.7	0.6	0.5	0.4	0.3	
Pressure p , mm Hg							
100	200	400	450	500	550	650	0.9
—	50	200	350	450	500	550	1.8
—	—	100	275	300	350	400	2.6
—	—	—	150	200	250	300	3.6

5. The specific heat of vaporization of a liquid L (in J/kg) at the pressure p can be determined by the equation

$$L = L_{std} \frac{M_{std}}{M} \left(\frac{T}{\theta} \right)^2 \frac{d\theta}{dT}$$
 (5-17)

with the use of the linearity rule permitting us to find the quantity $d\theta/dT$.

In Eq. (5-17):

L and L_{std} = specific heats of vaporization of the liquid being studied and the standard* one at the same pressure p , J/kg

M and M_{std} = molar masses of these liquids, kg/kmol

T and θ = their boiling points at the pressure p , K

$d\theta$, dT = differentials of the boiling points of the standard liquid and the liquid for which the heat of vaporization is being determined (the linearity rule allows us to replace the ratio of the differentials with the ratio of the differences in the boiling points at two pressures).

The specific heat of vaporization of non-polar liquids L (in J/kg) at atmospheric pressure can be calculated by Kistyakovsky's formula:

$$L = 19.2 \times 10^3 \frac{T}{M} (1.91 + \log T)$$
 (5-18)

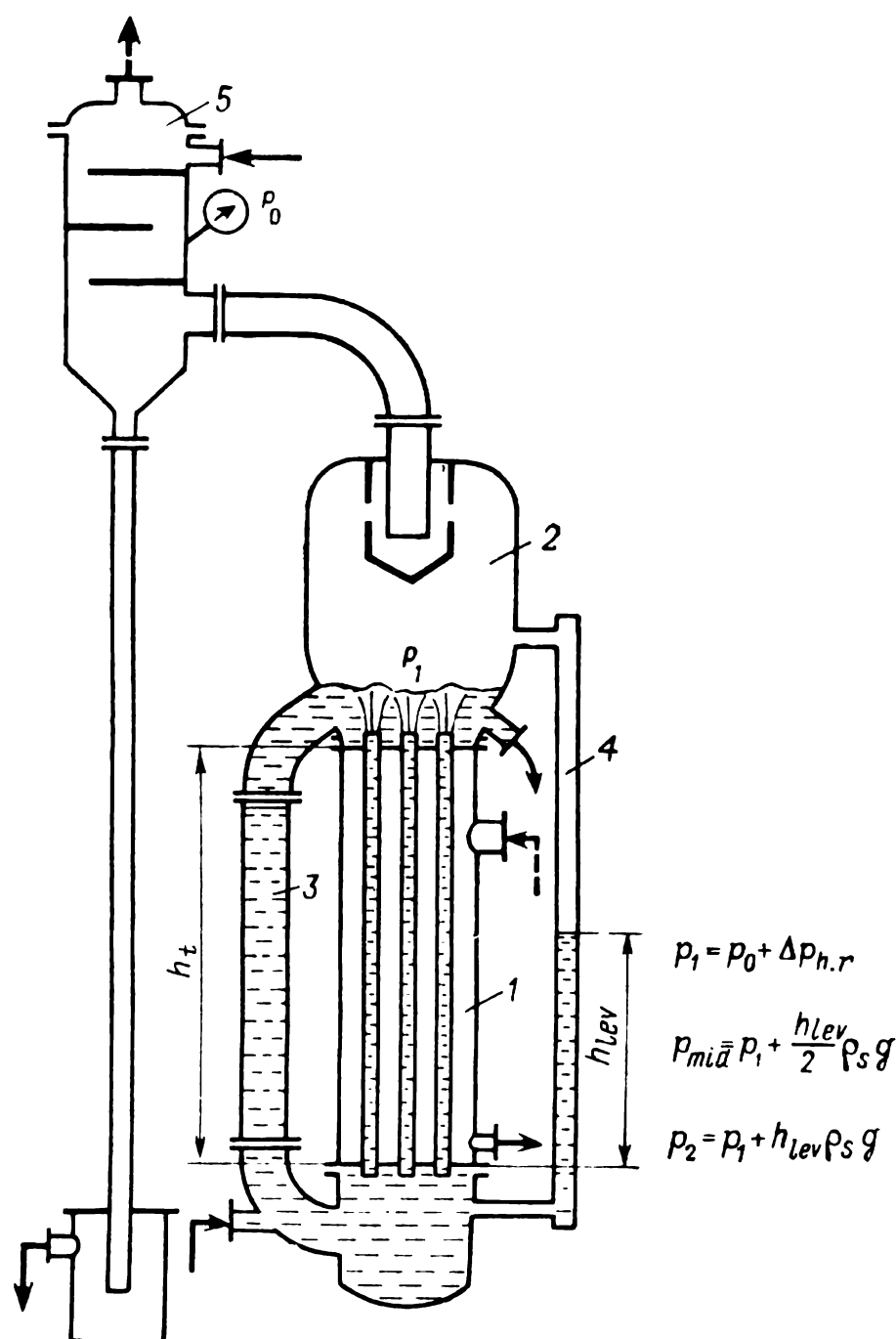
where T = boiling point, K

M = molar mass of the liquid, kg/kmol.

6. As follows from Fig. 5-1, the pressure in the middle layer of a solution being evaporated is:

$$p_{mid} = p_1 + \frac{h_{lev}}{2} \rho_s g = p_0 + \Delta p_{h.r} + \Delta p_{h.e}$$
 (5-19)

* See the footnote on the preceding page. The dependence of the specific heat of vaporization of hexane on the temperature is given in Fig. A-18.

Fig. 5-1. To determination of p_{mid} :

1 — heating chamber; 2 — separator; 3 — circulation pipe; 4 — water gauge glass; 5 — barometric condenser

where g = acceleration due to gravity

p_0 = pressure (abs.) of saturation (condensation) of secondary vapour in the space which it is delivered into from the evaporator (for example, in a barometric condenser)

$\Delta p_{h.r}$ = loss of pressure of the secondary vapour for overcoming the hydraulic resistance of the separator and the piping; it can be calculated by means of the equation [see Eq. (1-57)]:

$$\Delta p_{h.r} = \frac{\rho_v v^2}{2} \left(1 + \frac{\lambda L}{d} + \sum \zeta \right) \quad (5-20)$$

$\Delta p_{h.e} = \frac{h_{lev}}{2} \rho_s g$ = increase of the pressure in the liquid at a depth of $h_{lev}/2$ from the surface (the so-called hydrostatic effect)
 ρ_s = density of the solution, kg/m^3 .

The mean boiling point corresponding to the pressure in the middle layer is determined as follows:

$$t_b = t_{\text{mid}} + \Delta t_{\text{depr}} = t_0 + \Delta t_{\text{h.r}} + \Delta t_{\text{h.e}} + \Delta t_{\text{depr}} \quad (5-21)$$

where t_{mid} = boiling point of the water at the pressure p_{mid}

t_0 = boiling point of the water at the pressure p_0

Δt_{depr} = elevation of the boiling point of the solution in comparison with that of water at the same pressure p_{mid} (temperature depression) *

$\Delta t_{\text{h.r}}$ = elevation of the boiling point due to the increase in the pressure $\Delta p_{\text{h.r}}$ (hydraulic depression); in calculations of evaporators it is usually assumed that $\Delta t_{\text{h.r}} = 1$ to 1.5 K

$\Delta t_{\text{h.e}}$ = elevation of the boiling point in the middle layer of the solution due to the hydrostatic effect (hydrostatic depression).

The hydrostatic depression $\Delta t_{\text{h.e}}$ connected with the quantity $\Delta p_{\text{h.e}}$ depends on the height of the level of the solution determined according to a water gauge glass and on the density of the solution. The optimal height of the level when evaporating aqueous solutions in evaporators with natural circulation can be calculated by the formula [5-3]:

$$h_{\text{lev}} = [0.26 + 0.0014 (\rho_s - \rho_{\text{wat}})] h_1 \quad (5-22)$$

where h_{lev} = optimal height of the level according to the water gauge glass, m

h_1 = working height of the tubes, m

ρ_s and ρ_{wat} = densities of the solution (final concentration) and the water at the temperature t_b , kg/m³.

The quantity $\Delta t_{\text{h.e}}$ is determined by the equation:

$$\Delta t_{\text{h.e}} = t_{\text{mid}} - t_1 \quad (5-23)$$

where t_{mid} = boiling point of the water at the pressure p_{mid}

$$t_1 = t_0 + \Delta t_{\text{h.r}} \quad (5-24)$$

For the procedure used in calculating t_b see Example 5-11.

7. The difference between the condensation temperature of the heating steam t_{st} and the mean boiling point of the solution t_b

* Since in evaporators with natural or forced circulation the concentration of the boiling solution is close to the final one, then Δt_{depr} in them is taken for a solution of the final concentration. In the absence of experimental data, the value of Δt_{depr} can be approximately calculated using the rules given above—see Eqs. (5-14) to (5-16). Table A-36 gives the boiling points of aqueous solutions of selected salts at atmospheric pressure, and Fig. A-19 gives values of Δt_{depr} .

is called the efficient or overall temperature drop:

$$\Delta t_{ov} = t_{st} - t_b \quad (5-25)$$

The quantity Δt_{ov} expresses the mean driving force (Δt_m) in the equation of heat transfer (4-65) when determining the heating surface area of an evaporator.

From Eqs. (5-21) and (5-25), we get:

$$\Delta t_{ov} = t_{st} - t_0 - (\Delta t_{depr} + \Delta t_{h.e} + \Delta t_{h.r}) = \Delta t_{tot} - \sum \Delta t_i \quad (5-26)$$

where $\sum \Delta t_i = \Delta t_{depr} + \Delta t_{h.e} + \Delta t_{h.r}$ is called the sum of the temperature losses.

In multiple-effect evaporators, Δt_{tot} is the difference between the temperature of condensation of the heating steam in the first effect and the temperature of condensation of the secondary steam in the last effect; $\sum \Delta t_i = \Delta t_{depr} + \Delta t_{h.e} + \Delta t_{h.r}$ is the sum of the temperature losses in all the effects.

8. The overall temperature drop is distributed between the separate effects:

(a) when calculating for the minimum total surface area of all the effects—proportional to $(Q/K)^{1/2}$:

$$\Delta t_i = \frac{\Delta t_{ov} (Q_i/K_i)^{1/2}}{\sum_{i=1}^n (Q_i/K_i)^{1/2}} \quad (5-27)$$

(b) when calculating for equal surface areas of the effects—proportional to the ratio Q/K :

$$\Delta t_i = \frac{\Delta t_{ov} Q_i/K_i}{\sum_{i=1}^n \frac{Q_i}{K_i}} \quad (5-28)$$

where Q_i = heat load of an effect

K_i = overall coefficient of heat transfer in the effect.

9. The rate of flow of water in a condenser G_{wat} (in kg/s) is determined from the heat balance equation of the condenser:

$$G_{wat} = \frac{W (H'' - H')}{c (t_2 - t_1)} \quad (5-29)$$

here W = rate of flow of the secondary steam fed into the condenser, kg/s

H'' = specific enthalpy of this steam, J/kg

H' = specific enthalpy of the condensate when it leaves the condenser, J/kg

t_1 = initial temperature of the cooling water, °C or K

t_2 = temperature of the discharged water, °C or K

c = mean specific heat capacity of the water, J/kg·K.

In contact condensers, the temperature of the condensate equals the final temperature of the cooling water t_2 . Hence, in such condensers

$$G_{\text{wat}} = W \frac{H'' - ct_2}{c(t_2 - t_1)} \quad (5-30)$$

The amount of air exhausted by a vacuum pump from a barometric condenser G_{air} (in kg/s) is determined according to the empirical equation:

$$G_{\text{air}} = 0.000\,025 (W + G_{\text{wat}}) + 0.01W \quad (5-31)$$

where W = rate of flow of the secondary steam fed into the condenser, kg/s

G_{wat} = rate of flow of water in the condenser, kg/s.

The volume of air V saturated with steam (in m³/s) exhausted from a counterflow barometric condenser is:

$$V = \frac{RG_{\text{air}}T_{\text{air}}}{M_{\text{air}}(\rho_{\text{bc}} - \rho_v)} = \frac{287G_{\text{air}}T_{\text{air}}}{\rho_{\text{bc}} - \rho_v} \quad (5-32)$$

Here T_{air} = temperature of the air (in K) evacuated from the barometric condenser; it is determined by the approximate empirical formula

$$T_{\text{air}} = 273 + [t_{\text{wat, in}} + 0.1(t_{\text{wat, fin}} - t_{\text{wat, in}}) + 4] \quad (5-33)$$

$t_{\text{wat, in}}$ = initial temperature of the water, °C

$t_{\text{wat, fin}}$ = final temperature of the water, °C, taken to be at least by 3°C lower than the condensation temperature of the secondary steam

ρ_{bc} = pressure (abs.) in the barometric condenser, Pa

ρ_v = pressure of the saturated steam at the temperature T_{air} , Pa.

The height of a barometric tube h (in m) depends on the magnitude of the vacuum in the condenser and equals:

$$h = h_0 + h_{\text{h.r}} + 0.5 \quad (5-34)$$

$$\text{where } h_0 = 10.33 \frac{b}{760} \quad (5-35)$$

b = vacuum in the condenser, mm Hg

$$h_{\text{h.r}} = \frac{v^2}{2g} \left(1 + \lambda \frac{h}{d} + 0.5 \right) \quad (5-36)$$

v = velocity of the water in the tube, m/s

λ = dimensionless pipe resistance coefficient

d = diameter of the tube, m.

10. The mass of crystals formed G_{cr} (in kg) * is determined

* Sometimes the rate of flow (kg/h or kg/s) is used instead of the mass in the following equations.

from the equation of the material balance of the crystallizer:

$$G_{\text{cr}} = \frac{G_1 (\bar{x}_2 - \bar{x}_1) - W \bar{x}_2}{\bar{x}_2 - \bar{X}_{\text{cr}}} \quad (5-37)$$

where G_1 = amount of the initial solution, kg

\bar{x}_1 = concentration of the initial solution with respect to the anhydrous salt, mass fraction

\bar{x}_2 = concentration of the mother liquor with respect to the anhydrous salt after crystallization, mass fraction

W = amount of evaporated solvent, kg

$\bar{X}_{\text{cr}} = M/M_{\text{cr}}$ = ratio of the molar masses of the anhydrous solute and the crystallohydrate.

If a substance crystallizes in the anhydrous form, then $\bar{X}_{\text{cr}} = 1$.

When crystallization is conducted without removing part of the solvent ($W = 0$), we have

$$G_{\text{cr}} = \frac{G_1 (\bar{x}_1 - \bar{x}_2)}{\bar{X}_{\text{cr}} - \bar{x}_2} \quad (5-38)$$

11. The specific heat of solution L_s (in J/kg) of solids having a low solubility can be determined by the equation:

$$L_s = \frac{19.2 \times 10^3 \log \frac{c_1}{c_2}}{M \left(\frac{1}{T_2} - \frac{1}{T_1} \right)} \quad (5-39)$$

where c_1 and c_2 = solubility of the substance at the temperatures T_1 and T_2 (in K), respectively

M = molar mass of the solute, kg/kmol.

The specific heat of fusion L_f (in J/kg) in the absence of experimental data can be calculated by the following approximate relationships:

(a) for inorganic compounds

$$L_f = 25.1 \times 10^3 \frac{T_m}{M} \quad (5-40)$$

(b) for organic compounds

$$L_f = 41.9 \times 10^3 \frac{T_m}{M} \quad (5-41)$$

where T_m = melting point, K

M = molar mass of the compound, kg/kmol.

The quantity of heat Q (in J) liberated in crystallization without evaporation of part of the solvent is determined from the equation of the heat balance of the crystallizer:

$$Q = G_1 c (t_1 - t_2) + G_{\text{cr}} L \quad (5-42)$$

where G_1 = amount of the initial solution, kg
 c = specific heat capacity of the initial solution, J/kg·K
 t_1 and t_2 = initial and final temperatures of the solution, °C or K
 G_{cr} = amount of crystals formed, kg
 L = specific heat of crystallization, J/kg.

EXAMPLES

Example 5-1. An initial solution of sodium hydroxide contains 79 g of water per dm³. The density of the evaporated solution at 30 °C is 1.555 g/cm³. This corresponds to a concentration of 840 g per dm³ of solution. Determine the amount of water evaporated per ton of the initial solution.

Solution. The mass fraction of the solute in the initial solution is:

$$\bar{x}_{in} = \frac{79}{1000 + 79} = 0.0733$$

In the final solution it is:

$$\bar{x}_{fin} = \frac{840}{1555} = 0.54$$

The amount of water evaporated per ton of initial solution is:

$$W = G_{in} \left(1 - \frac{\bar{x}_{in}}{\bar{x}_{fin}} \right) = 1000 \left(1 - \frac{0.0733}{0.54} \right) = 865 \text{ kg}$$

Example 5-2. Find the specific heat capacity of a 25% aqueous solution of sodium salicylate.

Solution. Since the concentration of the solution exceeds 20%, we use Eq. (5-12) to calculate the specific heat capacity of the solution.

We first determine the specific heat capacity c_1 of dry sodium salicylate by Eq. (5-13), using the data of Table 5-1. The chemical formula of the salt is $C_6H_4(OH)COONa$, and $M = 160$. We have:

$$c_1 = \frac{7.5 \times 7 + 9.6 \times 5 + 16.8 \times 3 + 26.0}{160} = 1.11 \text{ kJ/kg} \cdot \text{K}$$

The specific heat capacity of a 25% solution is

$$c = 4190 (1 - \bar{x}) + c_1 \bar{x} = 4190 \times 0.75 + 1110 \times 0.25 = 3420 \text{ J/kg} \cdot \text{K}$$

Example 5-3. Using the linearity rule, determine the boiling point of aniline under a vacuum of 0.8 at, i. e. at a residual pressure of $p_{abs} = 0.2$ at. The saturated vapour pressure of aniline at a temperature of 160 °C is known to equal 390 mm Hg, and the boiling point of aniline is 184 °C at atmospheric pressure.

Solution. We take hexane as the standard liquid. According to the diagram in Fig. A-17, we find for hexane that at $p_1 = 390$ mm Hg $\theta_1 = 49.2^\circ\text{C}$, and at $p_2 = 760$ mm Hg $\theta_2 = 69^\circ\text{C}$. Hence, by Eq. (5-15),

$$K = \frac{t_1 - t_2}{\theta_1 - \theta_2} = \frac{160 - 184}{49.2 - 69} = 1.21$$

At a pressure of $p_{\text{abs}} = 0.2$ at $= 0.2 \times 735 = 147$ mm Hg, the boiling point of hexane is 24.5°C (Fig. A-17).

Consequently, by Eq. (5-15),

$$\frac{160 - t}{49.2 - 24.5} = 1.21$$

whence $t = 130.1^\circ\text{C}$.

If we solve the problem with the aid of a linearity rule diagram plotted with water as the standard liquid (Fig. A-15), we proceed as follows. From Table A-57, we find the boiling point of water at a pressure of $p_{\text{abs}} = 0.2$ at equal to about 60°C . In Fig. A-15, we find the point of intersection of the ordinate 60°C (for water) with line 12 (for aniline). On the axis of abscissas, a boiling point of aniline equal to 130°C corresponds to this point of intersection.

Example 5-4. Use the linearity rule to calculate the heat of vaporization of aniline at an absolute pressure of 0.2 at.

Solution. We use Eq. (5-17), taking hexane as the standard liquid. The boiling point of aniline at $p_{\text{abs}} = 0.2$ at is 130°C (see the preceding example). The boiling point of hexane at $p_{\text{abs}} = 0.2$ at is 24.5°C . We find the specific heat of vaporization of hexane at 24.5°C from the chart in Fig. A-18: $L = 366 \times 10^3$ J/kg.

We determine the ratio $d\theta/dT$ according to the data of the preceding example. Since the relationship between T and θ is linear at the same pressure, then $d\theta/dT$ is a constant quantity. Hence,

$$\frac{d\theta}{dT} = \frac{1}{K} = \frac{1}{1.21} = 0.826$$

Using all these values in Eq. (5-17), we get:

$$L = L_{\text{std}} \frac{M_{\text{std}}}{M} \left(\frac{T}{\theta} \right)^2 \frac{d\theta}{dT} = 366 \times 10^3 \times \frac{86}{93} \left(\frac{273 + 130}{273 + 24.5} \right)^2 0.826 = 515\,000 \text{ J/kg}$$

If we take water instead of hexane as the standard liquid, we get the following data.

The boiling point of water at $p_{\text{abs}} = 0.2$ at is 59.7°C , and its specific heat of vaporization (according to Table A-57) is $L = 2358 \times 10^3$ J/kg.

We find the ratio $d\theta/dT$ from Fig. A-15 as the slope of line 12:

$$\frac{d\theta}{dT} = \frac{110 - 20}{200 - 80} = 0.75$$

Introducing values into Eq. (5-17), we get:

$$L = 2358 \times 10^3 \times \frac{18}{93} \left(\frac{403}{333} \right)^2 0.75 = 503\,000 \text{ J/kg}$$

Example 5-5. Determine the rate of flow of the heating steam in a single-effect evaporator with continuous concentration of a solution of NaOH. The rate of flow of the initial solution is 2 ton/h, its concentration is 14.1% (mass), and its final concentration is 24.1% (mass). The temperature of the heating steam is 150 °C. The boiling point of the solution in the evaporator is 113 °C. The heat losses of the evaporator are 58 000 W.

Perform the calculations for three variants:

(a) The solution is fed in for evaporation with an initial temperature of 20 °C;

(b) the solution is fed in for evaporation at the boiling point in the evaporator;

(c) the solution is fed to the evaporator superheated to 130 °C.

Solution. To determine the rate of flow of the heating steam, we must calculate the total consumption of heat in the apparatus.

The rate of flow of the water being evaporated, by Eq. (5-3), is:

$$W = G_{\text{in}} \left(1 - \frac{\bar{x}_{\text{in}}}{\bar{x}_{\text{fin}}} \right) = 2000 \left(1 - \frac{14.1}{24.1} \right) = 829 \text{ kg/h}$$

We find the quantity of heat used to evaporate the water.

At 113 °C, the specific heat of vaporization of water is $L = 2225 \times 10^3 \text{ J/kg}$ (Table A-56); consequently, the quantity of heat used for evaporation is:

$$Q_e = WL = \frac{829 \times 2225 \times 10^3}{3600} = 510\,000 \text{ W}$$

We determine the consumption of heat for heating the solution. We calculate the specific heat capacity of the initial solution.

By Eq. (5-13), the specific heat capacity of solid NaOH is:

$$c_1 = \frac{16.8 + 9.6 + 26.0}{40} = 1.31 \text{ kJ/kg} \cdot \text{K}$$

The specific heat capacity of a 14.1% aqueous solution of NaOH is:

$$c = 1.31 \times 10^3 \times 0.141 + 4.19 \times 10^3 \times 0.859 = 3780 \text{ J/kg} \cdot \text{K}$$

We calculate the quantity of heat used to heat the solution for each variant:

(a) the initial temperature of the solution is 20°C

$$Q_h = \frac{2000 \times 3.78 \times 10^3}{3600} (113 - 20) = 195\,000 \text{ W}$$

(b) the initial temperature equals the boiling point

$$Q_h = 0$$

(c) the solution is supplied to the evaporator superheated to 130 °C

$$Q_h = \frac{2000 \times 3.78 \times 10^3}{3600} (113 - 130) = -35\,700 \text{ W}$$

We calculate the quantity of heat needed for dehydration of the NaOH to get an idea of the magnitude of this heat consumption item in the heat balance of an evaporator.

According to the *Reference Book of Physicochemical Quantities of the Technical Cyclopedia* (Vol. 7) (in Russian), the heat of formation of NaOH in a solution with n moles of water is.

n	3	5	7	9	13.5
kJ/mol	456.6	465.5	469.1	469.5	470.23

We calculate the number of moles of water per mole of NaOH.

Initial solution:

number of moles of NaOH in 1 kg of the solution

$$\frac{141}{40} = 3.52$$

number of moles of water in 1 kg of the solution

$$\frac{1000 - 141}{18} = \frac{859}{18} = 47.7$$

number of moles of water per mole of NaOH

$$\frac{47.7}{3.52} = 13.5$$

Final solution:

number of moles of NaOH per kg of the solution

$$\frac{241}{40} = 6.02$$

number of moles of water per kg of the solution

$$\frac{1000 - 241}{18} = \frac{759}{18} = 42.1$$

number of moles of water per mole of NaOH

$$\frac{42.1}{6.02} = 7.0$$

Hence, the heat of dehydration is the difference between the heats of formation of NaOH in solutions with 13.5 and 7.0 moles, i. e.

$$470.23 - 469.1 = 1.13 \text{ kJ/mol of NaOH}$$

The total heat of dehydration is:

$$Q_d = \frac{3.52 \times 2000 \times 1130}{3600} = 2220 \text{ W}$$

We draw up a summary table of the consumption of heat (in W) in the evaporator for all three variants (Table 5-3).

TABLE 5-3

Consumption of heat	Variant		
	a	b	c
For evaporation of water	510 000	510 000	510 000
For dehydration	2 220	2 220	2 220
Heat losses	58 000	58 000	58 000
For heating the solution	195 000	0	-35 700
Total	765 220	570 220	534 520

A glance at this table shows that the consumption of heat for dehydration is a small quantity that may be disregarded.

We determine the consumption of heating steam. From Table A-56 we find $H'' = 2753 \times 10^3 \text{ J/kg}$ and $H' = 633 \times 10^3 \text{ J/kg}$. Hence, by Eq. (5-8) with the dryness fraction $x = 1$, we have:

$$(a) \ G_{st} = \frac{765\,220}{2753 \times 10^3 - 633 \times 10^3} = 0.361 \text{ kg/s} = 1300 \text{ kg/h}$$

$$(b) \ G_{st} = \frac{570\,220}{2120 \times 10^3} = 0.272 \text{ kg/s} = 980 \text{ kg/h}$$

$$(c) \ G_{st} = \frac{534\,520}{2120 \times 10^3} = 0.255 \text{ kg/s} = 920 \text{ kg/h}$$

Example 5-6. A batch evaporator with a heating surface area of 40 m^2 is filled with 20 tons of a weak solution having a mass concentration of $\bar{x}_{in} = 5\%$ and an initial temperature of 20°C . The solution is evaporated under a vacuum to a concentration of $\bar{x}_{fin} = 50\%$ (mass). The dependence of the boiling point of the solution and the overall heat transfer coefficient in the evaporator on the concentration of the solution is given in Table 5-4.

TABLE 5-4

Mass concentration \bar{x} , %	5	10	20	30	40	50
Boiling point, °C	55	56	60	67	76	95
Overall heat transfer coefficient $K/1.16$, W/m ² ·K	1850	1500	970	640	420	240

The overall coefficient of heat transfer for the period of heating the weak solution to the beginning of boiling $K_1=300\times\times1.16$ W/m²·K. The temperature of the heating steam is 120 °C.

Determine the consumption of heating steam assuming its moisture content to be 5%, and the duration of the evaporation process.

Solution. We determine the consumption of heating steam. The quantity of evaporated water, by Eq. (5-3), is:

$$W = G_{in} \left(1 - \frac{\bar{x}_{in}}{\bar{x}_{fin}} \right) = 20\,000 \left(1 - \frac{5}{50} \right) = 18\,000 \text{ kg}$$

The consumption of heat (disregarding the losses of heat to the surroundings) is:

$$Q = G_{in}c_{in} (t_b - t_{in}) + WL$$

The specific heat capacity of a 5% aqueous solution is $c_{in}= =0.95\times4.19\times10^3$ J/kg·K, and from Table A-56 the specific heat of vaporization of water at the average temperature of $(55 + 95)/2 = 75$ °C is $L=554\times4.19\times10^3$ J/kg. Hence,

$$Q = 20\,000\times0.95\times4.19\times10^3 (55 - 20) + 18\,000\times554\times4.19\times10^3 = = (665\,000\times4.19\times10^3 + 10\,000\,000\times4.19\times10^3) \text{ J}$$

The consumption of the heating steam:
during the period of heating the weak solution to the beginning of boiling, by Eq. (5-8), is:

$$G'_{st} = \frac{Q_h}{(H'' - H')_{\bar{x}}} = \frac{665\,000\times4.19\times10^3}{526.7\times4.19\times10^3\times0.95} = 1330 \text{ kg}$$

during the period of evaporation

$$G''_{st} = \frac{10\,000\,000\times4.19\times10^3}{526.7\times4.19\times10^3\times0.95} = 20\,000 \text{ kg}$$

In the above two equations, $526.7\times4.19\times10^3$ is the specific heat of condensation of saturated steam at $t=120$ °C (Table A-56), J/kg.

We determine the duration of the period of heating the weak solution up to the beginning of boiling by the equation:

tau_1 = (Q_h / (K_1 A Delta t_1)) = (665 000 x 4.19 x 10^3 / (1.16 x 300 x 40 x 82.5)) = 2430 s = 0.67 h

where Delta t_1 is the mean (in time) temperature difference in the evaporator during the heating period

120 20 Delta t_gr = 100
120 55 Delta t_sm = 65

Since Delta t_gr/Delta t_sm < 2, then Delta t_1 = (100 + 65)/2 = 82.5 K.

We determine the duration of the period of evaporation proper. During this period, the concentration of the boiling solution x-bar, its boiling point t and the magnitude of the overall heat transfer coefficient K continuously change—see Table 5-4.

The equation of heat transfer for an infinitely small length of time dtau

dQ = K A (T - t) dtau

contains only two constant quantities—the condensation temperature of the heating steam T = 120 °C and the area of the heat exchange surface A = 40 m².

The latter equation yields

A dtau = (dQ / (K (T - t)))

Integration of the right-hand side of the equation

A tau_2 = integral from 0 to Q_2 of (dQ / (K (T - t)))

can be performed graphically.

Table 5-5 gives the quantities needed for graphical integration. They have been obtained by calculation from the data of Table 5-4.

TABLE 5-5

x-bar, %	5	10	20	30	40	50
W, kg	0	10 000	15 000	16 700	17 500	18 000
L x 4190^-1, J/kg	565	565	564	562	559	554
Q x 4190^-1 x 10^-6, J	0	5.65	8.47	9.4	9.8	10
T - t, K	65	64	60	53	44	25
(1 / (K (T - t))) x 1.16 x 10^6, m^2/W	8.3	10.4	17.2	29.5	54.2	166.7

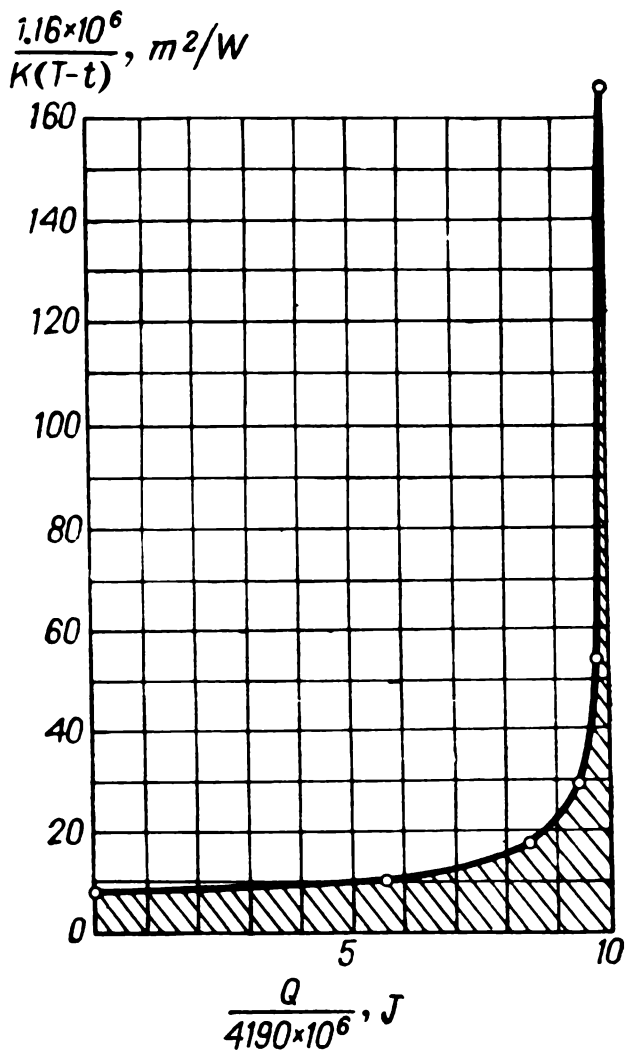


Fig. 5-2. To Example 5-6

We use the data of Table 5-5 to plot a graph (Fig. 5-2). Determining in this graph the value of the hatched area (for example, according to the law of trapeziums), we find

$$Q_2 = 10 \times 4190 \times 10^6 \int_0^{\frac{Q}{4190 \times 10^6}} \frac{dQ}{K(T-t)} = A\tau_2 = 547\,000 \text{ m}^2 \cdot \text{s}$$

whence

$$\tau_2 = \frac{547\,000}{40} = 13\,650 \text{ s} = 3.8 \text{ h}$$

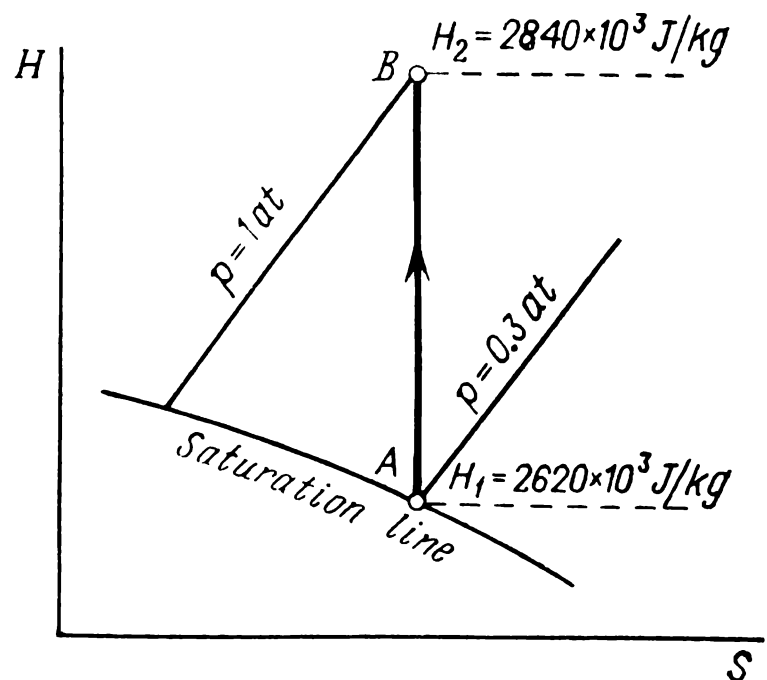
The total duration of the evaporation process is

$$\tau_1 + \tau_2 = 0.67 + 3.8 \approx 4.5 \text{ h}$$

Example 5-7. Compare the theoretical consumption of energy for two cases: (a) when the secondary steam is exhausted by a vacuum pump from an evaporator operating under a vacuum of 0.7 at, and (b) when the secondary steam is condensed in a condenser and the condensate is discharged by a pump. The capacity of the evaporator is 1000 kg of evaporated water an hour.

Solution. (a) We assume that the secondary steam is compressed adiabatically in the vacuum pump. In this case, the work done is determined by Eq. (2-18). We find the enthalpies from

Fig. 5-3. To Example 5-7



an H - S diagram for steam (Figs. 5-3 and A-25).

$$W = H_2 - H_1 = 2840 \times 10^3 - 2620 \times 10^3 = 220 \times 10^3 \text{ J/kg}$$

The required theoretical power (disregarding the efficiency of the vacuum pump) is:

$$P_{\text{th}} = \frac{220 \times 10^3 \times 1000}{3600} = 62.7 \times 10^3 \text{ W} = 62.7 \text{ kW}$$

(b) When the pump discharges a liquid (the condensate), we determine the required theoretical power of the pump (disregarding its efficiency) by the equation:

$$P_{\text{th}} = \frac{V \Delta p}{1000} = \frac{1000 \times 68.7 \times 10^3}{3600 \times 1000 \times 10^3} = 0.019 \text{ kW}$$

where

$$V = \frac{1000}{3600 \times 1000} = \frac{1}{3600} \text{ m}^3/\text{s}$$

$$\Delta p = 0.7 \text{ at} = 68.7 \times 10^3 \text{ Pa}$$

Inspection of the given example shows that it is not good practice to exhaust the secondary steam with the aid of a vacuum pump because this requires a greater expenditure of energy ($62.7/0.019 = 3300$ times more than when discharging the condensate). This is why the secondary steam is never exhausted, but is always condensed.

Example 5-8. Using Babo's law, determine the temperature depression Δt_{depr} for a 25% aqueous solution of calcium chloride at a pressure of $p_{\text{abs}} = 0.5 \text{ at}$.

Solution. From Table A-36, we find that a 25% aqueous solution of calcium chloride under atmospheric pressure (1.033 at) boils at a temperature of 107.5°C .

At this temperature, the saturated steam pressure (Table A-56) equals 1.345 at.

The ratio of the vapour pressures of the solution and water (at the same temperature of 107.5 °C) is:

$$\left(\frac{p}{p_0}\right)_{107.5} = \frac{1.033}{1.345} = 0.77$$

According to Babo's law [Eq. (5-16)], this ratio remains constant at all the boiling points of the solution. Hence, at the temperature t :

$$\left(\frac{p'}{p'_0}\right)_t = \frac{0.5}{p'_0} = 0.77$$

whence $p'_0 = 0.5/0.77 = 0.65$ at.

We find the temperature t —the boiling point of water at $p'_0 = 0.65$ at—equal to the boiling point of the calcium chloride solution at $p' = 0.5$ at.

From Table A-38 for $p_0 = 0.65$ at = 478 mm Hg, we find $t = 87.5$ °C.

The boiling point of water t_{wat} at $p_{\text{abs}} = 0.5$ at is 80.9 °C (Table A-57).

Hence, the temperature depression is

$$\Delta t_{\text{depr}} = t - t_{\text{wat}} = 87.5 - 80.9 = 6.6 \text{ K}$$

Table 5-2 gives no correction when $p/p_0 = 0.77$ and $p = 0.5$ at = 368 mm Hg.

Example 5-9. Calculate the hydrostatic depression $\Delta t_{\text{h.e}}$ when evaporating a 25% aqueous solution of calcium chloride under a vacuum in the evaporator with the optimal level of the solution in the tubes. The operating height of the tubes $h_t = 4$ m. The absolute pressure on the surface of the solution is $p_i = 0.36$ at (see Fig. 5-1).

Solution. At $p_i = 0.36$ at, the temperature of the water $t_i = 72.7$ °C (Table A-57).

The optimal height of the level according to the water gauge glass is determined by Eq. (5-22):

$$h_{\text{lev}} = [0.26 + 0.0014 (\rho_s - \rho_{\text{wat}})] h_t$$

Since the densities ρ_s and ρ_{wat} have to be taken at the boiling point of the solution which is meanwhile unknown, it becomes necessary to adopt its value. We assume that $t_b = 85$ °C.

The values of the densities are $\rho_s = 1195$ kg/m³ from Table A-3 taken approximately with a slight overstatement, and $\rho_{\text{wat}} = 969$ kg/m³ from Table A-39. Hence,

$$h_{\text{lev}} = [0.26 + 0.0014 (1195 - 969)] 4 = 2.3 \text{ m}$$

The pressure in the middle layer of the solution, by Eq. (5-19), is:

$$p_{\text{mid}} = p_1 + \frac{h_{\text{lev}}}{2} \rho_s g = 0.36 + \frac{2.3 \times 1195 \times 9.81}{2 \times 9.81 \times 10^4} = 0.5 \text{ at}$$

The boiling point of water at $p_{\text{mid}} = 0.5 \text{ at}$ is $t_{\text{mid}} = 80.9^\circ\text{C}$ (Table A-57).

The hydrostatic depression by Eq. (5-23) is:

$$\Delta t_{\text{h. e}} = t_{\text{mid}} - t_1 = 80.9 - 72.7 = 8.2 \text{ K}$$

We check the boiling point of the solution. With account taken of Δt_{depr} (see the preceding example), we have $t_b = t_{\text{mid}} + \Delta t_{\text{depr}} = 80.9 + 6.6 = 87.5^\circ\text{C}$ instead of the 85°C which we assumed. Since the densities of liquids change only slightly with the temperature, the approximate calculation of the quantity $\Delta t_{\text{h. e}}$ does not have to be repeated with greater accuracy.

Example 5-10. The secondary steam from an evaporator is fed into a barometric condenser through a steam pipe 150 mm in diameter and 14 m long. The velocity of the steam in the pipe is 50 m/s. The dimensionless pipe resistance coefficient is $\lambda = 0.03$. The pressure in the condenser (abs) is $p_0 = 0.3 \text{ at}$. The steam pipe has three 90-degree bends ($\zeta = 0.2$). Determine the hydraulic depression $\Delta t_{\text{h. r.}}$.

Solution. By Eq. (5-24), we have

$$\Delta t_{\text{h. r}} = t_1 - t_0$$

Table A-57 gives us $t_0 = 68.7^\circ\text{C}$ for $p_0 = 0.3 \text{ at}$. To determine the temperature t_1 , we must find the pressure on the surface of the solution being evaporated p_1 equal to $p_0 + \Delta p_{\text{h. r}}$ (see Fig. 5-1), where

$$\Delta p_{\text{h. r}} = \frac{v^2 \rho_{\text{st}}}{2} \left(1 + \frac{\lambda l}{d} + \sum \zeta \right)$$

The sum of the local resistance coefficients is:

Inlet to pipe	0.5 (Table A-13)
Bends	$3 \times 0.2 = 0.6$
	$\sum \zeta = 1.1$

We do not take into account the additional resistance of the separator.

From Table A-57 we find the density of the steam, considering it to be saturated— $\rho_{\text{st}} = 0.188 \text{ kg/m}^3$, and

$$\Delta p_{\text{h. r}} = \frac{50^2 \times 0.188}{2} \left(1 + \frac{0.03 \times 14}{0.15} + 1.1 \right) = 1129 \text{ Pa}$$

Hence,

$$p_1 = p_0 + \Delta p_{h.r} = 0.3 + \frac{1129}{9.81 \times 10^4} = 0.312 \text{ at}$$

For $p_1 = 0.312$ at, Table A-57 gives us $t_1 = 69.5^\circ\text{C}$.

The hydraulic depression is:

$$\Delta t_{h.r} = t_1 - t_0 = 69.5 - 68.7 = 0.8 \text{ K}$$

Example 5-11. Determine the required heating surface of a vacuum evaporator (see Fig. 5-1) and the rate of flow of the heating steam for evaporating a solution of calcium chloride from 15 to 25%. The capacity with respect to the initial (dilute) solution is 20 000 kg/h. The absolute pressure of the heating steam is 1.4 at, and its moisture content is 5%. The absolute pressure in the barometric condenser is $p_0 = 0.345$ at. The initial solution is fed into the evaporator at $t_{in} = 75^\circ\text{C}$.

Take the overall heat transfer coefficient equal to $1000 \text{ W/m}^2 \cdot \text{K}$ and the heat losses equal to 5% of the usefully spent heat.

Solution. We determine the mean boiling point of a 25% solution of calcium chloride in the evaporator by Eq. (5-21):

$$t_b = t_0 + \Delta t_{h.r} + \Delta t_{h.e} + \Delta t_{depr}$$

From Table A-57, we find $t_0 = 71.7^\circ\text{C}$ for $p_0 = 0.345$ at. We assume that $\Delta t_{h.r} = 1 \text{ K}$. Hence, $t_1 = t_0 + \Delta t_{h.r} = 72.7^\circ\text{C}$, which a pressure of $p_1 = 0.36$ at corresponds to (Table A-56).

We adopt the operating height of the tubes of 4 m. Therefore, $p_{mid} = 0.5$ at and $\Delta t_{h.e} = 8.2 \text{ K}$ (see Example 5-9).

The temperature depression Δt_{depr} at $p_{mid} = 0.5$ at equals 6.6 K (see Example 5-8).

The mean boiling point of the solution is:

$$t_b = 71.7 + 1 + 8.2 + 6.6 = 87.5^\circ\text{C}$$

We determine the consumption of heat in the evaporator.

The specific heat capacity of the initial solution, by Eq. (5-11), is:

$$c_{in} = 4190 (1 - \bar{x}_{in}) = 4190 (1 - 0.15) = 3560 \text{ J/kg} \cdot \text{K}$$

The consumption of heat for heating the initial solution, by Eq. (5-5), is:

$$Q_h = G_{in} c_{in} (t_b - t_{in}) = \frac{20\,000}{3600} \times 3560 (87.5 - 75) = 247\,000 \text{ W}$$

The rate of flow of the water being evaporated, by Eq. (5-3), is:

$$W = G_{in} \left(1 - \frac{\bar{x}_{in}}{\bar{x}_{fn}} \right) = \frac{20\,000}{3600} \left(1 - \frac{15}{25} \right) = 2.22 \text{ kg/s}$$

The specific heat of vaporization of water at $p_{\text{mid}} = 0.5$ at is $L = 2307 \times 10^3$ J/kg (Table A-57).

The consumption of heat for evaporation of the water is

$$Q_e = WL = 2.22 \times 2307 \times 10^3 = 5\,120\,000 \text{ W}$$

The total consumption of heat with account taken of the losses amounting to 5% is:

$$Q = 1.05 (Q_h + Q_e) = 1.05 (247\,000 + 5\,120\,000) = 5\,650\,000 \text{ W}$$

We determine the area of the heating surface of the evaporator by the equation:

$$A = \frac{Q}{K \Delta t_{\text{mid}}}$$

Here $\Delta t_{\text{mid}} = t_{\text{st}} - t_b = 108.7 - 87.5 = 21.2$ K, where $t_{\text{st}} = 108.7^\circ\text{C}$ is the temperature of the heating steam at $p = 1.4$ at (Table A-57). We have

$$A = \frac{5\,650\,000}{1000 \times 21.2} = 266 \text{ m}^2$$

We select an evaporator from a catalogue with a surface area from 15 to 20% greater than the calculated one.

The consumption of heating steam, by Eq. (5-8), is:

$$G_{\text{st}} = \frac{Q}{L_{\text{st}} x}$$

From Table A-57, the specific heat of condensation of the heating steam at $p = 1.4$ at is $L_{\text{st}} = 2237 \times 10^3$ J/kg, and the steam content (dryness fraction) of the heating steam is $x = 0.95$. Hence,

$$G_{\text{st}} = \frac{5\,650\,000}{2237 \times 10^3 \times 0.95} = 2.66 \text{ kg/s}$$

The specific consumption of the heating steam, by Eq. (5-9), is

$$g_{\text{st}} = \frac{G_{\text{st}}}{W} = \frac{2.66}{2.22} = 1.2 \text{ kg/kg of water}$$

Example 5-12. What amount of crystals will be liberated in a crystallizer when 10 tons of a saturated aqueous solution of potash are cooled from 80 to 35°C without evaporation of the water? The potash crystallizes with two molecules of water per molecule of K_2CO_3 .

Solution. We use Eq. (5-38). We find the concentrations of the saturated aqueous solutions of the potash from its solubility curve (Fig. A-20):

at 80°C 10 mol of K_2CO_3 per 1000 g of water
 at 35°C 8.15 mol of K_2CO_3 per 1000 g of water

The molar mass of K_2CO_3 is 138 kg/kmol. Consequently,

$$\bar{x}_{80} = \frac{10 \times 138}{1000 + 10 \times 138} = 0.58 \text{ kg/kg}$$

$$\bar{x}_{35} = \frac{8.15 \times 138}{1000 + 8.15 \times 138} = 0.53 \text{ kg/kg}$$

$$\frac{M}{M_{cr}} = \frac{138}{174} = 0.793$$

where 174 is the molar mass of $K_2CO_3 \cdot 2H_2O$, kg/kmol.

Using these values in Eq. (5-38), we get:

$$G_{cr} = \frac{10\,000(0.58 - 0.53)}{0.793 - 0.53} = 1900 \text{ kg}$$

Example 5-13. Determine the quantity of heat that must be withdrawn in a continuous crystallizer for cooling an aqueous solution of $NaNO_3$ from 90 to 40 °C at a rate of 5000 kg/h. The solution contains 16 moles of $NaNO_3$ per 1000 g of water at 90 °C. Take into account that when the solution is cooled in the crystallizer water simultaneously evaporates in an amount equal to 3% of the initial amount of the solution.

Solution. From the solubility curve of $NaNO_3$ (Fig. A-20), we find that the concentration of a saturated solution of $NaNO_3$ at 40 °C is 12.3 moles per 1000 g of water.

When the solution is cooled to 40 °C, the amount of crystals G_{cr} (in kg/s) that will freeze out, by Eq. (5-37), is:

$$G_{cr} = \frac{G_1(\bar{x}_2 - \bar{x}_1) - W\bar{x}_2}{\bar{x}_2 - \bar{X}_{cr}}$$

We calculate the concentrations in mass fractions:

$$\bar{x}_1 = \frac{16 \times 85}{1000 + 16 \times 85} = 0.576 \text{ kg/kg}$$

$$\bar{x}_2 = \frac{12.3 \times 85}{1000 + 12.3 \times 85} = 0.511 \text{ kg/kg}$$

where 85 is the molar mass of the $NaNO_3$, kg/kmol. The value of $\bar{X}_{cr} = 1$ because the $NaNO_3$ crystallizes in the anhydrous form. We have:

$$G_{cr} = \frac{5000(0.511 - 0.576) - 0.03 \times 5000 \times 0.511}{3600(0.511 - 1)} = 0.229 \text{ kg/s}$$

We calculate the quantity of heat that has to be withdrawn by the formula:

$$Q = G_1 c (t_1 - t_2) + G_{cr} L_{cr} - WL$$

where W = rate of flow of the evaporating water, kg/s

L = specific heat of vaporization of the water, J/kg.

We take the specific heat of crystallization of the $NaNO_3$ equal to $L_{cr} = 21\,100 \times 10^3 \text{ J/kmol}$ (Table A-37).

To determine the specific heat capacity of the solution c by Eq. (5-12), we first find the specific heat capacity of the solid salt c_1 by Eq. (5-13):

$$c_1 = \frac{26.0 + 26.0 + 3 \times 16.8}{85} = 1.2 \text{ kJ/kg} \cdot \text{K}$$

Hence, for $\bar{x} = 57.6\%$, we have:

$$c = 4190 \times 0.424 + 1200 \times 0.576 = 2470 \text{ J/kg} \cdot \text{K}$$

From Table A-56 for the mean temperature equal to $(90 + 40)/2 = 65^\circ\text{C}$, we find the specific heat of vaporization of water $L = 2345 \times 10^3 \text{ J/kg}$, and the quantity of withdrawn heat is:

$$Q = \frac{5000}{3600} \times 2470 (90 - 40) + 0.229 \times \frac{21\,100 \times 10^3}{85} - \frac{0.03 \times 5000 \times 2345 \times 10^3}{3600} = 130\,000 \text{ W}$$

Example 5-14. For the conditions of the preceding example, determine the required cooling surface area and the rate of flow of the water in the crystallizer. Assume that the overall heat transfer coefficient equals $100 \text{ W/m}^2 \cdot \text{K}$. The water enters the cooling jacket at 15°C and leaves it at 20°C . Counterflow cooling is used.

Solution. We determine the cooling surface area from Eq. (4-65):

$$A = \frac{Q}{K \Delta t_m}$$

The temperature scheme is:

$$\begin{array}{l} 90 \rightarrow 40 \\ 20 \leftarrow 15 \end{array}$$

whence by Eq. (4-70), we have

$$\Delta t_m = \frac{(90 - 20) - (40 - 15)}{2.3 \log \frac{90 - 20}{40 - 15}} = 36.2 \text{ K}$$

Consequently,

$$A = \frac{130\,000}{100 \times 36.2} = 36 \text{ m}^2$$

The rate of flow of the water is:

$$G_{\text{wat}} = \frac{130\,000}{4.19 \times 10^3 (20 - 15)} = 6.33 \text{ kg/s} = 22\,400 \text{ kg/h}$$

PROBLEMS

5-1. Calculate the specific rate of flow of dry saturated steam in evaporating water under atmospheric pressure ($p_{\text{abs}} = 1 \text{ at}$) and under a vacuum of 0.8 at . The pressure of the heating steam in both cases is $p_{\text{abs}} = 2 \text{ at}$. The water is

fed for evaporation (a) at a temperature of 15 °C, and (b) heated to its boiling point.

5-2. The capacity of an evaporator with respect to the feed solution is 2650 kg/h. The concentration of the feed solution is 50 g/dm³ of water. The concentration of the evaporated solution is 295 g/dm³ of solution. The density of the evaporated solution is 1189 kg/m³. Find the capacity of the evaporator with respect to the evaporated solution.

5-3. How will the capacity of an evaporator change if a layer of scale 0.5 mm thick is deposited on the walls of the heating tubes? The overall heat transfer coefficient K for clean tubes is 1390 W/m²·K. The dimensionless resistance coefficient of the scale is $\lambda = 1.16$ W/m·K.

5-4. The capacity of an evaporator heated with steam ($p_{\text{gauge}} = 1.5$ at) has to be raised from 1200 to 1900 kg/h (with respect to the feed solution). Evaporation is conducted under atmospheric pressure, the boiling point of the solution in the evaporator is 105 °C, the feed solution is heated to its boiling point. Determine the pressure of the heating steam. Disregard the heat losses, and consider that the overall coefficient of heat transfer and the final concentration of the solution are constant.

5-5. How much water has to be evaporated from 1500 kg of a calcium chloride solution to change its concentration from 8 to 30% (mass)?

5-6. What amount of water has to be evaporated from 1 m³ of sulphuric acid having a density of 1560 kg/m³ (a mass concentration of 65.2%) to get an acid with a density of 1840 kg/m³ (a mass concentration of 98.7%)? What volume does the obtained concentrated acid occupy?

5-7. An evaporator is supplied with 1.4 ton/h of a 9% solution that is evaporated under atmospheric pressure to a final concentration of 32% (mass). The feed solution has a temperature of 18 °C. The boiling point in the evaporator is 105 °C. The rate of flow of the heating steam ($p_{\text{gauge}} = 2$ at) is 1450 kg/h. The moisture content of the heating steam is 4.5%. Determine the quantity of heat lost by the evaporator to the surroundings.

5-8. Determine the specific heat capacity of a cooling mixture consisting of 2 dm³ of water, 8 kg of ice, and 5 kg of table salt.

5-9. A solution consists of 0.7 m³ of sulphuric acid (100%), 400 kg of blue vitriol (CuSO₄·5H₂O), and 1.4 m³ of water. Determine (a) the specific heat capacity of the solution, and (b) the quantity of dry saturated steam ($p_{\text{gauge}} = 1$ at) needed to heat the solution from 12 to 58 °C. The losses of heat by the apparatus during the time of heating the solution are 25 100 kJ.

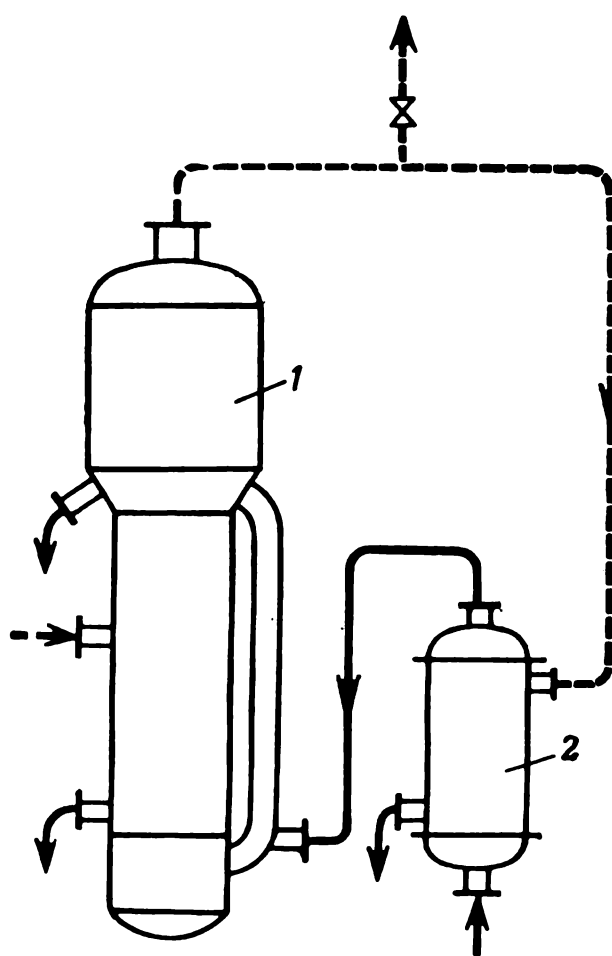
5-10. An evaporator is used to evaporate 2.69 ton/h of a 7% aqueous solution under atmospheric pressure. The boiling point in the evaporator is 103 °C. The initial temperature of the feed solution is 95 °C. The pressure of the heating steam is $p_{\text{gauge}} = 2$ at. The area of the heat exchange surface in the evaporator is 52 m², the overall heat transfer coefficient is 1000 W/m²·K. The quantity of heat lost by the evaporator to the surroundings is 110 000 W. Determine (a) the final concentration of the evaporated solution, and (b) the rate of flow of the heating steam if its moisture content is 5%.

5-11. A solution of potassium chloride is continuously concentrated from a mass concentration of 9.5 to 26.6% in an evaporator having a heat exchange surface area of 30 m² and operating under atmospheric pressure. The initial temperature of the solution is 18 °C. The pressure of the heating steam is $p_{\text{gauge}} = 2$ at. The capacity of the apparatus was originally 900 kg/h (of the feed solution), but after some time it dropped to 500 kg/h owing to the formation of scale. Ignoring the heat losses of the evaporator to the surroundings, determine the thickness of the layer of scale formed assuming that $\lambda = 1.4$ W/m·K for the scale. Disregard the hydrostatic effect.

5-12. For the conditions of Example 5-7, determine the consumption of energy in discharging the secondary steam with a vacuum pump and in discharging the condensate with a pump if the vacuum in the evaporator is 0.95 at.

5-13. A 12.5% solution of ammonium sulphate is fed into a continuous single-effect evaporator and is evaporated under a pressure of $p_{\text{abs}} = 1$ at to a

Fig. 5-4. To Problem 5-13:
1—evaporator; 2—heat exchanger



mass concentration of 30.6%. The concentrated solution flows out of the evaporator at a rate of 800 kg/h. The feed solution is heated in a heat exchanger from 24 to 80 °C by the secondary steam. The remaining amount of the secondary steam is used for heating other production equipment (Fig. 5-4). The heat losses of the evaporator are 6% of the usefully used quantity of heat, i. e. of the sum $Q_h + Q_e$. The sum $\Delta t_{h,e} + \Delta t_{h,r} = 3$ K.

Determine: (a) the rate of flow of the heating steam ($p_{\text{gauge}} = 2$ at) assuming its moisture content to be 5%; (b) the quantity of secondary steam tapped for heating the production equipment; and (c) the required surface area of the heat exchanger (heater) assuming the overall heat transfer coefficient in it to equal $K = 700$ W/m²·K.

5-14. Diphenyl (C_6H_5)₂ boils under atmospheric pressure at 255 °C. Calculate the specific heat of vaporization, and also the specific heat capacity of liquid diphenyl.

5-15. A 48% aqueous solution of sodium hydroxide boils under a pressure of 760 mm Hg at 140 °C, and under a pressure of $p_{\text{abs}} = 0.2$ at at 99 °C. Determine the specific heat of vaporization of the water from this solution at a pressure of 0.8 at, and also the specific heat capacity of the solution.

5-16. Determine the boiling point of bromobenzene under a pressure of $p_{\text{abs}} = 0.1$ at according to a linearity rule diagram and to the nomogram in Fig. A-14. Also determine the specific heat of vaporization of bromobenzene at this pressure.

5-17 Determine the saturated vapour pressure of benzaldehyde at 120 °C using a linearity rule diagram.

5-18 Using Babo's law and Table A-36, determine the boiling point of a 42.5% aqueous solution of ammonium nitrate at a pressure of $p_{\text{abs}} = 0.4$ at.

5-19. A vacuum evaporator (see Fig. 5-1) is supplied with 10 tons an hour of an 8% aqueous solution of ammonium nitrate at a temperature of 74 °C. The concentration of the evaporated solution is 42.5%. The pressure p_{abs} in the middle layer of the boiling solution is 0.4 at. The pressure of the heating steam is $p_{\text{gauge}} = 1$ at. The overall coefficient of heat transfer is 950 W/m²·K.

The heat losses are 3% of the sum $Q_h + Q_e$. Determine the required heating surface area of the evaporator.

5-20. Using the data of the preceding problem, determine the absolute pressure in a barometric condenser if the hydraulic depression $\Delta t_{h,r} = 1$ K and the hydrostatic depression $\Delta t_{h,e} = 6.7$ K.

5-21. An aqueous solution is evaporated from a mass concentration of 7 to 24% under atmospheric pressure at a rate of 2200 kg an hour. The solution is fed into the evaporator at 19 °C. The temperature depression is 3.5 K, the hydrostatic depression is 3.0 K, and the hydraulic depression is 1.0 K. The pressure of the heating steam is $p_{\text{gauge}} = 2$ at. The overall coefficient of heat transfer is 1100 W/m²·K. Determine the heat exchange surface area needed in the evaporator and the rate of flow of the heating steam assuming that the losses of heat to the surroundings form 5% of the sum $Q_h + Q_e$ and that the moisture content of the heating steam is 5%.

5-22. An evaporator operates under atmospheric pressure with the heating steam at $p_{\text{gauge}} = 1.2$ at. How will its capacity change if it operates under a vacuum of 0.7 at with the heating steam at $p_{\text{gauge}} = 0.6$ at? The hydrostatic effect for the middle layer is $\Delta p_{h,e} = 9.81 \times 10^3$ Pa; in both cases consider that the temperature depression is 4 K; the feed solution enters the evaporator at the boiling point. Consider that the overall heat transfer coefficient is constant. Disregard the heat losses.

5-23. An aqueous solution is concentrated from 14 to 30% (mass) in an evaporator. The heating steam has an absolute pressure of 0.9 at. The useful temperature drop is 11.2 K. Determine the rate of flow of the feed solution per hour if the heat exchange surface area is 40 m², and the overall heat transfer coefficient is 700 W/m²·K. The feed solution is heated to the boiling point. The mean pressure in the apparatus (absolute) is 0.4 at. Disregard the heat losses.

5-24. Determine the rate of flow of the heating steam ($p_{\text{abs}} = 2$ at) and the heating surface area of an evaporator used to evaporate 1.6 ton/h of solution from 10 to 40% (mass). The mean pressure in the evaporator (absolute) is 1 at. The initial solution is fed at 30 °C. The useful temperature drop is 12 K. The overall coefficient of heat transfer is 900 W/m²·K. Assume that the heat losses equal 5% of the usefully used quantity of heat $Q_h + Q_e$.

5-25. A potash solution is evaporated from 8 to 36% (mass) under a vacuum of 0.2 at. The initial solution flows in at a rate of 1500 kg/h. Determine the amount of water fed (a) into a barometric condenser, and (b) into a surface condenser assuming the temperature of the discharged condensate to be 5 °C lower than the condensation point. The water in both cases is heated from 15 to 35 °C.

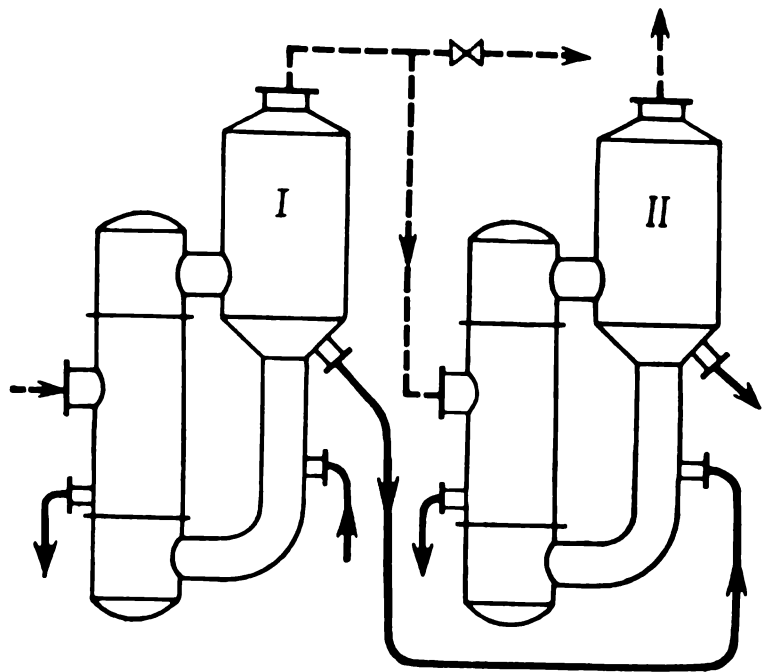
5-26. An evaporator concentrates an aqueous solution from 13 to 38% (mass) under a vacuum (in the condenser) of 600 mm Hg (see Fig. 5-1). The rate of flow of the cooling water in the barometric condenser is 40 m³/h, and the water is heated from 14 to 30 °C. Determine the hourly capacity of the evaporator with respect to the initial and the concentrated solution. Disregard the temperature depression. The atmospheric pressure is 747 mm Hg.

5-27. The vacuum in an evaporator is 0.7 at. The initial aqueous solution is fed at a rate of 2.4 ton/h, its concentration is 12% (mass). The final concentration is 32%. Cold water having a temperature of 12 °C is fed into the barometric condenser at a rate of 38.6 m³/h. Determine the temperature of the water at the outlet from the barometric condenser. Ignore the hydraulic resistance of the steam pipe and the temperature depression.

5-28. A triple-effect evaporator with forward feed (see Fig. 5-7) is used to evaporate 1300 kg/h of an aqueous solution having an initial mass concentration of 9% to a final concentration of 43%. Calculate the concentrations of the solutions by effects if it is known that 10% more water is evaporated in each following effect than in the preceding one.

5-29. What maximum number of effects can a multiple-effect evaporator have if the pressure of the heating steam in the first effect $p_{\text{gauge}} = 2.3$ at, and the residual pressure in the condenser is 147 mm Hg. Assume that the sum of the

Fig. 5-5. To Problem 5-33



temperature losses in all the effects is $\sum \Delta t_1 = 41$ K. The permissible useful temperature drop in each effect should be at least 8 K.

5-30. An aqueous solution having an initial concentration of 10% (mass) is evaporated at a rate of 1000 kg/h in a double-effect evaporator. The final concentration of the solution in the first effect is 15% and in the second 30%. The boiling points in the two effects are 108 and 95 °C, respectively. Determine the amount of water that evaporates in the second effect as a result of self-evaporation and what per cent it forms of the total amount of water evaporating in the second effect.

5-31. A double-effect evaporator with forward feed is supplied with 1000 kg/h of an aqueous solution having a concentration of 8% (mass). The concentration of the solution after the first effect is 12% (mass). Evaporation in the first effect is conducted at 105 °C under a pressure of $p_{\text{abs}} = 1$ at, in the second effect at 78 °C under a pressure of $p_{\text{abs}} = 0.3$ at. Determine the final concentration of the solution in the second effect if the secondary steam from the first effect is completely used for heating the second one (there is no tapping of extra steam). Disregard the heat losses.

5-32. The second effect of a double-effect evaporator with forward feed without the tapping of extra steam receives 500 kg/h of a 16% aqueous solution from the first effect. The boiling point in the first effect is 108 °C (the pressure is atmospheric) and in the second is 90 °C. The concentrated solution leaving the second effect with a mass concentration of 28% is used in a counterflow heat exchanger for heating the solution fed in for evaporation. Disregarding the heat losses and depression, determine (a) the concentration of the feed solution, and (b) by how many degrees should the feed be heated in the heat exchanger if the concentrated solution leaves it with a temperature of 32 °C. The specific heat capacity of the concentrated solution is 3.35×10^3 J/kg·K.

5-33. An aqueous solution is fed at a rate of 1000 kg/h into a double-effect evaporator with forward feed. The initial and final mass concentrations are 8 and 30%, respectively. In the first effect, evaporation proceeds under a pressure of $p_{\text{abs}} = 1$ at at 110 °C, and in the second under a pressure of 0.3 at at 80 °C. The secondary steam is used in the first effect at a rate of 400 kg/h. Part of this steam (Fig. 5-5) is tapped for use elsewhere (extra steam). Ignoring the heat losses, determine the amount of extra steam that is tapped.

5-34. An aqueous solution with a mass concentration of 5% is fed into a single-effect evaporator (Fig. 5-6) operating with a heat pump (compression of

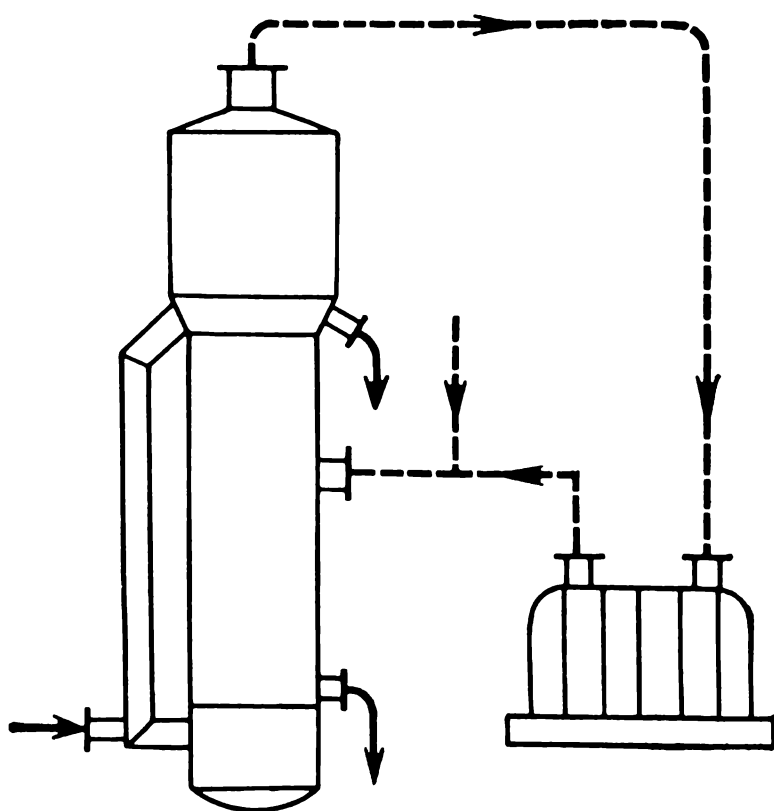


Fig. 5-6. To Problem 5-34

the secondary steam in a turbocompressor). A solution with a mass concentration of 15% leaves the evaporator at a rate of 550 kg/h. The temperature depression is 2.5 K. Disregard the hydrostatic effect and the hydraulic resistances. The turbocompressor compresses the secondary steam from 1 to 2 at. The heat losses are 5% of $Q_h + Q_e$. The initial temperature of the solution is 70 °C. Determine (a) how much heating steam has to be added (the steam is dry and saturated), and (b) what power is used by the turbocompressor if its total efficiency is 0.72.

5-35. To what temperature does a hot 40% aqueous solution of potassium nitrate have to be cooled for the concentration of the mother liquor to be half the initial one after it is cooled and the crystals freeze out?

5-36. How many kilograms of crystals freeze out when 4.2 tons of a soda solution containing 2.5 moles of soda per 1000 g of water are cooled from 30 to 15 °C? The soda crystallizes with 10 molecules of water per soda molecule.

5-37. Determine the required cooling surface area of a counterflow crystallizer used to cool 10 000 kg/h of a solution containing 7.0 moles of ammonium sulphate per 1000 g of water from 85 to 35 °C. During cooling, water evaporates (5% of the mass of the initial solution). The overall heat transfer coefficient is 127 W/m²·K. The cooling water is heated from 13 to 24 °C. Determine also its rate of flow.

EXAMPLE OF CALCULATING AND SELECTING A TRIPLE-EFFECT EVAPORATOR

Calculate and select a triple-effect evaporator with forward feed (Fig. 5-7) for concentrating 5 tons an hour of a 12% aqueous solution of sodium nitrate. The final mass concentration of the solution is 40%. The feed solution is heated to the boiling point which it will have in the evaporator. The pressure of the heating steam is $p_{\text{abs}} = 4$ at, and $t = 143$ °C. The vacuum in the barometric condenser is 0.8 at.

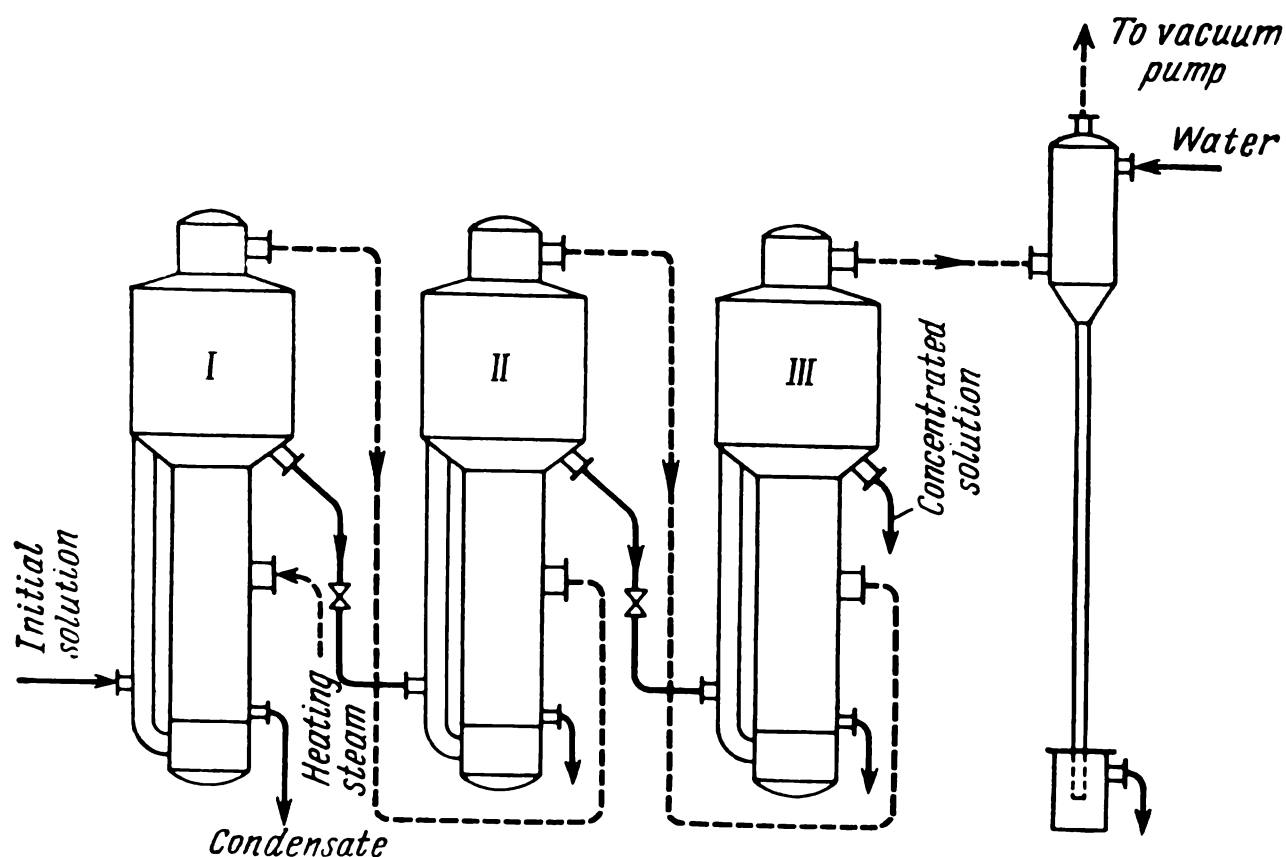


Fig. 5-7. Triple-effect evaporator

Solution *. 1. The amount of water evaporated in the three effects of the evaporator is:

$$W = \frac{5000}{3600} \left(1 - \frac{12}{40} \right) = 0.97 \text{ kg/s}$$

2. Distribution of the load by effects.

We distribute the load on the basis of practical data, adopting the following ratio of the mass quantities of evaporated water by effects: I:II:III = 1.0:1.1:1.2. Consequently, the amounts of evaporated water are:

$$\text{in effect I} \dots W_I = \frac{3500 \times 1}{3600 (1 + 1.1 + 1.2)} = 0.295 \text{ kg/s}$$

$$\text{in effect II} \dots W_{II} = \frac{3500 \times 1.1}{3600 \times 3.3} = 0.324 \text{ kg/s}$$

$$\text{in effect III} \dots W_{III} = \frac{3500 \times 1.2}{3600 \times 3.3} = 0.351 \text{ kg/s}$$

$$\text{Total} \dots W = 0.97 \text{ kg/s}$$

3. Calculation of the solution concentrations by effects.

The initial concentration of the solution is $\bar{x}_{in} = 12\%$. The rate of flow of the solution from effect I to effect II is:

$$G_1 = G_{in} - W_I = \frac{5000}{3600} - 0.295 = 1.39 - 0.295 = 1.09 \text{ kg/s}$$

* Several methods of calculating multi-effect evaporators have been described in publications on the subject. All of them include assumptions that diminish the accuracy of the final result. One of such simplified methods of calculation is used in the present example.

The concentration of the solution leaving effect I and flowing into effect II will be:

$$\bar{x}_1 = \frac{G_{in}\bar{x}_{in}}{G_{in}-W_1} = \frac{1.39 \times 12}{1.39 - 0.295} = 15.2\%$$

The rate of flow of the solution from effect II to III is:

$$G_2 = G_{in} - W_1 - W_{II} = 1.39 - 0.295 - 0.324 = 0.77 \text{ kg/s}$$

and the concentration of the solution is:

$$\bar{x}_2 = \frac{1.39 \times 12}{0.77} = 21.6\%$$

The rate of flow of the solution from effect III is:

$$G_{fin} = G_{in} - W = 1.39 - 0.97 = 0.42 \text{ kg/s}$$

and the concentration of the solution is:

$$\bar{x}_{fin} = \frac{1.39 \times 12}{0.42} = 40\%$$

which corresponds to the assignment.

4. Distribution of the pressure drop by effects.

The difference between the pressure of the heating steam (in the first effect) and the pressure of the steam in the barometric condenser is:

$$\Delta p = 4.0 - 0.2 = 3.8 \text{ at}$$

We preliminarily distribute this pressure drop between the effects uniformly, i. e. for each effect we assume that

$$\Delta p = \frac{3.8}{3} = 1.27 \text{ at}$$

Therefore, the absolute pressures by effects will be

in effect III.....

$p_3 = 0.2 \text{ at (given)}$

in effect II.....

$p_2 = 0.2 + 1.27 = 1.47 \text{ at}$

in effect I.....

$p_1 = 1.47 + 1.27 = 2.74 \text{ at}$

The pressure of the heating steam is:

$$p = 2.74 + 1.27 = 4 \text{ at}$$

We use tables of steam properties to find the temperatures of saturated steam and the specific heats of vaporization for the adopted pressures in the effects:

	Temperature of saturated steam, °C	Specific heat o. vaporization, kJ/kg
In effect I	130	2179
In effect II	110	2234
In effect III	60	2357
Heating steam (from boiler) . . .	143	2241

It is exactly these temperatures that will be the condensation points of the secondary steam by effects.

5. Calculation of the temperature losses by effects.

Due to Depression. We find the boiling points of the solutions at atmospheric pressure in reference tables (for example, in Table A-36):

	Concentration of NaNO_3 , %	Boiling point, $^{\circ}\text{C}$	Depres- sion, K
In effect I	15.2	102	2.0
In effect II	21.6	103	3.0
In effect III	40.0	107	7.0

To simplify our calculations, we do not determine the temperature depression more precisely (in connection with the pressure in the effects differing from atmospheric).

Consequently, for the three effects, we have:

$$\Delta t_{\text{depr}} = 2 + 3 + 7 = 12 \text{ K}$$

Due to the Hydrostatic Effect. We take the distance from the top level of the liquid in the evaporator to the middle of the heating surface around which the liquid flows (h_{mid}) equal to 0.4 m.

We use reference tables to find the densities of the solutions at 100°C :

	Concentration of NaNO_3 , %	Density, kg/m^3
In effect I	15.2	1056
In effect II	21.6	1102
In effect III	40.0	1255

Hence, the increase in the hydrostatic pressure in the first effect is:

$$\Delta p = 0.4 \times 1056 \times 9.81 = 4144 \text{ Pa} = 0.04 \text{ at}$$

i. e. the pressure in the middle layer is:

$$p_{\text{mid}} = p_1 + \Delta p = 2.74 + 0.04 = 2.78 \text{ at}$$

The boiling point of water at the pressure p_1 is 130.93°C , and at p_{mid} is 131.52°C . Hence,

$$\Delta t_{\text{h.e}} = t_{\text{mid}} - t_1 = 131.52 - 130.93 = 0.59 \text{ K}$$

Similar calculations give the temperature losses due to the hydrostatic effect in the second and third effects of the evaporator equal to 0.96 and 5.32°C , respectively. The sum of these losses for all three effects is:

$$\Delta t_{\text{h.e}} = 0.59 + 0.96 + 5.32 = 6.87 \text{ K}$$

Due to Hydraulic Resistances. We assume that the loss of the temperature drop in each interval between the effects is 1 K. There are altogether three intervals (I-II, II-III, III-condenser), consequently,

$$\Delta t_{\text{h.r}} = 1 \times 3 = 3 \text{ K}$$

The sum of all the temperature losses for the evaporator as a whole is:

$$\sum \Delta t_1 = 12 + 6.87 + 3 = 21.87 \text{ K}$$

6. Effective temperature drop.

The total temperature drop is $143 - 60 = 83 \text{ K}$, hence the effective temperature drop is:

$$\Delta t_{\text{eff}} = 83 - 21.87 = 61.13 \text{ K}$$

7. Determination of the boiling points of the solutions in the effects.

We use the equation

$$t_b = t_{\text{sec. st}} + \Delta t_{\text{depr}} + \Delta t_{\text{h. e}} + \Delta t_{\text{h. r}}$$

Hence,

$$\text{in effect III} \quad . . . \quad t_3 = 60 + 7 + 5.32 + 1 \approx 73^\circ\text{C} \quad (\sum \Delta t_1 = 13.32)$$

$$\text{in effect II} \quad . . . \quad t_2 = 110 + 3 + 0.96 + 1 \approx 115^\circ\text{C} \quad (\sum \Delta t_1 = 4.96)$$

$$\text{in effect I} \quad . . . \quad t_1 = 130 + 2 + 0.59 + 1 \approx 134^\circ\text{C} \quad (\sum \Delta t_1 = 3.59)$$

8. Calculation of the overall heat transfer coefficients by effects.

We use the found boiling points and concentrations of the solutions in the effects to select the physical constants of the solutions needed for calculations in reference tables (the density, thermal conductivity, heat capacity, viscosity). Next we set the diameter of the tubes and their length (depending on the type of evaporator).

According to these data, we calculate the coefficients of heat transfer for the condensing steam and boiling solution, and the overall coefficients of heat transfer (examples of such calculations are given in Chap. 4). Here account must be taken of a layer of scale of the order of magnitude of 0.5 mm.

On the basis of such preliminary calculations, we adopt:

$$\text{For effect I} \quad . . . \quad K_1 = 1700 \text{ W/m}^2 \cdot \text{K}$$

$$\text{For effect II} \quad . . . \quad K_2 = 990 \text{ W/m}^2 \cdot \text{K}$$

$$\text{For effect III} \quad . . . \quad K_3 = 580 \text{ W/m}^2 \cdot \text{K}$$

The approximate ratio of the overall heat transfer coefficients by effects when evaporating aqueous solutions of salts is $K_1:K_2:K_3 = 1:0.58:0.34$.

9. Drawing up of the heat balances by effects (without account taken of the heat losses).

According to the initial conditions, the feed solution is heated to the boiling point in the first effect.

Hence the consumption of heat in the first effect is:

$$Q_I = W_I L_1 = 0.295 \times 2179 \times 10^3 = 643\,000 \text{ W}$$

The solution is fed to the second effect in the superheated state, therefore Q_h is negative (the heat of self-evaporation), and the consumption of heat in the second effect is:

$$\begin{aligned} Q_{II} &= W_{II} L_2 - G_1 c_1 (t_1 - t_2) = 0.324 \times 2234 \times 10^3 - 1.09 \times 3.69 \times 10^3 (134 - 115) = \\ &= 647\,000 \text{ W} \end{aligned}$$

The quantity of heat liberated by the secondary steam of the first effect in condensation is $W_I L_1 = 643\,000 \text{ W}$. The difference between the input and consumption of heat in the heat balance of the second effect is less than 1%.

The consumption of heat in the third effect is:

$$\begin{aligned} Q_{III} &= W_{III} L_3 - G_2 c_2 (t_2 - t_3) = 0.351 \times 2357 \times 10^3 - 0.77 \times 3.43 \times 10^3 (115 - 73) = \\ &= 722\,000 \text{ W} \end{aligned}$$

The heat liberated by the secondary steam of the second effect in condensation (the heat supplied to the third effect) is:

$$W_{II} L_2 = 0.324 \times 2234 \times 10^3 = 724\,000 \text{ W}$$

10. The consumption of the heating steam in the first effect is:

$$G_{st} = \frac{643\,000}{2141 \times 10^3} = 0.3 \text{ kg/s}$$

The specific consumption of steam is

$$g = \frac{G_{st}}{W} = \frac{0.3}{0.97} = 0.31 \text{ kg/kg}$$

11. Distribution of the effective temperature drop by effects.

We distribute the effective temperature drop among the effects in two variants: using the condition of the minimum total surface area and the condition of an equal surface area of the effects, i.e. proportional to $(Q/K)^{1/2}$ and proportional to Q/K .

We find the proportionality factors:

	Ratio Q/K	$V(Q/K) \times 10^3$
Effect I	$\frac{643\,000}{1700} = 376$	612
Effect II	$\frac{647\,000}{990} = 656$	809
Effect III	$\frac{722\,000}{580} = 1242$	1114
	$\sum Q/K = 2274$	$\sum V(Q/K) \times 10^3 = 2535$

The effective temperature drops by effects are

Variant of minimum total surface area	Variant of equal surface area of the effects
$\Delta t_1 = \frac{61.13 \times 612}{2535} = 14.8$	$\Delta t_1 = \frac{61.13 \times 376}{2274} = 10.1$
$\Delta t_2 = \frac{61.13 \times 809}{2535} = 19.5$	$\Delta t_2 = \frac{61.13 \times 656}{2274} = 17.6$
$\Delta t_3 = \frac{61.13 \times 1114}{2535} = 26.8$	$\Delta t_3 = \frac{61.13 \times 1242}{2274} = 33.4$
Check: $\sum \Delta t = 61.1 \text{ K}$	Check: $\sum \Delta t = 61.1 \text{ K}$

12. Determination of the heating surface area.

Variant of minimum total surface area	Variant of equal surface area of the effects
$A_1 = \frac{643\,000}{1700 \times 14.8} = 25.3$	$A_1 = \frac{643\,000}{1700 \times 10.1} = 37.2$
$A_2 = \frac{647\,000}{990 \times 19.5} = 33.6$	$A_2 = \frac{647\,000}{990 \times 17.6} = 37.2$
$A_3 = \frac{722\,000}{580 \times 26.8} = 46.5$	$A_3 = \frac{722\,000}{580 \times 33.4} = 37.2$
$\sum A = 105.4 \text{ m}^2$	$\sum A = 111.6 \text{ m}^2$

Consequently, when the effects have equal surface area, the total heating surface area is greater by only 6%.

We therefore adopt the second variant with equal surface area of the effects, this ensuring the use of equipment of a single type.

We check the temperature of the secondary steam and the pressure by effects:

Effect	Boiling point, °C $t_b = t_{st} - \Delta t_{eff}$	Condensation point of secondary steam, °C $t_0 = t_b - \sum \Delta t_i$	Pressure p_{abs} , at
I	143.0 — 10.1 = 132.9	132.9 — 3.59 = 129.3	2.7
II	129.3 — 17.6 = 111.7	111.7 — 4.96 = 106.7	1.31
III	106.7 — 33.4 = 73.3	73.3 — 13.32 = 60	0.2

After this it is necessary, on the basis of the found areas of the effect surfaces, to calculate the evaporator more precisely, taking into account the losses of heat to the surroundings and the somewhat changed distribution of the temperatures and pressures by effects.

SYMBOLS

A area
C atomic heat capacity; constant
c solubility; specific heat capacity
d diameter
G mass, mass rate of flow
g acceleration due to gravity; specific rate of flow
H enthalpy
h height
K constant; overall heat transfer coefficient
L heat of solution; heat of vaporization; length
M molar mass
n number of atoms
P power
p pressure
Q amount of heat
R gas constant
T absolute temperature
t temperature, °C
V volume; volumetric rate of flow
v velocity
W amount of solvent or water; mass rate of flow of evaporated water; work
x steam content
 \bar{x} mass fraction

Greek Letters

α individual coefficient of heat transfer
 ζ dimensionless local resistance coefficient
 θ boiling point
 λ dimensionless pipe resistance coefficient
 τ time

MASS TRANSFER.
ABSORPTION

FUNDAMENTAL RELATIONSHIPS
AND FORMULAS FOR CALCULATIONS

1. The ways of expressing the composition of the phases of two-component liquid-gas (vapour) systems are set out in Table 6-1.

TABLE 6-1

Concentration	Symbol for concentration of component A	
	in liquid phase	in gaseous or vapour phase
Mole fraction, $\frac{\text{kmol of A}}{\text{kmol of A + B}}$	x	y
Mass fraction, $\frac{\text{kg of A}}{\text{kg of A + B}}$	\bar{x}	\bar{y}
Mole ratio, $\frac{\text{kmol of A}}{\text{kmol of B}}$	X	Y
Mass ratio, $\frac{\text{kg of A}}{\text{kg of B}}$	\bar{X}	\bar{Y}
Molar-volume concentration, $\frac{\text{kmol of A}}{\text{m}^3 \text{ of A + B}}$	c_x	c_y
Mass-volume concentration, $\frac{\text{kg of A}}{\text{m}^3 \text{ of A + B}}$	\bar{c}_x	\bar{c}_y

Formulas for conversion of the concentrations (in the liquid phase) are given in Table 6-2. The same relationships hold for the gaseous phase, but with y , Y and c_y used instead of x , X , and c_x , respectively.

2. The concentration of a component in the gaseous phase can also be expressed through its partial pressure. On the basis of Clapeyron's and Dalton's equations, the mole (volume) fraction y of any component i of a mixture of ideal gases is:

$$y = \frac{p_i}{p}$$

(6-1)

TABLE 6-2

Symbol for concentration of component A	x	\bar{x}	X	\bar{X}	c_x	\bar{c}_x
x	—	$\frac{\bar{x}M_{\text{mix}}}{M_A}$ or $\frac{\bar{x}/M_A}{\frac{\bar{x}}{M_A} + \frac{1-\bar{x}}{M_B}}$	$\frac{X}{1+X}$	$\frac{M_B\bar{X}}{M_B\bar{X} + M_A}$	$\frac{c_x M_{\text{mix}}}{\rho}$ or $\frac{c_x M_B}{\rho + c_x (M_B - M_A)}$	$\frac{\bar{c}_x M_{\text{mix}}}{\rho M_A}$ or $\frac{\bar{c}_x M_B}{\rho M_A + \bar{c}_x (M_B - M_A)}$
\bar{x}	$\frac{M_A x}{M_{\text{mix}}}$	—	$\frac{M_A X}{M_A X + M_B}$	$\frac{\bar{X}}{1 + \bar{X}}$	$\frac{M_{ACx}}{\rho}$	$\frac{\bar{c}_x}{\rho}$
X	$\frac{x}{1-x}$	$\frac{\bar{x}M_B}{M_A(1-\bar{x})}$	—	$\frac{M_B\bar{X}}{M_A}$	$\frac{M_{BCx}}{\rho - M_{ACx}}$	$\frac{M_{BCx}}{M_A(\rho - \bar{c}_x)}$
\bar{X}	$\frac{M_A x}{M_B(1-x)}$	$\frac{\bar{x}}{1-\bar{x}}$	$\frac{M_A X}{M_B}$	—	$\frac{M_{ACx}}{\rho - M_{ACx}}$	$\frac{\bar{c}_x}{\rho - \bar{c}_x}$
c_x	$\frac{\rho x}{M_{\text{mix}}}$	$\frac{\rho \bar{x}}{M_A}$	$\frac{\rho X}{M_A X + M_B}$	$\frac{\rho \bar{X}}{M_A(\bar{X} + 1)}$	—	$\frac{\bar{c}_x}{M_A}$
\bar{c}_x	$\frac{\rho M_A x}{M_{\text{mix}}}$	$\rho \bar{x}$	$\frac{M_A \rho X}{M_A X + M_B}$	$\frac{\rho \bar{X}}{\bar{X} + 1}$	M_{ACx}	—

M_A , M_B and M_{mix} = molar masses of the components in the mixture, kg/kmol
 $M_{\text{mix}} = M_A x + M_B (1 - x)$

ρ = density of the mixture, kg/m³.

For a mixture of ideal gases, we have:

$$\frac{\rho}{M_{\text{mix}}} = \frac{p}{RT} = \frac{pT_0}{22.4p_0T}$$

where p_i = partial pressure of the component i of the gas mixture
 $p = \sum p_i$ = total pressure of the mixture of gases or vapours equal to the sum of the partial pressures of all the components.

3. Laws of interphase equilibrium for ideal solutions.

(a) Henry's law:

$$p^* = kx \quad (6-2)$$

where p^* = partial pressure of a component in the gaseous phase over the liquid in equilibrium with the gas

x = mole fraction of the component in the liquid

k = Henry's coefficient (also called the Henry law constant) depending on the temperature and on the nature of the gas and the liquid.

The values of k for aqueous solutions of selected gases are given in Table A-41.

Using the value of $p^* = y^*p$ obtained according to Eq. (6-1) in Eq. (6-2), we get:

$$y^* = mx \quad (6-3)$$

where y^* = mole fraction of the component in the gaseous phase in equilibrium with the liquid

$m = k/p$ = dimensionless coefficient (distribution coefficient) constant for a given gas-liquid system at $t = \text{const}$ and $p = \text{const}$.

If for a two-phase three-component system (gas phase A + B, liquid phase A + C), we insert into equilibrium equation (6-3) the values of x and y^* for the component A expressed through its mole or mass ratios X and Y^* , \bar{X} and \bar{Y}^* (Table 6-2), we get

$$\frac{Y^*}{1 + Y^*} = m \frac{X}{1 + X} \quad (6-4)$$

$$\frac{\bar{Y}^*}{\bar{Y}^* + \frac{M_A}{M_B}} = m \frac{\bar{X}}{\bar{X} + \frac{M_A}{M_C}} \quad (6-5)$$

where M_A = molar mass of component A distributed between the phases

M_B = molar mass of the second component of the binary gas mixture

M_C = molar mass of the second component of the binary liquid mixture.

At low concentrations of the distributed component in the gas and the liquid, when $Y^* \ll 1$ and $X \ll 1$, the quantities Y^* and X may be disregarded in the denominators of Eq. (6-4):

$$Y^* = mX \quad (6-6)$$

Similarly, when $\bar{Y}^* \ll M_A/M_B$ and $\bar{X} \ll M_A/M_C$, from Eq. (6-5) we get:

$$\bar{Y}^* = m \frac{M_G}{M_B} \bar{X} \quad (6-7)$$

(b) Raoult's law:

$$p^* = p^\circ x \quad (6-8)$$

where p^* = partial pressure of a component in a vapour-gas mixture over the liquid in conditions of equilibrium

p° = saturated vapour pressure of the pure component—a single-valued function of the temperature: $p^\circ = f(T)$

x = mole fraction of the component in the liquid.

When substituting for p^* in Eq. (6-8) its value $p^* = y^*p$ from Eq. (6-1), we get:

$$y^* = \frac{p^\circ}{p} x \quad (6-9)$$

where y^* is the mole fraction of the component in the gaseous phase in equilibrium with the liquid.

For a two-component mixture when both of its components observe Raoult's law, Eq. (6-9) is reduced to the form

$$y_A^* = \frac{\alpha x_A}{1 + (\alpha - 1) x_A} \quad (6-10)$$

where $\alpha = p_A^\circ/p_B^\circ$ = coefficient of relative volatility

p_A° = saturated vapour pressure of the more volatile component (low boiler)

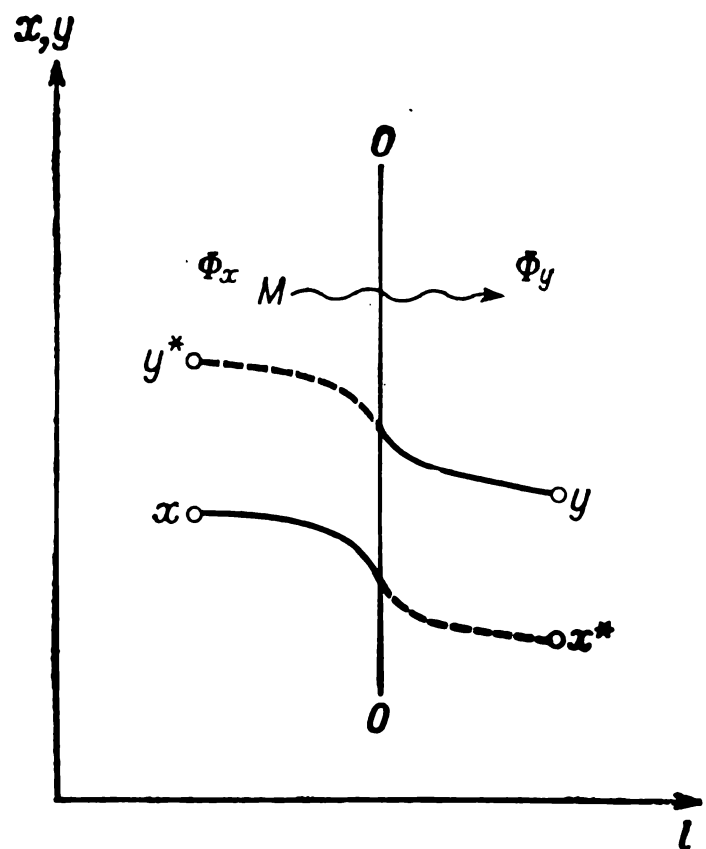
p_B° = saturated vapour pressure of the less volatile component (high boiler) at the same temperature.

4. Numerous experimental data on the equilibrium compositions of the liquid and vapour for various solutions are contained in the reference book [6-4]. Data on the equilibrium compositions of the liquid and vapour at $p = \text{const}$ for selected binary mixtures are given in Table A-47 and Figs. A-21 and A-22*.

5. The cause (driving force) of a mass transfer process—the transition of a component from one phase to another (for instance, from the liquid phase Φ_x having a molar concentration of the transferring component of x to the gaseous phase Φ_y having a molar concentration of y) is the lack of equilibrium between the phases in contact, their deviation from the state of dynamic equilibrium. It is established in chemical thermodynamics that

* Useful reference data can also be found in the following publications: Stull, D. R. "Vapour Pressure of Pure Substances", *Industrial and Engineering Chemistry*, Vol. 39, No. 4, pp. 517-550 (1947), and Horsley, L. H. *Azeotropic Data*, Am. Chem. Soc., 1952.

Fig. 6-1. Diagram of mass transfer process:
 OO — phase interface



the magnitude of this deviation with equal temperatures and pressures of the phases is determined by the difference between the chemical potentials of the transferring component ($\mu_x - \mu_y$). It is precisely this difference that is the driving force of the process of mass transfer. When the phases are in equilibrium, the potentials μ_x and μ_y are equal.

Substituting for the potential μ_x the potential of the equilibrium gaseous phase μ_y^* equal to it, we get the following expression for the driving force of a mass transfer process:

$$\mu_x - \mu_y = \mu_y^* - \mu_y \quad (6-11)$$

Similarly, substituting for the potential μ_y the potential of the equilibrium liquid phase μ_x^* equal to it, we get:

$$\mu_x - \mu_y = \mu_x - \mu_x^* \quad (6-12)$$

A glance at Eqs. (6-11) and (6-12) shows that the driving force of a process—the difference between the chemical potentials ($\mu_x - \mu_y$)—can be expressed in two ways: taken with respect to either the gaseous phase or the liquid phase. It must be remembered here that the gaseous phase with the potential μ_y^* and the concentration y^* and also the liquid phase with the potential μ_x^* and the concentration x^* are hypothetic phases; in a real process of mass transfer they are absent—see Fig. 6-1.

In practical calculations, the concentrations are used instead of the chemical potentials. The concentrations are simpler quantities and easy to determine. They can also be used to characterize the deviation of the phases Φ_x and Φ_y from the state of

equilibrium. Unlike the difference between the chemical potentials ($\mu_x - \mu_y$), however, which equals zero when the phases are in equilibrium, the difference between the concentrations ($x - y$), which in the general case does not equal zero upon equilibrium*, cannot be a measure of the deviation of the phases from the equilibrium state, i. e. cannot be the driving force of a mass transfer process. In practical calculations, the driving force of a mass transfer process—the deviation of the system from the state of equilibrium—is therefore expressed as the difference between the concentrations ($y^* - y$) or ($x - x^*$). Since the concentrations can be expressed in different units, the driving force of a mass transfer process can have different values—see Example 6-3.

6. In accordance with the two possible ways of expressing the driving force of a mass transfer process—with respect to the gaseous phase ($\Delta y = y^* - y$) or to the liquid phase ($\Delta x = x - x^*$), the equation of mass transfer, similar to that of heat transfer, can be written in two forms:

$$M = K_y \Delta y_m A \quad (6-13)$$

or

$$M = K_x \Delta x_m A \quad (6-14)$$

where M = rate of flow of the component passing from one phase to the other, kmol/s
 A = area of mass transfer surface, m²
 K_y = overall mass transfer coefficient related to the driving force Δy expressed through the mole fractions of the component in the gaseous phase, kmol/m²·s [the unit is sometimes written kmol/m²·s ($\Delta y = 1$) or kmol/m²·s·kmol·kmol⁻¹]
 K_x = overall mass transfer coefficient related to the driving force Δx expressed through the mole fractions of the component in the liquid phase, kmol/m²·s
 Δy_m and Δx_m = mean driving forces of the entire process, respectively (see below).

Equations (6-13) and (6-14) may contain the mass fractions and mass rates of flow instead of the relevant mole quantities, and ΔY_m , Δc_{y_m} and Δp_m instead of Δy_m . Accordingly, ΔX_m or Δc_{x_m} may be used instead of Δx_m .

7. With definite assumptions (the absence of diffusion resistance when a component passes through the phase interface, the existence of equilibrium at this interface, a linear nature of the equation of equilibrium $y^* = mx$ or $y^* = mx + b$), we get the fol-

* See, for example, the data on equilibrium concentrations in Table A-47.

lowing relationships between the overall mass transfer coefficients K_y and K_x and the phase coefficients of mass transfer β_y and β_x :

$$K_y = \frac{1}{\frac{1}{\beta_y} + \frac{m}{\beta_x}} \quad (6-15)$$

$$K_x = \frac{1}{\frac{1}{m\beta_y} + \frac{1}{\beta_x}} \quad (6-16)$$

where m is the slope of the equilibrium line. The individual and overall mass transfer coefficients are expressed in $\text{kmol/m}^2 \cdot \text{s}$.

The denominators in Eqs. (6-15) and (6-16) are the total diffusion resistance equal to the sum of the diffusion resistances of the gaseous and the liquid phases.

When the main diffusion resistance is concentrated in the gaseous phase, i. e. when $m/\beta_x \ll 1/\beta_y$, we have

$$K_y \approx \beta_y \quad (6-17)$$

When the main diffusion resistance is concentrated in the liquid phase, i. e. when $1/m\beta_y \ll 1/\beta_x$, we have

$$K_x \approx \beta_x \quad (6-18)$$

It follows from Eqs. (6-15) and (6-16) that

$$K_y = \frac{K_x}{m} \quad (6-19)$$

8. Principal dimensionless numbers of steady-state mass transfer processes.

The Nusselt number for mass transfer is:

$$Nu' = \frac{\beta l}{D} \quad (6-20)$$

The Peclet number for mass transfer is:

$$Pe' = \frac{vl}{D} \quad (6-21)$$

The Prandtl number for mass transfer is:

$$Pr' = \frac{Pe'}{Re} = \frac{\nu}{D} \quad (6-22)$$

Here β = mass transfer coefficient, m/s ($\text{kmol/m}^2 \cdot \text{s} \cdot \text{kmol} \cdot \text{m}^{-3}$ or $\text{kg/m}^2 \cdot \text{s} \cdot \text{kg} \cdot \text{m}^{-3}$)

l = characteristic linear dimension, m

D = coefficient of molecular diffusion, m^2/s

v = velocity of gas or liquid, m/s

ν = kinematic viscosity, m^2/s .

9. In the absence of experimental data, the coefficient of diffusion (molecular) of gas A in gas B (or of gas B in gas A) can be calculated by the formula:

$$D_g = \frac{4.3 \times 10^{-7} T^{3/2}}{p (V_A^{1/3} + V_B^{1/3})^2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}$$

(6-23)

where

D_g = diffusion coefficient, m²/s

T = temperature, K

p = pressure (absolute), at

M_A and M_B = molar masses of gases A and B

V_A and V_B = molar volumes of gases A and B determined as the sum of the atomic volumes of the elements which the gas consists of—see Example 6-6 (the atomic volumes of selected elements and the molar volumes of selected gases are given in Table 6-3).

TABLE 6-3

Atomic volume, cm ³ /atom		Molar volume, cm ³ /mol	
B	27.0	Air	29.9
C	14.8	Br ₂	53.2
Cl	24.6	Cl ₂	48.4
H	3.7	CO	30.7
I	37.0	CO ₂	34.0
N in primary amines	10.5	COS	51.5
N in secondary amines	12.0	H ₂	14.3
N with two saturated bonds	15.6	H ₂ O	18.9
O in acids	12.0	H ₂ S	32.9
O in aldehydes and ketones	7.4	I ₂	71.5
O in compounds with S, P, N	8.3	N ₂	31.2
O in esters	9.1	NH ₃	25.8
O in ethers	9.9	NO	23.6
O in higher ethers and esters	11.0	N ₂ O	36.4
O with two saturated bonds	7.4	O ₂	25.6
S	25.6	SO ₂	44.8

Structural Constants*

Benzene ring	—15
Naphthalene ring	—30
Anthracene ring	—47.5

* In calculating the molar volume of a chemical compound, the value of the relevant structural constant must be added to the sum of the atomic constants.

As follows from Eq. (6-23), if we know the value of the diffusion coefficient D_1 at the temperature T_1 and the pressure p_1 , then its value D_2 at the temperature T_2 and the pressure p_2 can

be found from the equation:

$$D_2 = D_1 \frac{p_1}{p_2} \left(\frac{T_2}{T_1} \right)^{3/2} \quad (6-24)$$

The values of the diffusion coefficients in air for selected gases and vapours are given in Table A-42.

10. The coefficient of diffusion in a liquid D_{lq} at 20 °C can be calculated by the approximate formula:

$$D_{lq} = \frac{1 \times 10^{-6}}{AB\mu^{1/2} (V_A^{1/3} + V_B^{1/3})^2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}} \quad (6-25)$$

where D_{lq} = diffusion coefficient, m^2/s

μ = dynamic viscosity of the liquid, $mPa \cdot s$

V_A and V_B = molar volumes of the solute and the solvent

M_A and M_B = molar masses of the solute and the solvent

A and B = coefficients depending on the properties of the solute and the solvent.

The values of the coefficients A for selected substances dissolved in water are:

gases	1
methyl alcohol	1.19
ethyl alcohol	1.24
acetic acid	1.27

The values of the coefficients B are:

water	4.7
ethyl alcohol	2.0
methyl alcohol	2.0
acetone	1.15
non-associated liquids	1.0

The coefficient of diffusion of a gas in a liquid D_t (at the temperature t) is related to the coefficient of diffusion D_{20} (at the temperature of 20 °C) by the following approximate expression:

$$D_t = D_{20} [1 + b(t - 20)] \quad (6-26)$$

in which the temperature coefficient b can be determined by the empirical formula:

$$b = 0.2\mu^{1/2}\rho^{-1/3} \quad (6-27)$$

where μ = dynamic viscosity of the liquid at 20 °C, $mPa \cdot s$

ρ = density of the liquid, kg/m^3 .

The values of the diffusion coefficients for selected gases in water are given in Table A-43.

The coefficient of diffusion in dilute solutions can also be calculated by the formula:

$$D_{lq} = 7.4 \times 10^{-12} \frac{(CM)^{1/2} T}{\mu V^{0.6}} \quad (6-28)$$

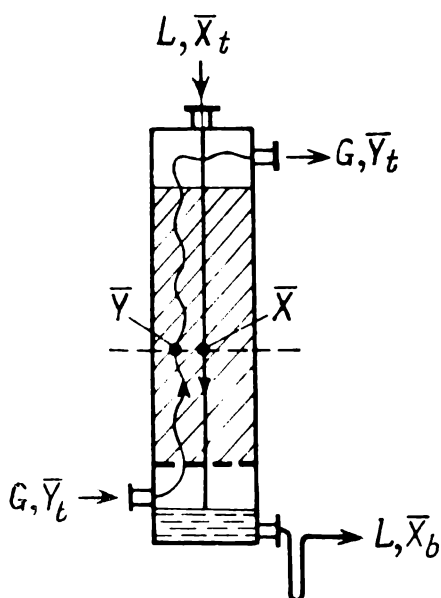


Fig. 6-2. Diagram of absorber

where D_{lq} = diffusion coefficient, m^2/s
 M = molar mass of the solvent
 V = molar volume of the diffusing substance
 T = temperature, K
 μ = dynamic viscosity of the solvent, $mPa \cdot s$
 C = coefficient taking into account the association of the solvent molecules.

The values of the coefficient C for selected substances are as follows:

water	2.6
methyl alcohol	1.9
ethyl alcohol	1.5
benzene, ether, heptane	1

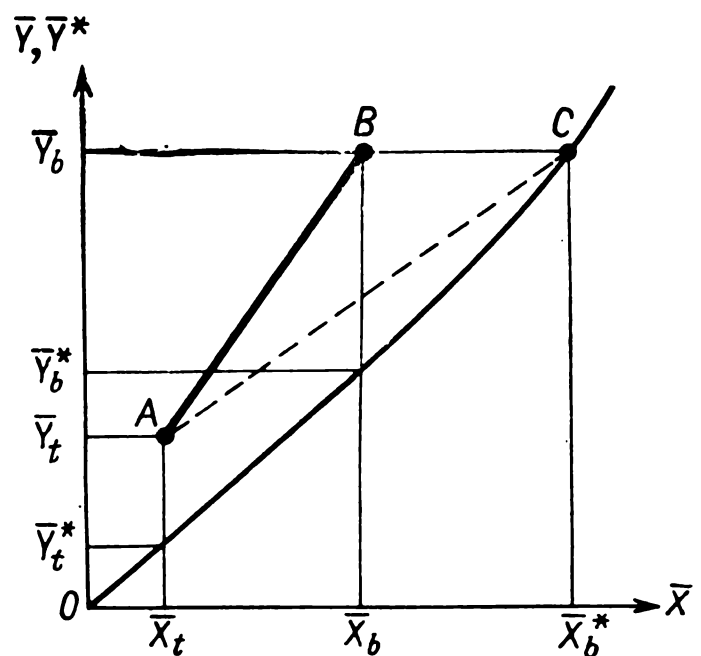
11. Material balance of an absorber (Fig. 6-2). When calculating an absorber with a non-volatile liquid absorbent, the mass (or molar) rates of flow of the absorber and the inert gas insoluble in the liquid will be constant along the height of the absorber. Expressing the concentration of the component being absorbed in the gas and liquid as a mass (or mole) ratio, we get an equation of the material balance (in mass units):

$M = G(\overline{Y}_b - \overline{Y}_t) = L(\overline{X}_b - \overline{X}_t)$ (6-29)

where M = rate of flow of the component being absorbed, kg/s
 G and L = rates of flow of the inert gas and the liquid absorbent, respectively, kg/s
 \overline{Y}_b and \overline{Y}_t = concentrations of the component being absorbed in the gas at the bottom and at the top of the absorber, respectively, kg/kg of inert gas
 \overline{X}_b and \overline{X}_t = ditto in the liquid at the bottom and at the top of the absorber, respectively, kg/kg of liquid absorbent.

In the coordinates \overline{Y} versus \overline{X} , Eq. (6-29), with G and L being constant, expresses a section of the straight line passing through points $B[\overline{X}_b, \overline{Y}_b]$ and $A[\overline{X}_t, \overline{Y}_t]$ —see Fig. 6-3.

Fig. 6-3. Operating and equilibrium lines of an absorber:
 AB —operating line for rate of flow of absorber L ; AC —ditto, L_{\min} ; OC —equilibrium line $\bar{Y}^* = f(\bar{X})$



The equation of this line—the operating line, is:

$$\bar{Y} = \bar{Y}_b + \frac{L}{G} (\bar{X} - \bar{X}_b) \quad (6-30)$$

or

$$\bar{Y} = \bar{Y}_t + \frac{L}{G} (\bar{X} - \bar{X}_t) \quad (6-31)$$

where \bar{Y} and \bar{X} are the concentrations of the component being absorbed in the contacting non-equilibrium streams of the gas and the liquid in the given cross section of the absorber—Fig. 6-2. Their values vary along the height of the column.

The rate of flow of the liquid absorbent is:

$$L = \varphi L_{\min} \quad (6-32)$$

where $\varphi > 1$ = excess absorbent coefficient

L_{\min} = theoretically minimum rate of flow of the absorbent, determined graphically—see Fig. 6-3—or analytically by the equation:

$$L_{\min} = \frac{M}{\bar{X}_b^* - \bar{X}_t} \quad (6-33)$$

By the absorption (or extraction) factor is meant the quantity:

$$e_a = \frac{\bar{Y}_b - \bar{Y}_t}{\bar{Y}_b} = \frac{Y_b - Y_t}{Y_b} \quad (6-34)$$

12. Mean driving force in an absorber with continuous contact of the phases. The general equation of mass transfer is:

$$A = \frac{M}{K_y \Delta \bar{Y}_m} \quad (6-35)$$

where A = surface of mass transfer in the absorber, m^2

M = rate of flow of the component being absorbed, kg/s

K_y = overall coefficient of mass transfer, $kg/m^2 \cdot s (kg/kg \text{ of inert gas})$

$\Delta \bar{Y}_m$ = mean driving force.

To solve Eq. (6-35) the value of $\Delta \bar{Y}_m$ is determined as follows.

If within the limits from \bar{X}_t to \bar{X}_b (Fig. 6-3), the equilibrium line is straight, then

$$\Delta \bar{Y}_m = \frac{\Delta \bar{Y}_b - \Delta \bar{Y}_t}{2.3 \log \frac{\Delta \bar{Y}_b}{\Delta \bar{Y}_t}} \quad (6-36)$$

where $\Delta \bar{Y}_b = \bar{Y}_b - \bar{Y}_b^*$ = terminal driving force at the bottom of the absorber when $\bar{X} = \bar{X}_b$

$\Delta \bar{Y}_t = \bar{Y}_t - \bar{Y}_t^*$ = ditto at the top of the absorber when $\bar{X} = \bar{X}_t$.

When $0.5 \leq \Delta \bar{Y}_b / \Delta \bar{Y}_t \leq 2$, the following simpler formula can be used to calculate the mean driving force in an absorber:

$$\Delta \bar{Y}_m = \frac{\Delta \bar{Y}_b + \Delta \bar{Y}_t}{2} \quad (6-37)$$

If the equilibrium line is not straight, then

$$\Delta \bar{Y}_m = \frac{\bar{Y}_b - \bar{Y}_t}{\int_{\bar{Y}_t}^{\bar{Y}_b} \frac{d\bar{Y}}{\bar{Y} - \bar{Y}^*}} \quad (6-38)$$

The value of the integral in the denominator of Eq. (6-38) is found by graphical construction or the method of graphical integration. Another method of calculation when the equilibrium line is a curve is as follows. The equilibrium line is divided into sections that are assumed to approximately be lengths of straight lines, and the mean driving force is determined for each section separately by Eq. (6-36) or (6-37).

When calculating absorbers, the driving force is often expressed in units of pressure—see Example 6-9.

13. Determination of the diameter of a packed absorber. The diameter of an absorption column or tower D (in m) is calculated by Eq. (1-25) of the rate of flow for a gas stream:

$$D = \sqrt{\frac{Q_v}{0.785v}} \quad (6-39)$$

where Q_v = volumetric rate of flow of the gas passing through the absorber, m^3/s

v = velocity of the gas related to the total cross-sectional area of the column (fictitious), m/s

The velocity of the gas v can be found as follows.

First we compute the fictitious velocity of the gas v_n at the point of flooding (inversion) by the equation (with $\rho_{lq} \gg \rho_g$):

$$\log \left(\frac{v_n^2 \sigma \rho_g \mu_{lq}^{0.16}}{g V_{free}^3 \rho_{lq}} \right) = C - 1.75 \left(\frac{L}{G} \right)^{0.25} \left(\frac{\rho_g}{\rho_{lq}} \right)^{0.125} \quad (6-40)$$

where σ = unit surface area of packing, m^2/m^3

g = acceleration due to gravity, m/s^2

V_{free} = free volume of the packing, m^3/m^3

ρ_g and ρ_{lq} = densities of the gas and the liquid, respectively, kg/m^3

μ_{lq} = dynamic viscosity of the liquid, $mPa \cdot s$

L and G = mass rates of flow of the liquid and the gas, respectively, kg/s

$C = 0.022$ for a packing of rings or spirals [6-5].

Next the operating velocity of the gas v (fictitious) is determined, assuming for absorbers operating in film conditions that:

$$v = (0.75 \text{ to } 0.9) v_n \quad (6-41)$$

The fictitious velocity of a gas in packed columns operating in the so-called optimal hydrodynamic conditions (the conditions of the beginning of suspension) can be found from the equation:

$$Re_g = 0.045 Ar^{0.57} \left(\frac{G}{L} \right)^{0.43} \quad (6-42)$$

$$\text{where } Re_g = \frac{v d_{eq} \rho_g}{V_{free} \mu_g} = \frac{4v \rho_g}{\sigma \mu_g} \quad (6-43)$$

v/V_{free} = actual velocity of the gas in the free section of the packing, m/s

$$Ar = \frac{d_{eq}^3 \rho_g \rho_{lq} g}{\mu_g^2} \quad (6-44)$$

$$d_{eq} = \frac{4V_{free}}{\sigma} \text{ [see Eq. (1-65)]} \quad (6-45)$$

μ_g = dynamic viscosity of the gas, $Pa \cdot s$.

14. Determination of the height of a packed absorber.

(a) Through the height of a transfer unit (HTU).

The surface area of contact of the phases in an absorber with film conditions of operation is

$$A = h_p A_c \sigma \psi \quad (6-46)$$

where h_p = height of the packing layer, m

$A_c = \frac{\pi D^2}{4}$ = cross-sectional area of the column, m^2

D = diameter of the column, m

σ = unit surface area of the dry packing, m^2/m^3

ψ = dimensionless coefficient of packing wetting; for the calculation of this coefficient see [6-5].

The height of the packing layer is:

$$h_p = \frac{G}{K_y A_c \sigma \psi} \int_{\bar{Y}_t}^{\bar{Y}_b} \frac{d\bar{Y}}{\bar{Y} - \bar{Y}^*} = h_{oy} n_{oy} \quad (6-47)$$

where G = constant rate of flow of the inert gas along the height of the column, kg/s or kmol/s

K_y = mean overall coefficient of mass transfer, $kg/m^2 \cdot s$ (kg/kg of inert gas) or $kmol/m^2 \cdot s$ (kmol/kmol of inert gas)

$h_{oy} = \frac{G}{K_y A_c \sigma \psi}$ = height of a transfer unit, m

$n_{oy} = \int_{\bar{Y}_t}^{\bar{Y}_b} \frac{d\bar{Y}}{\bar{Y} - \bar{Y}^*}$ = total number of transfer units.

It follows from Eq. (6-38) that

$$n_{oy} = \frac{\bar{Y}_b - \bar{Y}_t}{\Delta \bar{Y}_m} \quad (6-48)$$

With a linear equilibrium relationship, the mean driving force $\Delta \bar{Y}_m$ is calculated by Eq. (6-36) or (6-37); with a curved equilibrium relationship, the number of transfer units n_{oy} is found by graphical construction or by the method of graphical integration—see Example 6-10.

The volume overall mass transfer coefficient K_{yV} is defined as the quantity

$$K_{yV} = K_y \sigma \psi = K_y a \quad (6-49)$$

where $a = \sigma \psi$ is the unit wetted (active) surface area of the packing, m^2/m^3 ; when $\psi = 1$ we have $a = \sigma$.

Introducing the volume overall coefficient of mass transfer, we get for the height of a transfer unit (HTU):

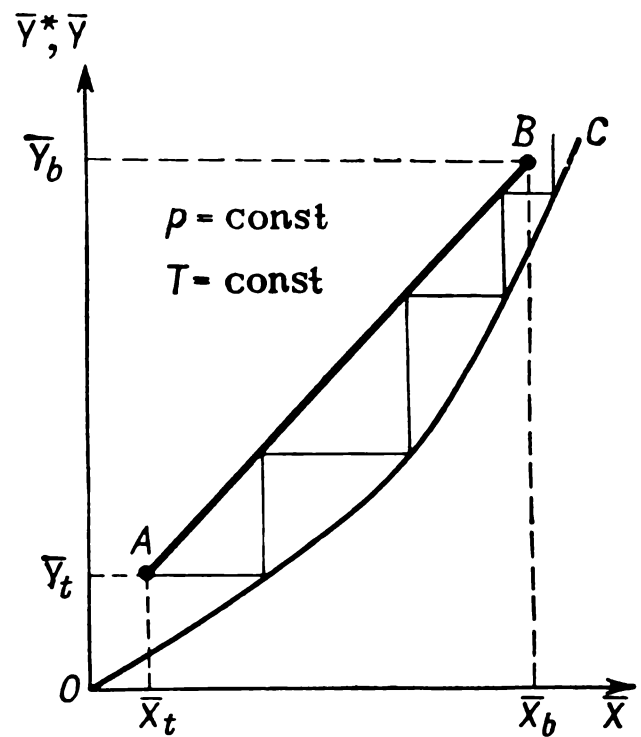
$$h_{oy} = \frac{G}{K_y A_c \sigma \psi} = \frac{G}{K_{yV} A_c} \quad (6-50)$$

(b) Through the height equivalent to a theoretical plate (HETP).

The height of the packing layer h_p can also be calculated by the equation:

$$h_p = h_{eq} n_{th} \quad (6-51)$$

Fig. 6-4. Graphical determination of the number of stages of the changes in the concentration (theoretical plates) in an absorber



where h_{eq} = height of an equivalent theoretical plate (HETP) or of a theoretical stage (HETS), m (it is determined according to experimental data)

n_{th} = number of theoretical plates (stages of changes in the concentration).

The number of theoretical plates in an absorber is usually determined graphically (Fig. 6-4).

In this figure, AB is the operating line constructed according to Eq. (6-30) or (6-31), and OC is the equilibrium line.

15. Dimensionless formulas for calculating the individual mass transfer coefficients in packed absorbers with random packing in film conditions.

(a) For the gaseous phase:

$$Nu'_g = 0.407 Re_g^{0.655} (Pr'_g)^{0.33} \quad (6-52)$$

where:

$$Nu'_g = \frac{\beta_g d_{eq}}{D_g} \quad (6-53)$$

$$Re_g = \frac{4v\rho_g}{\sigma\mu_g}, \quad \text{Eq. (6-43)}$$

$$Pr'_g = \frac{\mu_g}{\rho_g D_g} \quad (6-54)$$

β_g = individual mass transfer coefficient for the gas, $\text{kmol/m}^2 \times \text{s}(\text{kmol/m}^3) = \text{m/s}$

D_g = coefficient of diffusion of the component being absorbed in the gas, m^2/s .

For the remaining symbols, see Eqs. (6-40) and (6-42).

Equation (6-52) holds for values of Re_g from 10 to 10 000.

(b) For the liquid phase:

$$Nu'_{lq} = 0.0021 Re_{lq}^{0.75} (Pr'_{lq})^{0.5} \quad (6-55)$$

where

$$Nu'_{lq} = \frac{\beta_{lq} \delta_{red}}{D_{lq}} \quad (6-56)$$

$$Re_{lq} = \frac{4L}{A_c \sigma \psi \mu_{lq}} \quad (6-57)$$

$$Pr'_{lq} = \frac{\mu_{lq}}{\rho_{lq} D_{lq}} \quad (6-58)$$

β_{lq} = individual mass transfer coefficient for the liquid, m/s

$\delta_{red} = (\mu_{lq}^2 / \rho_{lq}^2 g)^{1/3}$ = so-called reduced liquid film thickness, m

D_{lq} = coefficient of diffusion of the component being absorbed in the liquid, m²/s

L = mass rate of flow of the liquid, kg/s.

Equation (6-57) for Re_{lq} has been obtained as follows [for the symbols see Eqs. (6-40) and (6-46)].

We find the perimeter of the absorber section around which the liquid flows from Eq. (6-46):

$$P = \frac{A}{h_p} = A_c \sigma \psi \quad (6-59)$$

The velocity with which the film of liquid flows through the packing is:

$$v_{lq, f} = \frac{L}{\rho_{lq} P \delta} = \frac{L}{\rho_{lq} A_c \sigma \psi \delta} \quad (6-60)$$

where δ is the mean thickness of the film, m.

The equivalent diameter of the liquid film is:

$$d_f = \frac{4P\delta}{P} = 4\delta \quad (6-61)$$

Using these values in the expression for the dimensionless number Re_{lq} , we get Eq. (6-57), namely,

$$Re_{lq} = \frac{v_{lq, f} d_f \rho_{lq}}{\mu_{lq}} = \frac{4L}{A_c \sigma \psi \mu_{lq}}$$

16. The diameter and height of a plate absorption column are determined in the same way as for plate rectification columns — see Chap. 7. The diameter of a plate absorber is calculated by Eqs. (7-16) and (7-17) or (7-18). The height of the plate part of an absorber H_{p1} is calculated by Eq. (7-19). The required number of plates is found graphically with the use of kinetic relationships for calculating the overall coefficients of mass transfer or the HTU. In approximate calculations for determining the number of plates,

the number of stages of the change in the concentration is found graphically (see Fig. 6-4), and then the number of plates n by Eq. (7-20).

EXAMPLES

Example 6-1. A liquid mixture contains 58.8% (molar) of toluene and 41.2% (molar) of carbon tetrachloride (ctc). Determine the mass ratio of the toluene \bar{X} (in kg of toluene/kg of ctc) and its mass-volume concentration \bar{c}_x (in kg/m³).

Solution. The mass ratio of the toluene (see Table 6-2) is:

$$\bar{X} = \frac{M_{\text{tol}} x}{M_{\text{ctc}} (1 - x)}$$

where M_{tol} = molar mass of the toluene (92 kg/kmol)
 M_{ctc} = ditto, carbon tetrachloride (154 kg/kmol)
 x = mole fraction of the toluene.

We have

$$\bar{X} = \frac{92 \times 0.588}{154 \times 0.412} = 0.853 \text{ kg of toluene/kg of ctc}$$

To calculate the mass-volume concentration of the toluene \bar{c}_x , we must know the density of the mixture ρ . To find the latter, we first determine the mass fraction of the toluene \bar{x} . From Table 6-2

$$\bar{x} = \frac{\bar{X}}{1 + \bar{X}} = \frac{0.853}{1.853} = 0.461$$

Next we use Table A-3 to find the density of toluene $\rho_{\text{tol}} = 870 \text{ kg/m}^3$ and the density of carbon tetrachloride $\rho_{\text{ctc}} = 1630 \text{ kg/m}^3$.

Assuming that no change in volume occurs upon mixing, i.e. that the volume of the mixture equals the sum of the volumes of the components, we find the volume of 1 kg of the mixture:

$$\frac{0.461}{870} + \frac{0.539}{1630} = 0.862 \times 10^{-3} \text{ m}^3$$

whence the density of the mixture is:

$$\rho = \frac{1}{0.862 \times 10^{-3}} = 1160 \text{ kg/m}^3$$

The mass-volume concentration of the toluene is:

$$\bar{c}_x = \rho \bar{x} = 1160 \times 0.461 = 535 \text{ kg/m}^3$$

Example 6-2. Air at atmospheric pressure and a temperature of 34 °C is saturated with water vapour. Determine the partial pressure of the air, the volume (mole) and mass fractions of the

vapour in the air-vapour mixture and its mass ratio considering both components of the mixture to be ideal gases. The atmospheric pressure is 745 mm Hg. Also determine the density of the air-vapour mixture and compare it with the density of dry air.

Solution. From Table A-38, we find that at $t = 34^\circ\text{C}$ the pressure of saturated water vapour is 39.9 mm Hg. This pressure is the partial pressure of the water vapour p_v in the air-vapour mixture, and the partial pressure of the air is:

$$p_a = p - p_v = 745 - 39.9 \approx 705 \text{ mm Hg}$$

The mole (volume) fraction of the water vapour in the mixture is:

$$y = \frac{p_v}{p} = \frac{39.9}{745} = 0.0535$$

The mass fraction of the vapour is:

$$\bar{y} = \frac{M_v y}{M_v y + M_a (1 - y)} = \frac{18 \times 0.0535}{18 \times 0.0535 + 29 \times 0.9465} = 0.0339$$

The mass ratio is:

$$\bar{Y} = \frac{\bar{y}}{1 - \bar{y}} = \frac{0.0339}{0.9661} = 0.0351 \text{ kg of vapour/kg of air}$$

We calculate the density of the air-vapour mixture as the sum of the densities of the components, each taken at its partial pressure:

$$\begin{aligned} \rho_{\text{mix}} &= \rho_a + \rho_v = \frac{M_a p_a T_0}{22.4 T p_0} + \frac{M_v p_v T_0}{22.4 T p_0} = \frac{T_0}{22.4 T p_0} (M_a p_a + M_v p_v) = \\ &= \frac{273}{22.4 \times 307 \times 760} (29 \times 705 + 18 \times 39.9) = 1.105 \text{ kg/m}^3 \end{aligned}$$

We can calculate the density of the mixture in a different way. The molar mass of the mixture is:

$$M_{\text{mix}} = M_v y + M_a (1 - y) = 18 \times 0.0535 + 29 \times 0.9465 = 28.4 \text{ kg/mol}$$

The density of the mixture at $p = 745 \text{ mm Hg}$ and $t = 34^\circ\text{C}$ is:

$$\rho_{\text{mix}} = \frac{M_{\text{mix}} p T_0}{22.4 p_0 T} = \frac{28.4 \times 745 \times 273}{22.4 \times 760 \times 307} = 1.105 \text{ kg/m}^3$$

The density of the dry air at the same pressure and temperature is:

$$\rho_{\text{dry air}} = \frac{M_a p T_0}{22.4 T p_0} = \frac{29 \times 745 \times 273}{22.4 \times 307 \times 760} = 1.13 \text{ kg/m}^3$$

Example 6-3. Air at a pressure of 765 mm Hg containing 14% (by volume) of acetylene (C_2H_2) and water containing dissolved acetylene in an amount of (a) $0.29 \times 10^{-3} \text{ kg}$ per kg of water, and

(b) 0.153×10^{-3} kg per kg of water are brought into contact at a temperature of 25°C . Determine (1) the initial and final phases of the acetylene; (2) the driving force of this process of phase transition at the initial moment of time (in mole ratios). The equilibrium concentrations of the acetylene in the gaseous and liquid phases are determined by Henry's law.

Solution. Henry's law expressed by Eq. (6-2) is:

$$p^* = kx$$

We find from Table A-41 that Henry's coefficient at $t = 25^\circ\text{C}$ is $k = 1.01 \times 10^6$ mm Hg.

The partial pressure of the acetylene in air by Eq. (6-1) is

$$p_{ac} = yp = 0.14 \times 765 = 107 \text{ mm Hg}$$

(a) The mole fraction of the acetylene in water at $\bar{X} = 0.29 \times 10^{-3}$ kg of acetylene/kg of water (Table 6-2) is:

$$x = \frac{\bar{X}}{\bar{X} + \frac{M_{ac}}{M_w}} = \frac{0.29 \times 10^{-3}}{0.29 \times 10^{-3} + \frac{26}{18}} \approx \frac{18 \times 0.29 \times 10^{-3}}{26} = 0.2 \times 10^{-3}$$

The answers to the questions can be obtained in two ways.

I. In conditions of equilibrium, the partial pressure of the acetylene in the gaseous phase over the liquid with $x = 0.2 \times 10^{-3}$ according to Henry's law should be

$$p^* = kx = 1.01 \times 10^6 \times 0.2 \times 10^{-3} = 202 \text{ mm Hg}$$

The actual partial pressure of the acetylene over this liquid is lower: $p_{ac} = 107$ mm Hg. For the system gas-liquid to approach the state of equilibrium in the process of mass transfer, the partial pressure of the acetylene in the gaseous phase should increase, i.e. the acetylene will pass from the water to the air.

The driving force of this transition process (the deviation from the state of equilibrium) at the initial moment of time is: in units of the partial pressure of the acetylene

$$\Delta p_{ac} = p^* - p_{ac} = 202 - 107 = 95 \text{ mm Hg}$$

in mole fractions

$$\Delta y = y^* - y = \frac{202}{765} - 0.14 = 0.264 - 0.14 = 0.124$$

in mole ratios

$$\begin{aligned} \Delta Y = Y^* - Y &= \frac{y^*}{1 - y^*} - \frac{y}{1 - y} = \frac{0.264}{1 - 0.264} - \frac{0.14}{1 - 0.14} = \\ &= 0.359 - 0.163 = 0.196 \frac{\text{kmol of acetylene}}{\text{kmol of air}} \end{aligned}$$

II. In conditions of equilibrium with the gaseous phase in which the partial pressure of the acetylene equals 107 mm Hg, the water according to Henry's law should have an acetylene concentration (in mole fractions) of:

$$x^* = \frac{p_{ac}}{k} = \frac{107}{1.01 \times 10^6} = 0.106 \times 10^{-3}$$

The actual mole fraction of the acetylene in the water is greater: $x = 0.2 \times 10^{-3}$. For the system to approach the state of equilibrium in the process of mass transfer, the mole fraction of the acetylene in the water should diminish, i.e. the acetylene will pass from the water to the air.

The driving force of this transfer process at the initial moment of time (according to the concentrations in the liquid phase) is: in mole fractions

$$\Delta x = x - x^* = 0.2 \times 10^{-3} - 0.106 \times 10^{-3} = 0.094 \times 10^{-3}$$

in mole ratios

$$\Delta X = X - X^* = \frac{x}{1-x} - \frac{x^*}{1-x^*}$$

Since in the given example x and x^* are both much smaller than unity, they may be disregarded in the denominators of the last equation, and

$$\Delta X = x - x^* = 0.094 \times 10^{-3} \frac{\text{kmol of acetylene}}{\text{kmol of water}}$$

(b) The mole fraction of the acetylene in water:

$$x \approx \frac{18 \times 0.153 \times 10^{-3}}{26} = 0.106 \times 10^{-3}$$

There will be no transition of the acetylene from one phase to the other because the contacting phases are in equilibrium:

$$x = x^* = 0.106 \times 10^{-3}$$

$$y = y^* = 0.14$$

Example 6-4. In a mass exchange apparatus operating at a pressure of $p_{abs} = 3.1$ at, the individual mass transfer coefficients have the following values: $\beta_y = 1.07$ kmol/m²·h ($\Delta y = 1$) and $\beta_x = 22$ kmol/m²·h ($\Delta x = 1$). The equilibrium compositions of the gaseous and liquid phases are characterized by the equation of Henry's law $p^* = 0.08 \times 10^6 x$. Determine (a) the overall mass transfer coefficients K_y and K_x , and (b) how many times the diffusion resistance of the liquid phase differs from that of the gaseous phase.

Solution. We transform the equilibrium equation to the form $y^* = mx$:

$$y^* = \frac{p^*}{p} = \frac{0.08 \times 10^6}{3.1 \times 735} x = 35.1x$$

We find the overall coefficients of mass transfer:

$$K_y = \frac{1}{\frac{1}{\beta_y} + \frac{m}{\beta_x}} = \frac{1}{\frac{1}{1.07} + \frac{35.1}{22}} = \frac{1}{0.935 + 1.595} = 0.396 \text{ kmol/m}^2 \cdot \text{h} (\Delta y = 1)$$

$$K_x = \frac{1}{\frac{1}{m\beta_y} + \frac{1}{\beta_x}} = \frac{1}{\frac{1}{35.1 \times 1.07} + \frac{1}{22}} = \frac{1}{0.0266 + 0.0455} = 13.9 \text{ kmol/m}^2 \cdot \text{h} (\Delta x = 1)$$

Check:

$$\frac{K_x}{K_y} = \frac{13.9}{0.396} = 35.1 = m$$

The ratio of the diffusion resistances of the liquid and gaseous phases with a driving force of Δy is:

$$\frac{m}{\beta_x} : \frac{1}{\beta_y} = \frac{35.1}{22} : \frac{1}{1.07} = 1.71$$

We get the same ratio for the driving force Δx .

The diffusion resistance of the liquid phase is 1.71 times greater than that of the gaseous phase.

Example 6-5. The overall coefficient of mass transfer in a mass transfer apparatus—an absorber—is $K_y = 10.4 \text{ kmol/m}^2 \cdot \text{h} \cdot \text{kmol} \cdot \text{m}^{-3}$. The inert gas (which does not pass into the liquid) is nitrogen. The pressure p_{abs} in the apparatus is 760 mm Hg, the temperature is 20°C. Determine the values of the overall coefficient of mass transfer K_y in the following units: (1) $\text{kmol/m}^2 \cdot \text{h} (\Delta y = 1)$; (2) $\text{kmol/m}^2 \cdot \text{h} \cdot \text{mm Hg}$; and (3) $\text{kg/m}^2 \cdot \text{h} (\text{kg/kg of inert gas})$.

Solution. We write the equations:

$$M = K_y \Delta c_y A = K'_y \Delta y A = K''_y \Delta p_i A$$

where M is the molar rate of flow of the component passing into the liquid, kmol/h .

Hence:

$$(1) K_y \Delta c_y = K'_y \Delta y, \text{ and } K'_y = K_y \frac{\Delta c_y}{\Delta y}.$$

According to Table 6-2, we have:

$$c_y = \frac{\rho}{M_{\text{mix}}} y = \frac{\rho T_0}{22.4 p_0 T} y$$

In the given example, $p = p_0$, and

$$\frac{\Delta c_y}{\Delta y} = \frac{T_0}{22.4T} = \frac{273}{22.4 \times 293} = 0.0416$$

$$K'_y = K_y \frac{\Delta c_y}{\Delta y} = 10.4 \times 0.0416 = 0.433 \text{ kmol/m}^2 \cdot \text{h} (\Delta y = 1)$$

$$(2) K'_y \Delta y = K''_y \Delta p_i.$$

By Eq. (6-1), we have:

$$y = \frac{p_i}{p}; \quad \Delta y = \frac{\Delta p_i}{p}$$

$$K''_y = K'_y \frac{\Delta y}{\Delta p_i} = \frac{K'_y}{p} = \frac{0.433}{760} = 5.69 \times 10^{-4} \text{ kmol/m}^2 \cdot \text{h} \cdot \text{mm Hg}$$

or

$$\frac{5.69 \times 10^{-4}}{3600 \times 133.3} = 1.19 \times 10^{-9} \text{ kmol/m}^2 \cdot \text{s} \cdot \text{Pa}$$

(3) From the equations

$$W = M_c M = M_c K'_y \Delta y A = K'''_y \Delta \bar{Y} A$$

where W is the mass rate of flow of the transferring component, kg/h, we find:

$$K'''_y = K'_y M_c \frac{\Delta y}{\Delta \bar{Y}}$$

According to Table 6-2, we have:

$$\bar{Y} = \frac{M_c y}{M_{i.g} (1-y)}$$

For low values of y , we get:

$$\bar{Y} \approx \frac{M_c}{M_{i.g}} y$$

Hence

$$\frac{\Delta y}{\Delta \bar{Y}} \approx \frac{M_{i.g}}{M_c}$$

$$K'''_y = K'_y \frac{M_c \Delta y}{\Delta \bar{Y}} = K'_y M_{i.g} = 0.433 \times 28 = 12.1 \text{ kg/m}^2 \cdot \text{h} \text{ (kg/kg of inert gas)}$$

Example 6-6. Calculate the diffusion coefficient for hydrogen sulphide in water at 40°C.

Solution. First we calculate the diffusion coefficient for 20°C by Eq. (6-25):

$$D_{20} = \frac{1 \times 10^{-6}}{AB \sqrt{\mu} (V_A^{1/3} + V_B^{1/3})^2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}$$

For hydrogen sulphide

$$\begin{aligned} A &= 1 \\ V_A &= 2 \times 3.7 + 25.6 = 33.0 \text{ (Table 6-3)} \\ M_A &= 34 \end{aligned}$$

For water

$$\begin{aligned} B &= 4.7 \\ \mu &= 1 \text{ cP} = 1 \text{ mPa} \cdot \text{s} \\ V_B &= 2 \times 3.7 + 7.4 = 14.8 \\ M_B &= 18 \end{aligned}$$

We use these values in Eq. (6-25):

$$D_{20} = \frac{1 \times 10^{-6}}{4.7 \times 1 (14.8^{1/3} + 33^{1/3})^2} \sqrt{\frac{1}{18} + \frac{1}{34}} = 1.93 \times 10^{-9} \text{ m}^2/\text{s}$$

We calculate the temperature coefficient b by Eq. (6-27):

$$b = \frac{0.2 \sqrt{\mu}}{\sqrt[3]{\rho}} = \frac{0.2 \sqrt{1}}{\sqrt[3]{1000}} = 0.02$$

The required diffusion coefficient, by Eq. (6-26), is:

$$D_{40} = 1.93 \times 10^{-9} [1 + 0.02 (40 - 20)] = 2.7 \times 10^{-9} \text{ m}^2/\text{s}$$

For purposes of comparison, we calculate the diffusion coefficient for hydrogen sulphide in water at 40°C by Eq. (6-28), after finding the dynamic viscosity of water at 40°C from Table A-6, namely $\mu = 0.656 \text{ cP}$:

$$D_{40} = \frac{7.4 \times 10^{-12} (2.6 \times 18)^{0.5} 313}{0.656 \times 33^{0.6}} = 2.96 \times 10^{-9} \text{ m}^2/\text{s}$$

Example 6-7. Determine the consumption of sulphuric acid for drying air in the following conditions. The capacity of the scrubber is 500 m³/h (calculating for dry air in standard conditions). The initial content of moisture in the air is 0.016 kg/kg of dry air, the final content is 0.006 kg/kg of dry air. The initial content of water in the acid is 0.6 kg/kg of monohydrate, the final content is 1.4 kg/kg of monohydrate. The air is dried under atmospheric pressure.

Solution. The density of air in standard conditions is 1.293 kg/m³, and the mass rate of flow of the air is:

$$G = 500 \times 1.293 = 646 \text{ kg/h}$$

The rate of flow of the sulphuric acid (monohydrate), by Eq. (6-29), is:

$$L = G \frac{\bar{Y}_b - \bar{Y}_t}{\bar{X}_b - \bar{X}_t} = 646 \times \frac{0.016 - 0.006}{1.4 - 0.6} = 8.1 \text{ kg/h}$$

Example 6-8. A scrubber for absorbing acetone vapour from air is irrigated with water at a rate of 3000 kg/h. The mean temperature in the scrubber is 20°C. A mixture of air with acetone vapour containing 6% by volume of acetone is passed

through the scrubber at atmospheric pressure. This mixture contains 1400 m³/h of pure air (reduced to standard conditions). The scrubber absorbs 98% of the acetone.

The equation of the equilibrium line is:

$$Y^* = 1.68X$$

where X and Y^* are expressed in kilomoles of acetone per kilomole of the second component, i.e. water or air.

Find the diameter and height of a scrubber packed with ceramic rings $25 \times 25 \times 3$ mm in size. Assume that the velocity of the gas is 75% of the flooding velocity.

The overall coefficient of mass transfer is $K_y = 0.4$ kmol of acetone/m²·h (kmol of acetone/kmol of air). Assume that the packing is wetted completely.

Solution. The quantity of acetone absorbed is:

$$M = \frac{Q_{v, i.g} y_b e_a}{(1 - y_b) 22.4} = \frac{1400 \times 0.05 \times 0.98}{0.94 \times 22.4} = 3.9 \text{ kmol/h}$$

where $e_a = 0.98$ is the absorption factor.

The initial concentration of the acetone in the water fed to the top of the scrubber is $X_t = 0$.

The final concentration of the acetone in the water flowing out of the scrubber from its bottom is:

$$X_b = \frac{M}{\frac{L}{M_w}} = \frac{3.9}{\frac{3000}{18}} = 0.0234 \frac{\text{kmol of acetone}}{\text{kmol of water}}$$

The initial concentration of the acetone in the air entering the scrubber at the bottom is:

$$Y_b = \frac{y_b}{1 - y_b} = \frac{0.06}{0.94} = 0.0639 \frac{\text{kmol of acetone}}{\text{kmol of air}}$$

The final concentration of the acetone in the air leaving the scrubber is:

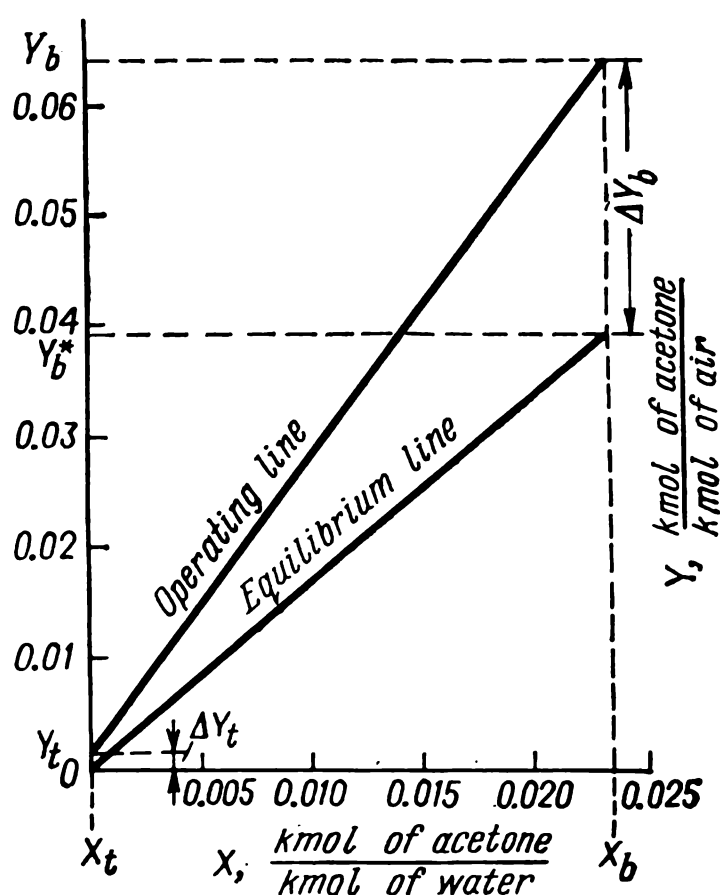
$$Y_t = \frac{y_b (1 - e_a)}{1 - y_b} = \frac{0.06 \times 0.02}{0.94} = 0.00128 \frac{\text{kmol of acetone}}{\text{kmol of air}}$$

These points have been used to plot the operating line in the Y - X diagram (Fig. 6-5). Below it is the equilibrium line according to the equation $Y^* = 1.68X$.

We find the driving force of absorption at the bottom of the scrubber:

$$\Delta Y_b = Y_b - Y_b^* = 0.0639 - 0.0393 = 0.0246 \frac{\text{kmol of acetone}}{\text{kmol of air}}$$

Fig. 6-5. To Example 6-8



We find the value of Y_b^* according to the equation of the equilibrium line for X_b corresponding to the bottom of the scrubber:

$$Y_b^* = 1.68X_b = 1.68 \times 0.0234 = 0.0393 \frac{\text{kmol of acetone}}{\text{kmol of air}}$$

The driving force of absorption at the top of the scrubber is:

$$\Delta Y_t = Y_t - Y_t^* = 0.00128 - 0 = 0.00128 \frac{\text{kmol of acetone}}{\text{kmol of air}}$$

The mean driving force is:

$$\Delta Y_m = \frac{\Delta Y_b - \Delta Y_t}{2.3 \log \frac{\Delta Y_b}{\Delta Y_t}} = \frac{0.0246 - 0.00128}{2.3 \log \frac{0.0246}{0.00128}} = 0.0079 \frac{\text{kmol of acetone}}{\text{kmol of air}}$$

We find the required mass transfer surface area by an equation similar to Eq. (6-35):

$$A = \frac{M}{K_y \Delta Y_m} = \frac{3.9}{0.4 \times 0.0079} = 1230 \text{ m}^2$$

The volume of the layer of ceramic rings needed for creating this area, with $\psi = 1$ [see Eq. (6-46)], is:

$$V = h_p A_c = \frac{A}{\sigma} = \frac{1230}{204} = 6 \text{ m}^3$$

where $\sigma = 204 \text{ m}^2/\text{m}^3$ is the unit surface area of the packing (Table A-17).

We determine the cross-sectional area of the scrubber column.

We use Eq. (6-40) to calculate the fictitious velocity of the gas v_n at the point of inversion, ignoring the slight content of acetone in the liquid and the gas.

The values of the quantities in the equation are:

$$L = 3000 \text{ kg/h}, \quad \rho_g = \rho_0 \frac{T_0}{T} = 1.29 \times \frac{273}{293} = 1.2 \text{ kg/m}^3$$

$$G = 1400 \times 1.29 = 1810 \text{ kg/h}; \quad \rho_{lq} = 1000 \text{ kg/m}^3$$

$$\frac{L}{G} = \frac{3000}{1810} = 1.66; \quad \frac{\rho_g}{\rho_{lq}} = \frac{1.2}{1000} = 0.0012$$

$$\mu_{lq} = 1 \text{ mPa}\cdot\text{s}; \quad V_{\text{free}} = 0.74 \text{ m}^3/\text{m}^3 \text{ (Table A-17)}$$

We insert these values into Eq. (6-40):

$$\log \left(\frac{v_n^2 \times 204 \times 0.0012}{9.81 \times 0.74^3} \times 1^{0.16} \right) = 0.022 - 1.75 \times 1.66^{0.25} \times 0.0012^{0.125}$$

or

$$\log 0.0616 v_n^2 = -0.825$$

whence $v_n = 1.56 \text{ m/s}$.

According to the initial conditions, we take the operating fictitious velocity equal to 75% of the flooding one:

$$v = 0.75 v_n = 0.75 \times 1.56 = 1.17 \text{ m/s}$$

The cross-sectional area of the scrubber is:

$$A_c = \frac{G}{3600 v \rho_g} = \frac{1810}{3600 \times 1.17 \times 1.2} = 0.358 \text{ m}^2$$

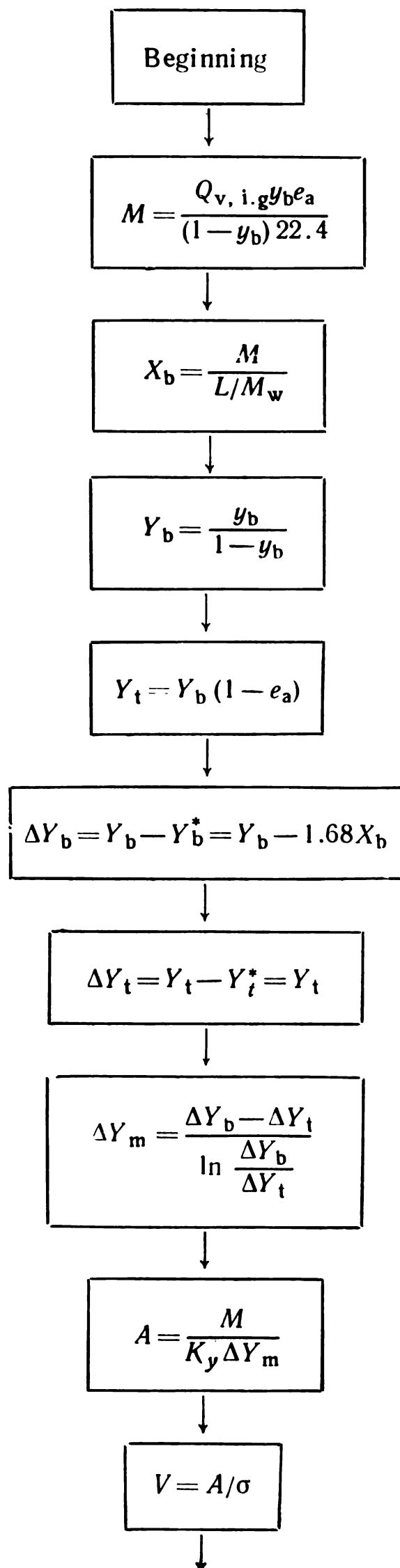
Hence, the diameter of the scrubber is:

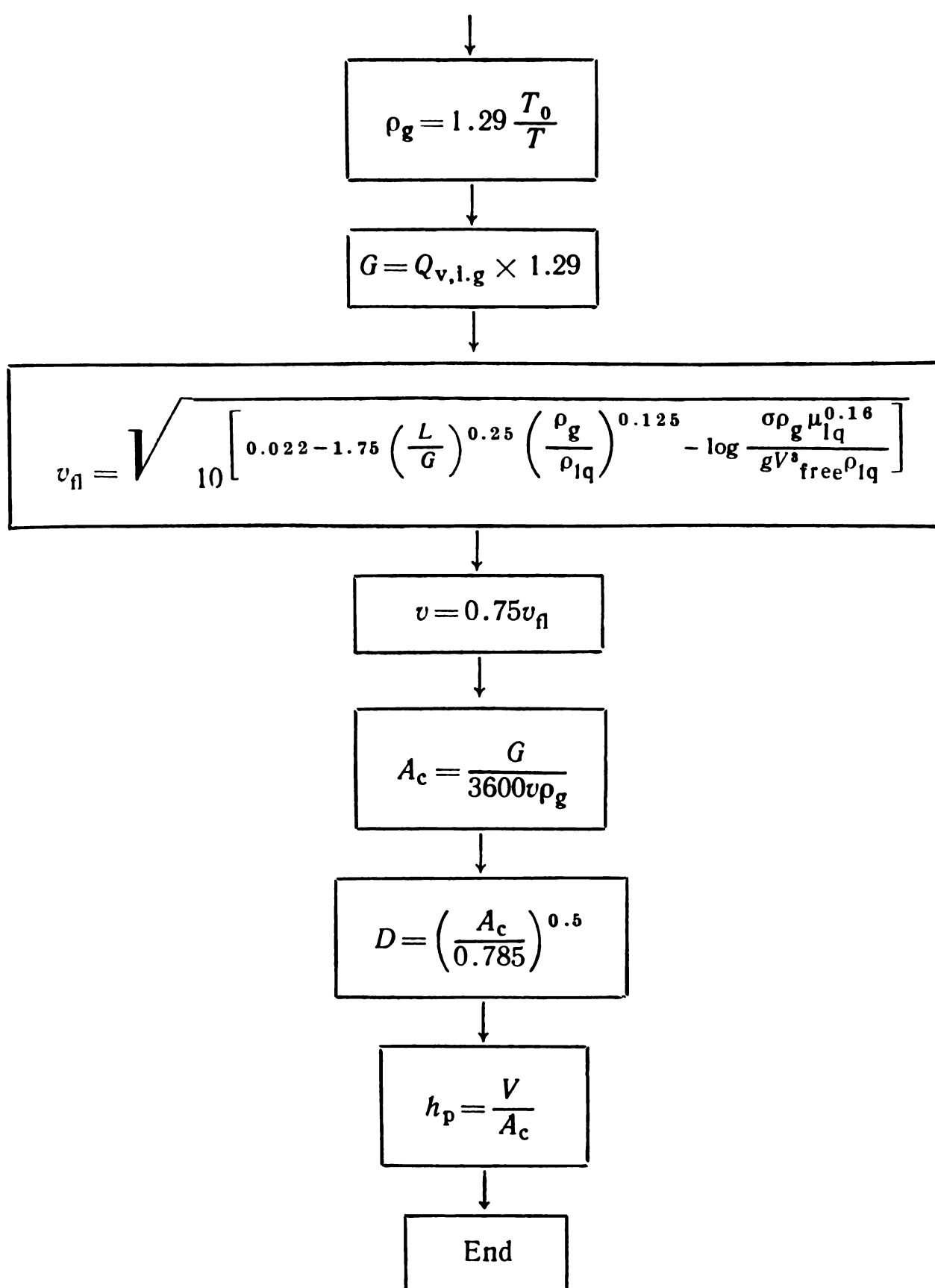
$$D = \sqrt{\frac{0.358}{0.785}} = 0.675 \text{ m}$$

The required height of the packing layer is:

$$h_p = \frac{V}{A_c} = \frac{6}{0.358} = 16.8 \text{ m}$$

The flow chart of the algorithm for solving Example 6-8 is as follows:





The table of identifiers is given below (Table 6-4).

The ALGOL record of the programme is as follows:

Begin real M, QV, yb, e, XB, L, YB, YT, DYB, DYT, DYM, A, ky, V,
 sigma, r, T, G, vf, mul, VF, v, AC, D, h;
Input (QV, yb, e, L, ky, sigma, T, mul, VF);
 M:= QV × yb × e/(1-yb)/22.4;
 XB:= M/L/18;
 YB:= yb/(1-yb); YT:= YB × (1-e);
 DYB:= YB-1.68 × XB; DYT:= YT;

```
DYM:= DYB-DYT/ln (DYB/DYT); A:= M/ky/DYM;
V:= A/sigma; r:= 1.29 × 273/T; G:= 1.29 × QV;
vf:= sqrt (10 ↑ (.022-1.75 × (L/G) ↑ .25 × (r/1000) ↑ .125-
ln (sigma × r × mul ↑ .16/9.81/VF ↑ 3/1000)/2.3));
v:= .75 × vf; AC:= G/3600/v/r;
D:= sqrt (AC/.785); h:= V/AC;
Output (h, D); end.
```

TABLE 6-4

Quantity	M	$Q_{v,i.g}$	y_b	e_a	X_b	L	Y_b	Y_t	ΔY_b	ΔY_t
Identifier	M	QV	yb	e	XB	L	YB	YT	DYB	DYT

ΔY_m	A	K_y	v	σ	ρ_g	T	G	v_{fl}	μ_{lq}	V_{free}	v	A_c	D	h_p
DYM	A	ky	V	sigma	r	T	G	vf	mul	VF	v	AC	D	h

The numerical information fed into an electronic digital computer with the aid of punched cards will have the form:

1400, .06, .98, 3000, .4, 204, 293, 1, .74

Example 6-9. Determine the overall mass transfer coefficient in a water scrubber when carbon dioxide is absorbed from a gas in the following conditions. The scrubber is supplied with 5000 m³/h of a gas mixture, considering it to be at atmospheric pressure and the operating temperature. Pure water is fed to the scrubber at a rate of 650 m³/h. The initial content of carbon dioxide in the gas is 28.4% (by volume), the final content (at the top of the scrubber) is 0.2% (by volume). The pressure in the scrubber is $p_{abs}=16.5$ at. The temperature is 15°C. The bottom part of the scrubber is packed with 3 tons of ceramic rings 50×50×5 mm in size. Above them are 17 tons of rings 35×35×4 mm in size. Assume that the coefficient of packing wetting equals unity.

Solution. We calculate the total surface area of all the rings. The surface area of the 50×50×5-mm rings is:

$$A_1 = \frac{G_1}{\rho_1} \sigma_1 = \frac{3000}{530} \times 87.5 = 495 \text{ m}^2$$

where $\rho_1=530$ kg/m³= bulk density of the packing of the 50×50×5-mm rings
 $\sigma_1=87.5$ m²/m³=unit surface area of the packing (Table A-17).

In a similar way, we calculate the surface area of the rings $35 \times 35 \times 4$ mm in size:

$$A_2 = \frac{G_2}{\rho_2} \sigma_2 = \frac{17\,000}{505} \times 140 = 4717 \text{ m}^2$$

The total surface of all the rings is:

$$A = A_1 + A_2 = 495 + 4717 = 5212 \text{ m}^2$$

We determine the amount of carbon dioxide (cd) absorbed by the water.

The initial amount of carbon dioxide in the gas (at the bottom of the scrubber) is:

$$Q_{v, b, cd} = Q_{v, b, mix} y_b = 5000 \times 0.284 = 1420 \text{ m}^3/\text{h}$$

The amount of carbon dioxide in the discharged gas (at the top of the scrubber) is:

$$\begin{aligned} Q_{v, t, cd} &= Q_{v, t, mix} y_t = \frac{Q_{v, b, mix} - Q_{v, b, cd}}{1 - y_t} y_t = \\ &= \frac{5000 - 1420}{1 - 0.002} \times 0.002 = 7.2 \text{ m}^3/\text{h} \end{aligned}$$

The amount absorbed by the water is ($p = 1 \text{ at} = 0.1 \text{ mPa}$, and $t = 15^\circ\text{C}$):

$$Q_{v, a} = Q_{v, b, cd} - Q_{v, t, cd} = 1420 - 7.2 = 1412.8 \text{ m}^3/\text{h}$$

or

$$G_a = \frac{Q_{v, a} T_0}{T} \rho_0 = \frac{1412.8 \times 273}{273 + 15} \times 1.976 = 2630 \text{ kg/h}$$

$$\text{i. e. } \frac{2630}{44} = 60 \text{ kmol/h}$$

Here $1.976 \text{ kg/m}^3 = \text{density of CO}_2 \text{ in standard conditions}$
 $44 \text{ kg/kmol} = \text{molar mass of CO}_2$.

We find the driving force of the absorption process at the bottom of the scrubber.

The total pressure in the scrubber is $1620 = 16.5 \times 98.1 \text{ kPa}$, and the partial pressure of the carbon dioxide at the scrubber inlet is:

$$p_b = p y_b = 1620 \times 0.284 = 460 \text{ kPa}$$

The mole fraction of the CO_2 in the water at the scrubber outlet is:

$$x_b = \frac{\frac{G_{cd}}{M_{cd}}}{\frac{G_{cd}}{M_{cd}} + \frac{G_{wat}}{M_{wat}}} = \frac{\frac{2630}{44}}{\frac{2630}{44} + \frac{550\,000}{18}} = 0.001\,66$$

We find Henry's coefficient for carbon dioxide at 15°C from Table A-41: $k = 0.93 \times 10^6 \text{ mm Hg} = 0.124 \times 10^6 \text{ kPa}$; hence, the

partial pressure of the carbon dioxide in the gas in equilibrium with the liquid flowing out of the scrubber [Eq. (6-2)] is:

$$p_b^* = kx_b = 0.00166 \times 0.124 \times 10^6 = 206 \text{ kPa}$$

The driving force of the absorption process at the bottom of the scrubber is:

$$\Delta p_b = p_b - p_b^* = 460 - 206 = 254 \text{ kPa}$$

We determine the driving force of the absorption process at the top of the scrubber.

The partial pressure of the carbon dioxide in the gas discharged from the scrubber at its top is:

$$p_t = py_t = 1620 \times 0.002 = 3.24 \text{ kPa}$$

Since pure water is supplied for irrigation of the scrubber, the partial pressure of the carbon dioxide in the gas in equilibrium with the water equals zero; hence, the driving force of the absorption process at the top of the scrubber is:

$$\Delta p_t = p_t - p_t^* = 3.24 - 0 = 3.24 \text{ kPa}$$

The mean driving force for the entire process is:

$$\Delta p_m = \frac{254 - 3.24}{2.3 \log \frac{254}{3.24}} = 57.4 \text{ kPa}$$

The overall coefficient of mass transfer is:

$$K_{\Delta p} = \frac{G_a}{A \Delta p_m} = \frac{2630}{5212 \times 57.4} = 0.0088 \frac{\text{kg}}{\text{m}^2 \cdot \text{h} \cdot \text{kPa}} = 0.86 \frac{\text{kg}}{\text{m}^2 \cdot \text{h} \cdot \text{at}}$$

The overall mass transfer coefficient related to the pressure drop Δp (in mm Hg) is:

$$K_{\Delta p} = \frac{2630}{5212 \times \frac{57.4}{0.133}} = 0.0012 \frac{\text{kg}}{\text{m}^2 \cdot \text{h} \cdot \text{mm Hg}}$$

Example 6-10. Ammonia is absorbed from a gas by water in a scrubber under atmospheric pressure. The initial ammonia content in the gas is 0.03 kmol per kmol of inert gas. The extraction factor is 90%. The water leaving the scrubber contains 0.02 kmol of ammonia per kmol of water. A constant temperature is maintained in the scrubber by removing heat from it.

Data on the equilibrium concentrations of the ammonia in the liquid and the gas at the absorption temperature are given in Table 6-5.

Determine the required number of transfer units n_{oy} (1) by graphical construction, and (2) by graphical integration.

TABLE 6-5

$X, \frac{\text{kmol of ammonia}}{\text{kmol of water}}$	$Y^*, \frac{\text{kmol of ammonia}}{\text{kmol of inert gas}}$
0	0
0.005	0.0045
0.010	0.0102
0.0125	0.0138
0.015	0.0183
0.020	0.0273
0.023	0.0327

Solution. (I) We use the data of Table 6-5 to construct equilibrium line AB (Fig. 6-6). Operating line CD is shown on the same graph. It passes through point C with the coordinates $X_t=0$, $Y_t=0.03(1-0.9)=0.003$ (the top of the scrubber) and point D with the coordinates $X_b=0.02$, $Y_b=0.03$ (the bottom of the scrubber).

We find the number of transfer units n_{oy} as follows. We divide the lengths of the ordinates between the operating and the equilibrium lines in half and draw an auxiliary dash line through their middles. Next, beginning from point C , we construct a series of steps so that for each of them $ab=bc$. Each of these steps is a transfer unit, i.e. such a section of the apparatus corresponds to each step in which the change in the operating concentration (Y_1-Y_2) equals the mean driving force in this section $(Y-Y^*)_m$.

Altogether, 5.82 steps or stages have been obtained (the last

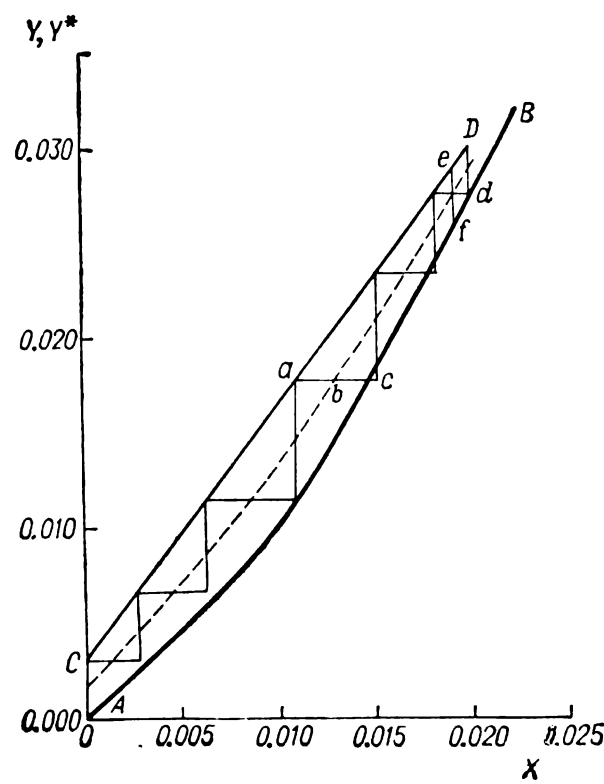


Fig. 6-6. To Example 6-10

incomplete step equals the ratio of the lengths $Dd/ef=0.82$):
 $n_{ou}=5.82$

Inspection of the graph shows that on the lower portion of the equilibrium curve where its slope is less than that of the operating line, a transfer unit is less than a step of the change in the concentration; on the upper portion of the equilibrium line where its slope is greater than that of the operating line, the reverse picture is observed.

(2) To determine the number of transfer units by graphical integration, we use the data of Table 6-5 and Fig. 6-6 to compile Table 6-6.

TABLE 6-6

X	Y	Y*	Y-Y*	$\frac{1}{Y-Y^*}$
0	0.003	0	0.003	333
0.005	0.0097	0.0045	0.0052	193
0.010	0.0165	0.0102	0.0063	159
0.0125	0.0200	0.0138	0.0062	161
0.015	0.0234	0.0183	0.0051	196
0.020	0.0300	0.0273	0.0027	371

According to the data of this table, we plot a graph of $1/(Y-Y^*)=f(Y)$ —Fig. 6-7. We calculate the hatched area on this graph, for instance by the method of trapeziums (see Example 4-21). The value of this area (5.83) gives the value of the

integral $\int_{Y_t=0.003}^{Y_b=0.03} \frac{dY}{Y-Y^*}$, i.e. the number of transfer units n_{oy} .

Example 6-11. Determine the theoretically minimum rate of flow of a liquid absorbent with a molar mass of 224 kg/kmol needed to completely extract propane and butane from 1000 m³/h of a gas mixture (in standard conditions). The content of the propane and butane in the gas is 15 and 10% by volume, respectively. The temperature in the absorber is 30 °C, the absolute pressure is 3 at (294 kPa). The solubilities of butane and propane in the absorbent are characterized by Raoult’s law.

Solution. The maximum concentration (mole fraction) of the propane in the absorbent leaving the absorber (in equilibrium with the entering mixture) is determined by Eq. (6-9):

$$x_p^* = \frac{p}{p_p} y_p = \frac{294}{981} \times 0.15 = 0.045$$

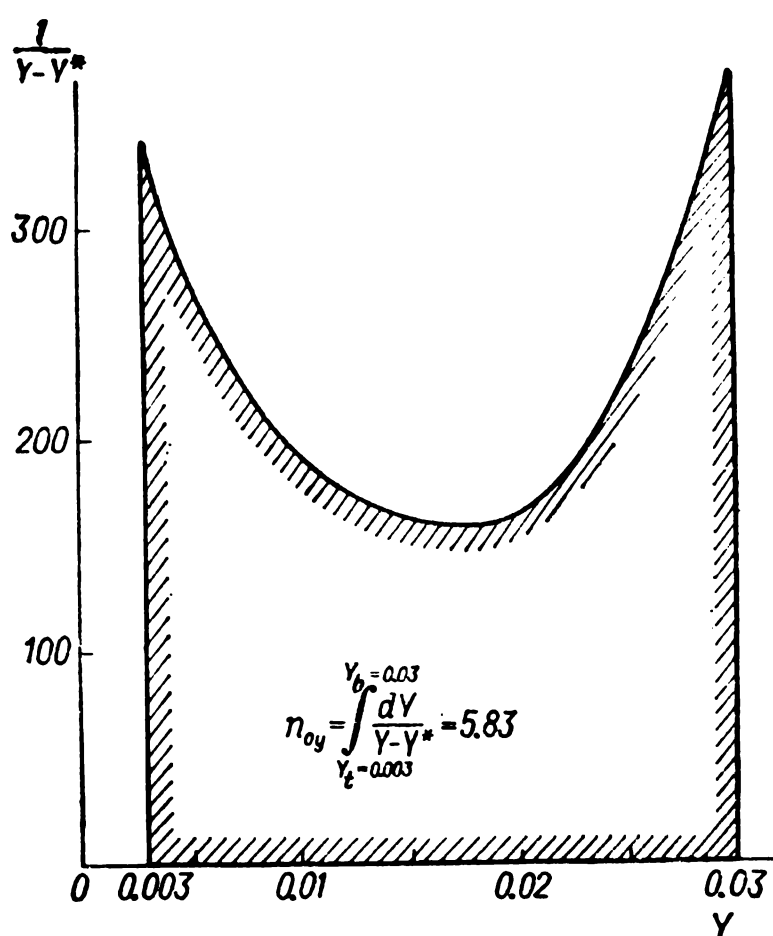


Fig. 6-7. Determination of the number of transfer units by graphical integration (to Example 6-10)

where $p_p = 981 \text{ kPa}$ (10 at) is the saturated vapour pressure of propane at 30°C .

The amount of propane in the gas mixture that has to be absorbed is:

$$G_p = \frac{Q_v y_p}{22.4} = \frac{1000 \times 0.15}{22.4} = 6.7 \text{ kmol/h}$$

The minimum rate of flow of the absorbent needed to absorb the propane is determined from the equation:

$$\frac{L_{\min} x_p^*}{1 - x_{i1}^*} = G_p$$

whence

$$L_{\min} = \frac{G_p (1 - x_p^*)}{x_p^*} = \frac{6.7 \times 0.955}{0.045} = 142 \text{ kmol/h} \quad 142 \times 224 = 31\,800 \text{ kg/h}$$

The maximum possible concentration of the butane in the absorbent leaving the absorber at its bottom is:

$$x_{\text{but}}^* = \frac{p}{p_{\text{but}}} y_{\text{but}} = \frac{294}{265} \times 0.1 = 0.11$$

where $p_{\text{but}} = 265 \text{ kPa}$ (2.7 at) is the saturated vapour pressure of butane at 30°C .

The amount of butane absorbed is:

$$G_{\text{but}} = \frac{Q_v y_{\text{but}}}{22.4} = \frac{1000 \times 0.1}{22.4} = 4.47 \text{ kmol/h}$$

The minimum rate of flow of the absorbent needed to absorb the butane is:

$$L'_{\min} = \frac{G_{\text{but}} (1 - x_{\text{but}})}{x_{\text{but}}} = \frac{4.47 \times 0.89}{0.11} = 36.1 \text{ kmol/h}$$

The minimum rate of flow of the absorbent for the complete absorption of the butane is considerably lower than for the absorption of the propane, therefore the amount of the absorbent found above (142 kmol/h) will be sufficient to completely absorb the butane.

Example 6-12. Determine the individual mass transfer coefficient for the gaseous phase in a packed absorber used to absorb sulphur dioxide from an inert gas (nitrogen) under atmospheric pressure. The temperature in the absorber is 20°C and it operates in film conditions. The velocity of the gas in the absorber (fictitious) is 0.35 m/s. The absorber is filled with lumps of coke ($\sigma = 42 \text{ m}^2/\text{m}^3$, $V_{\text{free}} = 0.58 \text{ m}^3/\text{m}^3$).

Solution. By Eq. (6-52), we have

$$Nu'_g = 0.407 Re_g^{0.655} (Pr'_g)^{0.33}$$

where

$$Re_g = \frac{4v\rho_g}{\sigma\mu_g} = \frac{4 \times 0.35 \times 1.16}{42 \times 0.0175 \times 10^{-3}} = 2210$$

$$\rho_g = \frac{28 \times 273}{22.4 \times 293} = 1.16 \text{ kg/m}^3$$

$$\mu_g = 0.0175 \times 10^{-3} \text{ Pa}\cdot\text{s} \text{ (Fig. A-6)}$$

We assume that the diffusion coefficient is the same as in air:

$$D_g = 10.3 \times 10^{-6} \left(\frac{293}{273} \right)^{1.6} = 11.45 \times 10^{-6} \text{ m}^2/\text{s} \text{ (Table A-42)}$$

$$Pr'_g = \frac{\mu_g}{\rho_g D_g} = \frac{0.0175 \times 10^{-3}}{1.16 \times 11.45 \times 10^{-6}} = 1.32$$

The Nusselt number for mass transfer is:

$$Nu'_g = 0.407 \times 2210^{0.655} \times 1.32^{0.33} = 69$$

From Eq. (6-53), we have

$$\beta_g = \frac{Nu'_g D_g}{d_{\text{eq}}}$$

The equivalent diameter, by Eq. (6-45), is:

$$d_{\text{eq}} = \frac{4V_{\text{free}}}{\sigma} = \frac{4 \times 0.58}{42} = 0.055 \text{ m}$$

and the individual mass transfer coefficient is

$$\beta_g = \frac{69 \times 11.45 \times 10^{-6}}{0.055} = 144 \times 10^{-4} \text{ m/s}$$

Example 6-13. Use dimensionless formula (6-52) to derive an equation for determining the height of a transfer unit with respect to the gaseous phase.

Solution. From Eq. (6-15), we have

$$\frac{1}{K_y} = \frac{1}{\beta_y} + \frac{m}{\beta_x}$$

in which K_y , β_y , and β_x are expressed in $\text{kmol}/\text{m}^2 \cdot \text{s}$; after multiplying both sides of the latter equation by $G/A_c \sigma$ and the last term by L/L , we get

$$\frac{G}{K_y A_c \sigma} = \frac{G}{\beta_y A_c \sigma} + \frac{mG}{L} \frac{L}{\beta_x A_c \sigma}$$

or in accordance with Eq. (6-50) with $\psi = 1$:

$$h_{oy} = h_y + \frac{mG}{L} h_x$$

where G and L = molar rates of flow of the gas and the liquid, kmol/s

A_c = cross-sectional area of the absorber column, m^2

σ = unit surface area of the packing, m^2/m^3

$h_y = \frac{G}{\beta_y A_c \sigma}$ = height of a transfer unit for the gaseous phase, m

$h_x = \frac{L}{\beta_x A_c \sigma}$ = ditto for the liquid phase, m.

By Eq. (6-53), we have

$$Nu'_g = \frac{\beta_g d_{eq}}{D_g}$$

where the individual mass transfer coefficient β_g is expressed in $\text{kmol}/\text{m}^2 \cdot \text{s} \cdot \text{kmol} \cdot \text{m}^{-3} = \text{m}/\text{s}$. We find the relationship between β_y and β_g from the equation

$$\beta_y \Delta y = \beta_g \Delta c_y$$

whence

$$\beta_y = \beta_g \frac{\Delta c_y}{\Delta y} = \beta_g \frac{\rho_g}{M_{\text{mix}}} \quad (\text{see Table 6-2})$$

Therefore,

$$h_y = \frac{G}{\beta_y A_c \sigma} = \frac{GM_{\text{mix}}}{\beta_g \rho_g A_c \sigma} = \frac{v}{\beta_g \sigma}$$

and

$$\beta_g = \frac{v}{h_y \sigma}$$

where $v = \frac{GM_{\text{mix}}}{\rho_g A_c}$ is the fictitious velocity of the gas, m/s .

Introducing the found value of β_g into Eq. (6-53), multiplying the numerator and denominator by $4\mu_g\rho_g$, rearranging the quantities, and substituting for the first two multipliers the dimensionless numbers Re_g and Pr'_g according to Eqs. (6-43) and (6-54), respectively, we get:

$$Nu'_g = \frac{vd_{eq}}{h_y\sigma D_g} \frac{4\mu_g\rho_g}{4\mu_g\rho_g} = \frac{4v\rho_g}{\sigma\mu_g} \frac{\mu_g}{D_g\rho_g} \frac{d_{eq}}{4h_y} = \frac{Re_g Pr'_g d_{eq}}{4h_y}$$

Equating the right-hand side of this equation to the right-hand side of Eq. (6-52):

$$Nu' = 0.407 Re_g^{0.655} (Pr'_g)^{0.33}$$

and solving the resulting equation with respect to h_y , we obtain:

$$h_y = 0.615 d_{eq} Re_g^{0.345} (Pr'_g)^{0.67}$$

Example 6-14. A scrubber with randomly packed ceramic rings $50 \times 50 \times 5$ mm in size is used to absorb carbon dioxide with water from a gas under a pressure of $p_{abs} = 16$ at (1.57 MPa) at a temperature of 22°C . The mean molar mass of the gas is 20.3 kg/kmol, the dynamic viscosity of the gas in the operating conditions is 1.31×10^{-5} Pa·s, the diffusion coefficient of CO_2 in the inert portion of the gas is 1.7×10^{-6} m²/s. The mean fictitious velocity of the gas in the scrubber is 0.041 m/s, the irrigation density (the fictitious velocity of the liquid) is 0.064 m³/m²·s. Determine the total height of a transfer unit h_{0y} , assuming that the packing is completely wetted ($\psi = 1$).

Solution. The total height of a transfer unit (see the preceding problem) is:

$$h_{0y} = h_y + \frac{mG}{L} h_x$$

We find h_y — the height of a transfer unit for the gaseous phase:

$$h_y = 0.615 d_{eq} Re_g^{0.345} (Pr'_g)^{0.67}$$

We take the characteristics of the packing $V_{free} = 0.785$ m³/m³ and $\sigma = 87.5$ m²/m³ from Table A-17. Thus,

$$d_{eq} = \frac{4V_{free}}{\sigma} = \frac{4 \times 0.785}{87.5} = 0.0359 \text{ m}$$

The density of the gas is:

$$\rho_g = \frac{M_g p_0 T_0}{22.4 p_0 T} = \frac{20.3 \times 16 \times 273}{22.4 \times 295} = 13.4 \text{ kg/m}^3$$

Hence, by Eq. (6-43)

$$Re_g = \frac{4v\rho_g}{\sigma\mu_g} = \frac{4 \times 0.041 \times 13.4}{87.5 \times 1.31 \times 10^{-5}} = 1920$$

and by Eq. (6-54):

$$Pr'_g = \frac{\mu_g}{\rho_g D_g} = \frac{1.31 \times 10^{-5}}{13.4 \times 1.7 \times 10^{-6}} = 0.575$$

The height of a transfer unit for the gaseous phase is:

$$h_y = 0.615 \times 0.0359 \times 1920^{0.345} \times 0.575^{0.67} = 0.205 \text{ m}$$

We find h_x —the height of a transfer unit for the liquid phase—by the equation:

$$h_x = 119 \delta_r Re_{lq}^{0.25} (Pr'_{lq})^{0.5}$$

which has been obtained from Eq. (6-55) in the same way as we obtained the expression for h_y from Eq. (6-52) in the preceding example.

The values of the physicochemical constants for water at 22 °C are $\rho_{lq} = 1000 \text{ kg/m}^3$, $\mu_{lq} = 0.958 \times 10^{-3} \text{ Pa}\cdot\text{s}$ (Table A-6), and $D_{lq} = 1.87 \times 10^{-9} \text{ m}^2/\text{s}$ (Table A-43).

The reduced thickness of the liquid film is:

$$\delta_r = \left(\frac{\mu_{lq}^2}{\rho_{lq}^2 g} \right)^{1/3} = \left(\frac{0.958^2 \times 10^{-6}}{10^6 \times 9.81} \right)^{1/3} = 4.55 \times 10^{-5} \text{ m}$$

According to the initial conditions, the irrigation density is:

$$\frac{L_m}{A_c \rho_{lq}} = 0.064 \text{ m}^3/\text{m}^2 \cdot \text{s}$$

where L_m is the mass rate of flow of the liquid, kg/s.

The mass irrigation density is:

$$\frac{L_m}{A_c} = 0.064 \rho_{lq} = 64 \text{ kg/m}^2 \cdot \text{s}$$

By Eq. (6-62), we have:

$$Re_{lq} = \frac{4L_m}{A_c \sigma \psi \mu_{lq}} = \frac{4 \times 64}{87.5 \times 0.958 \times 10^{-3}} = 3060$$

and by Eq. (6-58):

$$Pr'_{lq} = \frac{\mu_{lq}}{\rho_{lq} D_{lq}} = \frac{0.958 \times 10^{-3}}{1000 \times 1.87 \times 10^{-9}} = 512$$

The height of a transfer unit for the liquid phase is:

$$h_x = 119 \times 4.55 \times 10^{-5} \times 3060^{0.25} \times 512^{0.5} = 0.91 \text{ m}$$

We find the ratio of the molar rates of flow of the gas and the liquid G/L . From the equation of the rate of flow of a gas $v = GM_g/\rho_g A_c$ [see Eq. (1-23)], we get:

$$\frac{G}{A_c} = \frac{v \rho_g}{M_g} = \frac{0.041 \times 13.4}{20.3} = 0.0271 \text{ kmol/m}^2 \cdot \text{s}$$

For a liquid, we have:

$$\frac{L}{A_c} = \frac{L_m}{A_c M_{lq}} = \frac{64}{18} = 3.56 \text{ kmol/m}^2 \cdot \text{s}$$

Hence,

$$\frac{G}{L} = \frac{0.0271}{3.56} = 0.00761$$

The distribution coefficient m in Eq. (6-15) is [see Eq (6-3)]:

$$m = \frac{k}{p}$$

By interpolating the data of Table A-41, we get Henry's coefficient $k = 1.144 \times 10^6$ mm Hg (at 22°C), and

$$m = \frac{1.144 \times 10^6}{16 \times 735} = 97.3$$

The total height of a transfer unit is:

$$h_{0y} = h_y + \frac{mG}{l} h_x = 0.205 + 97.3 \times 0.00761 \times 0.91 = 0.88 \text{ m}$$

Example 6-15. Using the data of Example 6-8, determine the number of transfer units in the absorber with account taken of reverse (longitudinal) agitation.

Solution. The number of transfer units for the conditions of ideal displacement, i. e. without account taken of reverse agitation, is:

$$n_{0y} = \frac{Y_b - Y_t}{\Delta Y_m} = \frac{0.0639 - 0.00128}{0.0079} = 7.93$$

We find the number of transfer units with account taken of reverse agitation by the equation *

$$\frac{1}{n'_{0y}} = \frac{1}{n_{0y}} - \frac{1}{n_{\text{rev}}}$$

in which the correction for reverse agitation n_{rev} is:

$$n_{\text{rev}} = \frac{C \ln C}{C - 1} \Phi + Pe_{\text{red}}$$

where:

$$C = \frac{L}{mG}$$

$$\Phi = 1 - \frac{0.05}{(Pe'_{\text{red}})^{0.25}} \left(\frac{A}{n'_{0y}} \right)^{0.5}$$

* Migauchi, T. and Vermeulen, T. *Ind. Eng. Chem. Fund.* 2, 2: 113 (1963).

The value of the dimensionless number Pe'_{red} is calculated by the equation:

$$\frac{1}{Pe'_{red}} = \frac{1}{Cf_g Pe'_{m,g}} + \frac{1}{f_{lq} Pe'_{m,lq}}$$

Here

$$f_g = \frac{n'_{0y} + 6.8C^{0.5}}{n'_{0y} + 6.8C^{1.5}} \quad \text{and} \quad f_{lq} = \frac{n'_{0y} + 6.8C^{0.5}}{n'_{0y} + 6.8C^{-0.5}}$$

$$Pe'_{m,g} = \frac{v_g h}{E_g} \quad \text{and} \quad Pe'_{m,lq} = \frac{v_{lq} h}{E_{lq}} = \text{modified Peclet numbers for the gas and the liquid, respectively}$$

v_g and v_{lq} = velocities of the gas and liquid streams, m/s
 E_g and E_{lq} = corresponding coefficients of reverse (longitudinal) agitation, m²/s

h = operating length of the apparatus—the height of the packing layer, m.

According to the data of Example 6-8, we obtain:

$$\frac{L'}{G'} = \frac{Y_b - Y_t}{X_b - X_t} = \frac{0.0639 - 0.00128}{0.0234 - 0} = 2.67$$

$$m = \frac{dY^*}{dX} = 1.68$$

$$C = \frac{L'}{mG'} = \frac{2.67}{1.68} = 1.59$$

We preliminary assume that $n'_{0y} = 9$. Hence,

$$f_g = \frac{9 + 6.8 \times 1.59^{0.5}}{9 + 6.8 \times 1.59^{1.5}} = 0.78 \quad \text{and} \quad f_{lq} = \frac{9 + 6.8 \times 1.59^{0.5}}{9 + 6.8 \times 1.59^{-0.5}} = 1.22$$

To determine the velocities of the gas and the liquid v_g and v_{lq} , we must find the fraction of the cross section of the absorber occupied by each stream separately. The fraction of the packing volume δ occupied by the liquid is calculated by the equation [6-5]:

$$\delta = 4.83 \times 10^{-4} \frac{\sigma S^{0.435}}{d_{eq}^{0.24}} \quad (a)$$

in which:

$$S = \frac{L_{sec}}{A_c \sigma} = \frac{3000}{3600 \times 0.358 \times 204} = 0.0114 \text{ kg/s} \cdot \text{m}$$

$$d_{eq} = \frac{4V_{free}}{\sigma} = \frac{4 \times 0.74}{204} = 0.0145 \text{ m}$$

Using these values in Eq. (a), we get:

$$\delta = \frac{4.83 \times 10^{-4} \times 204 \times 0.0114^{0.435}}{0.0145^{0.24}} = 0.039$$

The velocity of the liquid in the packing layer is:

$$v_{1q} = \frac{L_{\text{sec}}}{\rho_{1q} A_c \delta} = \frac{3000}{3600 \times 1000 \times 0.358 \times 0.039} = 0.06 \text{ m/s}$$

The velocity of the gas is:

$$v_g = \frac{G_{\text{sec}}}{\rho_g A_c (V_{\text{free}} - \delta)} = \frac{1810}{3600 \times 1.2 \times 0.358 (0.74 - 0.039)} = 1.67 \text{ m/s}$$

The values of the coefficients of reverse agitation E_{1q} and E_g are found experimentally—see Example 1-37. For their approximate determination in a packed absorber, we use the dimensionless equations*:

for the liquid phase:

$$\frac{v_{1q} d_p}{E_{1q}} = 7.58 \times 10^{-3} Re_{1q}^{0.703}$$

for the gaseous phase:

$$\frac{v_g d_p}{E_g} = 2.4 Re_g^{-0.2} \times 10^{-0.002} Re_{1q}$$

In these equations:

$$Re_{1q} = \frac{d_p L_{\text{sec}}}{A_c \mu_{1q}} \text{ and } Re_g = \frac{d_p G_{\text{sec}}}{A_c \mu_g}$$

where d_p is the nominal size of the packing elements, m.

In our case, we have:

$$Re_{1q} = \frac{0.025 \times 3000}{3600 \times 0.358 \times 1 \times 10^{-3}} = 58$$

$$Re_g = \frac{0.025 \times 1810}{3600 \times 0.358 \times 0.0182 \times 10^{-3}} = 1930$$

For the liquid phase

$$\frac{v_{1q} d_p}{E_{1q}} = 7.58 \times 10^{-3} \times 58^{0.703} = 130 \times 10^{-3}$$

The coefficient of reverse agitation in the liquid phase is:

$$E_{1q} = \frac{v_{1q} d_p}{0.13} = \frac{0.06 \times 0.025}{0.13} = 0.0116 \text{ m}^2/\text{s}$$

and

$$Pe'_{m,1q} = \frac{v_{1q} h}{E_{1q}} = \frac{0.06 \times 16.8}{0.0116} = 87$$

* Sater, V. E. and Levenspiel, O. *Ind. Eng. Chem. Fund.*, **5**, 1: 86 (1966).

For the gaseous phase:

$$\frac{v_g d_p}{E_g} = \frac{2.4}{Re_g^{0.2} \times 10^{0.002 Re_{lq}}} = \frac{2.4}{1930^{0.2} \times 10^{0.002 \times 58}} = 0.403$$

The coefficient of reverse agitation in the gaseous phase is:

$$E_g = \frac{v_g d_p}{0.403} = \frac{1.67 \times 0.025}{0.403} = 0.104 \text{ m}^2/\text{s}$$

and

$$Pe'_{m,g} = \frac{v_g h}{E_g} = \frac{1.67 \times 16.8}{0.104} = 270$$

The reduced Peclet dimensionless number is:

$$Pe'_{red} = \left(\frac{1}{C f_g Pe'_{m,g}} + \frac{1}{f_{lq} Pe'_{m,lq}} \right)^{-1} = \\ = \left(\frac{1}{1.59 \times 0.78 \times 270} + \frac{1}{1.22 \times 87} \right)^{-1} = 80.5$$

Hence

$$\Phi = 1 - \frac{0.05}{(Pe'_{red})^{0.25}} \left(\frac{C}{n'_{oy}} \right)^{0.5} = 1 - \frac{0.05}{80.5^{0.25}} \left(\frac{1.59}{9} \right)^{0.5} = 1 - 0.007 = 0.993$$

The correction for reverse agitation is:

$$n_{rev} = \frac{C 2.3 \log C}{C - 1} \Phi + Pe'_{red} = \frac{1.59 \times 2.3 \log 1.59}{0.59} \times 0.993 + 80.5 = 81.7$$

The number of transfer units with account taken of reverse agitation is:

$$n'_{oy} = \left(\frac{1}{n_{oy}} - \frac{1}{n_{rev}} \right)^{-1} = \left(\frac{1}{7.93} - \frac{1}{81.7} \right)^{-1} = 8.8$$

which is close to the value $n'_{oy} = 9$ adopted at the beginning of our calculations.

PROBLEMS

6-1. Two equal volumes of benzene and nitrobenzene have been mixed. Considering that the volume of the liquid mixture equals the sum of the volumes of the components, determine the density of the mixture, the mass ratio \bar{X} of the nitrobenzene, and its molar-volume concentration c_x .

6-2. A liquid mixture has the following composition in mole per cent: chloroform—20, acetone—40, and carbon disulphide—40. Determine the density of the mixture considering that no change in volume occurs upon mixing.

6-3. Air is saturated with the vapour of ethyl alcohol. The total pressure of the air-vapour mixture is 600 mm Hg, the temperature is 60 °C. Assuming that both components of the mixture are ideal gases, determine the mass ratio \bar{Y} of the ethyl alcohol in the mixture and the density of the mixture.

6-4. A gas has the following composition in mole per cent: hydrogen—26, methane—60, ethylene—14. Its pressure is $p_{\text{abs}}=30$ at, and its temperature is 20°C . Considering the components of the mixture to be ideal gases, determine their mass-volume concentrations \bar{c}_y (in kg/m^3).

6-5. Show that in the equation

$$y = \frac{c_y M_B}{\rho + c_y (M_B - M_A)} \quad (\text{see Table 6-2})$$

y cannot be negative at any values of M_B and M_A .

6-6. For the conditions of Example 6-3(a), determine the driving force of the mass transfer process at the initial moment with respect to the gaseous and the liquid phases in molar-volume and mass-volume concentrations.

6-7. The vapour of the binary mixture chloroform-benzene containing 50 mole per cent each of chloroform and benzene enters into contact with a liquid containing 44 mole per cent of chloroform and 56 mole per cent of benzene. The pressure is atmospheric. Determine (a) the initial and final phases of the chloroform and benzene, and (b) the driving force of the mass transfer process with respect to the vapour and the liquid phases at the entry of the vapour into the liquid (in mole fractions). For data on the equilibrium compositions see Table A-47.

6-8. A mixture of air and carbon tetrachloride vapour compressed to an absolute pressure of 10 at is cooled in a tubular water cooler. Condensation of the carbon tetrachloride begins at 40°C . Determine its mass fraction (in per cent) in the air in the initial mixture and the factor of extraction from the gas mixture after cooling it to 27°C . For the saturated vapour pressure of carbon tetrachloride see Fig. A-14 or A-24.

6-9. A gas mixture containing 0.8% (by volume) of octane is compressed by a compressor to $p_{\text{abs}}=5$ at and is then cooled to 25°C . Determine the extraction factor of the octane. How will it change when the compressed gas mixture is cooled with cooler brine to 0°C ? For the saturated vapour pressure of octane see Fig. A-14, point 31.

6-10. Calculate the coefficients of molecular diffusion under atmospheric pressure (a) of benzene vapour in toluene vapour at a temperature of 100°C , and (b) of ethyl alcohol vapour in water vapour at a temperature of 92°C .

6-11. Determine the overall mass transfer coefficient in an absorber irrigated with water for which $\beta_y=2.76 \times 10^{-3} \text{ kmol}/\text{m}^2 \cdot \text{h} \cdot \text{kPa}$, $\beta_x=1.17 \times 10^{-4} \text{ m/s}$ and the pressure is $p_{\text{abs}}=1.07$ at. The equation of the equilibrium line in mole fractions is $y^*=1.02x$.

6-12. Determine the mean driving force and the total number of transfer units n_{0y} when benzene vapour is absorbed from a gas with oil. The initial concentration of the benzene in the gas is 4% (by volume), 80% of the benzene is extracted. The concentration of the benzene in the oil leaving the scrubber is 0.02 kmol of benzene per kmol of pure oil. The oil fed into the scrubber contains no benzene. The equation of the equilibrium line in mole ratios is:

$$Y^*=0.126X$$

Express the driving force in units of Y (kmol of benzene per kmol of inert gas).

6-13. Sulphur dioxide is absorbed by water from an inert gas (nitrogen) in a scrubber under atmospheric pressure (760 mm Hg). The initial content of the sulphur dioxide in the gas is 5% (by volume). The temperature of the water is 20°C , and its rate of flow is 20% greater than the minimum theoretical value. The amount of SO_2 extracted from the gas is 90%. Determine (1) the rate of flow of the water needed to absorb 1000 kg/h of sulphur dioxide, (2) the mean driving force of the process, and (3) the total number of transfer units n_{0y} . The equilibrium line may be assumed to be a straight line; the coordinates of two of its points are: (1) partial pressure of the SO_2 in the

gaseous phase $p = 39$ mm Hg, $\bar{X} = 0.007$ kg of SO_2 /kg of water; and (2) $p = 26$ mm Hg, $\bar{X} = 0.005$ kg of SO_2 /kg of water.

6-14. A packed absorber is used to absorb the vapour of methyl alcohol with water from a gas under atmospheric pressure at a mean temperature of 27°C . The content of the methyl alcohol in the gas fed into the scrubber is 100 g per m^3 of the inert gas (taking the volume of the gas in operating conditions). At the outlet from the scrubber, the water has a concentration of 67% of the maximum possible one, i. e. of the equilibrium one with the incoming gas. The equation of solubility of methyl alcohol in water in mole ratios is $Y^* = 1.15X$. The fraction of the initial amount of the alcohol extracted by the water is 98%. The overall mass transfer coefficient is $K_x = 0.5$ kmol of alcohol/ $\text{m}^2 \cdot \text{h}$ (kmol of alcohol/kmol of water). The rate of flow of the inert gas is $1200 \text{ m}^3/\text{h}$ (in operating conditions). The absorber is filled with ceramic ring packing having a unit surface area of $190 \text{ m}^2/\text{m}^3$. The coefficient of wetting of the packing is $\psi = 0.87$. The fictitious velocity of the gas in the absorber is $v = 0.4$ m/s. Determine the rate of flow of the water and the required height of the packing layer.

6-15. A scrubber with a diameter of 0.5 m is supplied with $550 \text{ m}^3/\text{h}$ of air (at 760 mm Hg and 20°C) containing 2.8% (by volume) of ammonia which is absorbed by water under atmospheric pressure. The extraction factor is 0.95. The rate of flow of the water is 40% greater than the theoretically minimum value. Determine: (1) the rate of flow of the water, (2) the total number of transfer units n_{0y} , and (3) the depth of the packing layer of ceramic rings $50 \times 50 \times 5$ mm in size. The overall mass transfer coefficient is $K_y = 0.001$ kmol of ammonia/ $\text{m}^2 \cdot \text{s}$ (kmol of ammonia/kmol of air). Take the data on the equilibrium concentrations of the liquid and gas from Example 6-10. The coefficient of wetting of the packing is $\psi = 0.9$.

6-16. Derive a formula for determining the height of a transfer unit in a packed absorber for the liquid phase h_x from dimensionless equation (6-55).

6-17. Air with a mixture of ammonia is passed through a scrubber irrigated with water and packed with rings having a unit surface area of $89.5 \text{ m}^2/\text{m}^3$. The free volume of the packing is $0.79 \text{ m}^3/\text{m}^3$. The temperature of absorption is 28°C , the absolute pressure is 1 at. The mean content of the ammonia in the gas mixture is 5.8% (by volume). The mass rate of flow of the gas related to the total cross section of the scrubber is $1.1 \text{ kg}/\text{m}^2 \cdot \text{s}$. Determine the individual coefficient of mass transfer for the gas considering that the scrubber is operating in film conditions.

6-18. Calculate the individual mass transfer coefficient for the liquid phase in a packed absorber in which carbon dioxide is absorbed by water at a temperature of 20°C . The density of irrigation is $60 \text{ m}^3/\text{m}^2 \cdot \text{h}$. The packing is dumped ceramic rings $35 \times 35 \times 4$ mm in size. The coefficient of wetting of the packing is $\psi = 0.86$.

6-19. Determine the individual mass transfer coefficient for a gas in a scrubber when benzene vapour is being absorbed from coke gas in the following conditions: grid packing with bars 12.5×100 mm in size having a spacing of $b = 25$ mm is used (for such packing $d_{eq} = 2b = 0.05$ m); the velocity of the gas calculated for the entire cross section of the scrubber is 0.95 m/s; the density of the gas is $0.5 \text{ kg}/\text{m}^3$; the dynamic viscosity of the gas is $0.013 \text{ mPa} \cdot \text{s}$; the coefficient of diffusion of benzene in the gas is $16 \times 10^{-6} \text{ m}^2/\text{s}$. Consider that film conditions prevail in the column.

6-20. Determine the diameter and height of a plate absorber for absorbing ammonia with water from an air-ammonia mixture at atmospheric pressure and a temperature of 20°C . The initial content of the ammonia in the gas mixture is 7% (by volume). The extraction factor is 90%. The rate of flow of the inert gas (air) is $10\,000 \text{ m}^3/\text{h}$ in the operating conditions. Consider the equilibrium line to be straight; its equation in mass ratios is $\bar{Y}^* = 0.61\bar{X}$. The fictitious velocity of the gas in the absorber is 0.8 m/s. The distance between

plates is 0.6 m. The mean efficiency of the plates is 0.62. The excess absorbent coefficient is $\varphi = 1.3$.

6-21. Using the conditions of the preceding problem, determine (1) the height of an absorber packed with ceramic rings $50 \times 50 \times 5$ mm in size, assuming that h_{eq} —the height of the packing layer equivalent to a theoretical plate (HETP) equals 0.85 m, and (2) the overall mass transfer coefficient K_y kg of ammonia/ $m^2 \cdot s$ (kg of ammonia/kg of air), considering the coefficient of packing wetting ψ to equal 0.9.

6-22. Using the data of problems 6-20 and 6-21, determine the depth of the packed section through the total number of transfer units n_{oy} and the height of one transfer unit (HTU) h_{oy} .

6-23. An absorber for removing benzene vapour from a vapour and gas mixture is irrigated with absorbing oil having a molar mass of 260 kg/kmol. The mean pressure in the absorber is $p_{abs} = 800$ mm Hg, and the temperature is $40^\circ C$. The rate of flow of the vapour and gas mixture is $3600 m^3/h$ in the operating conditions. The concentration of the benzene in the gas mixture at the inlet to the absorber is 2% (by volume), and 95% of the benzene is extracted. The content of the benzene in the absorbing oil entering the absorber after regeneration is 0.2 mole per cent. The rate of flow of the absorbing oil is 1.5 times greater than the theoretical minimum value. To calculate the equilibrium compositions, assume that the solubility of benzene in the oil is determined by Raoult's law. Consider that the equilibrium relationship $Y^* = f(X)$ is linear at benzene concentrations in the liquid up to $X = 0.1$ kmol per kmol of oil.

Determine (1) the rate of flow of the absorbing oil in kg/h, (2) the concentration of the benzene in the absorbing oil leaving the absorber, (3) the diameter and height of a packed absorber for a fictitious velocity of the gas in it of 0.5 m/s and a transfer unit height (HTU) of $h_{oy} = 0.9$ m, and (4) the height of a plate absorber for a mean plate efficiency of 0.67 and a plate spacing of 0.4 m.

6-24. Sulphur dioxide is absorbed from air by water in a packed absorber with a diameter of 1 m. The initial content of the SO_2 in the entering mixture is 7% (by volume). The extraction factor is 0.9. The water leaving the absorber contains 0.0072 kg of SO_2 /kg of water. The overall mass transfer coefficient in the absorber is $K_y = 0.005$ kg of SO_2 / $m^2 \cdot s$ (kg of SO_2 /kg of air). The packing is of ceramic rings $50 \times 50 \times 5$ mm in size. The packing is completely wetted ($\psi = 1$). The height of a transfer unit is $h_{oy} = 1.17$ m. Determine the rate of flow of the water in the absorber.

6-25. Benzene is absorbed at a rate of 300 kg/h from a vapour-gas mixture in an absorber under atmospheric pressure at a temperature of $20^\circ C$. The initial content of the benzene vapour in the mixture is 4% (by volume). The extraction factor is 0.85. The liquid absorbent fed into the absorber after regeneration contains 0.0015 kmol of benzene per kmol of absorbent. The fictitious velocity of the gas in the absorber is 0.9 m/s. The equation of the equilibrium line is $Y^* = 0.2X$, where Y^* and X are expressed in kmol of benzene/kmol of inert gas and kmol of benzene/kmol of absorbent, respectively. The excess absorbent coefficient is $\varphi = 1.4$. Determine the diameter of the absorber and the concentration of the benzene in the absorbent leaving the absorber.

SYMBOLS

- A area; coefficient
- Ar Archimedes dimensionless number
- a unit wetted area of packing
- B coefficient
- b coefficient

C	coefficient; constant
c	molar-volume concentration
\bar{c}	mass-volume concentration
D	diameter; diffusion coefficient
d	diameter
E	coefficient of longitudinal mixing
e	absorption (extraction) factor
G	rate of flow of gas
g	acceleration due to gravity
h	height
K	overall mass transfer coefficient
k	Henry's coefficient (Henry law constant)
L	rate of flow of liquid
l	characteristic linear dimension
M	mass rate of flow; molar mass
m	distribution coefficient; slope of equilibrium line
Nu'	Nusselt dimensionless number for mass transfer
P	perimeter
Pe'	Peclet dimensionless number for mass transfer
Pr'	Prandtl dimensionless number for mass transfer
p	pressure
p^*	partial pressure
p°	pressure of pure component
Q_v	volumetric rate of flow
S	shape factor
T	absolute temperature
t	temperature, °C
V	volume; volumetric rate of flow
X	mole ratio, liquid phase
\bar{X}	mass ratio, liquid phase
x	mole fraction, liquid phase
\bar{x}	mass fraction, liquid phase
Y	mole ratio, gaseous phase
\bar{Y}	mass ratio, gaseous phase
y	mole fraction, gaseous phase
\bar{y}	mass fraction, gaseous phase

Greek Letters

α	coefficient of relative fugacity
β	individual (phase) coefficient of mass transfer
δ	fraction of volume
μ	chemical potential; dynamic viscosity
ν	kinematic viscosity
ρ	density
σ	unit surface area
Φ	phase
φ	excess absorbent coefficient
ψ	dimensionless coefficient of packing wetting

CHAPTER 7

DISTILLATION AND RECTIFICATION

FUNDAMENTAL RELATIONSHIPS AND FORMULAS FOR CALCULATIONS

The present chapter treats the separation of binary (two-component) mixtures by distillation and rectification. According to the phase rule, a two-phase two-component system has two degrees of freedom.

1. The equation of simple distillation is:

$$\ln \frac{F}{W} = \int_{x_W}^{x_F} \frac{dx}{y^* - x} \quad (7-1)$$

where F = feed (the initial amount of the mixture being distilled)
 W = bottom product (the residue of the liquid in the still or column after distillation)

y^* and x = equilibrium concentrations of the low-boiling component (the low boiler) in the vapour and in the liquid, respectively

x_F = content of the low boiler in the feed

x_W = content of the low boiler in the bottom product.

The average composition of the distilled liquid (overhead product) is:

$$x_D = \frac{Fx_F - Wx_W}{F - W} \quad (7-2)$$

Calculations by Eqs. (7-1) and (7-2) are performed using either molar amounts and molar concentrations, or mass amounts and mass concentrations.

2. When a liquid is distilled with steam (or an inert gas), the consumption of the steam is calculated by the equation:

$$G_w = G \frac{M_w (p_{\text{tot}} - p)}{M p \varphi} \quad (7-3)$$

where G_w = amount of the steam carried off with the vapour of the liquid being distilled, kg

G = amount of the liquid being distilled, kg

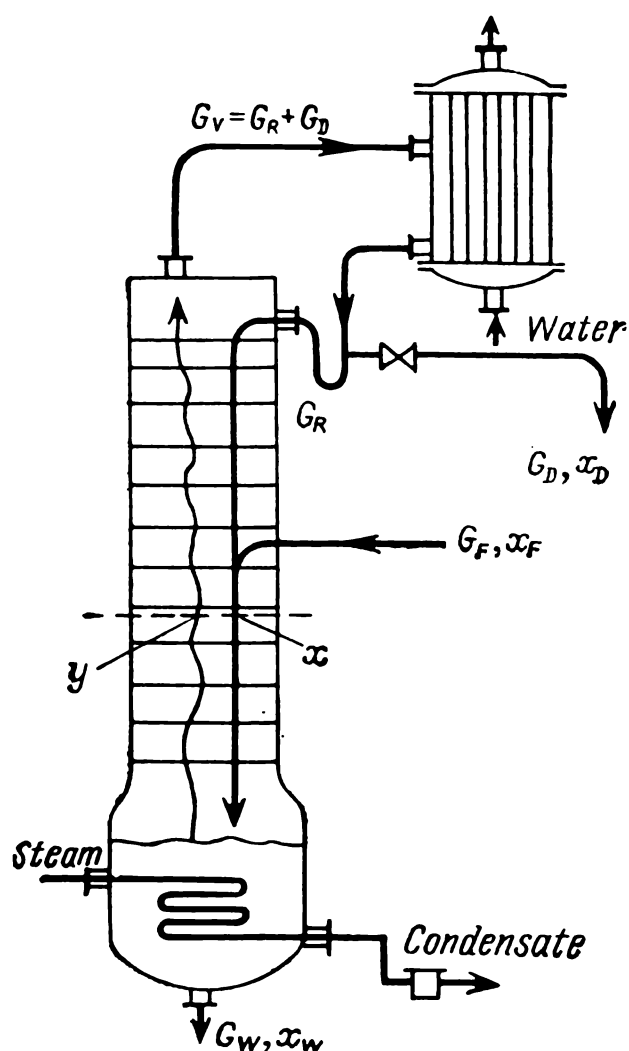


Fig. 7-1. Diagram of rectification column

M_w and M = molar masses of the water and the liquid being distilled, respectively

p = pressure of the saturated vapour of the liquid being distilled at the temperature of distillation

p_{tot} = total pressure of the mixture of vapour and steam

φ = coefficient taking into account the incomplete saturation of the steam with the vapour of the substance being distilled (in approximate calculations it is assumed that $\varphi = 0.7$ to 0.8).

3. The material balance equations of a continuous rectification column (Fig. 7-1) are*:

$$G_F = G_D + G_W \quad (7-4)$$

$$G_F x_F = G_D x_D + G_W x_W \quad (7-5)$$

where G_F , G_D , G_W = mass or molar consumptions of the feed, overhead product (distillate), and bottom product

x_F , x_D , x_W = mass or mole fractions of the low boiler in the feed, overhead and bottom products.

* The equations given below hold for a rectification column heated with indirect steam. When aqueous mixtures are being rectified with direct (live) steam, the equations of the material balance and the operating line of the bottom part of the column change—see Example 7-16.

4. The equations of the operating lines of a rectification column are:

(a) for the top (rectifying) section:

$$y = \frac{R}{R+1} x + \frac{x_D}{R+1} \quad (7-6)$$

(b) for the bottom (stripping) section:

$$y = \frac{R+F}{R+1} x - \frac{F-1}{R+1} x_W \quad (7-7)$$

Theoretical prerequisites indicate that in a rectification column, unlike an absorption one, the total molar rates of flow of the vapour and liquid are assumed to be constant along the height of the column. Accordingly, the molar rates of flow and concentrations are used in Eqs. (7-6) and (7-7).

In the top section of the column above the inlet of the liquid feed mixture, the molar rate of flow of the liquid constant along the height of the column equals G_R , and in the bottom section of the column it equals $(G_R + G_F)$ —see Fig. 7-1. The molar rate of flow of the vapour constant along the height of the column G_V is identical in its top and bottom sections.

The following notation is used in Eqs. (7-6) and (7-7): y and x are the non-equilibrium concentrations (mole fractions) of the low boiler in the vapour and the liquid in a given column cross section; they vary along the height of the column. For plate columns, y is the mole fraction of the low boiler in the vapour entering a plate from below; x is the mole fraction of the low boiler in the liquid flowing down from this plate; $R = G_R/G_D$ is the reflux ratio; $f = G_F/G_D = (x_D - x_W)/(x_F - x_W)$ is the relative (per kmole of overhead product) molar rate of flow of the feed.

When using the relative rates of flow, the equations of the material balance of a column (7-4) and (7-5) become:

$$f = 1 + w \quad (7-8)$$

$$fx_F = x_D + wx_W \quad (7-9)$$

where $w = G_W/G_D = (x_D - x_F)/(x_F - x_W)$ = relative molar rate of flow of the bottom product

x_F, x_D, x_W = mole fractions of the low boiler in the feed, overhead and bottom products.

5. The minimum reflux ratio R_{\min} in a continuous rectification column when the equilibrium curve has no points of inflection (hollows) is determined by the equation:

$$R_{\min} = \frac{x_D - y_F^*}{y_F^* - x_F} \quad (7-10)$$

where x_D = mole fraction of the low boiler in the overhead product
 x_F = ditto in the feed of the column
 y_F^* = ditto in the vapour in equilibrium with the feed liquid.

The operating (actual) reflux ratio is:

$$R = \psi R_{\min} \quad (7-11)$$

where $\psi > 1$ is the excess reflux coefficient.

In calculations of rectification columns, the operating reflux ratio is often determined by the formula:

$$R = 1.3R_{\min} + 0.3 \quad (7-12)$$

For details on the economically optimal reflux ratio see [6-3, 7-3].

If the equilibrium curve has a hollow, the quantity R_{\min} is determined graphically—see Example 7-16.

6. The consumption of heat in the reboiler of a continuous rectification column is determined from the heat balance equation for a column with a reflux condenser (see Fig. 7-1):

$$Q_b + G_F H_F = Q_{r.c} + G_D H_D + G_W H_W + Q_1 \quad (7-13)$$

where Q_b = amount of heat received by the boiling liquid from the condensing heating steam in the reboiler, W

$Q_{r.c}$ = amount of heat taken away by the cooling water from the vapour condensing in the reflux condenser, W

Q_1 = heat losses of the column to the surroundings, W

G_F, G_D, G_W = mass rates of flow of the feed, overhead and bottom products, kg/s

H_F, H_D, H_W = corresponding specific enthalpies, J/kg.

From Eq. (7-13), we get:

$$Q_b = Q_{r.c} + G_D c_D t_D + G_W c_W t_W - G_F c_F t_F + Q_1 \quad (7-14)$$

where c_D, c_W, c_F = specific heat capacities, J/kg·K

t_D, t_W, t_F = corresponding temperatures, K.

The amount of heat given up to the cooling water in the reflux condenser is:

$$Q_{r.c} = G_D (1 + R) L_D \quad (7-15)$$

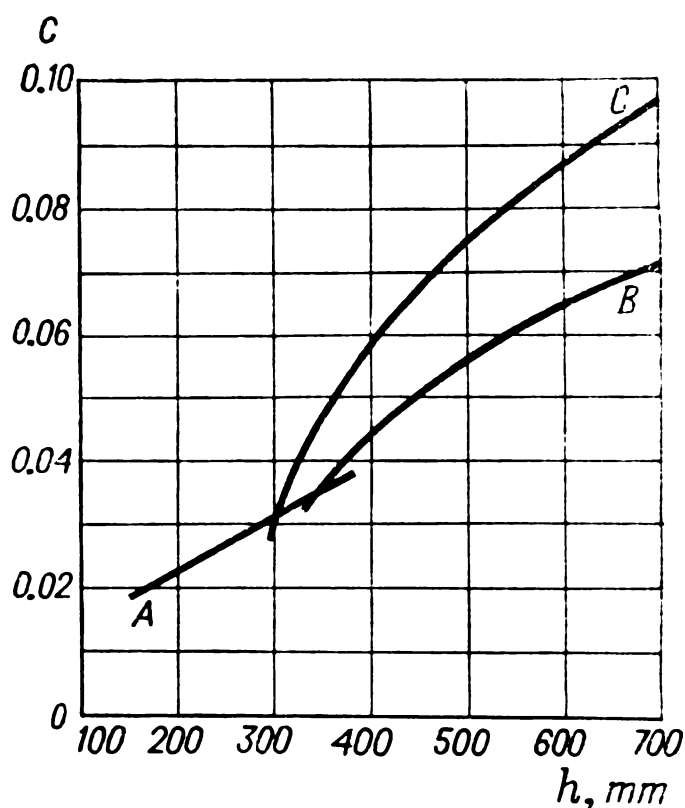
where R = reflux ratio

L_D = specific heat of condensation of vapour in the reflux condenser, J/kg.

7. The diameter of a plate rectification column is determined by the equation:

$$D = \sqrt{\frac{Q_v}{0.785v}} \quad (7-16)$$

Fig. 7-2. Values of coefficient C :
 A , B —bubble-cap plates with round caps;
 C —sieve plates



where Q_v = volumetric rate of flow of the vapour through the column, m^3/s

v = velocity of the vapour related to the total cross-sectional area of the column, m/s .

The permissible optimal velocity of the vapour in the column is calculated by the formula:

$$v = C \sqrt{\frac{\rho_{lq} - \rho_v}{\rho_v}} \quad (7-17)$$

where C = coefficient depending on the design of the plates, the distance between the plates, the operating pressure in the column, and the load on the column with respect to the liquid

ρ_{lq} and ρ_v = densities of the liquid and the vapour, respectively, kg/m^3 .

When $\rho_{lq} \gg \rho_v$, we have:

$$v = C \sqrt{\frac{\rho_{lq}}{\rho_v}} \quad (7-18)$$

The dependence of the coefficient C on the distance h between the plates for rectification columns that operate under atmospheric pressure and with medium loads with respect to the liquid are given in Fig. 7-2 according to data taken from [7-1].

After finding the diameter of the column by Eq. (7-16), it is determined more precisely in accordance with the standards in force.

8. The height of a plate rectification column (the distance Z_{p1} between the top and the bottom plates) is determined by the

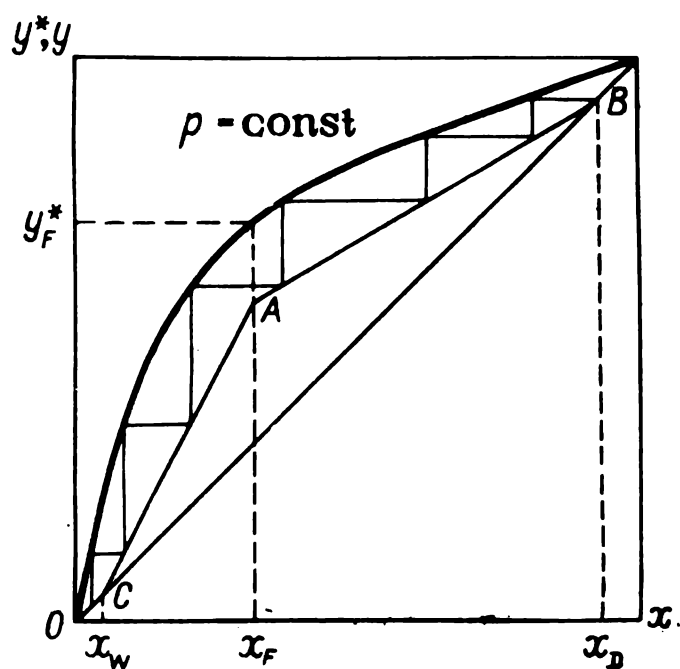


Fig. 7-3. Graphical determination of the number of stages of concentration change (theoretical plates) in a rectification column:

AB — operating line of the top section of the column; AC — operating line of the bottom section of the column

equation:

$$Z_{pl} = (n - 1)h \quad (7-19)$$

where n = number of plates in the column

h = distance between the plates.

The required number of plates n is determined graphically using kinetic laws—the equation of mass transfer and the relationships for calculating the individual mass transfer coefficients (or the numbers of transfer units) in the vapour and liquid phases on a plate [7-1, 7-3].

In approximate calculations, a theoretically less substantiated, but simpler method of determining the number of plates is used.

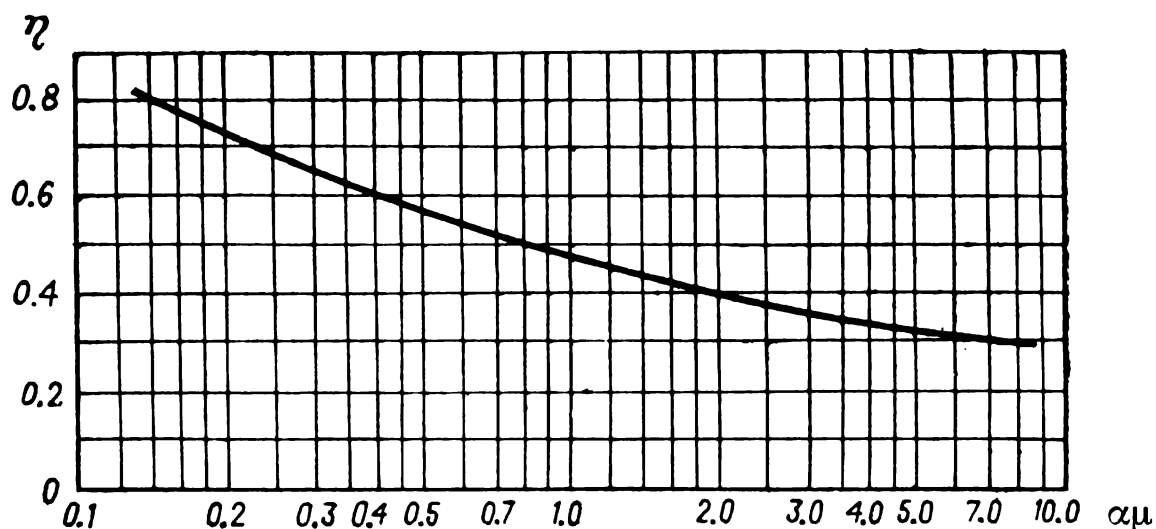


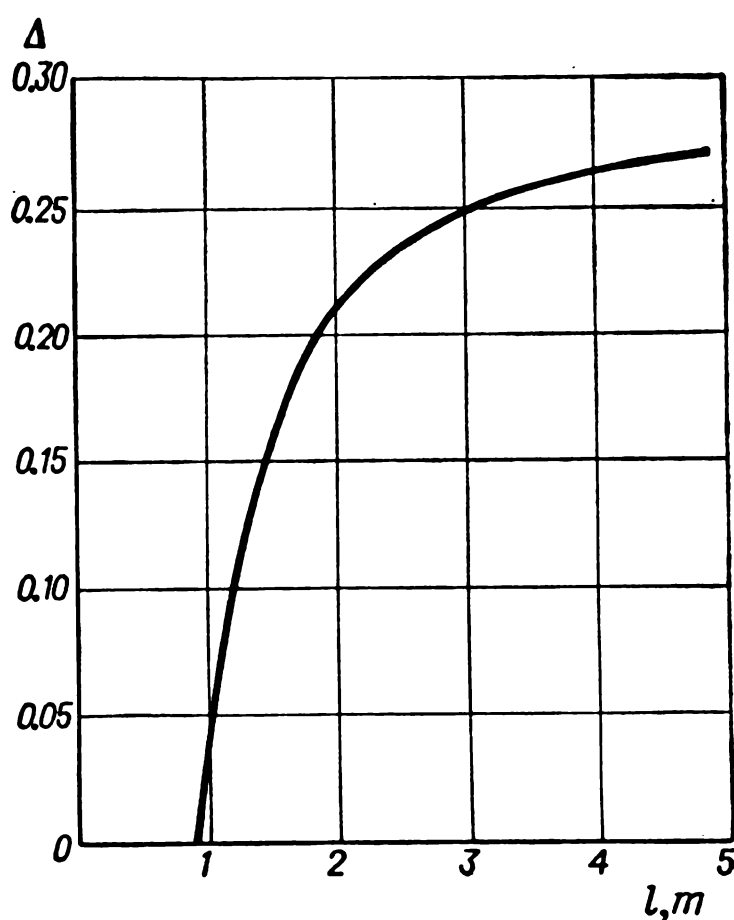
Fig. 7-4. Diagram for approximate determination of the overall plate efficiency

It employs the so-called overall plate efficiency (column efficiency) η :

$$n = \frac{n_{th}}{\eta} \quad (7-20)$$

where n_{th} is the number of theoretical (ideal) plates—stages of concentration change which is found by graphical construction

Fig. 7-5. Dependence of the correction Δ on the length l of the path of the liquid along a plate



between the equilibrium and the operating lines in a y - x diagram (see Fig. 7-3).

The value of the overall plate efficiency η which considers the real conditions of mass exchange on the plates depends on many variable quantities (the design and dimensions of the plates, hydrodynamic factors, the physicochemical properties of the vapour and liquid). The values of η are determined according to experimental data, and they are generally within the range from 0.3 to 0.8.

On plates with a cross flow of the vapour and liquid, an increase in the length of the path of the liquid over a plate, i.e. in the time spent by the liquid on the plate, results in better mass exchange, and the value of the overall efficiency η grows.

Figure 7-4 gives values of the overall plate efficiency obtained according to experimental data for industrial rectification columns of a comparatively small diameter. The efficiency is plotted against the product of the coefficient of relative volatility α of the components being separated [see Eq. (6-10)] and the dynamic viscosity of the feed liquid μ (in mPa·s) at the mean temperature in the column.

When determining the overall plate efficiency for columns having a large diameter (with the length l of the path travelled by the liquid exceeding 0.9 m), it is recommended to add the correction Δ to the values found in Fig. 7-4:

$$\eta_l = \eta(1 + \Delta) \quad (7-21)$$

The values of the correction Δ for mixtures with the product α_{μ} ranging from 0.1 to 1.0 are given in Fig. 7-5.

9. By the Murphree efficiency of a separate plate is meant the quantity

$$\eta_0 = \frac{y_2 - y_1}{y_1^* - y_1} \quad (7-22)$$

where y_1 = mole fraction of the low boiler in the vapour entering the plate from below

y_2 = the same in the vapour leaving the plate

y_1^* = the same in the vapour in equilibrium with the liquid flowing down from the plate.

If the liquid on the plate is completely miscible, then y_1^* is the mole fraction of the low boiler in the vapour in equilibrium with the liquid on the plate.

10. The diameter and height of a packed rectification column are determined in the same way as for packed absorption columns—see Chap. 6.

The diameter of a packed column is calculated by Eq. (6-39). The fictitious velocity of the vapour at the flooding point v_n when $\rho_{lq} \gg \rho_v$ is found by Eq. (6-40), but with a different value of the constant C . For rectification columns, $C = -0.125$.

The fictitious velocity of the vapour in packed rectification columns operating in conditions of the beginning of suspension is determined by Eq. (6-42)

The depth of the packed section h_p for film conditions of column operation is found separately for the top and for the bottom sections of the column by the equation:

$$h_p = \frac{G}{K_y A_c \sigma \psi} \int_{y_1}^{y_2} \frac{dy}{y^* - y} = h_{0y} n_{0y} \quad (7-23)$$

similar to Eq. (6-47).

In Eq. (7-23):

G = molar rate of flow of the vapour constant along the height of the column, kmol/s

K_y = overall mass transfer coefficient, kmol/m²·s ($\Delta y = 1$)

$A_c = \pi D^2/4$ = cross-sectional area of the column, m²

σ = unit surface area of the packing, m²/m³

ψ = dimensionless coefficient of packing wetting

y^* and y = equilibrium and operating mole fractions of the low boiler in the vapour

The equivalent depth of the packing h_{eq} (at the point of inversion) for packed rectification columns operating in emulsifying

conditions can be determined by the equation [6-3]:

$$\frac{h_{eq}}{d_{eq}} = 5.2 Re_v^{0.2} \left(\frac{G}{L} \right)^{0.35} \left(\frac{\rho_{liq}}{\rho_v} \right)^{0.2} \frac{\log \frac{L}{mG}}{1 - m \frac{G}{L}} \quad (7-24)$$

where $d_{eq} = 4V_{free}/\sigma$ = equivalent diameter of the packing, m

$Re_v = 4v\rho_v/\sigma\mu_v$, see Eq. (6-43)

G/L = ratio of flow rates of the vapour and the liquid (for the top section of a column $G/L = (R + 1)/R$, and for the bottom section $G/L = (R + 1)/(R + F)$ — see Eqs. (7-6) and (7-7)

m = slope of the equilibrium line.

The depth of the packed section h_p is found by Eq. (6-51).

EXAMPLES

Example 7-1. Calculate the composition of the equilibrium vapour phase at 50°C for a liquid consisting of a mixture of hexane and water, assuming them to be completely immiscible.

Solution. The saturated vapour pressure of hexane at 50°C is 400 mm Hg (Fig. A-17) and that of water at 50°C is 92.5 mm Hg (Table A-38). With complete mutual immiscibility of the components, the partial pressure p_i of each component equals the pressure of its saturated vapour p_i° .

The total pressure of the mixture of vapours is:

$$p = p_w + p_h = p_w^\circ + p_h^\circ = 400 + 92.5 = 492.5 \text{ mm Hg}$$

We find the mole fraction of the hexane in the vapour phase by Eq. (6-1):

$$y_h = \frac{p_h}{p} = \frac{400}{492.5} = 0.812$$

The mole fraction of the water is:

$$y_w = \frac{p_w}{p} = 1 - y_h = 0.188$$

Example 7-2. Determine the boiling point of a liquid mixture of toluene and water under atmospheric pressure, taking into account that they are completely immiscible.

Solution. The given mixture will boil at a temperature at which the sum of the saturated vapour pressures of the toluene and water equals 760 mm Hg.

Inspection of the diagram in Fig. A-24 shows that the intersection of the saturated vapour pressure curve for toluene with the relevant curve for water (descending from 760 mm Hg) corresponds to a temperature of 84°C. Since the point of intersection

of these curves corresponds to the temperature at which the sum of the saturated vapour pressures is 760 mm Hg, it is exactly this temperature that will be the boiling point of the mixture.

Example 7-3. Calculate the composition of the equilibrium vapour phase at 60°C for a liquid mixture consisting of 40 mole per cent of benzene and 60 mole per cent of toluene, considering that the given mixture is characterized by Raoult's law. Also find the composition of a liquid mixture of benzene and toluene that boils at 90°C under a pressure of 760 mm Hg.

Solution. We determine the saturated vapour pressures of benzene and toluene at 60°C from Fig. A-24: for benzene $p_b^\circ = 385$ mm Hg, for toluene $p_t^\circ = 140$ mm Hg.

We determine the partial pressures of the benzene and the toluene by Eq. (6-8):

$$p_b = p_b^\circ x_b = 385 \times 0.4 = 154 \text{ mm Hg}$$

$$p_t = p_t^\circ x_t = p_t^\circ (1 - x_b) = 140 (1 - 0.4) = 84 \text{ mm Hg}$$

The total pressure is:

$$p = p_b + p_t = 154 + 84 = 238 \text{ mm Hg}$$

We determine the composition of the vapour phase by Eq. (6-1):

$$y_b = \frac{p_b}{p} = \frac{154}{238} = 0.648$$

Thus, the equilibrium vapour contains 64.8 mole per cent of benzene and 35.2 mole per cent of toluene.

To find the composition of the liquid boiling under a pressure of 760 mm Hg at 90°C, we write the equation:

$$p = p_b^\circ x_b + p_t^\circ x_t \text{ or } 760 = 1013x_b + 408(1 - x_b)$$

whence $x_b = 58.3\%$ and $x_t = 41.7\%$.

Here 1013 and 408 are the saturated vapour pressures of pure benzene and toluene, respectively, at 90°C, mm Hg.

Example 7-4. Calculate the equilibrium phase compositions and construct equilibrium diagrams in the coordinates t against x and y , and y^* against x for a benzene-toluene mixture under atmospheric pressure, considering that the mixture is characterized by Raoult's law.

Solution. To calculate the equilibrium phase compositions, we use Eq. (6-8):

$$p_b = p_b^\circ x$$

$$p_t = p_t^\circ (1 - x)$$

According to Dalton's law, we have:

$$p = p_b + p_t = p_b^\circ x + p_t^\circ (1 - x)$$

whence

$$x = \frac{p - p_t^{\circ}}{p_b^{\circ} - p_t^{\circ}}$$

By Eq. (6-9), we have:

$$y^* = \frac{p_b^{\circ}}{p} x$$

Here x and y^* are the mole fractions of the benzene in the liquid and in the vapour in equilibrium with it.

All the calculations are given in Table 7-1.

TABLE 7-1

$t, ^\circ\text{C}$	p_b° mm Hg	p_t° mm Hg	p_t mm Hg	$x = \frac{p - p_t^{\circ}}{p_b^{\circ} - p_t^{\circ}}$	$y^* = \frac{p_b^{\circ}}{p} x$
80	760	300.0	760	1	1
84	852	333.0	760	$\frac{760 - 333}{852 - 333} = 0.823$	$\frac{852}{760} 0.823 = 0.922$
88	957	379.5	760	$\frac{760 - 379.5}{957 - 379.5} = 0.659$	$\frac{957}{760} 0.659 = 0.830$
92	1078	432.0	760	$\frac{760 - 432}{1078 - 432} = 0.508$	$\frac{1078}{760} 0.508 = 0.720$
96	1204	492.5	760	$\frac{760 - 492.5}{1204 - 492.5} = 0.376$	$\frac{1204}{760} 0.376 = 0.596$
100	1344	559.0	760	$\frac{760 - 559.0}{1344 - 559.0} = 0.256$	$\frac{1344}{760} 0.256 = 0.453$
104	1495	625.5	760	$\frac{760 - 625.5}{1495 - 625.5} = 0.155$	$\frac{1495}{760} 0.155 = 0.304$
108	1659	704.5	760	$\frac{760 - 704.5}{1659 - 704.5} = 0.058$	$\frac{1659}{760} 0.058 = 0.128$
110	1748	760.0	760	0	0

We plot the data obtained in the form of curves in the coordinates t against x and y (Fig. 7-6) and in the coordinates y^* against x (Fig. 7-7).

Example 7-5. Use the diagram of t against x and y in Fig. 7-6 to determine the composition of the equilibrium vapour and the boiling point for a liquid containing 55 mole per cent of benzene and 45 mole per cent of toluene.

Solution. Drawing line $ABCD$ in Fig. 7-6, we find (a) that the boiling point is about 91°C , and (b) that the composition of the equilibrium vapour is 75 mole per cent of benzene and 25 mole per cent of toluene.

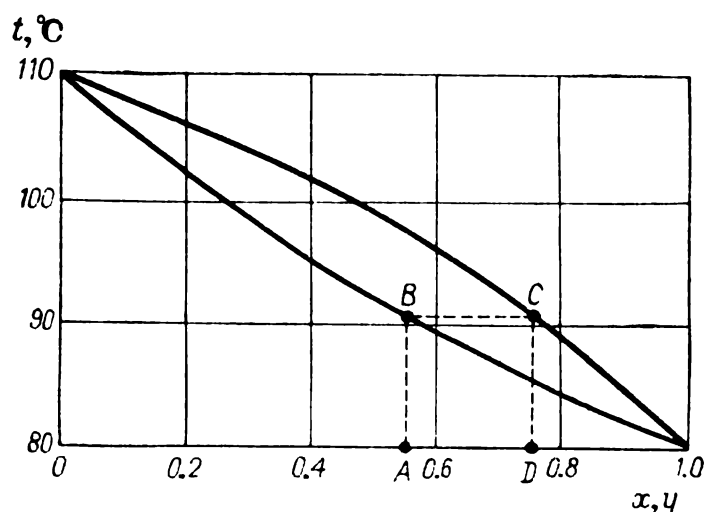


Fig. 7-6. To Examples 7-4 and 7-5

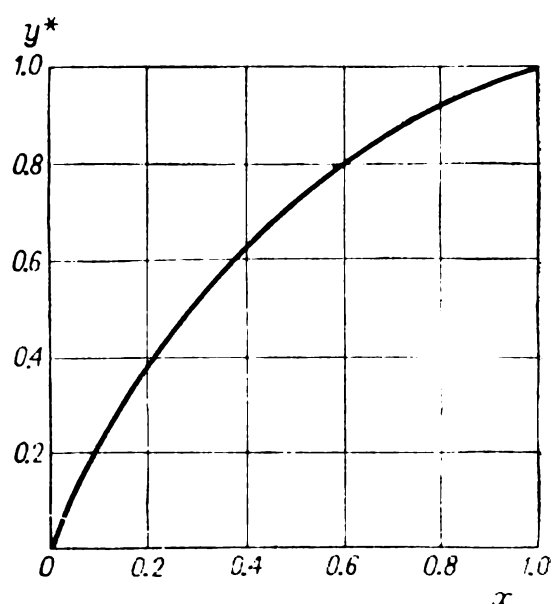


Fig. 7-7. To Example 7-4

Example 7-6. Contaminated turpentine in an amount of 2500 kg is distilled under atmospheric pressure (760 mm Hg) in a stream of saturated steam. The initial mixture contains 88 mass per cent of turpentine, 12 mass per cent of water, and is fed into the apparatus at 30 °C. No external heating is used.

The pressure in the steam pipe is $p_{\text{abs}} = 1.5$ atm. The coefficient of saturation of the steam with the turpentine is $\varphi = 0.7$.

Determine: (a) the consumption of steam for distillation, (b) the mass and molar compositions of the distilled vapour, and (c) the partial pressures of the turpentine and water in the vapour mixture. Assume that the losses of heat to the surroundings are one-tenth of the usefully consumed heat. The molar mass of the turpentine is 136 kg/kmol, the specific heat capacity of the liquid turpentine is 1.76×10^3 J/kg·K, and its specific heat of vaporization is 310×10^3 J/kg.

Solution. We determine the distillation temperature and the saturated vapour pressure of turpentine from Fig. A-24. The distillation temperature is 96 °C, and $p_{\text{turp}}^s = 115$ mm Hg.

We find the amount of steam withdrawn together with the turpentine vapour by Eq. (7-3):

$$G_w = G_{\text{turp}} \frac{M_w (p - p_{\text{turp}}^s)}{M_{\text{turp}} p_{\text{turp}}^s \varphi} = 2500 \times 0.88 \times \frac{18 (760 - 115)}{136 \times 115 \times 0.7} = 2330 \text{ kg}$$

The mass fractions of the components in the vapour mixture are:

turpentine

$$\bar{y}_{\text{turp}} = \frac{G_{\text{turp}}}{G_{\text{turp}} + G_w} = \frac{2500 \times 0.88}{2500 \times 0.88 + 2330} = 0.485$$

water

$$\bar{y}_w = 1 - \bar{y}_{\text{turp}} = 1 - 0.485 = 0.515$$

The mole fractions are (see Table 6-2):

$$y_{\text{turp}} = \frac{\bar{y}_{\text{turp}}/M_{\text{turp}}}{\frac{\bar{y}_{\text{turp}}}{M_{\text{turp}}} + \frac{\bar{y}_{\text{w}}}{M_{\text{w}}}} = \frac{0.485/136}{\frac{0.485}{136} + \frac{0.515}{18}} = 0.110$$

$$y_{\text{w}} = 1 - y_{\text{turp}} = 1 - 0.11 = 0.89$$

The partial pressures in the vapour mixture at the total pressure $p = 760$ mm Hg are:

$$p_{\text{turp}} = p y_{\text{turp}} = 760 \times 0.11 = 84 \text{ mm Hg}$$

$$p_{\text{w}} = p - p_{\text{turp}} = 760 - 84 = 676 \text{ mm Hg}$$

The consumption of heat is:

(a) for heating the initial mixture from 30°C to the distillation temperature of 96°C :

turpentine

$$G_{\text{turp}} c_{\text{turp}} (t_{\text{dist}} - t_{\text{in}}) = 2500 \times 0.88 \times 1.76 \times 10^3 (96 - 30) = 256\,000 \times 10^3 \text{ J}$$

water

$$G_{\text{w}} c_{\text{w}} (t_{\text{dist}} - t_{\text{in}}) = 2500 \times 0.12 \times 4.19 \times 10^3 (96 - 30) = 83\,000 \times 10^3 \text{ J}$$

$$Q_{\text{h}} = (256 + 83) \times 10^6 = 339 \times 10^6 \text{ J}$$

(b) for evaporation of the turpentine being distilled:

$$Q_{\text{v}} = G_{\text{turp}} L_{\text{turp}} = 2500 \times 0.88 \times 310 \times 10^3 = 682\,000 \times 10^3 \text{ J}$$

(c) to compensate for the losses of heat to the surroundings:

$$Q_{\text{l}} = 0.1 (Q_{\text{h}} + Q_{\text{v}}) = (339\,000 + 682\,000) \times 10^3 \times 0.1 = 102\,000 \times 10^3 \text{ J}$$

The total consumption of heat is:

$$Q_{\text{tot}} = Q_{\text{h}} + Q_{\text{v}} + Q_{\text{l}} = (339\,000 + 682\,000 + 102\,000) \times 10^3 = 1\,123\,000 \times 10^3 \text{ J} = 1123 \times 10^3 \text{ kJ}$$

We determine the consumption of steam. From Table A-57, we find that a temperature of 110.7°C corresponds to the absolute pressure of 1.5 at of the saturated steam in the pipe. The amount of steam withdrawn with the turpentine being distilled is 2330 kg at the distillation temperature of 96°C . Hence, the withdrawn steam is cooled from 110.7 to 96°C , the amount of heat being liberated is:

$$Q_{\text{lib}} = G_{\text{w}} c_{\text{st}} (t_{\text{st}} - t_{\text{dist}})$$

From Table A-27, we find the specific heat of the steam:

$$c_{\text{st}} = \frac{35.5 \times 10^3}{18} = 1.97 \times 10^3 \text{ J/kg} \cdot \text{K}$$

and

$Q_{lib} = 2330 \times 1.97 \times 10^3 (110.7 - 96) = 67\,900 \times 10^3 \text{ J} = 67\,900 \text{ kJ}$

The remaining amount of heat:

$Q_{rem} = Q_{tot} - Q_{lib} = (1\,123\,000 - 67\,900) 10^3 = 1055 \times 10^3 \text{ kJ}$

must be supplied as a result of condensing a certain additional amount of the steam and cooling the condensate obtained to the distillation temperature.

The consumption of the condensing (heating) steam is:

$G_{cond} = \frac{Q_{rem}}{H'' - H'}$

The specific enthalpy of saturated steam at 1.5 at is $2700 \times 10^3 \text{ J/kg}$, and that of water at 96°C is $402 \times 10^3 \text{ J/kg}$. Hence,

$G_{cond} = \frac{1\,055\,000 \times 10^3}{(2700 - 402) 10^3} = 460 \text{ kg}$

The total consumption of steam is:

$G_{tot} = 2330 + 460 = 2790 \text{ kg}$

Example 7-7. A simple still is used to distill 1000 kg of a mixture containing 60 mass per cent of ethyl alcohol and 40 mass per cent of water. After distillation, the bottom product contains 5 mass per cent of the alcohol. Determine the composition of the overhead product, its mass, and the mass of the bottom product. The data on the equilibrium compositions are given in Table 7-2, where \bar{x} stands for the mass fraction of the ethyl alcohol in the liquid, and \bar{y}^* for the mass fraction of the ethyl alcohol in the equilibrium vapour.

TABLE 7-2

\bar{x}	\bar{y}^*	$\bar{y}^* - \bar{x}$	$\frac{1}{\bar{y}^* - \bar{x}}$	\bar{x}	\bar{y}^*	$\bar{y}^* - \bar{x}$	$\frac{1}{\bar{y}^* - \bar{x}}$
0.025	0.225	0.200	5.00	0.45	0.754	0.304	3.29
0.05	0.360	0.310	3.22	0.50	0.767	0.267	3.74
0.10	0.516	0.416	2.40	0.55	0.778	0.228	4.38
0.15	0.600	0.450	2.22	0.60	0.789	0.189	5.29
0.20	0.655	0.455	2.20	0.65	0.800	0.150	6.66
0.25	0.690	0.440	2.27	0.70	0.810	0.110	9.09
0.30	0.710	0.410	2.44	0.75	0.835	0.085	11.75
0.35	0.728	0.378	2.64	0.80	0.855	0.055	18.20
0.40	0.740	0.340	2.94				

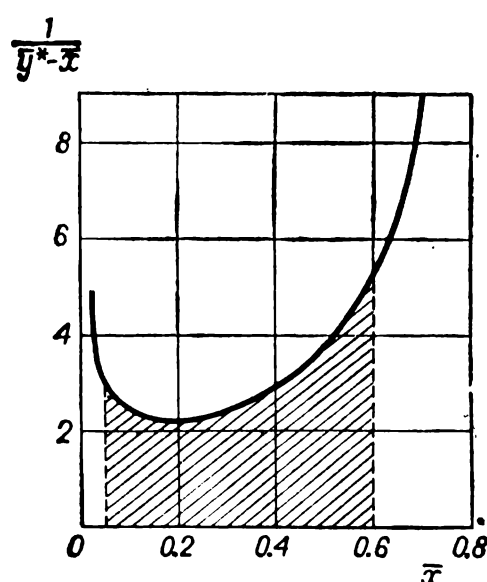


Fig. 7-8. To Example 7-7

Solution. We use Eq. (7-1):

$$\ln \frac{F}{W} = \int_{\bar{x}_2}^{\bar{x}_1} \frac{d\bar{x}}{\bar{y}^* - \bar{x}}$$

It is impossible to perform integration analytically because the analytical form of the relationship between \bar{y}^* and \bar{x} is unknown. We therefore solve the integral graphically. To do this, we plot a graph showing how $1/(\bar{y}^* - \bar{x})$ depends on \bar{x} for the alcohol-water mixture.

The curve is plotted in Fig. 7-8 according to the data of Table 7-2. The scale used is 1 mm = 0.02 along the axis of abscissas and 1 mm = 0.2 along the axis of ordinates. Hence 1 mm² = 0.02 × 0.2 = 0.004.

The value of the integral within the limits from $\bar{x}_1 = 0.6$ to $\bar{x}_2 = 0.05$ is shown by the hatched area. This area equals 403 mm². Hence

$$\int_{0.05}^{0.6} \frac{d\bar{x}}{\bar{y}^* - \bar{x}} = 403 \times 0.004 = 1.612^*$$

Consequently,

$$\ln \frac{F}{W} = 2.3 \log \frac{F}{W} = 1.612, \text{ and } \frac{F}{W} = 5.01$$

According to the initial conditions, $F = 1000$ kg. Hence,

$$W = \frac{1000}{5.01} = 200 \text{ kg}$$

whence the mass of the overhead product is:

$$D \quad F - W = 1000 - 200 = 800 \text{ kg}$$

* The value of the integral can be found approximately using the formula of trapeziums—see Example 4-21.

The amount of the alcohol in the bottom product after distillation is:

$$G_W = W \bar{x}_W = 200 \times 0.05 = 10 \text{ kg}$$

The amount of alcohol withdrawn with the overhead product is:

$$G_D = F \bar{x}_F - G_W = 1000 \times 0.6 - 10 = 590 \text{ kg}$$

The mass per cent of the alcohol in the overhead product is:

$$\bar{x}_D = \frac{G_D}{D} \times 100 = \frac{590}{800} \times 100 = 73.7$$

Example 7-8. Find the consumption of heating steam ($p_{\text{abs}} = 2$ at) for the distillation process in the conditions of the preceding example. The feed is supplied to the still at the temperature of the beginning of boiling. Assume that the losses of heat to the surroundings are 10% of the usefully consumed amount of heat. It has been found from a diagram of t against x and y that the temperature of the beginning of distillation, when the boiling mixture contains 60% (mass) of alcohol, is 81°C , and the temperature of the end of distillation, when the boiling mixture contains 5% (mass) of alcohol, is 94.9°C . The moisture content of the heating steam is 5%.

Solution. The mean distillation temperature is:

$$t = \frac{81 + 94.9}{2} = 88^\circ\text{C}$$

The consumption of heat for heating the feed from 81 to 88°C is:

$$Q_1 = (1000 \times 0.6 \times 3.35 \times 10^3 + 1000 \times 0.4 \times 4.19 \times 10^3) (88 - 81) = 25\,800 \times 10^3 \text{ J}$$

where 3.35×10^3 and $4.19 \times 10^3 \text{ J/kg}\cdot\text{K}$ are the specific heat capacities of alcohol and water (Fig. A-11).

The specific heats of vaporization of alcohol and water are 855×10^3 and $2290 \times 10^3 \text{ J/kg}$, respectively, and the consumption of heat for vaporization at 88°C is:

$$Q_2 = 590 \times 855 \times 10^3 + (800 - 590) 2290 \times 10^3 = 985\,000 \times 10^3 \text{ J}$$

The consumption of heat for heating the bottom product from 88 to 94.9°C is:

$$Q_3 = (200 \times 0.05 \times 3.35 \times 10^3 + 200 \times 0.95 \times 4.19 \times 10^3) (94.9 - 88) = 5700 \times 10^3 \text{ J}$$

The total consumption of heat with account taken of the losses is:

$$Q = (25\,800 + 985\,000 + 5700) 10^3 \times 1.1 = 1\,120\,000 \times 10^3 \text{ J}$$

The consumption of the heating steam is:

$$G_{st} = \frac{1\,120\,000 \times 10^3}{2208 \times 10^3 \times 0.95} = 535 \text{ kg}$$

where 2208×10^3 J/kg is the specific heat of condensation of steam at $p_{abs} = 2$ at.

Example 7-9. In designing a bubble-cap rectification column, the plate spacing was taken equal to 300 mm. Vapour flows through the column at a rate of 3200 m³/h. The density of the vapour is 1.25 kg/m³ (the density and the rate of flow are given for standard conditions). The density of the liquid is 430 kg/m³. Determine the required diameter of the column if the absolute pressure in it is 1.2 at, and the mean temperature is -40°C .

Solution. To determine the diameter of the column, we must find the permissible operating velocity of the vapour in it.

The density of the vapour in the operating conditions is:

$$\rho_v = \frac{\rho_0 T_0 p}{T p_0} = \frac{1.25 \times 273 \times 1.2}{233 \times 1} = 1.75 \text{ kg/m}^3$$

By Eq. (7-18), we have:

$$v = C \sqrt{\frac{\rho_{li}}{\rho_v}} = 0.0315 \sqrt{\frac{430}{1.75}} = 0.495 \text{ m/s}$$

where $C = 0.0315$ has been found from Fig. 7-2.

The volumetric rate of flow of the vapour in the column in operating conditions is

$$Q_v = \frac{Q_{v,0} T p_0}{T_0 p \times 3600} = \frac{3200 \times 233}{273 \times 1.2 \times 3600} = 0.632 \text{ m}^3/\text{s}$$

The required cross-sectional area of the column is:

$$A_c = \frac{Q_v}{v} = \frac{0.632}{0.495} = 1.28 \text{ m}^2$$

and the diameter of the column is:

$$D = \sqrt{\frac{A_c}{0.785}} = \sqrt{\frac{1.28}{0.785}} = 1.275 \text{ m}$$

In accordance with a catalogue for column apparatuses, we adopt $D = 1200$ mm.

Example 7-10. Determine the number of plates in a continuous rectification column for separating a mixture of methyl alcohol and water under atmospheric pressure. The content of the methyl alcohol in the feed is 31.5 mole per cent. It is required to obtain an overhead product containing 97.5 mole per cent of the alcohol; the bottom product may contain up to 1.1 mole per cent of the alcohol. The excess reflux coefficient

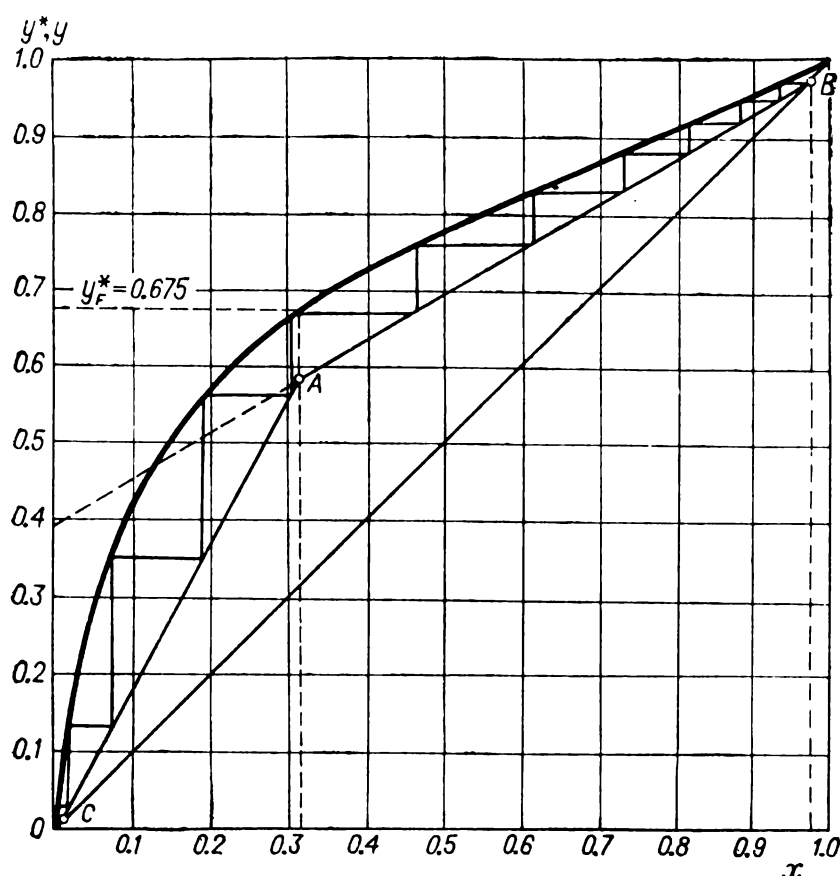


Fig. 7-9. To Example 7-10

is $\phi = 1.77$. The number of plates equivalent to one stage of the change in the concentration is 1.7. The column is heated with indirect steam.

Solution. We construct an equilibrium curve in the coordinates y^* against x (Fig. 7-9) according to the data of Table A-47 for a mixture of methyl alcohol and water at $p_{\text{abs}} = 760$ mm Hg.

We find the minimum reflux ratio by Eq. (7-10):

$$R_{\min} = \frac{x_D - y_F^*}{y_F^* - x_f} = \frac{0.975 - 0.675}{0.675 - 0.315} = 0.835$$

where the value of $y_F^* = 0.675$ has been found on the equilibrium curve in Fig. 7-9.

The actual (operating) reflux ratio is:

$$R = \phi R_{\min} = 1.77 \times 0.835 = 1.48$$

The equation of the operating line of the top section of the column is:

$$y = \frac{1.48}{1.48 + 1} x + \frac{0.975}{1.48 + 1} = 0.598x + 0.393$$

Laying off 0.393 along the axis of ordinates, we plot operating line AB for the top section of the column. We draw the operating line for the bottom section of the column through points A and C .

After constructing the broken line in the diagram (beginning from point B), we find the required number of stages or steps

of the change in the concentration: it is about 7 for the top section of the column and about 4 for the bottom part, altogether 11.

The number of actual plates is $1.7 \times 7 \approx 12$ for the top section and $1.7 \times 4 \approx 7$ for the bottom section of the column, altogether 19 plates.

Example 7-11. Construct an equilibrium curve for a mixture of carbon tetrachloride CCl_4 and sulphur dichloride SCl_2 at a pressure of 760 mm Hg.

Solution. In the absence of experimental data, we construct the equilibrium curve following the assumption that the mixture obeys Raoult's law.

The saturated vapour pressures of CCl_4 for different temperatures are:

$t, ^\circ\text{C}$	40	50	60	70	75	80
$p, \text{ mm Hg}$	214	315	439	621	715	843

We have two points for SCl_2 , namely, the boiling point at a pressure of 760 mm Hg equal to 59°C , and the boiling point at 400 mm Hg equal to 41°C .

We use the rule of linearity to find the saturated vapour pressures of SCl_2 at other temperatures. From Table A-38, the boiling point of water, which we take as the standard liquid, at 400 mm Hg is 83°C . By Eq. (5-15), we calculate the constant K for SCl_2 and water:

$$K = \frac{59 - 41}{100 - 83} = 1.06$$

We use the same equation to calculate the boiling points of SCl_2 at different pressures:

$$59 - t = 1.06 (100 - \theta)$$

whence

$$t = 1.06\theta - 47$$

where t and θ are the boiling points of SCl_2 and water, respectively, at the same pressure, $^\circ\text{C}$.

The values of t for SCl_2 at different pressures calculated according to the latter equation are given below:

$p, \text{ mm Hg}$	760	880	1030	1175	1320	1470
$\theta, ^\circ\text{C}$	100	104.2	108.7	112.7	116.3	119.6
$t, ^\circ\text{C}$	59	63.5	68.2	72.5	76.3	79.8

These data and those given above for carbon tetrachloride are used to plot the temperature dependence of the saturated vapour pressure for SCl_2 and CCl_4 —see Fig. 7-10.

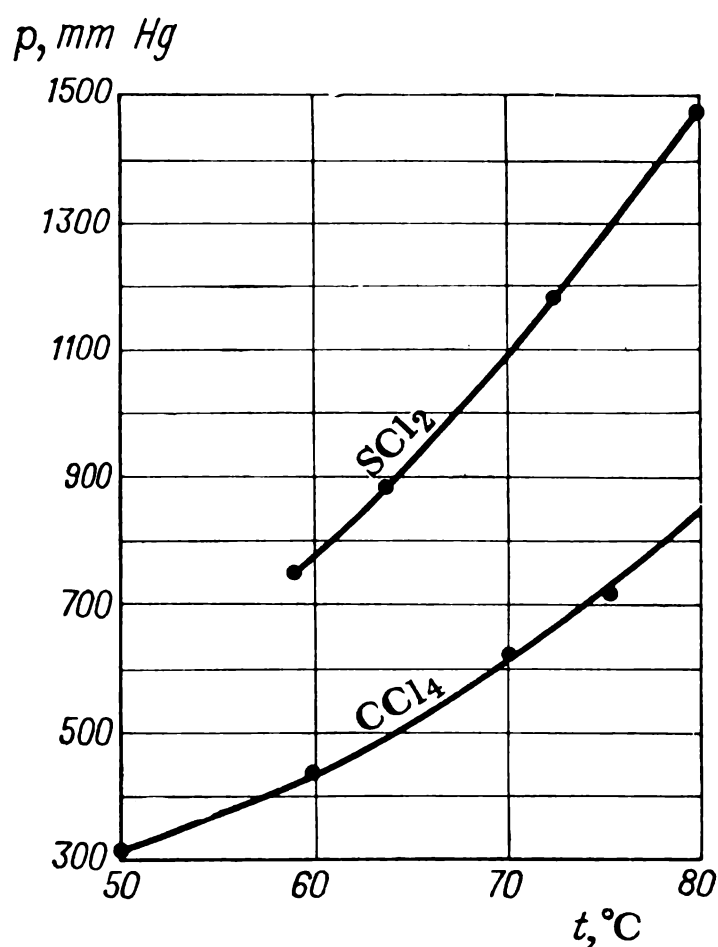


Fig. 7-10. To Example 7-11

Having at our disposal data on the saturated vapour pressures of the pure components at various temperatures, we can construct an equilibrium diagram using Raoult's law. For the procedure used see Example 7-4. Figure 7-11 gives the relevant equilibrium diagram in the coordinates y^* against x .

Example 7-12. A batch plate rectification column is used to fractionate a liquid mixture of carbon tetrachloride CCl_4 and sulphur dichloride SCl_2 containing 50 mole per cent of SCl_2 under atmospheric pressure. The overhead product is to contain 90 mole per cent and the bottom product after fractionation 15 mole per cent of SCl_2 .

Determine the number of plates needed if one stage of change in the concentration requires 1.7 plates. Also determine the ratio of the minimum reflux ratios at the end and the beginning of fractionation.

Solution. We use the data on the equilibrium of a mixture of carbon tetrachloride and sulphur dichloride obtained in the preceding example.

We use Eq. (7-10) to find the minimum reflux ratio R_{\min} for the final moment of fractionation when the bottom product contains 15% of SCl_2 :

$$R_{\min} = \frac{x_D - y_F^*}{y_F^* - x_F} = \frac{0.9 - 0.25}{0.25 - 0.15} = 6.5$$

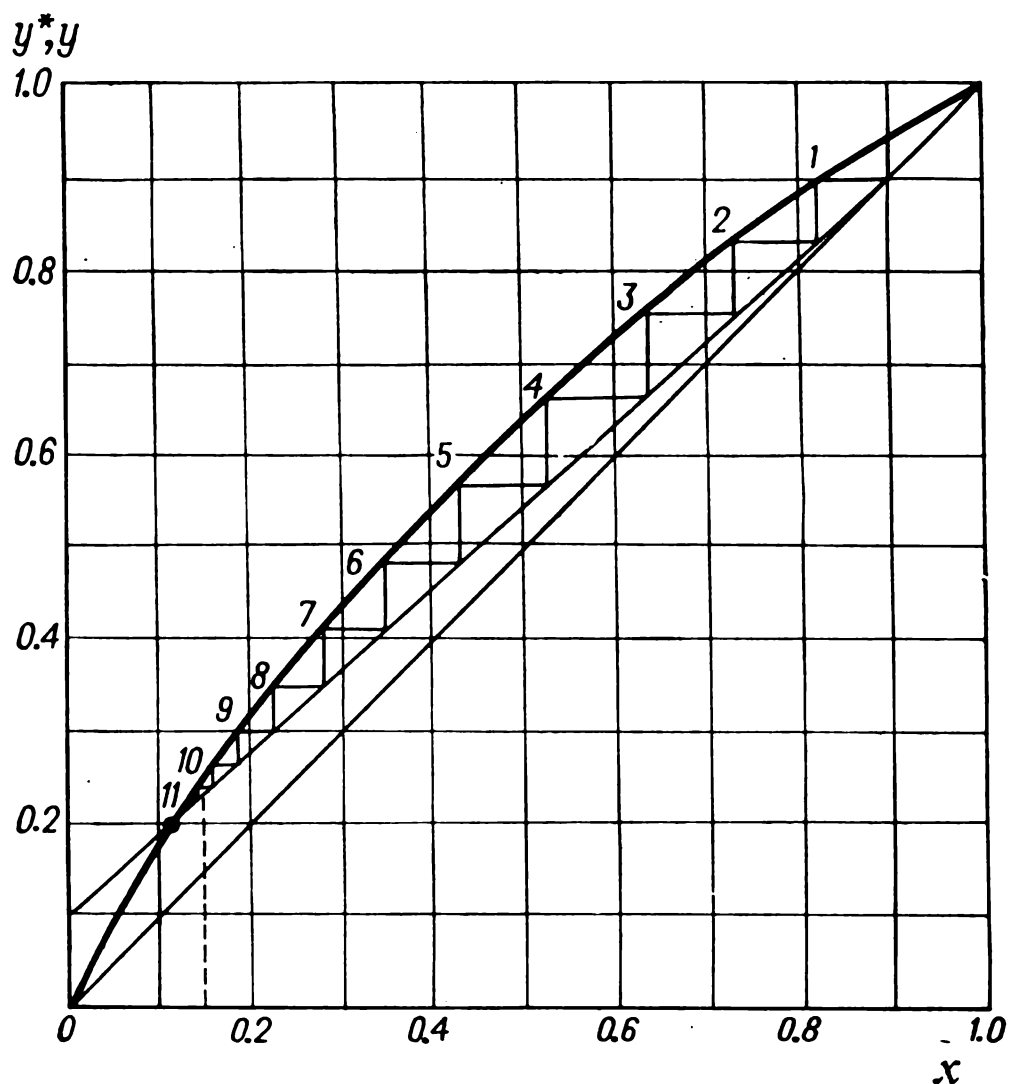


Fig. 7-11. To Examples 7-11 and 7-12

Here 0.25—the mole fraction of SCl_2 in the vapour in equilibrium with the bottom product containing 15% of SCl_2 —has been found from the equilibrium diagram.

For the actual process, we take a reflux ratio that is 20% greater than R_{\min} :

$$R = 1.2 \times 6.5 = 7.8$$

Equation (7-6) of the operating line for the final moment of fractionation is:

$$y = \frac{7.8}{7.8+1} x + \frac{0.9}{7.8+1} = 0.887x + 0.102$$

We construct the operating line and determine the number of stages of the change in concentration graphically. It follows from Fig. 7-11 that 11 steps or stages correspond to the reflux ratio $R=7.8$.

The number of actual plates needed for fractionation is:

$$n = 1.7 \times 11 \approx 19$$

For the initial moment of fractionation, when $x_F = 0.5$, the minimum reflux ratio is:

$$R'_{\min} = \frac{0.9 - 0.64}{0.64 - 0.5} = 1.86$$

where $y_F^* = 0.64$ has been found from the equilibrium diagram in Fig. 7-11.

The ratio of the minimum reflux ratios at the end and the beginning of fractionation is:

$$\frac{R_{\min}}{R'_{\min}} = \frac{6.5}{1.86} = 3.5$$

Example 7-13. Construct a diagram of H against x in the saturation region for a mixture of CH_4 and N_2 at $p_{\text{abs}} = 10$ at.

Solution. For the procedure used in constructing and using an H - x diagram see [6-3, 11-2].

To determine the equilibrium relationships in the system CH_4 - N_2 , we use experimental data.

We use phase diagrams of H against $\log p$ for N_2 and CH_4 to determine the enthalpies of the pure components per kmol of vapour and liquid within the limits of the temperatures at which the system CH_4 - N_2 exists at 10 at (Table 7-3).

We construct the coordinate grid of the diagram: we lay off the content of N_2 along the axis of abscissas, and the enthalpy in kJ/kmol along the axis of ordinates (Fig. 7-12).

Next we plot the enthalpies of the pure gaseous N_2 and CH_4 (Table 7-3) on the ordinate axis for the temperatures 170, 160, and 150 K (points A , B , and C for CH_4 , and points A' , B' , and C' for N_2). Straight lines AA' , BB' , and CC' are the corresponding isotherms. We assume that the heat of mixing equals zero.

Below the temperature 150 K, gaseous methane does not exist at a pressure of 10 at. We extrapolate the enthalpy of CH_4 to below this temperature, however, and assume that in the region of extrapolation $\Delta H/\Delta T$ has the same value as above 150 K. As a result, we get isotherms of the vapour down to 100 K, i. e. to the condensation point of pure nitrogen at 10 at. Figure 7-12 shows several isotherms of the vapour plotted in this way beginning from the isotherm for 145 K.

Next, using the equilibrium data for the system CH_4 - N_2 , we determine the temperatures of the beginning of condensation for different vapour mixtures and plot the corresponding points on the H - x diagram. We obtain condensation line CD by joining these points.

We construct isotherms for the liquid following the same principle as for the vapour. Complications are introduced by

TABLE 7-3

T, K	Vapour		Liquid	
	H_{CH_4}	H_{N_2}	H_{CH_4}	H_{N_2}
	kJ/kmol	kJ/kmol	kJ/kmol	kJ/kmol
170	11 100	9420	4690	—
160	10 700	9100	3930	—
150	10 270	8800	3310	—
145	—	8640	3040	—
140	—	8480	2820	—
135	—	8330	2550	—
130	—	8160	2310	—
125	—	7990	2050	—
120	—	7810	1830	—
115	—	7630	1580	3460
110	—	7420	1360	3160
105	—	7210	—	2890
100	—	—	—	2610

the fact that part of the equilibrium temperatures are above the critical temperature of nitrogen (126 K), and this makes it impossible to get the enthalpy of liquid nitrogen at these temperatures. To surmount this obstacle, we proceed as follows. We calculate the mean heat capacity of liquid nitrogen for the interval, say, of from 110 to 120 K and assume conditionally that further, at a temperature above the critical one, the heat capacity of the nitrogen in the mixture remains the same. Part of the isotherms for the liquids constructed in this way are shown by dash lines in Fig. 7-12 (in the region of saturation).

The adopted assumption cannot lead to large errors because for high-temperature isotherms (above 125 K) the error is reduced owing to the circumstance that these isotherms are intersected by the boiling line at a great fraction of the methane whose enthalpy has been found without any assumptions being made. Therefore, the deviation due to the inaccuracy in determining the enthalpy of nitrogen has an insignificant influence because of its low fraction. For the low-temperature isotherms (below 125 K), the enthalpies of liquid nitrogen have been found without any assumptions.

The data on phase equilibrium of the system CH₄-N₂ are used to plot the points of the beginning of boiling on the diagram for liquid mixtures having different compositions. Joining these points gives boiling line EF in the H-x diagram. Isotherms called tie lines are drawn in the region of saturation connecting

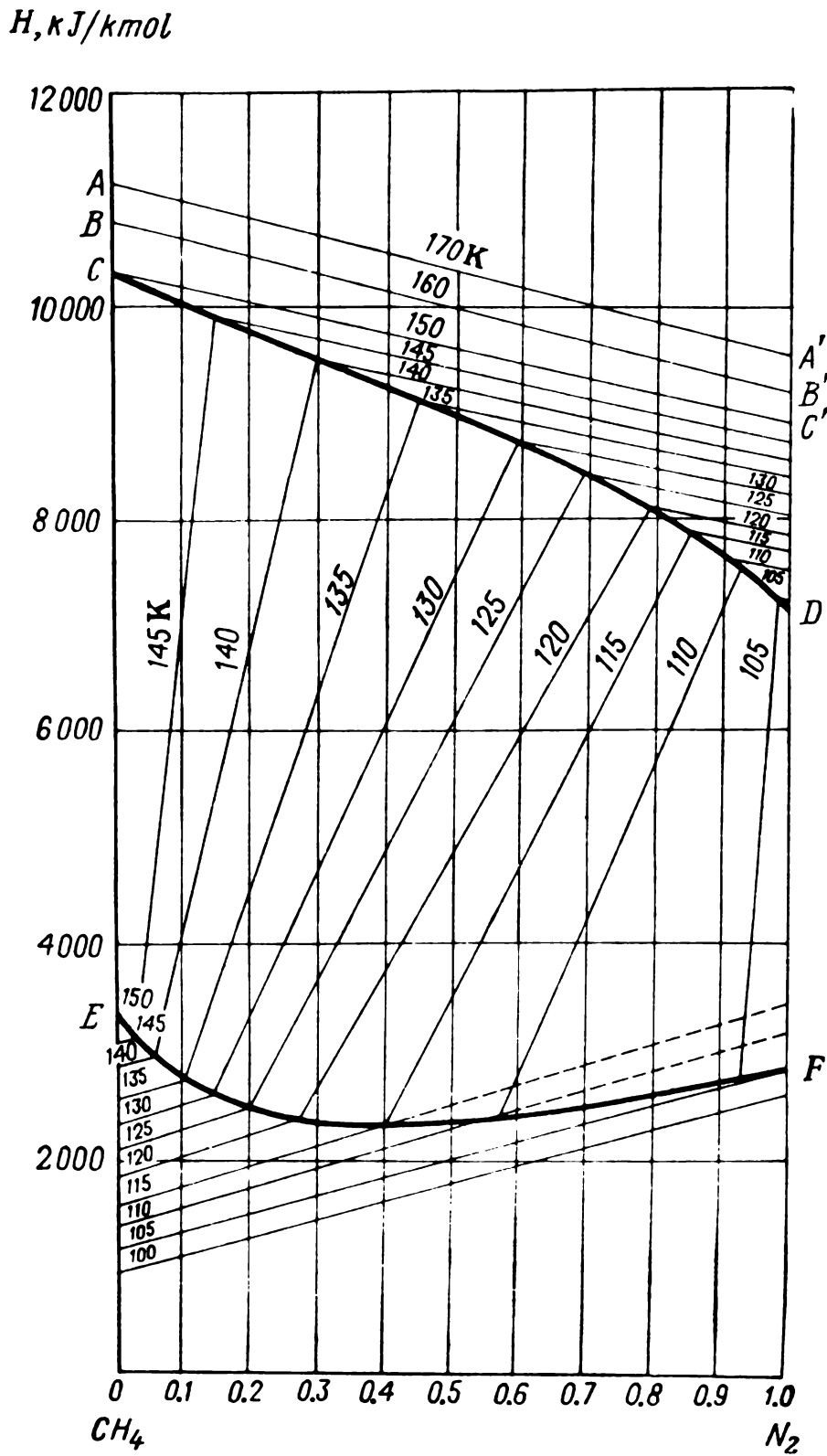


Fig. 7-12. To Example 7-13:
CD—condensation line; EF—boiling line

the compositions of the liquid on the boiling curve with the equilibrium vapour compositions on the condensation line. This terminates construction of the H - x diagram.

Example 7-14. A mixture consisting of 30 mole per cent of CH_4 and 70 mole per cent of N_2 is fed into a continuous rectification column. The feed is supplied at a rate of $1000 \text{ m}^3/\text{h}$ in the vapour state at $p_{\text{abs}}=10$ at and the saturation temperature. Here and below in the present example, all the volumetric rates of flow are indicated (in m^3/h) for standard conditions.

As a result of fractionation, gaseous products are obtained

in the column: a methane fraction containing 3% of N_2 and a nitrogen fraction containing 5% of CH_4 .

Determine, using an H - x diagram, (a) the amounts of the fractions obtained, (b) the minimum reflux ratio R_{\min} , (c) the number of stages of the change in the concentration for an excess reflux coefficient of 1.25, (d) the amount of heat supplied in the reflux condenser of the column, and (e) the amount of heat supplied in the still (if the methane fraction is withdrawn from the column still in the liquid state).

Solution. We use the equilibrium diagram of the system CH_4 - N_2 for 10 at in the coordinates H - x constructed in the preceding example.

(a) To determine the amounts of the fractions obtained, we draw horizontal line $III-I-II$ at the top of the diagram (Fig. 7-13). Point I characterizes the feed (70% of N_2), point II —the nitrogen fraction (95% of N_2), and point III the methane fraction (3% of N_2).

The ratio of the segments $(I-II)/(III-II) = (0.95 - 0.7)/(0.95 - 0.03)$ gives the amount of the methane fraction obtained per kmol of the feed.

The volumetric rate of flow of the methane fraction is:

$$Q_{v, \text{meth}} = \frac{0.25}{0.92} \times 1000 = 272 \text{ m}^3/\text{h}$$

and that of the nitrogen fraction is:

$$Q_{v, \text{nit}} = 1000 - 272 = 728 \text{ m}^3/\text{h}$$

(b) We determine R_{\min} . Point F corresponds to the state of the feed. It must be on the main straight line (called the overall enthalpy line) joining the poles of the rectifying and stripping parts of the column. To determine the lowest position of the pole of the rectifying section π_r corresponding to R_{\min} , we must make the overall enthalpy line coincide with the isotherm passing through point F (line FO). Continuation of this line to its intersection with the vertical lines drawn from the values of the concentrations of the fractions obtained gives us the poles π_r and π_s corresponding to R_{\min} .

The ratio of the segments

$$\frac{FF'}{OK} = \frac{\pi_r F'}{\pi_r K} = \frac{R_{\min}}{R_{\min} + 1}$$

In our case, we have:

$$\frac{FF'}{OK} = \frac{0.95 - 0.7}{0.95 - 0.217} = 0.341$$

Hence, $R_{\min} = 0.518$.

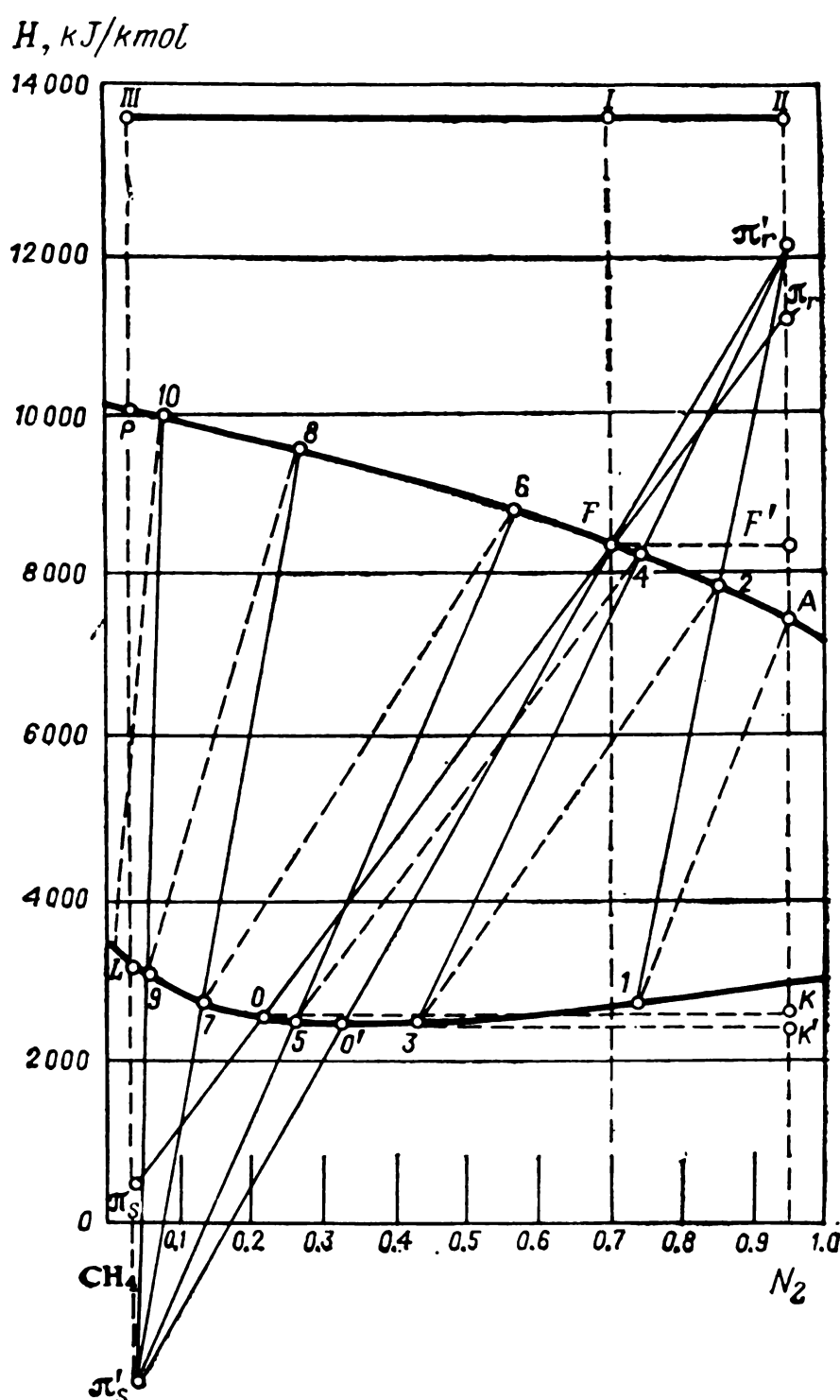


Fig. 7-13. To Example 7-14

According to the initial conditions, the actual reflux ratio is:
 $R = 1.25R_{\min} = 1.25 \times 0.518 = 0.647$

We determine the position of the actual pole π_r' corresponding to a reflux ratio of $R = 0.647$.

For this purpose from the ratio

$$\frac{FF'}{O'K'} = \frac{R}{R+1}$$

we find the magnitude of segment $O'K'$:

$$O'K' = FF' \frac{R+1}{R} = \frac{(0.95-0.7)1.647}{0.647} = 0.637$$

and then the abscissa of point O' : $0.95 - 0.637 = 0.313$.

By drawing a straight line through points F and O' up to its intersection with vertical lines erected from the abscissas x_D and x_W , we get the overall enthalpy line for $R = 0.647$. The points of intersection of this line with the vertical ones are the actual poles π'_r and π'_s .

(c) Next we determine the number of stages of the change in concentration graphically. For this purpose, we find the composition of the liquid in equilibrium with the vapour containing 95% of N_2 (point A). This will be point 1 joined to point A by a tie line (isotherm). Next we draw ray $1-\pi'_r$. Intersection of this ray with the vapour curve gives point 2. We repeat the construction from point 2. After getting point 5 on the liquid curve to the left of the overall enthalpy line, we perform further construction with the aid of the bottom pole π'_s . The number of such steps gives the number of stages of the change in the concentration. As a result, we get two stages (slightly more) in the rectifying section (above the feed) and four stages (slightly less) in the stripping section (below the feed).

(d) The amount of heat removed in the reflux condenser is determined by the difference between the enthalpies at points π'_r and A , i. e.

$$q_{r.c} = 12\,230 - 7330 = 4900 \text{ kJ/kmol}$$

Since in fractionation we get 728 m³/h of the nitrogen fraction, the total amount of heat transferred in the reflux condenser is:

$$Q_{r.c} = 4900 \times \frac{728}{22.4} = 159\,000 \text{ kJ/h} = \frac{159\,000 \times 10^3}{3600} = 44\,200 \text{ W}$$

(e) The amount of heat transferred in the still of the column q_{st} when the methane fraction is withdrawn in the gaseous state is determined by the difference between the enthalpies at points P and π_s . When the methane fraction is withdrawn in the liquid state, no heat is spent for evaporation of this fraction. In this case, the heat q_{st} is determined by the difference between the enthalpies at points L and π'_s , which amounts to 5150 kJ per kmol of the methane fraction.

The total amount of heat transferred in the still of the column is:

$$Q_{st} = \frac{5150 \times 10^3}{3600} \times \frac{272}{22.4} = 17\,300 \text{ W}$$

Example 7-15. Determine the height and diameter of the top (rectifying) section of a packed rectification column for separating a mixture of methyl alcohol and water under atmospheric pressure.

The column is supplied with 1500 kg/h of a feed consisting of 40% of alcohol and 60% of water. The concentration of the distillate (overhead product) is 97.5% alcohol. The bottom product contains 2% of alcohol (all percentages are molar). The reflux ratio is $R=1.48$. Rings $25 \times 25 \times 3$ mm in size are used as the packing. The column operates in emulsifying conditions. Indirect steam is used for heating.

Solution. We find the velocity of the vapour in a packed rectification column operating in emulsifying conditions by Eq. (6-40). Here the coefficient $C = -0.125$

$$\log \left(\frac{v^2 \sigma \rho_v \mu_{lq}^{0.16}}{g V_{free}^3 \rho_{lq}} \right) = -0.125 - 1.75 \left(\frac{L}{G} \right)^{0.25} \left(\frac{\rho_v}{\rho_{lq}} \right)^{0.125}$$

In our case, we have: $\sigma = 204 \text{ m}^2/\text{m}^3$ (Table A-17), $V_{free} = 0.74 \text{ m}^3/\text{m}^3$, and $L/G = R/(R+1) = 1.48/2.48 = 0.6$.

The mean composition of the liquid is:

$$x_m = \frac{x_F + x_D}{2} = \frac{40 + 97.5}{2} = 70\% \text{ (mole) of alcohol}$$

For this composition of the liquid, we get $\rho_{lq} \approx 840 \text{ kg/m}^3$ and $\mu_{lq} \approx 0.5 \times 10^{-3} \text{ Pa} \cdot \text{s}$.

The mean composition of the vapour is:

$$y_m = \frac{y_F + y_D}{2} = \frac{63 + 97.5}{2} = 80\% \text{ (mole) of alcohol}$$

Here $y_F = 63$ mole per cent is the content of the alcohol in the vapour entering the top section of the column (Fig. 7-9; in Example 7-10 the operating line of the top section of the column has the same position as in our case). The mean temperature in the top section of the column is about 72°C . We have:

$$\rho_v = \frac{[M_{alc} y_m + M_w (1 - y_m)] T_0}{22.4 T} = \frac{(0.8 \times 32 + 0.2 \times 18) 273}{22.4 \times 345} = 1.03 \text{ kg/m}^3$$

$$\mu_v = 1.11 \times 10^{-5} \text{ Pa} \cdot \text{s}$$

Introducing these values into Eq. (6-40), we get:

$$\log \left(\frac{v^2 \times 204 \times 1.03 \times 0.5^{0.16}}{9.81 \times 0.74^3 \times 840} \right) = -0.125 - 1.75 \times 0.6^{0.25} \left(\frac{1.03}{840} \right)^{0.125}$$

Thus, $\log(0.0565v^2) = -0.788$, and

$$v = 1.7 \text{ m/s}$$

We calculate the equivalent depth of the packing h_{eq} by Eq. (7-24):

$$\begin{aligned}
 h_{eq} &= 5.2 d_{eq} Re_v^{0.2} \left(\frac{G}{L} \right)^{0.35} \left(\frac{\rho_{lq}}{\rho_v} \right)^{0.2} \frac{\log \frac{L}{mG}}{1 - \frac{mG}{L}} = \\
 &= 5.2 \frac{4V_{free}}{\sigma} \left(\frac{4v\rho_v}{\sigma\mu_v} \right)^{0.2} \left(\frac{R+1}{R} \right)^{0.35} \left(\frac{\rho_{lq}}{\rho_v} \right)^{0.2} \frac{\log \frac{R}{m(R+1)}}{1 - \frac{m(R+1)}{R}} = \\
 &= \frac{5.2 \times 4 \times 0.74}{204} \left(\frac{4 \times 1.7 \times 1.03}{204 \times 1.11 \times 10^{-5}} \right)^{0.2} \left(\frac{2.48}{1.48} \right)^{0.35} \left(\frac{840}{1.03} \right)^{0.2} \frac{\log \frac{1.48}{0.45 \times 2.48}}{1 - \frac{0.45}{0.6}} = \\
 &= 0.86 \text{ m}
 \end{aligned}$$

It follows from Fig. 7-9 that the number of stages of the change in concentration within the limits from $x_D = 0.975$ to $x_F = 0.4$ is 6.4*.

Consequently, the required depth of the packing by Eq. (6-51) is:

$$h_p = h_{eq} n_{th} = 0.86 \times 6.4 = 5.5 \text{ m}$$

To determine the diameter of the column, we first find the rate of flow of the overhead product. The molar rate of flow of the feed is:

$$G_F = \frac{\dot{G}_F}{x_F M_{alc} + (1 - x_F) M_w} = \frac{1500}{0.4 \times 32 + 0.6 \times 18} = 63.6 \text{ kmol/h}$$

From the equations of the material balance of the column:

$$G_D + G_W = 63.6$$

$$0.975 G_D + 0.02 G_W = 0.4 \times 63.6$$

we find the rate of flow of the overhead product $G_D = 25.3 \text{ kmol/h}$.

The volumetric rate of flow of the vapour in the top section of the column is:

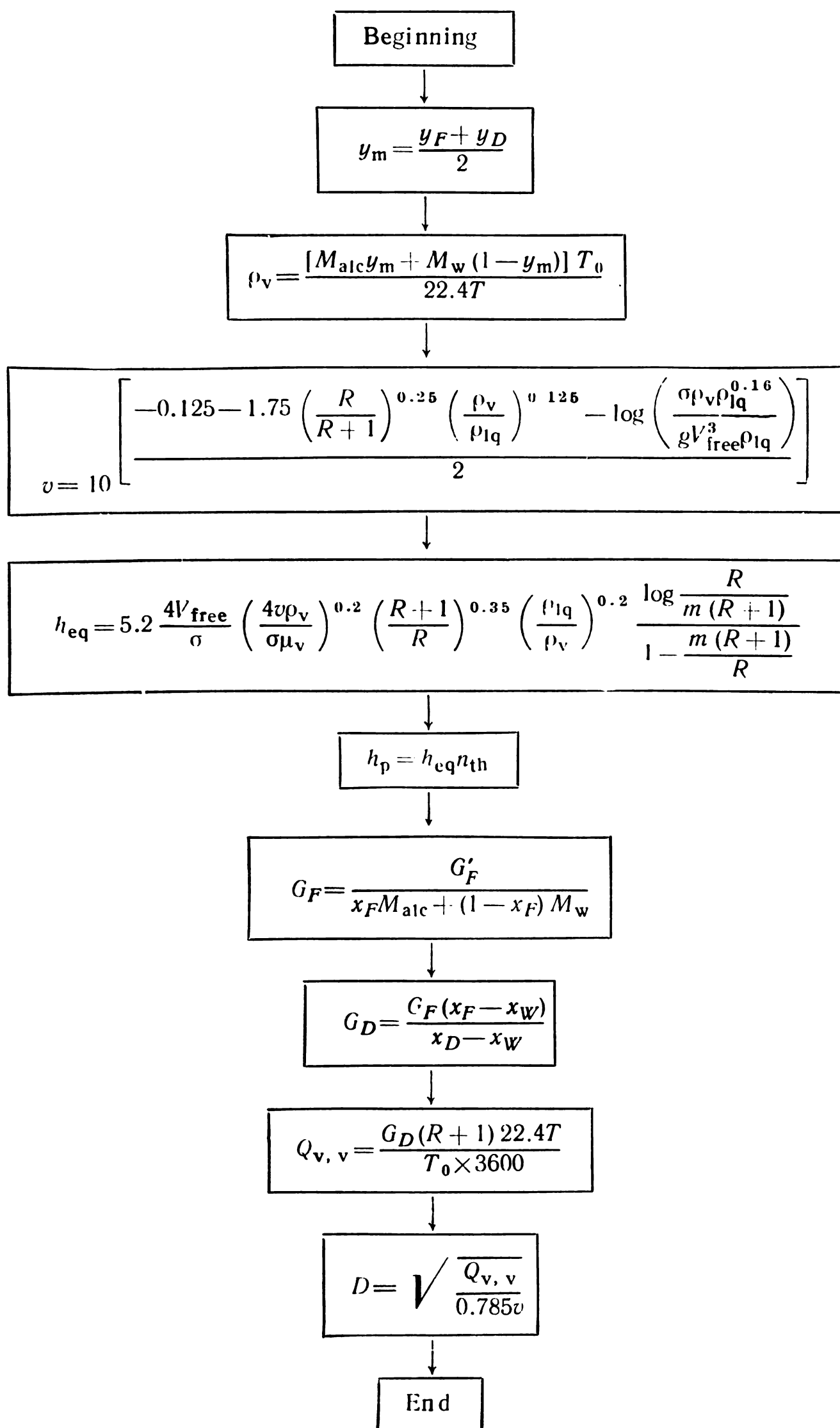
$$Q_{v, v} = \frac{G_D (R+1) 22.4T}{T_0 \times 3600} = \frac{25.3 (1.48 + 1) 22.4 \times 345}{273 \times 3600} = 0.493 \text{ m}^3/\text{s}$$

The required diameter of the column is:

$$D = \sqrt{\frac{Q_{v, v}}{v \times 0.785}} = \sqrt{\frac{0.493}{1.7 \times 0.785}} = 0.61 \text{ m}$$

The flow chart of the algorithm for solving Example 7-15 is as follows:

* Upon a more accurate construction using a greater scale, the number of stages of the change in concentration obtained is slightly greater than 7.



The identifiers are given in Table 7-4.

TABLE 7-4

Quantity	y_m	y_F	y_D	ρ_v	T_0	M_{alc}	M_w	T	v	R	ρ_{lq}	σ	μ_{lq}
Identifier	YM	YF	YD	RV	TN	MAL	MW	T	V	R	RL	SIGMA	ML
V_{free}	h_{eq}	μ_v	m	h_p	n_{th}	G_F	G'_F	x_F	G_D	x_W	x_D	$Q_{v,v}$	D
VF	HE	MV	M	HP	NT	GF	GF1	XF	GD	XW	XD	QV	D

The FORTRAN-IV record of the programme is as follows:

```
1 FORMAT (E 10.2)
3 FORMAT (30X, 'INITIAL DATA',/10X, ((5E10.23X))
  READ 1, YF, YD, TN, MAL, MW, T, RL, SIGMA, ML,
* VF, MV, M, NT, GF1, XF, XW, XD, R
  WRITE 3, YF, YD, TN, MAL, MW, T, RL, SIGMA, ML,
* VF, MV, M, NT, GF1, XF, XW, XD, R
  YM=(YF+YD)*.5
  RV=(MAL*YM+MW*(1.-YM))*TN/(22.4*T)
  V=10.*((-.125-1.75*(R/(R+1.))**.25*(RV/RL)**.125-
* ALOG (SIGMA*RV*ML**.16/9.81/VF**3./RL))/2.)
  HE=5.2*4.*VF*(4.*V*RV/SIGMA/MV)**.2*((R+1.)/R)**.35*
* (RL/RV)**.2*ALOG(R/M/(R+1.)/(1.-M*(R+1.)/R)/SIGMA
  HP=HE*NT
  GF=GF1/(XF*MAL+(1.-XF)*MW)
  GD=GF*(XF-XW)/(XD-XW)
  QV=GD*(R+1.)*22.4*T/TN/3600
  D=SQRT(QV/.785/V)
  WRITE 4, HP, D
4 FORMAT (10X, 'CALCULATION RESULTS',/10X, 'HP=', E12.4,
* 'D=', E12.4)
  STOP
  END
```

The numerical information can be written in the form:

0.63, 0.975, 273., 32., 18., 345., 840., 204., 0.5, 0.74,
0.0000111, 0.45, 6.4, 1500., 0.4, 0.02, 0.975, 1.48

The following text will be printed on the result output form:

INITIAL DATA
YF=0.63 YD=0.975 TN=273. MAL=32. MW=18. T=345.
RL=840. SIGMA=204. ML=0.5 VF=0.74 MV=0.0000111
M=0.45 NT=6.4 GF1=1500. XF=0.4 XW=0.02 XD=0.975
R=1.48
RESULTS OF CALCULATION
HP=5.5 D=0.61

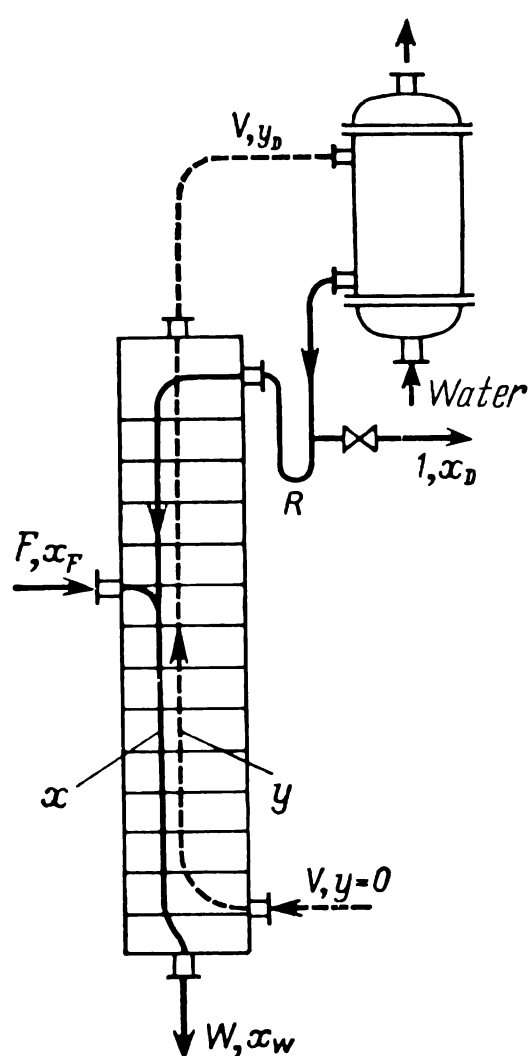


Fig. 7-14. To Example 7-16

Example 7-16. A mixture of ethyl alcohol and water containing 25 mole per cent of the alcohol is fractionated in a continuous rectification column under atmospheric pressure. The required concentration of the overhead product is 80 mole per cent, and the permissible alcohol content in the bottom product is not over 0.1 mole per cent. The still of the column is supplied with direct steam. Determine the operating reflux ratio and the relative molar rates of flow of the feed, the direct steam supplied, and the bottom product per kmol of overhead product. Plot the operating lines on a diagram of y against x .

Solution. Figure 7-14 gives a diagram of the material streams (the relative molar rates of flow) in a rectification column whose still is supplied with direct steam. From the equations of the material balance of the column

$$F + V = 1 + W$$

$$Fx_F = x_D + Wx_W$$

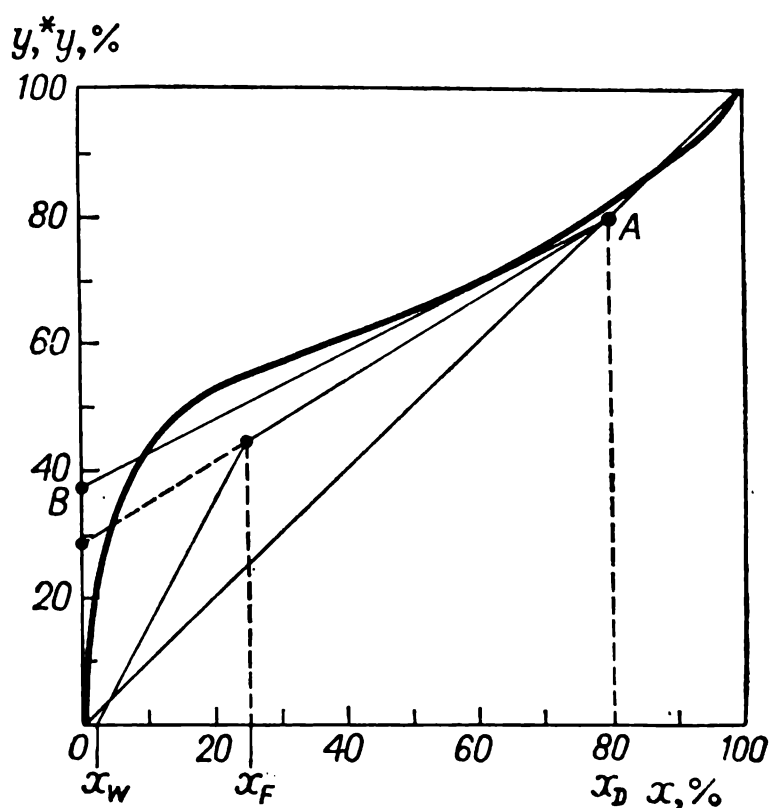
when $W = F + R$, we get:

$$F = \frac{x_D + Rx_W}{x_F - x_W}$$

$$W = \frac{x_D + Rx_F}{x_F - x_W}$$

$$V = R + 1$$

Fig. 7-15. To Example 7-16



The equations of the operating lines are:
for the top section of the column

$$y = \frac{R}{R+1} x + \frac{x_D}{R+1}$$

for the bottom section of the column

$$y = \frac{R+F}{R+1} x - \frac{R+F}{R+1} x_W$$

In the above equations $F = G_F/G_D$; $W = G_W/G_D$; $V = G_V/G_D$, where G_F , G_D , G_W , and G_V are the molar rates of flow of the feed, the overhead product (distillate), the bottom product, and the supplied steam, respectively, kmol/s.

The operating line for the top section of the column is the same as when a column is heated with indirect steam—see Eq. (7-6). It intersects the diagonal of the diagram of y against x at $x = x_D$ and the axis of ordinates at $y = x_D/(R+1)$.

The operating line for the bottom section of the column intersects the axis of abscissas at $x = x_W$. Both operating lines intersect at $x = x_F$.

We find the operating reflux ratio by Eq. (7-12):

$$R = 1.3R_{\min} + 0.3$$

The equilibrium curve constructed according to the data of [7-9] has a depression (Fig. 7-15). To determine R_{\min} , we draw tangent AB to the equilibrium curve from point A . Tangent AB

intercepts a segment equal to 37.5 on the ordinate axis. Hence,

$$\frac{x_D}{R_{\min} + 1} = 0.375 \text{ and } R_{\min} = \frac{0.8 - 0.375}{0.375} = 1.13$$

The operating reflux ratio is $R = 1.3 \times 1.13 + 0.3 = 1.77$.

The segment intercepted by the operating line of the top section of the column on the ordinate axis at $R = 1.77$ is:

$$\frac{x_D}{R + 1} = \frac{0.8}{1.77 + 1} = 0.289$$

We plot the operating lines on the diagram in Fig. 7-15 (the point x_W on the axis of abscissas is not drawn to scale in the figure). We find the relative molar rates of flow:

$$F = \frac{x_D + Rx_W}{x_F - x_W} = \frac{80 + 1.77 \times 0.1}{25 - 0.1} = 3.21$$

$$W = \frac{x_D + Rx_F}{x_F - x_W} = \frac{80 + 1.77 \times 25}{24.9} = 4.98$$

$$V = R + 1 = 1.77 + 1 = 2.77$$

Checking of the material balance:

$$F + V = 1 + W$$

$$3.21 + 2.77 = 1 + 4.98$$

The rate of flow of the direct steam supplied to the column should be checked according to the equation of the heat balance of the column:

$$G'_V H_{st} + G'_F c_F t_F = Q_{r.c} + G'_D c_D t_D + G'_W c_W t_W + Q_1$$

where

G'_V, G'_F, G'_D, G'_W = mass rates of flow of the steam, feed, overhead product, and bottom product, kg/s

$$G'_W = G'_V + G'_F - G'_D$$

c_F, c_D, c_W = respective specific heat capacities, J/kg·K

t_F, t_D, t_W = temperatures, °C

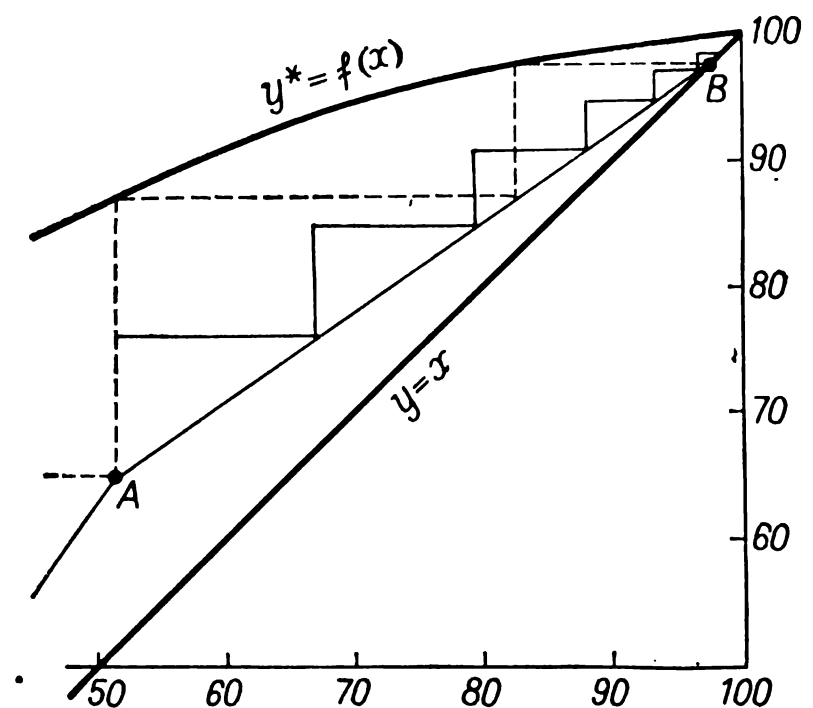
H_{st} = specific enthalpy of the steam, J/kg

$Q_{r.c}$ = amount of heat given up to the cooling water in the reflux condenser, W, Eq. (7-15)

Q_1 = heat losses to the surroundings, W.

Example 7-17. A plate rectification column fed with a liquid mixture containing 51.5 mole per cent of a low boiler is to be used to obtain an overhead product with a concentration of 97.5 mole per cent. According to experimental data, the Murphree efficiency of a plate of the adopted design η_0 [Eq. (7-22)] for the given mixture within a concentration range from 50 to 100

Fig. 7-16. To Example 7-17



mole per cent does not virtually depend on the composition of the liquid and for the given vapour velocity equals 0.5 (the same for all the plates). Determine the number of plates needed in the top section of the column in two ways: (a) through the number of stages of the change in concentration considering that the overall plate efficiency η [Eq. (7-20)] is the same as that of the separate plates; and (b) by direct graphical construction of the number of actual plates with account taken of $\eta_0 = 0.5$ (the method of the kinetic curve).

Solution. Figure 7-16 shows the top section of the diagram of y versus x . Here AB is the operating line for the top section of the column.

(a) We determine the number of stages of the change in the concentration n_{th} by a construction beginning from point A (dash line). We get $n_{th} = 2$, and by Eq. (7-20) we find $n = n_{th}/\eta = 2/0.5 = 4$.

(b) We construct, beginning from the same point A , stages corresponding to actual plates for which $\eta_0 = 0.5$, i. e. we divide into halves the segments of the ordinates between the equilibrium and the operating lines. This gives us $n = 6$.

The results obtained in these two different ways are not the same. The second way gives a more reliable result.

PROBLEMS

7-1. Cresol ($\text{CH}_3\text{C}_6\text{H}_4\text{OH}$) is distilled with steam (a) under atmospheric pressure, and (b) under a pressure of 300 mm Hg. Determine the distillation temperature, the mass composition of the mixture obtained, the volume per cent of the cresol in the vapour, and its partial pressure. Assume that $\phi = 0.8$. For the saturated vapour pressure of cresol see Fig. A-14 (*m*-cresol).

7-2. Glycerine is purified by distillation with superheated steam at 230 °C under a vacuum of 590 mm Hg. The degree of saturation of the steam with

glycerine is 0.75. Determine the amount of steam withdrawn with one ton of glycerine. The crude glycerine is fed in at the distillation temperature. The apparatus has external heating. How will the composition of the vapour mixture change if the vacuum is changed to 620 mm Hg? The boiling point of pure glycerine at a pressure of 760 mm Hg is 290 °C, and at a pressure of 50 mm Hg it is 205 °C. Use the linearity rule taking water as the standard liquid (Table A-38).

7-3. A mixture of benzene and toluene boils at 95 °C under a pressure of 760 mm Hg. At 95 °C, the saturated vapour pressure of benzene is $p_b = 1167$ mm Hg, and that of toluene is $p_t = 480$ mm Hg. Find the composition of the boiling liquid assuming that the mixture obeys Raoult's law.

If the liquid contains half the quantity of toluene, under what pressure will it boil at the same temperature?

7-4. Determine the equilibrium compositions of the liquid and the vapour for a mixture of methyl alcohol and water at a temperature of 50 °C (a) under a pressure of 300 mm Hg, and (b) under a pressure of 500 mm Hg assuming that the mixture obeys Raoult's law.

Explain the result obtained for case (b).

7-5. Construct an equilibrium curve of x against y^* for a mixture of hexane and heptane under a total pressure of 2 at, assuming that Raoult's law holds. Take the saturated vapour pressures of the pure components from the nomogram in Fig. A-14.

7-6. Determine the composition of the equilibrium vapour over a liquid mixture consisting of 10 mole per cent of water, 50 mole per cent of acetic acid, and 40 mole per cent of acetone at $t = 80$ °C, assuming that the mixture components obey Raoult's law.

7-7. One ton (1000 kg) of a binary mixture of benzene and toluene containing 30 mass per cent of benzene is subjected to simple distillation under atmospheric pressure. Determine the amount and the composition of the overhead product if the content of benzene in the bottom product is 18 mass per cent. Use the data of Table 7-1.

7-8. A mixture of acetic acid and water amounting to 2600 kg is subjected to simple distillation under atmospheric pressure. The feed contains 10 mole per cent of acetic acid, the bottom product 50 mole per cent of acetic acid. Determine the mass of the bottom and the overhead products and the composition of the overhead product. For data on the equilibrium compositions see Table A-47.

7-9. The feed of a continuous rectification column contains 24 mole per cent of a low boiler. The concentration of this component in the overhead and the bottom products is 95 and 3 mole per cent, respectively. Steam is supplied to the reflux condenser at a rate of 850 kmol/h, and 670 kmol/h of the reflux is delivered to the column from the condenser. What amount of the bottom product is obtained?

7-10. Determine analytically the abscissas of the points of intersection of the operating lines of a rectification column with the diagonal of a y - x diagram and with each other.

7-11. A continuous rectification column is used to fractionate a mixture of ethyl alcohol and water. The equation of the operating line of the bottom section of the column is $y = 1.28x - 0.0143$. Determine the mass per cent of the alcohol in the bottom product. The column is heated with indirect steam.

7-12. A continuous column processes a mixture of benzene and chloroform. Rectification results in an overhead product containing 95% (mass) of the low boiler. The feed liquid contains 40% of this component. Find the slope of the operating line for the top section of the column if the reflux ratio is known to be double the minimum ratio. For data on the equilibrium compositions see Table A-47.

7-13. A mixture of water and acetic acid is fractionated in a continuous rectification column under atmospheric pressure at a rate of 340 kmol/h. The

ordinate of the point of intersection of the operating lines is 0.48. The equation of the operating line for the top section of the column is $y = 0.84x + 0.15$. The amount of steam fed into the reflux condenser is 550 kmol/h. Determine the amount of bottom product (in kg/h) and the mass fraction of the acetic acid in it.

7-14. A continuous rectification column is fed with 1000 kmol/h of a mixture containing 30% (mole) of pentane and 70% (mole) of hexane, respectively. The overhead product contains 95% (mole) of pentane, the bottom product 90% (mole) of hexane. Determine the amount of the overhead and bottom products (in kg/h) and also the amount of steam condensing in the reflux condenser if the slope of the operating line of the top (rectifying) section of the column equals 0.75.

7-15. A rectification column discharges 1100 kg/h of the overhead product containing 98.5% (mass) of the low boiler and 3650 kg/h of the bottom product containing 96.6% (mass) of the second component. The reflux ratio is 2.94. Determine: (a) the mass per cent of the low boiler in the column feed, and (b) the amount of steam (in kg/h) entering the reflux condenser from the column.

7-16. A mixture of nitrogen and oxygen is boiling on one of the plates of a rectification column in its top (rectifying) section. The concentration of the nitrogen in the liquid flowing down from the plate is 50 mole per cent. Find the nitrogen concentration in the liquid flowing onto the given plate from above if one stage of the change in the concentration corresponds to one plate. The reflux ratio is 2.3. Assume that the overhead product is pure nitrogen. For data on the equilibrium concentrations see Table A-47.

7-17. A liquid containing 65 mole per cent of the low boiler is on one of the plates of the top section of a rectification column. The latter operates with a reflux ratio of $R = 2.5$. The overhead product contains 98% of the low boiler.

Determine the compositions of the vapour entering the given plate and leaving it if the Murphree efficiency of the plate is $\eta_0 = 0.75$, the mixture obeys Raoult's law, and the coefficient of relative volatility is $\alpha = 2.5$. The liquid on the plate mixes completely.

7-18. In a continuous rectification column, $x_D = 90\%$ (mole), $x_F = 30\%$ (mole), $x_W = 3\%$ (mole), and $R = 8$.

Determine the composition of the vapour entering the plate where the liquid contains (a) 75, and (b) 15 mole per cent of the low boiler.

7-19. A continuous rectification column produces 200 kg/h of acetic acid with a concentration of 70 mole per cent. The column processes a mixture of acetic acid and water, which is fed into the column at its boiling point. The content of the acetic acid in the feed is 31 mole per cent. The overhead product is water containing 8 mole per cent of acetic acid. The pressure in the column is atmospheric. Determine the number of stages of the change in concentration with a reflux ratio of 4. Also determine the consumption of heating steam ($p_{\text{abs}} = 4$ at) in the reboiler of the column if the steam has a moisture content of 5%. The heat losses are 4% of the usefully spent heat. For data on the equilibrium compositions see Table A-47.

7-20. A rectification column is fed with 5000 kg/h of a mixture consisting of 29 mass per cent of methyl alcohol and 71 mass per cent of water. The equation of the operating line for the top (rectifying) section of the column is $y = 0.73x + 0.264$. The bottom product amounts to 3800 kg/h.

Determine: (a) the mass per cent of the methyl alcohol in the bottom product, (b) the amount of steam (in kg/h) passing from the column to the reflux condenser, and (c) the rate of flow of the water in the condenser if it is heated in it by 12 K.

7-21. The equations of the operating lines of a rectification column for separating a mixture of benzene and toluene under atmospheric pressure are:

$$y = 0.723x + 0.263$$

$$y = 1.25x - 0.0188$$

The column is fed with 75 kmol/h of the mixture at its boiling point. The heating steam in the column reboiler has a gauge pressure of 3 at. Determine the required heating surface area in the reboiler and the consumption of heating steam having a moisture content of 5%. The overall coefficient of heat transfer is $K = 580 \text{ W/m}^2 \cdot \text{K}$. Disregard the heat losses. Assume that the boiling point of the liquid in the reboiler equals that for pure toluene.

7-22. A continuous rectification column is fed with a mixture of water and ethyl alcohol at 70°C containing 10% (mass) of the alcohol. Determine the consumption of heat in the column reboiler and the amount of heat withdrawn in the reflux condenser per kilogram of overhead product containing 94% (mass) of alcohol if the bottom product contains virtually no alcohol. The rectifying section of the column operates with a reflux ratio of 4. Disregard the heat losses. Indirect steam is used for heating.

7-23. The capacity of a rectification column for separating a mixture of methyl alcohol and water is 1500 kg/h of the overhead product. The column operates under atmospheric pressure. The heat exchange surface area of the reflux condenser is 60 m^2 , the overall coefficient of heat transfer in it is $810 \text{ W/m}^2 \cdot \text{K}$. Determine the reflux ratio and the consumption of the cooling water in the condenser if it is heated from 15 to 35°C .

7-24. Determine the required surface area of and the consumption of water in the reflux condenser of a rectification column for the fractionation of a benzene-toluene mixture in the following conditions: the amount of the overhead product is 600 kg/h, the reflux ratio is 3.75, the initial and final temperatures of the cooling water are 20 and 45°C , the overall heat transfer coefficient is $700 \text{ W/m}^2 \cdot \text{K}$. Assume that the overhead product is pure benzene. The pressure in the column is atmospheric.

7-25. Determine the required number of plates in a batch rectification column for fractionating a mixture of chloroform and benzene under atmospheric pressure. The feed contains 38% (mole) of chloroform, the overhead product should contain 97% (mole), and the bottom product 10% (mole) of chloroform. The reflux ratio is 2. For data on the equilibrium compositions see Table A-47. The number of plates corresponding to one stage of the change in concentration is 1.4.

7-26. A mixture of methyl alcohol and water is fractionated in a continuous rectification column at a rate of 5000 kg/h. The mass fraction of the methyl alcohol in the feed is 20% and in the overhead product 90%. The reflux ratio is 1.8. Water is supplied to the reflux condenser at a rate of $40 \text{ m}^3/\text{h}$ and is heated in it from 20 to 40°C . Determine the amount of methyl alcohol discharged with the bottom product an hour.

7-27. Steam having a moisture content of 5% and an absolute pressure of 1 at is available for heating the reboiler of a rectification column into which 6 ton/h of a benzene-toluene mixture is fed for separation. The feed contains 32 mass per cent of benzene. The required concentration of the overhead product is 97 mass per cent of benzene, and that of the bottom product is 95 mass per cent of toluene.

Determine: (a) the mass rates of flow of the overhead and bottom products; (b) pressure in the column; (c) the required number of plates with a reflux ratio of 3.1 and with an overall plate efficiency of $\eta = 0.71$; and (d) the consumption of heating steam and of water in the reflux condenser if the water is heated in it by 15 K.

The mixture obeys Raoult's law. Take the heat losses equal to 3% of the usefully spent heat. The feed is supplied to the column at its boiling point. Assume that the temperature difference in the column reboiler is 10 K.

EXAMPLE OF CALCULATING AND SELECTING A PLATE RECTIFICATION COLUMN

Calculate and select a continuous rectification column with sieve plates for the fractionation under atmospheric pressure of 10 ton/h of a liquid mixture containing 50 mass per cent each of benzene and toluene. The required content of the benzene in the overhead product is 96 mass per cent and that of the toluene in the bottom product is 98 mass per cent. The feed is heated to its boiling point before being supplied to the column. The heating steam has a pressure of $p_{\text{gauge}} = 3$ at.

The rectification plant is shown schematically in Fig. 7-17.

I. *Material Balance.* We denote the mass rate of flow of the overhead product by G_D (kg/h) and that of the bottom product by G_W (kg/h).

From material balance equations (7-4) and (7-5), i.e.

$$\begin{aligned} G_D + G_W &= 10\,000 \\ G_D \times 0.96 + G_W \times 0.02 &= 10\,000 \times 0.5 \end{aligned}$$

we find $G_D = 5110$ kg/h and $G_W = 4890$ kg/h.

For further calculations, we express the concentrations of the feed, overhead product, and bottom product in mole fractions.

Feed:

$$x_F = \frac{\bar{x}_F / M_b}{\frac{\bar{x}_F}{M_b} + \frac{100 - \bar{x}_F}{M_t}} = \frac{50/78}{\frac{50}{78} + \frac{50}{92}} = 0.542$$

Overhead product:

$$x_D = \frac{\bar{x}_D / M_b}{\frac{\bar{x}_D}{M_b} + \frac{100 - \bar{x}_D}{M_t}} = \frac{96/78}{\frac{96}{78} + \frac{4}{92}} = 0.965$$

Bottom product:

$$x_W = \frac{\bar{x}_W / M_b}{\frac{\bar{x}_W}{M_b} + \frac{100 - \bar{x}_W}{M_t}} = \frac{2/78}{\frac{2}{78} + \frac{98}{92}} = 0.023$$

The relative molar rate of flow of the feed is

$$f = \frac{x_D - x_W}{x_F - x_W} = \frac{0.965 - 0.023}{0.542 - 0.023} = 1.82$$

The equilibrium curve (Fig. 7-7)* has no points of inflection.

We determine the minimum reflux ratio by Eq. (7-10):

$$R_{\min} = \frac{x_D - y_F^*}{y_F^* - x_F} = \frac{0.965 - 0.74}{0.74 - 0.542} = 1.135$$

where $y_F^* = 0.74$ is the mole fraction of the benzene in the vapour in equilibrium with the feed liquid; we determine it from a diagram of y^* against x .

* Experimental data on phase equilibrium are generally used in engineering calculations [6-4]. In the present example, equilibrium relationships are used that have been obtained analytically on the basis of Raoult's law. For the system benzene-toluene, they are close to experimental data.

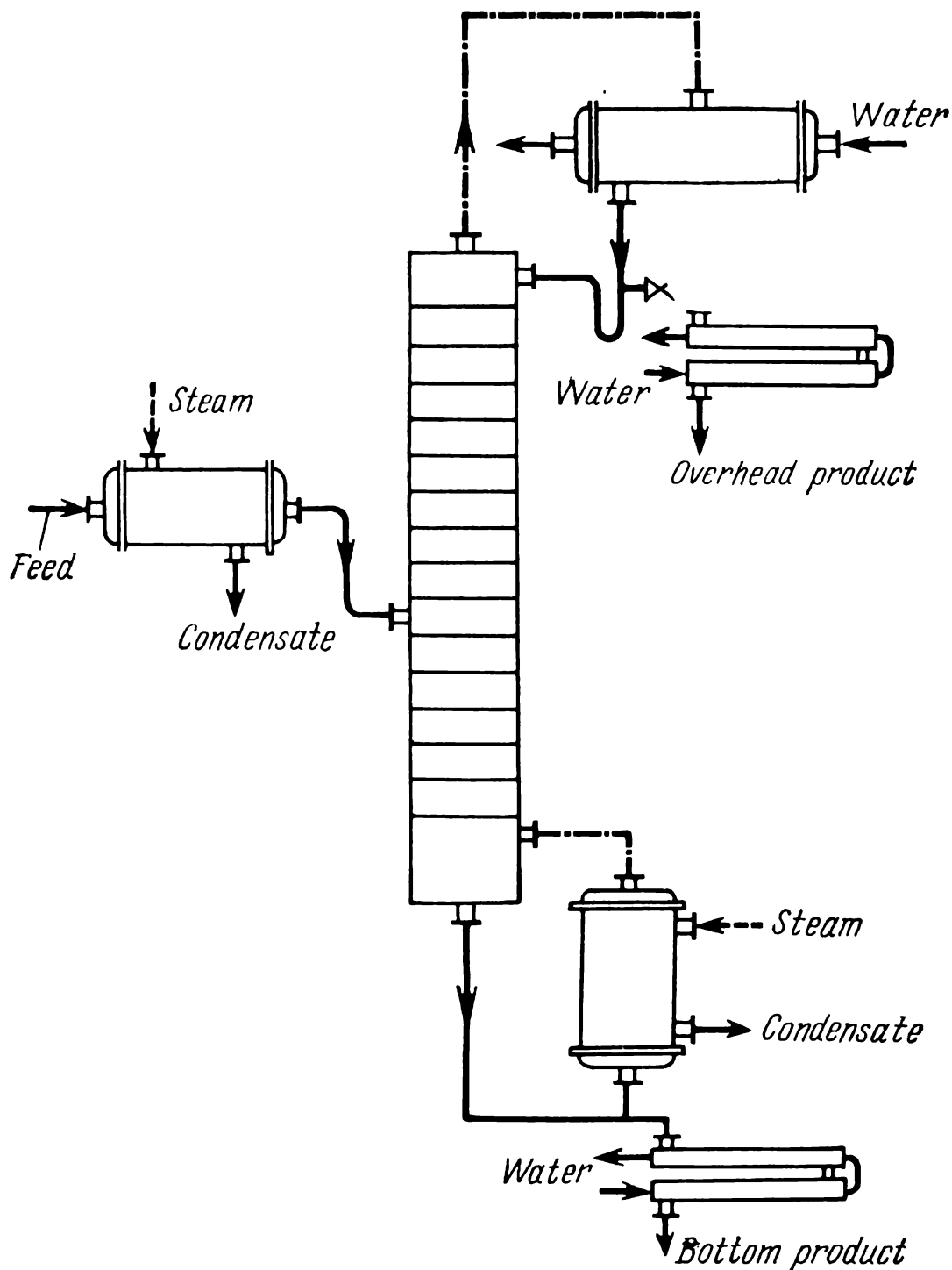


Fig. 7-17. Rectification plant

The operating reflux ratio by Eq. (7-12) is:

$$R = 1.3R_{\min} + 0.3 = 1.3 \times 1.135 + 0.3 = 1.78$$

The equations of the operating lines are:

(a) for the top (rectifying) section of the column:

$$y = \frac{R}{R+1}x + \frac{x_D}{R+1} = \frac{1.78}{2.78}x + \frac{0.965}{2.78}$$

$$y = 0.64x + 0.347$$

(b) for the bottom (stripping) section of the column:

$$y = \frac{R+F}{R+1}x - \frac{F-1}{R+1}x_W = \frac{1.78+1.82}{2.78}x - \frac{1.82-1}{2.78} \times 0.023$$

$$y = 1.3x - 0.0068$$

11. *Determination of the Vapour Velocity and the Column Diameter.* The mean concentrations of the liquid are:

(a) in the top section of the column:

$$x'_m = \frac{x_F + x_D}{2} = \frac{0.542 + 0.965}{2} = 0.754$$

(b) in the bottom section of the column:

$$x''_m = \frac{x_F + x_W}{2} = \frac{0.542 + 0.023}{2} = 0.283$$

We find the mean concentrations of the vapour according to the equations of the operating lines:

(a) in the top section of the column:

$$y'_m = 0.64x'_m + 0.347 = 0.64 \times 0.754 + 0.347 = 0.829$$

(b) in the bottom section.

$$y''_m = 1.3x''_m - 0.0068 = 1.3 \times 0.283 - 0.0068 = 0.361$$

We determine the mean temperatures of the vapour from a diagram of t against x and y (see Fig. 7-6):

(a) for $y'_m = 0.829$ we find $t'_m = 88^\circ\text{C}$

(b) for $y''_m = 0.361$ we find $t''_m = 103^\circ\text{C}$.

The mean molar masses and densities of the vapour are:

$$(a) M'_m = 0.829 \times 78 + 0.171 \times 92 = 80.3 \text{ kg/kmol}$$

$$\rho'_m = \frac{M'_m T_0}{22.4 T'_m} = \frac{80.3 \times 273}{22.4 \times 361} = 2.71 \text{ kg/m}^3$$

$$(b) M''_m = 0.361 \times 78 + 0.639 \times 92 = 87$$

$$\rho''_m = \frac{M''_m T_0}{22.4 T''_m} = \frac{87 \times 273}{22.4 \times 376} = 2.82 \text{ kg/m}^3$$

The mean density of the vapour in the column is:

$$\rho_v = \frac{\rho'_m + \rho''_m}{2} = \frac{2.71 + 2.82}{2} = 2.77 \text{ kg/m}^3$$

The densities of liquid benzene and toluene are close to each other. The temperature at the top of the column with $y_D = 0.965$ is 82°C , and in the re-boiler with $x_W = 0.023$ it is 109°C (see Fig. 7-6).

The density of liquid benzene at 82°C is $\rho_b = 813 \text{ kg/m}^3$, and that of liquid toluene at 109°C is $\rho_t = 783 \text{ kg/m}^3$ [4-22].

We adopt the mean density of the liquid in the column equal to

$$\rho_{lq} = \frac{813 + 783}{2} \approx 800 \text{ kg/m}^3$$

We find the velocity of the vapour in the column by Eq. (7-17). We take a plate spacing of $h = 300 \text{ mm}$ according to a catalogue of column apparatuses. For sieve plates, the graph in Fig. 7-2 gives us $C = 0.032$.

The velocity of the vapour in the column by Eq. (7-18) is:

$$v = C \sqrt{\frac{\rho_{lq}}{\rho_v}} = 0.032 \sqrt{\frac{800}{2.77}} = 0.54 \text{ m/s}$$

The volumetric rate of flow of the vapour passing through the column at the mean temperature in it of $t_m = (88 + 103)/2 \approx 96^\circ\text{C}$ is:

$$Q_v = \frac{G_D (R + 1) 22.4 \times T_m}{M_D T_0 \times 3600} = \frac{5110 (1.78 + 1) 22.4 \times 369}{78.5 \times 273 \times 3600} = 1.52 \text{ m}^3/\text{s}$$

where M_D is the molar mass of the overhead product equal to:

$$M_D = 0.965 \times 78 + 0.035 \times 92 = 78.5 \text{ kg/kmol}$$

The diameter of the column is:

$$D = \sqrt{\frac{Q_v}{0.785v}} = \sqrt{\frac{1.52}{0.785 \times 0.54}} = 1.89 \text{ m}$$

We take a diameter of $D = 1800 \text{ mm}$ according to a catalogue of column apparatuses. Consequently, the velocity of the vapour in the column will be:

$$v = \frac{Q_v}{0.785D^2} = \frac{1.52}{0.785 \times 1.8^2} = 0.6 \text{ m/s}$$

III. *Hydraulic Calculations of the Plates.* We adopt the following dimensions of a sieve plate: hole diameter $d_h = 4 \text{ mm}$, height of liquid-overflow weir $h_w = 40 \text{ mm}$. The free section of the plate (total area of the holes) is 8% of the total area of the plate. The area occupied by the two segmental downcomers is 20% of the total area of the plate.

We calculate the hydraulic resistance of a plate in the top and the bottom sections of the column by Eq. (1-69):

$$\Delta p = \Delta p_{\text{dry}} + \Delta p_\sigma + \Delta p_{\text{fr}}$$

(a) Top section of the column.

The hydraulic resistance of a dry plate, by Eq. (1-70), is:

$$\Delta p_{\text{dry}} = \zeta \frac{v_h^2 \rho_v}{2}$$

The dimensionless resistance coefficient of unirrigated sieve plates with a free section of from 7 to 10% is $\zeta = 1.82$, and the velocity of the vapour in the holes of the plate $v_h = 0.6/0.08 = 7.5 \text{ m/s}$. Introducing values into Eq. (1-70), we get:

$$\Delta p_{\text{dry}} = 1.82 \times \frac{7.5^2 \times 2.71}{2} = 138 \text{ Pa}$$

The resistance due to the forces of surface tension, by Eq. (1-71), is:

$$\Delta p_\sigma = \frac{4\sigma}{d_h}$$

The surface tension of the liquid at the mean temperature in the top section of 88°C [4-22] is $\sigma = 20.5 \times 10^{-3} \text{ N/m}$ (benzene and toluene have virtually the same surface tension), and the diameter of a plate hole $d_h = 4 \text{ mm} = 0.004 \text{ m}$. Hence,

$$\Delta p_\sigma = \frac{4 \times 20.5 \times 10^{-3}}{0.004} = 20.5 \text{ Pa}$$

The resistance of the froth (vapour-liquid layer) on a plate is:

$$\Delta p_{\text{fr}} = 1.3 h_{\text{fr}} k \rho_{\text{fr}} g$$

The height of the froth layer (Fig. 7-18) is:

$$h_{\text{fr}} = h_w + \Delta h$$

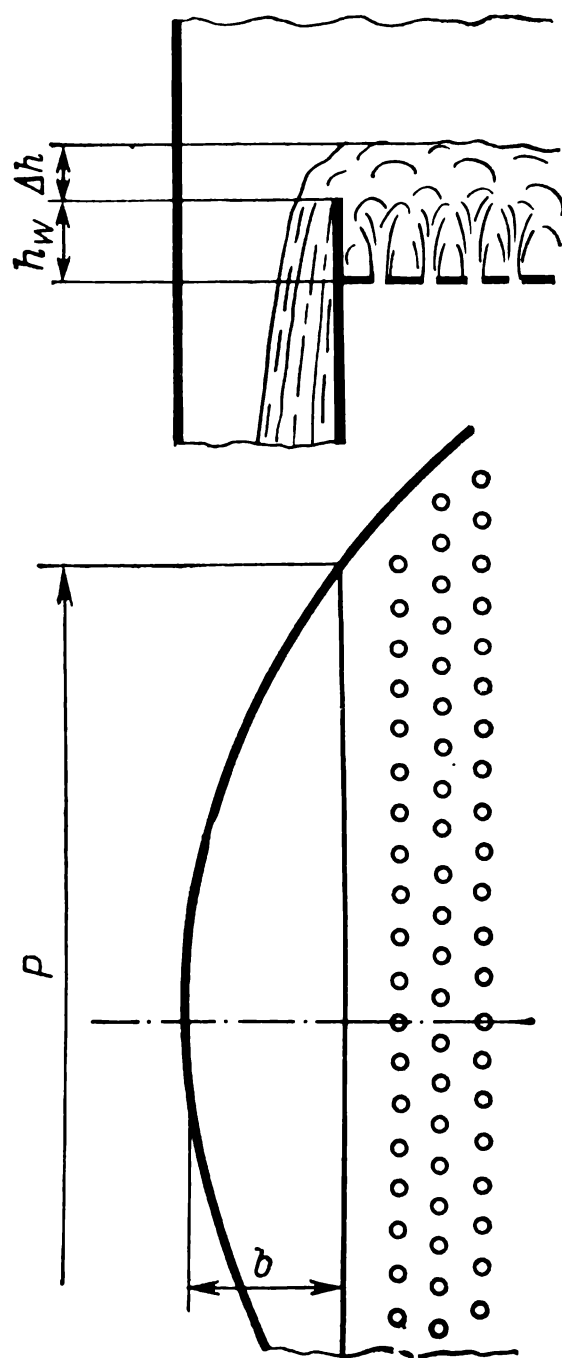


Fig. 7-18. Diagram of a sieve plate

We calculate the value of Δh —the weir crest (the height of the layer above the weir) by Eq. (1-74):

$$\Delta h = \left(\frac{Q_{v, lq}}{1.85 P k} \right)^{2/3}$$

where $Q_{v, lq}$ = volumetric rate of flow of the liquid, m^3/s

P = perimeter of the weir, m

$k = \rho_{fr}/\rho_{lq}$ = ratio of the density of the froth layer to that of the liquid, taken approximately equal to 0.5.

The volumetric rate of flow of the liquid in the top section of the column is:

$$Q_{v, lq} = \frac{G_D R M_m}{M_D \rho_{lq}} = \frac{5110 \times 1.78 \times 81.4}{3600 \times 78.5 \times 800} = 0.00328 \text{ m}^3/\text{s}$$

where $M_m = 0.754 \times 78 + 0.246 \times 92 = 81.4$ is the mean molar mass of the liquid, kg/kmol .

The perimeter P of the weir (Fig. 7-18) is found by solving the system of equations:

$$\left(\frac{P}{2} \right)^2 + (R - b)^2 = R^2$$

$$0.1\pi R^2 = \frac{2}{3} P b$$

where $R = 0.9$ m = radius of a plate

$\frac{2}{3} Pb$ = approximate value of the area of a segment*.

Solution of this system of equations yields $P = 1.32$ m and $b = 0.289$ m. We find Δh :

$$\Delta h = \left(\frac{0.00328}{1.85 \times 1.32 \times 0.5} \right)^{2/3} = 0.0193 \text{ m}$$

The height of the froth layer on a plate is:

$$h_{fr} = h_w + \Delta h = 0.04 + 0.0193 = 0.0593 \text{ m}$$

The resistance of the froth layer is:

$$\Delta p_{fr} = 1.3 h_{fr} k \rho_{lq} g = 1.3 \times 0.0593 \times 0.5 \times 800 \times 9.81 = 302 \text{ Pa}$$

The total hydraulic resistance of a plate in the top section of the column is:

$$\Delta p' = \Delta p_{dry} + \Delta p_{\sigma} + \Delta p_{fr} = 138 + 20.5 + 302 = 461 \text{ Pa}$$

(b) Bottom section of the column:

$$\Delta p_{dry} = \frac{1.82 \times 7.5^2 \times 2.82}{2} = 144 \text{ Pa}$$

$$\Delta p_{\sigma} = \frac{4 \times 18.8 \times 10^{-3}}{0.004} = 18.8 \text{ Pa}$$

where 18.8×10^{-3} N/m is the surface tension of the liquid at $t_m = 103$ °C.

The volumetric rate of flow of the liquid is:

$$Q_{v, lq} = \left(\frac{G_D R}{M_D} + \frac{G_F}{M_F} \right) \frac{M_m}{\rho_{lq}}$$

The molecular masses are $M_F = 0.542 \times 78 + 0.458 \times 92 = 84.4$ kg/kmol and $M_m = 0.283 \times 78 + 0.717 \times 92 = 88$ kg/kmol. Therefore,

$$Q_{v, lq} = \left(\frac{5110 \times 1.78}{78.5} + \frac{10000}{84.4} \right) \frac{88}{3600 \times 800} = 0.00717 \text{ m}^3/\text{s}$$

Next, we find:

$$\Delta h = \left(\frac{0.00717}{1.85 \times 1.32 \times 0.5} \right)^{2/3} = 0.0325 \text{ m}$$

$$h_{fr} = 0.04 + 0.0325 = 0.0725 \text{ m}$$

$$\Delta p_{fr} = 1.3 \times 0.0725 \times 0.5 \times 800 \times 9.81 = 369 \text{ Pa}$$

The total hydraulic resistance of a plate in the bottom section of the column is:

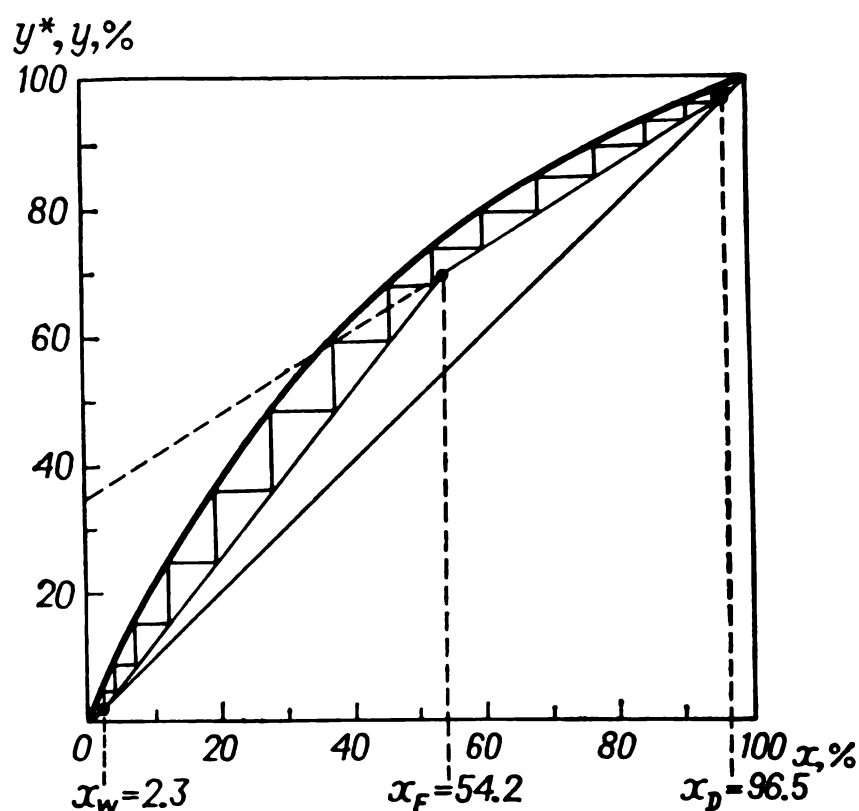
$$\Delta p'' = 144 + 18.8 + 369 = 532 \text{ Pa}$$

We check whether a plate spacing of $h = 0.3$ m ensures the condition needed for normal operation of the plate that

$$h > 1.8 \frac{\Delta p}{\rho_{lq} g}$$

* Vygodsky, M. Ya *Mathematical Handbook. Elementary Mathematics*. Moscow, Mir Publishers (1975).

Fig. 7-19. Determination of the number of stages of the change in concentration



For the plates of the bottom section of the column for which the hydraulic resistance Δp is greater than for the plates of the top section, we have:

$$\frac{1.8\Delta p''}{\rho_{lq}g} = \frac{1.8 \times 532}{800 \times 9.81} = 0.122 \text{ m}$$

Consequently, the above condition is observed.

We check the uniformity of operation of the plates—we calculate the minimum velocity of the vapour in the holes $v_{h,\min}$ sufficient for a sieve plate to operate with all its holes [0-47]:

$$v_{h,\min} = 0.67 \sqrt{\frac{g\rho_{lq}h_{fr}}{\zeta\rho_v}} = 0.67 \sqrt{\frac{9.81 \times 800 \times 0.0725}{1.82 \times 2.82}} = 7.05 \text{ m/s}$$

The calculated velocity $v_{h,\min}$ is less than $v_h = 7.5 \text{ m/s}$, therefore the plates will operate with all their holes.

IV. *Determination of the Number of Plates and the Column Height.* We plot the operating lines for the top and the bottom sections of the column in a diagram of y against x (Fig. 7-19) and find the number of stages of the change in the concentration n_{th} . For the top section of the column, we get $n'_{th} \approx 7$, for the bottom section $n''_{th} \approx 8$, altogether 15 stages.

We calculate the number of plates by Eq. (7-20):

$$n = \frac{n_{th}}{\eta}$$

To determine the overall plate efficiency η , we find the coefficient of relative volatility of the components being separated $\alpha = p_b/p_t$ and the dynamic viscosity of the feed μ at the mean temperature in the column, which equals 96°C .

At this temperature, the saturated vapour pressure of benzene is $p_b = 1204 \text{ mm Hg}$, and of toluene is $p_t = 492.5 \text{ mm Hg}$ (Table 7-1), whence $\alpha = 1204/492.5 = 2.45$.

The dynamic viscosities of benzene and toluene at 96°C are 0.27 cP and 0.29 cP , respectively. We take the dynamic viscosity of the feed $\mu = 0.28 \text{ cP} = 0.28 \times 10^{-3} \text{ Pa}\cdot\text{s}$.

Consequently, $\alpha\mu = 2.45 \times 0.28 = 0.685$.

From the graph in Fig. 7-4, we find $\eta = 0.53$. The length of the path of the liquid on a plate (Fig. 7-18) is:

$$l = D - 2b = 1.8 - 2 \times 0.289 = 1.22 \text{ m}$$

We use the graph in Fig. 7-5 to find the value of the correction for the length of the path $\Delta = 0.105$. The mean overall efficiency of the plates by Eq. (7-21) is

$$\eta_L = \eta(1 + \Delta) = 0.53(1 + 0.105) = 0.59$$

For purposes of comparison, let us calculate the Murphree efficiency of a plate η_0 according to a dimensionless formula obtained by the statistical processing of numerous experimental data for bubble and sieve plates*:

$$\eta_0 = 0.068 C_1^{0.1} C_2^{0.115}$$

In this formula, the dimensionless complexes are:

$$C_1 = \frac{Re_v}{A_{\text{free}}} Pr'_{lq} \frac{\mu_v}{\mu_{lq}} = \frac{vh_w \rho_v}{A_{\text{free}} \mu_v} \frac{\mu_{lq}}{\rho_{lq} D_{lq}} \frac{\mu_v}{\mu_{lq}} = \frac{vh_w \rho_v}{A_{\text{free}} \rho_{lq} D_{lq}}$$

$$C_2 = \frac{Re_v}{We} Pr'_{lq} \frac{\nu_v}{\nu_{lq}} = \frac{vh_w \sigma}{\nu_v \rho_{lq} v^2} \frac{\nu_{lq} \nu_v}{h_w D_{lq} \nu_{lq}} = \frac{\sigma}{v \rho_{lq} D_{lq}}$$

where v = velocity of the vapour in the column, m/s

A_{free} = relative area of the free section of a plate

h_w = height of a weir, m

ρ_v and ρ_{lq} = densities of the vapour and the liquid, respectively, kg/m³

D_{lq} = coefficient of diffusion of the low boiler in the feed determined by Eq. (6-28), m²/s

σ = surface tension of the feed liquid, N/m

We = Weber dimensionless number.

The physicochemical constants are related to the mean temperature in the column. We preliminarily find the diffusion coefficient D_{lq} by Eq. (6-28):

$$D_{lq} = 7.4 \times 10^{-12} \times \frac{(CM)^{0.5} T}{\mu_{lq} V^{0.6}}$$

In our case, $C = 1$, $\mu_{lq} = 0.28 \text{ cP} = 0.28 \times 10^{-3} \text{ Pa} \cdot \text{s}$, $M = M_F = 84.4 \text{ kg/kmol}$, $V = 6 \times 14.8 + 6 \times 3.7 = 15 = 96$, and $T = 96 + 273 = 369 \text{ K}$. Hence, the diffusion coefficient is:

$$D_{lq} = \frac{7.4 \times 10^{-12} \times 84.4^{0.5} \times 369}{0.28 \times 96^{0.6}} = 5.8 \times 10^{-9} \text{ m}^2/\text{s}$$

The dimensionless complexes are:

$$C_1 = \frac{vh_w \rho_v}{A_{\text{free}} \rho_{lq} D_{lq}} = \frac{0.6 \times 0.04 \times 2.77}{0.08 \times 800 \times 5.8 \times 10^{-9}} = 1.79 \times 10^5$$

$$C_2 = \frac{\sigma}{v \rho_{lq} D_{lq}} = \frac{19.7 \times 10^{-3}}{0.6 \times 800 \times 5.8 \times 10^{-9}} = 0.71 \times 10^4$$

The Murphree efficiency of a plate is

$$\eta_0 = 0.068 C_1^{0.1} C_2^{0.115} = 0.068 (1.79 \times 10^5)^{0.1} (0.71 \times 10^4)^{0.115} = 0.63$$

which is close to the found value of η_L .

* MacFarland, S. A., Sigmund, P. M., and Van Winkle, M. *Hydrocarb. Process. and Petrol. Refiner*, 51, 7: 111 (1972).

The number of plates is:
in the top section of the column:

$$n' = \frac{n'_{th}}{\eta_L} = \frac{7}{0.59} = 12$$

in the bottom section of the column:

$$n'' = \frac{n''_{th}}{\eta_L} = \frac{8}{0.59} = 14$$

The total number of plates $n = 26$; with a reserve we adopt $n = 30$, of which 14 are in the top section and 16 in the bottom section of the column.

The height of the plate part of the column, by Eq. (7-19), is:

$$Z_{pl} = (n - 1)h = (30 - 1)0.3 = 8.7 \text{ m}$$

The total hydraulic resistance of the plates is:

$$\Delta p = \Delta p' n' + \Delta p'' n'' = 461 \times 14 + 532 \times 16 = 14\,950 \text{ Pa} \approx 0.15 \text{ at}$$

V. *Thermal Calculations of the Plant.* We find the amount of heat given up to the cooling water in the reflux condenser by Eq. (7-15):

$$Q_{r.c} = G_D (1 + R) L_D$$

Here

$$L_D = \bar{x}_D L_b + (1 - \bar{x}_D) L_t = 0.96 \times 392.4 \times 10^3 + 0.04 \times 377.8 \times 10^3 = 392 \times 10^3 \text{ J/kg}$$

where L_b and L_t are the specific heats of condensation of benzene and toluene at 82 °C. Hence,

$$Q_{r.c} = \frac{5110}{3600} (1 + 1.78) 392 \times 10^3 = 1\,550\,000 \text{ W}$$

The amount of heat obtained in the reboiler from the heating steam is calculated by Eq. (7-14):

$$\begin{aligned} Q_{reb} &= Q_{r.c} + G_D c_D t_D + G_W c_W t_W - G_F c_F t_F + Q_1 = \\ &= 1.03 \left(1\,550\,000 + \frac{5110}{600} \times 0.46 \times 4190 \times 82 + \frac{4890}{3600} \times 0.45 \times 4190 \times 109 - \right. \\ &\quad \left. - \frac{10\,000}{3600} \times 0.455 \times 4190 \times 91.5 \right) = 1\,615\,000 \text{ W} \end{aligned}$$

Here the heat losses Q_1 have been taken equal to 3% of the usefully spent heat; the specific heat capacities have been taken for $t_D = 82$ °C, $t_W = 109$ °C, and $t_F = 91.5$ °C, respectively; the boiling point of the feed $t_F = 91.5$ °C has been determined from Fig. 7-6.

The consumption of heat in the steam heater of the feed is:

$$Q = 1.05 G_F c_F (t_F - t_{in}) = 1.05 \times \frac{10\,000}{3600} \times 0.425 \times 4190 (91.5 - 18) = 382\,000 \text{ W}$$

Here the heat losses have been taken equal to 5%, the specific heat capacity of the feed $c_F = (0.5 \times 0.43 + 0.5 \times 0.42) 4190 \text{ J/kg} \cdot \text{K}$ has been taken for the mean temperature of $(91.5 + 18)/2 = 55$ °C.

The amount of heat given up to the cooling water in the water cooler of the overhead product is:

$$Q = G_D c_D (t_D - t_{fin}) = \frac{5110}{3600} \times 0.43 \times 4190 (82 - 25) = 145\,500 \text{ W}$$

where the specific heat capacity of the overhead product $c_D = 0.43 \times 4190 \text{ J/kg} \cdot \text{K}$ has been taken for the mean temperature of $(82 + 25)/2 \approx 54^\circ\text{C}$.

The amount of heat given up to the cooling water in the water cooler of the bottom product is:

$$Q = G_W c_W (t_W - t_{\text{fin}}) = \frac{4890}{3600} \times 0.425 \times 4190 (109 - 25) = 203\,000 \text{ W}$$

where the specific heat capacity of the bottom product $c_W = 0.425 \times 4190 \text{ J/kg} \cdot \text{K}$ has been taken for the mean temperature of $(109 + 25)/2 = 67^\circ\text{C}$.

The rate of flow of the heating steam having a pressure of $p_{\text{abs}} = 4 \text{ at}$ and a moisture content of 5% is:

(a) in the reboiler:

$$G_{\text{st}} = \frac{Q_{\text{reb}}}{L_{\text{st}} x} = \frac{1\,615\,000}{2141 \times 10^3 \times 0.95} = 0.8 \text{ kg/s}$$

where $L_{\text{st}} = 2141 \times 10^3 \text{ J/kg}$ is the specific heat of condensation of the heating steam;

(b) in the feed heater:

$$G_{\text{st}} = \frac{382\,000}{2141 \times 10^3 \times 0.95} = 0.19 \text{ kg/s}$$

The total rate of flow of the heating steam is $0.8 + 0.19 = 0.99 \text{ kg/s} = 3.6 \text{ ton/h}$.

The volumetric rate of flow of the cooling water when heated by 20 K is:

(a) in the reflux condenser:

$$Q_{v,w} = \frac{Q_{r,c}}{c_W (t_{\text{fin}} - t_{\text{in}}) \rho_w} = \frac{1\,550\,000}{4190 \times 20 \times 1000} = 0.0185 \text{ m}^3/\text{s}$$

(b) in the overhead product water cooler:

$$Q_{v,w} = \frac{145\,500}{4190 \times 20 \times 1000} = 0.001\,74 \text{ m}^3/\text{s}$$

(c) in the bottom product water cooler:

$$Q_{v,w} = \frac{203\,000}{4190 \times 20 \times 1000} = 0.002\,42 \text{ m}^3/\text{s}$$

The total rate of flow of the cooling water is $0.0185 + 0.001\,74 + 0.002\,42 = 0.022\,66 \approx 0.0227 \text{ m}^3/\text{s} = 82 \text{ m}^3/\text{h}$.

SYMBOLS

A	area
C	coefficient depending on plate design; coefficient taking into account the association of the solvent molecules; dimensionless complex
c	specific heat capacity
D	diameter; mass of distillate (overhead product); diffusion coefficient
d	diameter
F	feed (amount)
f	relative rate of flow of feed
G	amount of substance; rate of flow of gas
G'	mass rate of flow
g	acceleration due to gravity
H	enthalpy

h	depth; distance between plates
L	latent heat of condensation; rate of flow of liquid
l	length
M	molar mass
m	slope of equilibrium line
n	number of plates
P	perimeter
p	pressure
Pr	Prandtl dimensionless number
Q	amount of heat
Q_v	volumetric rate of flow
R	radius; reflux ratio
Re	Reynolds dimensionless number
T	absolute temperature
t	temperature, °C
V	ratio G_V/G_D ; volume
v	velocity
W	bottom product
w	relative molar rate of flow of bottom product
We	Weber dimensionless number
x	mole fraction, liquid phase
y	mole fraction, gaseous phase
Z	height of column

Greek Letters

α	coefficient of relative fugacity
Δ	correction to Murphree efficiency
η	overall plate efficiency
μ	dynamic viscosity
ν	kinematic viscosity
π	pole
ρ	density
φ	coefficient taking into account incomplete steam saturation with vapour; excess reflux coefficient

**FUNDAMENTAL RELATIONSHIPS
AND FORMULAS FOR CALCULATIONS**

Extraction is the name given to the process of separating one or more components from a liquid mixture with the aid of a solvent that preferentially dissolves the required components. Leaching is the name given to the process of separating soluble matter from its mixture with an insoluble solid. After extraction or leaching, the target component is separated from its mixture with the solvent by distillation or rectification.

The expediency of using extraction is determined (1) by the impossibility of separating a mixture by rectification owing to the formation of azeotropic mixtures, the non-volatility, or the insufficient thermal stability of the components, (2) by the saving of heat if its expenditure for rectification of the feed owing to the close values of the boiling points of the components, low concentrations, or other reasons is greater than that for extraction and distillation of the solvent from the separation products.

Problems in the statics of extraction and leaching are mainly solved graphically with the aid of a phase (a triangular or a rectangular) diagram.

1. The features of a triangular diagram are as follows:

(a) the vertexes of the triangle (Fig. 8-1) correspond to the pure components A, B, and C, the sides AB , BC and AC of the triangle correspond to two-component mixtures of A and B, B and C, and A and C, respectively, and points inside the triangle correspond to three-component systems (for example, point g is the figurative, i.e. characterizing point for a mixture of 70 mass per cent of A, 20 of B and 10 mass per cent of C);

(b) rays (for instance, Aa , Bb , Cc) drawn from the vertexes of the triangle are the loci of figurative points of mixtures with a constant ratio of the contents of the other two components x_B/x_C , x_A/x_C , and x_A/x_B , respectively;

(c) lines dd , ee , ff parallel to the sides of the triangle AC , BC , and AB are the loci of figurative points of mixtures having a constant content of the components B, A, and C, respectively.

Fig. 8-1. Triangular diagram

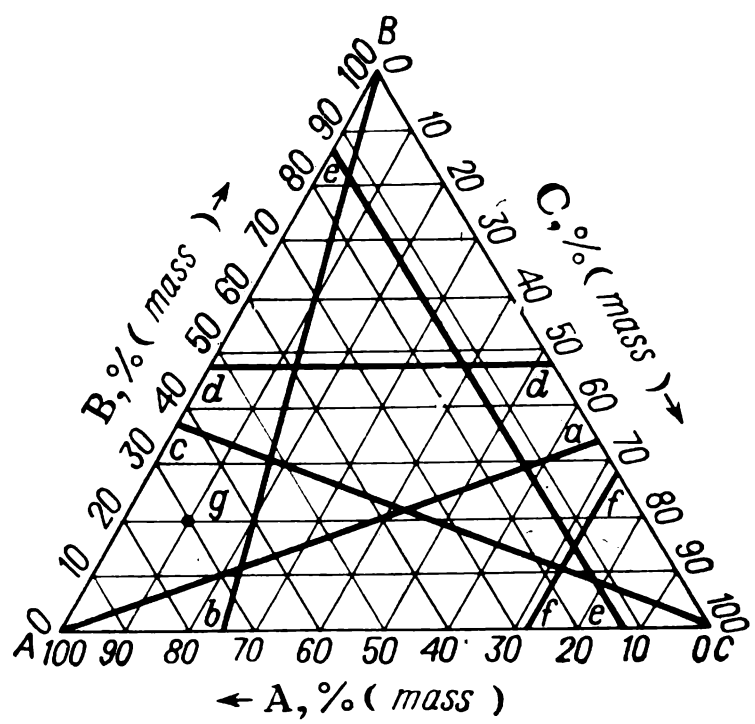
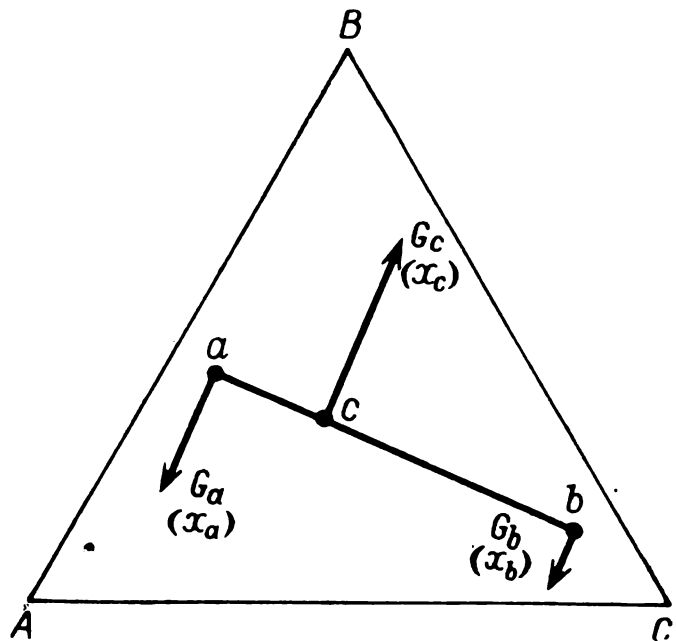


Fig. 8-2. The lever rule



2. The lever rule (a particular case of the centre-of-gravity principle) states that when two solutions are mixed whose compositions are characterized in the diagram by any points a and b , the total composition of the mixture will be shown by point c lying on straight line ab connecting these points. Segments ac and bc are inversely proportional to the amounts of solutions taken (Fig. 8-2):

$$G_a + G_b = G_c$$

where $\bar{x}_a + \bar{x}_b \neq \bar{x}_c$.

Hence,

$$\left. \begin{aligned} G_a \overline{ac} &= G_b \overline{bc}; & \frac{G_a}{\overline{bc}} &= \frac{G_b}{\overline{ac}} \\ G_c \overline{ac} &= G_b \overline{ab} \\ G_c \overline{bc} &= G_a \overline{ab}; & \frac{G_c}{\overline{ab}} &= \frac{G_b}{\overline{ac}} = \frac{G_a}{\overline{bc}} \end{aligned} \right\} \tag{8-1}$$

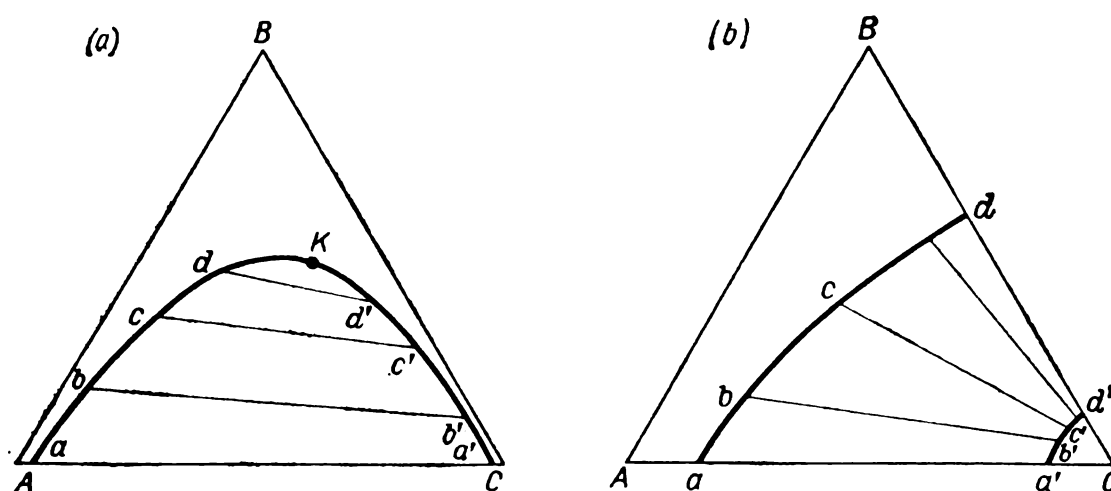


Fig. 8-3. Liquid-liquid system with one (a) and two (b) pairs of partly miscible components (with $t = \text{const}$)

where G_a , G_b , G_c = masses of the solutions a and b , and the mixture c , respectively, kg

x_a , x_b , x_c = contents of any component (A, B, or C) in the solutions a and b , and the mixture c , mass per cent.

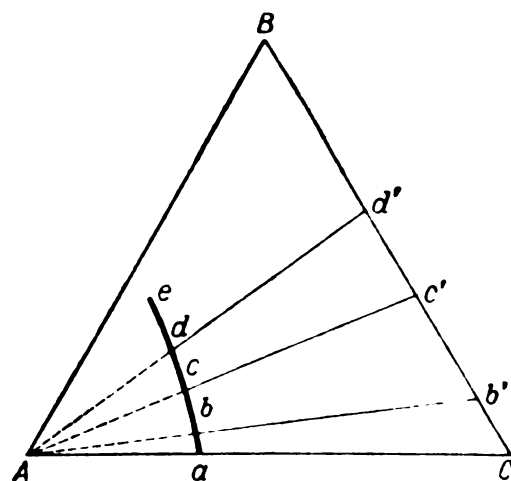
The same relationships hold when the mixture c becomes stratified into two coexisting phases a and b .

3. Typical ternary systems having the greatest importance are shown in Figs. 8-3 (liquid extraction) and 8-4 (leaching). Here vertex A of the triangle corresponds to the primary (feed) solvent (liquid or solid), vertex B to the required solute (liquid or solid), and vertex C to the secondary (leaching or extracting) solvent (liquid).

In Fig. 8-3, line $abcdKd'c'b'a'$ is a boundary (binodal) curve. The region within it is that of mixtures stratifying into two coexisting phases whose composition is expressed by points on the curve; the region without the curve is that of non-stratifying (homogeneous) solutions. Point K is the critical (plait) point. The left-hand part of the boundary curve (from point K) is the branch of raffinates (what remains of the feed after extraction of the solute—the desired substance—from it); the right-hand part of the curve is the branch of extracts. Segments bb' , cc' and dd' connecting the figurative points of coexisting phases are equilibrium chords called tie lines*.

In Fig. 8-4, line $abcde$ is the boundary curve. The region to the left of it is that of three-component heterogeneous mixtures; the region to the right is that of stratification. Side BC of the triangle characterizes the compositions of the overflow (the solution of the solute in the solvent). The boundary curve character-

* For a description of the graphical interpolation of the tie lines in a triangular diagram see Example 8-1.

Fig. 8-4. Solid-liquid system (with $t = \text{const}$)

izes the compositions of the underflow (a heterogeneous mixture of the solid insoluble substance, the solute, and the solvent retained in the pores of the solid substance). Tie lines bb' , cc' , and dd' when continued pass through vertex A of the triangle.

4. The coefficient of distribution of the desired component B between the extract and raffinate phases is expressed by the relationship:

$$k = \frac{y_B}{x_B}; \quad k \geq 1 \quad (8-2)$$

where y_B = content of the desired component B in the extract phase, mass per cent

x_B = equilibrium content of the desired component B in the raffinate phase, mass per cent.

The distribution coefficient usually depends on the concentrations, therefore analytical calculations [8-6] give only an approximate result.

5. **Rectangular diagrams.** If the miscibility of the primary (A) and secondary (C) solvents may be disregarded, then a more convenient rectangular diagram in the coordinates x'_B against y'_B (the mass ratios) is used for graphical calculations. Here:

$$\left. \begin{aligned} x'_B &= \frac{x_B}{100 - x_B} \quad \frac{\text{kg of extract}}{\text{kg of feed solvent}} \quad \text{in raffinate phase} \\ y'_B &= \frac{y_B}{100 - y_B} \quad \frac{\text{kg of extract}}{\text{kg of extracting solvent}} \quad \text{in extract phase} \end{aligned} \right\} \quad (8-3)$$

6. A rectangular diagram is also used when a triangular one does not provide sufficient accuracy because of the density of the lines in it.

The ratio of the content of one of the components of a ternary system to the sum of the contents of the other two components (called the base mixture) is laid off along the axis of ordinates, and the ratio of the content of the other component to the same

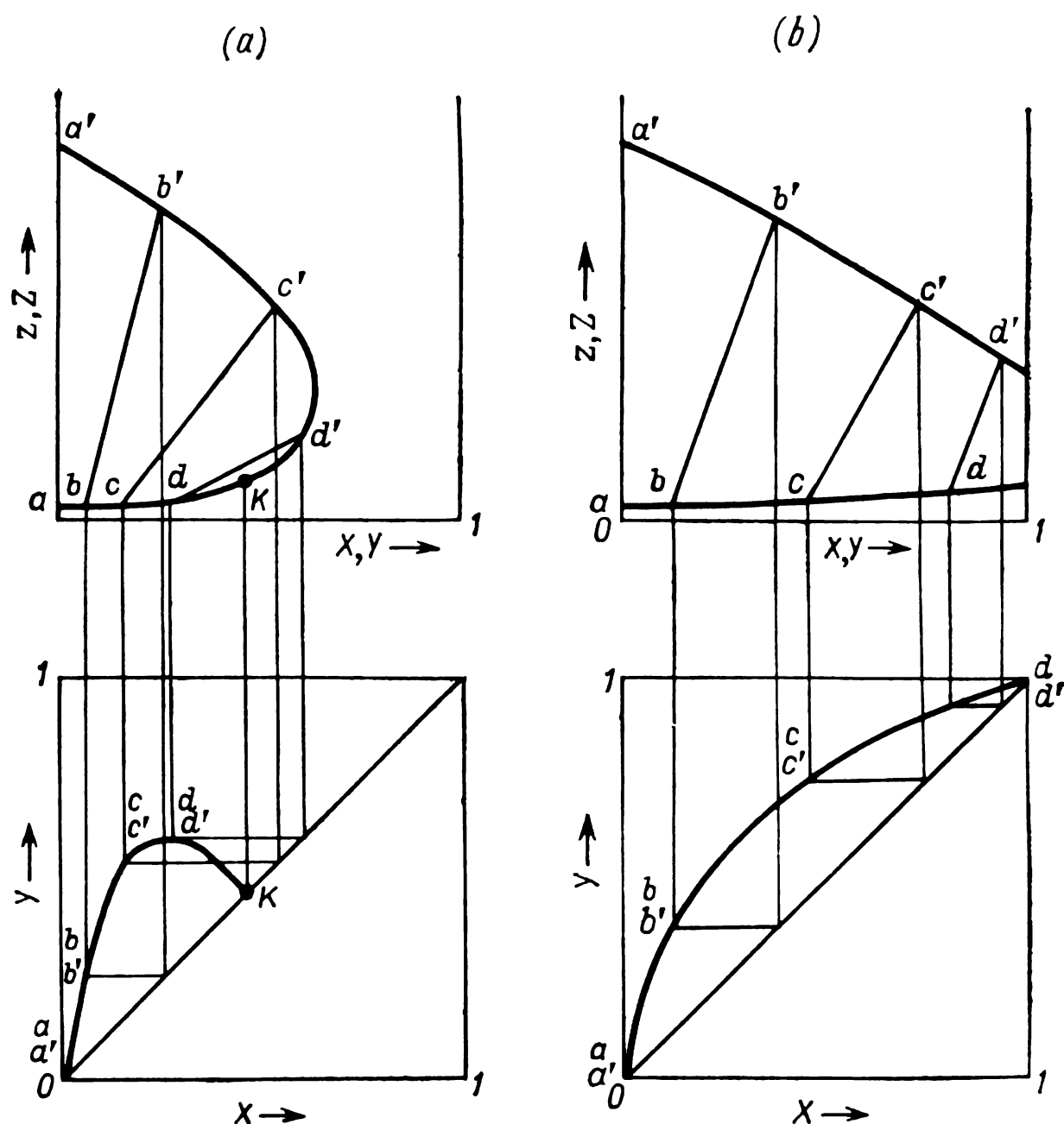


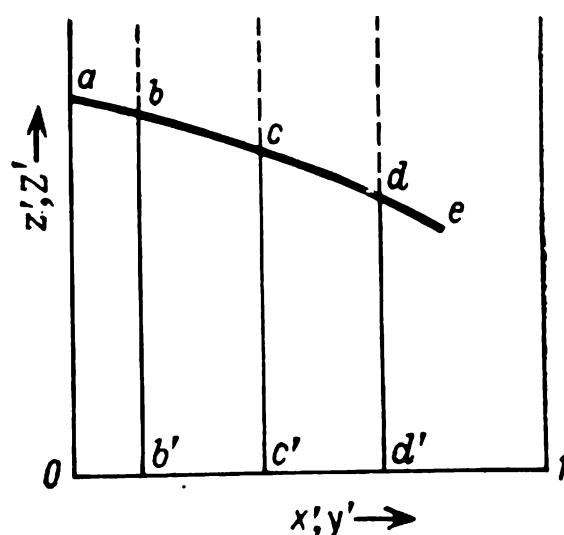
Fig. 8-5. Liquid-liquid system with one (a) and two (b) pairs of partly miscible components (with $t = \text{const}$)

base mixture along the axis of abscissas. The following coordinates are often used, for example:

(a) the coordinates X , Y against z , Z , and X against Y^* for the system liquid-liquid (Fig. 8-5; the symbols are the same as in Fig. 8-3):

$$\begin{aligned}
 X &= \frac{x_B}{x_A + x_B} = \frac{x_B}{100 - x_C} \quad \frac{\text{kg of B}}{\text{kg of A + B}} && \text{in raffinate phase} \\
 Y &= \frac{y_B}{y_A + y_B} = \frac{y_B}{100 - y_C} \quad \frac{\text{kg of B}}{\text{kg of A + B}} && \text{in extract phase} \\
 z &= \frac{x_C}{x_A + x_B} = \frac{x_C}{100 - x_C} \quad \frac{\text{kg of C}}{\text{kg of A + B}} && \text{in raffinate phase} \\
 Z &= \frac{y_C}{y_A + y_B} = \frac{y_C}{100 - y_C} \quad \frac{\text{kg of C}}{\text{kg of A + B}} && \text{in extract phase}
 \end{aligned} \quad (8-4)$$

* An auxiliary diagram for finding the tie lines.

Fig. 8-6. Solid-liquid system (with $t = \text{const}$)

The formulas for the reverse calculations are:

$$\left. \begin{aligned} x_A &= \frac{1-X}{1+z}; & y_A &= \frac{1-Y}{1+Z} \\ x_B &= \frac{X}{1+z}; & y_B &= \frac{Y}{1+Z} \\ x_C &= \frac{z}{1+z}; & y_C &= \frac{Z}{1+Z} \end{aligned} \right\} \quad (8-5)$$

(b) the coordinates X' , Y' against z' , Z' for a solid-liquid system (Fig. 8-6; the symbols are the same as in Fig. 8-4);

$$\left. \begin{aligned} X' &= \frac{x_B}{x_B + x_C} = \frac{x_B}{100 - x_A} \frac{\text{kg of B}}{\text{kg of B} + \text{C}} && \text{in underflow} \\ Y' &= \frac{y_B}{y_B + y_C} = \frac{y_B}{100 - y_A} \frac{\text{kg of B}}{\text{kg of B} + \text{C}} && \text{in overflow} \\ z' &= \frac{x_A}{x_B + x_C} = \frac{x_A}{100 - x_A} \frac{\text{kg of A}}{\text{kg of B} + \text{C}} && \text{in underflow} \\ Z' &= \frac{y_A}{y_B + y_C} = \frac{y_A}{100 - y_A} \frac{\text{kg of A}}{\text{kg of B} + \text{C}} && \text{in overflow} \end{aligned} \right\} \quad (8-6)$$

In these formulas:

x_A , x_B , x_C = content of the components A, B, and C in the raffinate phase (in the underflow), mass per cent

y_A , y_B , y_C = equilibrium content of the components A, B, and C in the extract phase (in the overflow), mass per cent.

7. Crosscurrent (simple multistage) liquid extraction. The equation of the overall material balance for the n -th extraction stage* (Fig. 8-7a) is:

$$G_{R, n-1} + G_{S, n} = G_{R, n} + G_{E, n} \quad (8-7)$$

* The subscripts stand for: F = feed; S = extracting solvent; R = raffinate, and E = extract.

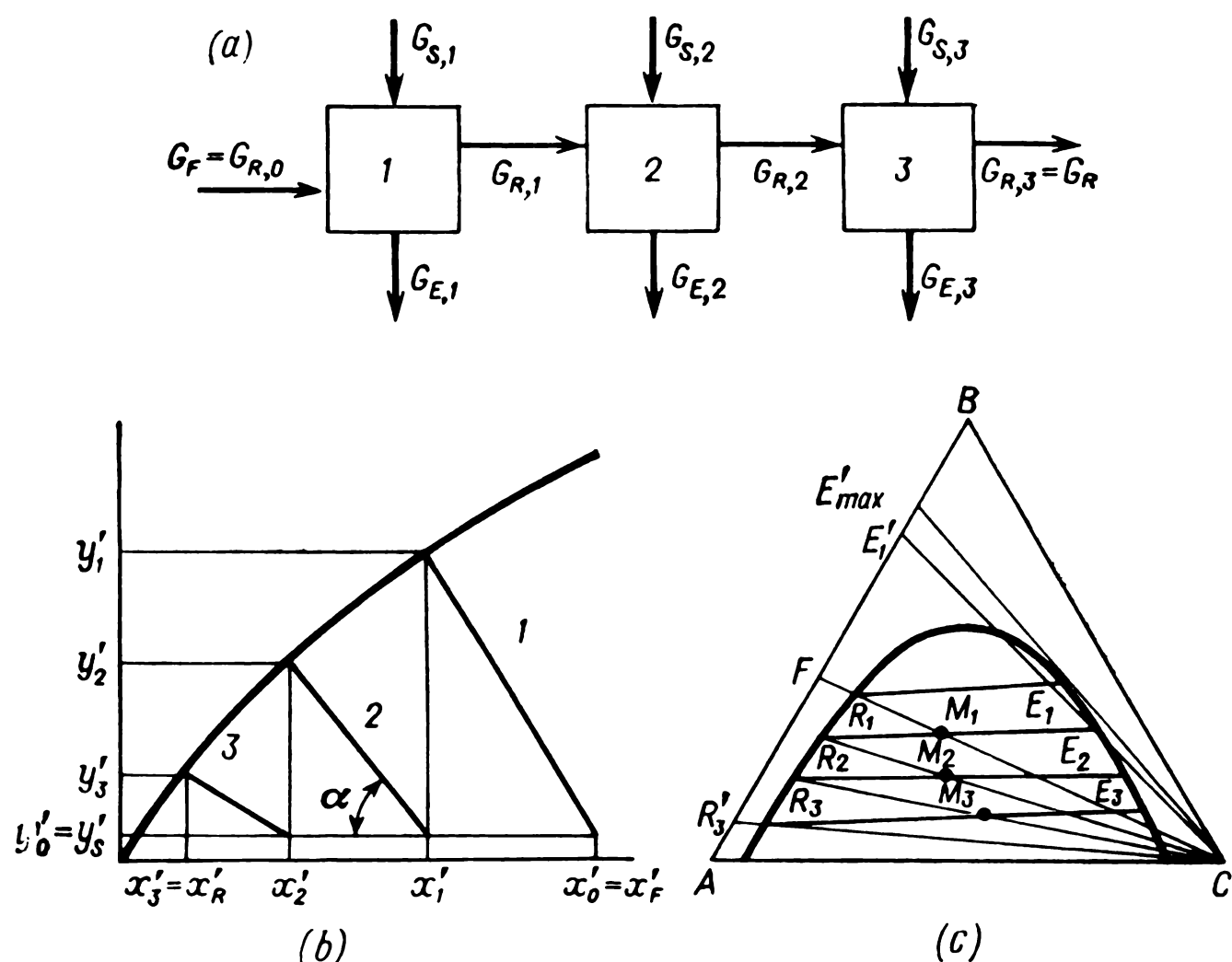


Fig. 8-7. Liquid extraction in a crosscurrent

The equation of the material balance for the n -th stage with respect to the desired component is:

$$G_{R, n-1}x_{n-1} + G_{S, n}y_S = G_{R, n}x_n + G_{E, n}y_n \quad (8-8)$$

When the miscibility of the feed solvent and the extracting solvent may be disregarded, a rectangular diagram in the coordinates x' against y' is used for the calculations (paragraph 5).

The amount of feed solvent (diluent) A (in kg or kg/s) in the feed is:

$$G_A = G_F \frac{100 - x_F}{100} \quad (8-9)$$

The amount of the extracting solvent C_n (in kg or kg/s) in the extract is:

$$G_{C_n} = G_S \frac{100 - y_S}{100} \quad (8-10)$$

The equation of the operating line of the n -th stage is:

$$y'_n = -\frac{G_A}{G_{C_n}} (x'_n - x'_{n-1}) + y'_S \quad (8-11)$$

The slope of the operating line is characterized by the expression:

$$\tan \alpha = \frac{G_A}{G_{C_n}} \quad (8-12)$$

The number of stages of the change in the concentration (the number of extraction stages) is determined by the number of operating lines in the diagram (Fig. 8-7b).

When the feed and extracting solvents are partly miscible, a triangular diagram is used (Fig. 8-7c).

The position of point M_n characterizing the general composition of the mixture at the n -th stage is determined according to the lever rule from the ratio of the flows $G_{R, n-1}/G_{S, n-1}$.

The compositions of the raffinate x_n and the extract y_n leaving the n -th stage are determined by the ends of the tie line (R_n and E_n) drawn through point M_n . The amounts of the raffinate and extract are also determined according to the lever rule.

The required number of theoretical extraction stages is determined by the number of tie lines $E_n R_n$ obtained in the construction in the diagram up to reaching the preset composition of the raffinate x_R .

The composition and amount of the extract or raffinate after distillation of the solvent are determined by the point of intersection of a ray drawn through vertex C of the triangle and the figurative point of the crude extract or raffinate with side AB .

For the procedure to be followed in the calculations and graphical construction, see Examples 8-2 and 8-3.

8. Countercurrent liquid extraction. The equation of the overall material balance for an n -stage extraction plant (Fig. 8-8a) is:

$$G_F + G_S = G_R + G_E \quad (8-13)$$

The equation of the material balance with respect to the solute is:

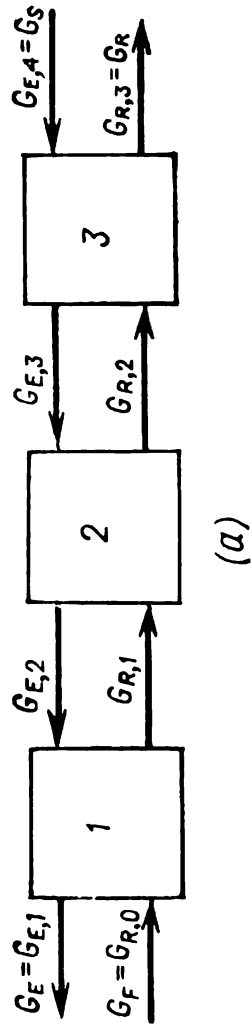
$$G_F x_F + G_S y_S = G_R x_R + G_E y_E \quad (8-14)$$

When the miscibility of the feed solvent and the extracting solvent may be disregarded, the amounts of the pure feed solvent G_A and extracting solvent G_C will be the same for all the stages of the plant. The equation of the material balance with respect to the solute therefore becomes:

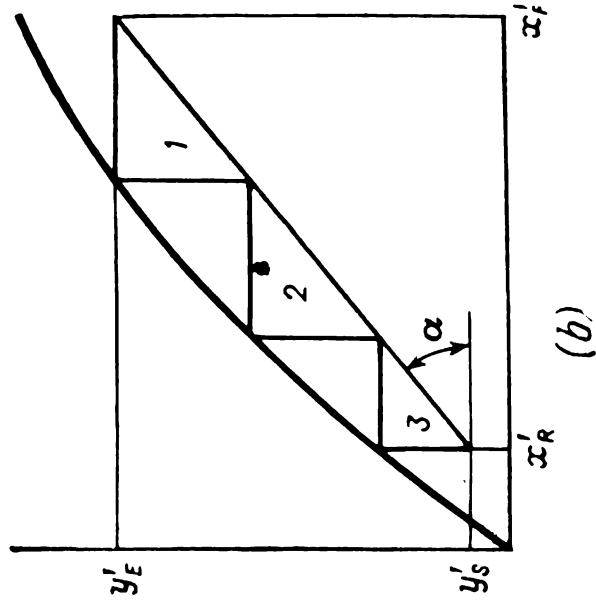
$$G_A (x'_F - x'_R) = G_C (y'_E - y'_S) \quad (8-15)$$

The equation of the operating line is:

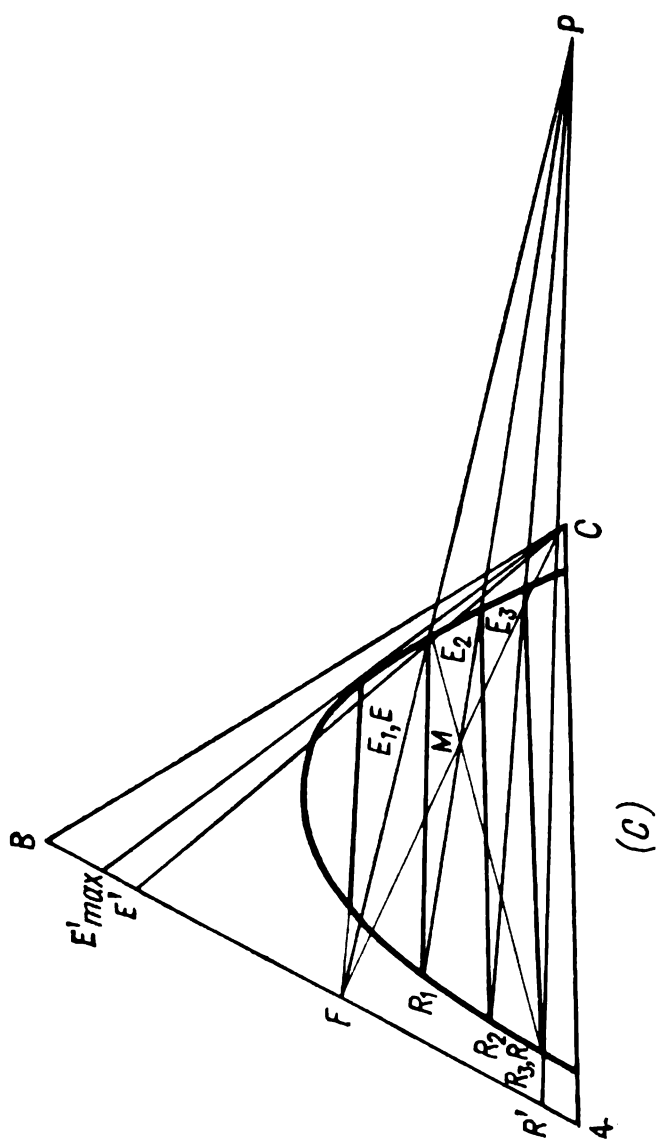
$$y'_{n+1} = \frac{G_A}{G_C} (x'_n - x'_F) + y'_E \quad (8-16)$$



(a)



(b)



(c)

Fig. 8-8. Countercurrent liquid extraction

The slope of the operating line— $\tan \alpha$ (Fig. 8-8b) is determined by the expression:

$$\tan \alpha = \frac{G_A}{G_C} = \frac{y'_E - y'_S}{x'_F - x'_R} \quad (8-17)$$

The required number of theoretical extraction stages is determined graphically in the same way as in absorption and rectification.

Should a process be calculated with the aid of a triangular diagram (Fig. 8-8c), the amounts of the currents are expressed by the position of point M characterizing the fictitious (conditional) overall composition of the mixture in the extraction plant and are determined according to the lever rule from the relationships:

$$\begin{aligned} G_F + G_S &= G_R + G_E = G_M \\ \frac{G_F}{G_S} &= \frac{\overline{MC}}{\overline{FM}}; \quad \frac{G_E}{G_R} = \frac{\overline{MR}}{\overline{ME}} \end{aligned} \quad (8-18)$$

The required number of theoretical extraction stages is determined by the number of tie lines $E_n R_n$ obtained in the construction in the diagram up to reaching the preset composition of the raffinate x_R .

Point P called the extraction pole is the point of intersection of the straight lines drawn through points F and E , R and C , R_n and E_{n+1} and is used to find the figurative points of the extracts on the extract branch of the boundary curve. The pole may be at either side of the triangle. For the procedure to be followed in the calculations and graphical construction, see Example 8-4.

9. Countercurrent liquid extraction using reflux*. The overall material balance of the extraction plant (Fig. 8-9a) is:

$$G_F = G'_E + G'_R \quad (8-19)$$

(we assume here and below that the flows $G_{S,0}$, $G_{S,-1}$, and $G_{S,n+1}$ are the pure solvent C , and the flows G'_E and G'_R contain no solvent).

The material balance with respect to the solute B is:

$$G_F X_F = G'_E Y_E + G'_R X_R \quad (8-20)$$

The minimum extract and raffinate reflux ratios $R_{E_{\min}}$ and $R_{R_{\min}}$ are determined (Fig. 8-9b) by the points of intersection $P_{E_{\min}}$ and $P_{R_{\min}}$ (the poles of the extract-enriching section and raffinate-stripping section of the column) of tie line ab ,

* The reflux of either one or both products may be used depending on the conditions.

An infinitely great number of extraction stages corresponds to the minimum refluxes. The operating reflux ratios are:

$$\left. \begin{aligned} R_E &= \beta R_{E_{\min}} = \frac{\overline{P_E E_1}}{\overline{E_1 E}} \\ R_R &= \beta R_{R_{\min}} = \frac{\overline{P_R R}}{\overline{R E_{n+1}}} \end{aligned} \right\} \quad (8-22)$$

where β is a surplus reflux factor always greater than unity.

The amounts of the flows and the consumption of the solvent are calculated by sequential computations:

$$\begin{aligned} G_{S-1} &= G'_E z_E \\ G_E &= G'_E + G_{S-1} = G'_E (1 + z_E) \end{aligned} \quad (8-23)$$

$$\begin{aligned} G_{R,0} &= R_E G_E \\ G_{R,0} + G_E &= G_E (1 + R_E) = G'_E (1 + z_E) (1 + R_E) \\ G_{S,1} &= \frac{G_{R,0} + G_E}{1 + z_F} (Z_{E,1} - z_E) = G'_E (1 + R_E) (Z_{E,1} - z_E) \end{aligned} \quad (8-24)$$

$$G_{E,1} = G_{S,0} + (G_{R,0} + G_E) \text{ and so on} \quad (8-25)$$

$$\begin{aligned} G_{S,n+1} &= G'_R z_R \\ G_R &= G'_R + G_{S,n+1} = G'_R (1 + z_R) \\ G_{R,n+1} &= R_R G_R = R_R G'_R (1 + z_R) \\ G_{C,n+1} &= R_R G'_R z_R \text{ and so on.} \end{aligned} \quad (8-26)$$

Here $G_{C,n+1}$ is the amount of the solvent C in the raffinate $G_{R,n+1}$ directed into the mixer.

The total amount of circulating solvent is:

$$G_{S_{\text{tot}}} = G'_E [(1 + R_E) (Z_{E,1} - z_E) + z_E] + G'_R (1 + R_R) z_R \quad (8-27)$$

The required number of theoretical extraction stages is determined graphically with the aid of diagrams of X, Y against z, Z and X against Y (Fig. 8-9b). The procedure followed in plotting the graphs is described in Examples 8-10 and 8-11.

10. Multistage cocurrent (parallel) leaching*. The equations for the material balances of the n -th stage are the same as for liquid crosscurrent extraction [Fig. 8-10a and Eqs. (8-7) and (8-8)].

If pure solvent C is used for treatment, the fraction of the solute remaining unextracted (the ratio of the amount of the solute in the residue to that in the feed) can be calculated by the formula

$$\varphi = \frac{G_{R,n} x_n}{G_F x_F} = \frac{1}{(1 + a_1) (1 + a_2) (1 + a_3) \dots (1 + a_i) \dots (1 + a_n)} \quad (8-28)$$

* The diagram of the process is similar to that of liquid crosscurrent extraction.

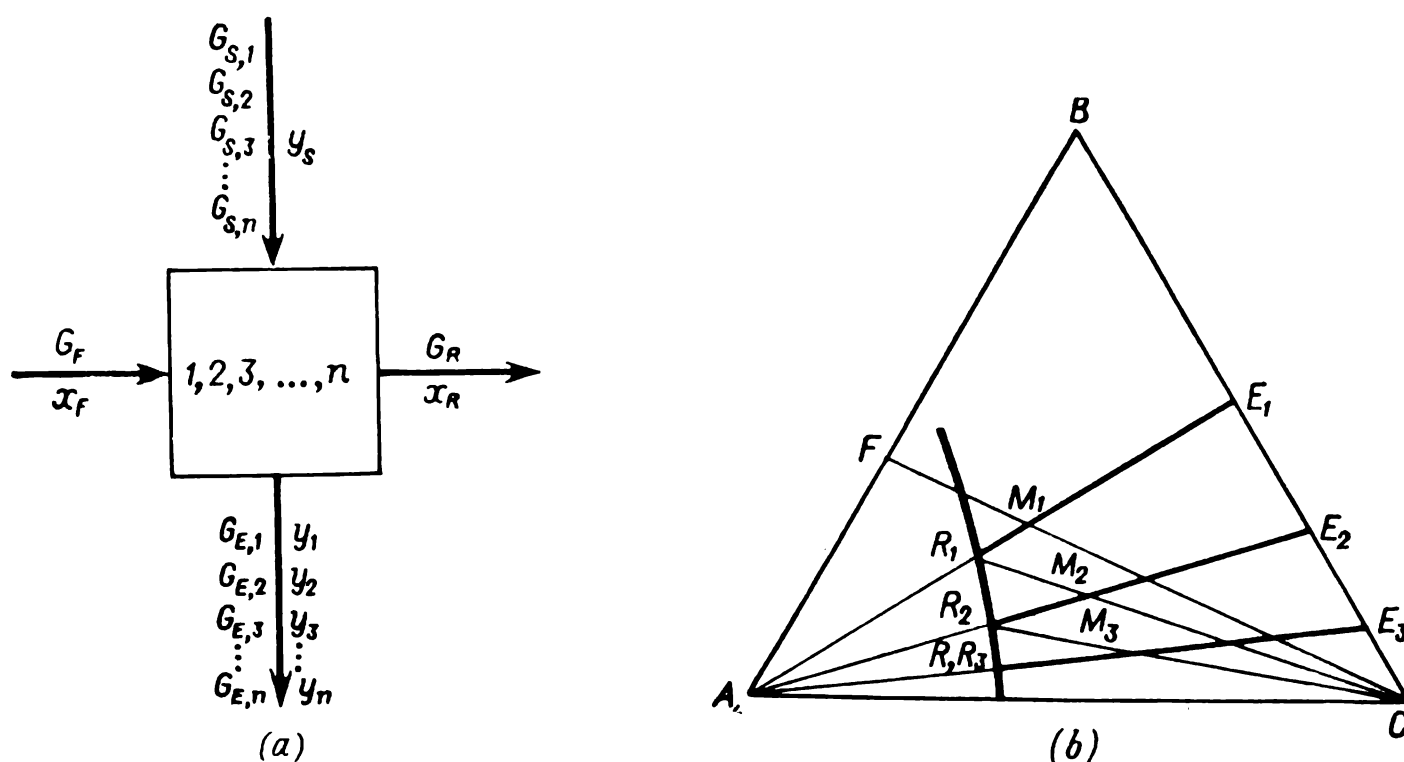


Fig. 8-10. Leaching with periodical changing of the solvent

where $a_i = \frac{G_{E,i}}{G'_{R,i}}$ = ratio of the flows: the mass of the separated solution to that of the solution retained by the solid (the ratio of the volumes of the solutions may also be used)

$G'_{R,i} = G_{R,i} (1 - x_A)$ = mass of components B and C.

If the ratio of the flows is constant, i. e. $a_1 = a_2 = a_3 \dots = a_n = \text{constant}$, then Eq. (8-28) becomes simplified:

$$\varphi = \frac{1}{(1 + a)^{n_s}} \quad (8-29)$$

where n_s is the number of leaching stages.

Calculations with the aid of a triangular diagram (Fig. 8-10b) are similar to those for liquid crosscurrent extraction.

11. Countercurrent leaching. The equations for the material balances are the same as for countercurrent liquid extraction [Fig. 8-11a and Eqs. (8-13) and (8-14)].

When the ratio of the flows for all the stages except the first one is constant, i. e. $a_2 = a_3 = \dots = a = \text{const}$, the fraction of the solute unextracted can be determined by the equation:

$$\varphi = \frac{1}{1 + a_1 (1 + a + a^2 + \dots + a^{n-1}) - \frac{G_{sys}}{G_{R,n} x_n} [1 + a_1 (1 + a + a^2 + \dots + a^{n-2})]} \quad (8-30)$$

When the pure solvent is used ($y_S = 0$), this expression becomes simplified:

$$\varphi = \frac{1}{1 + a_1 (1 + a + a^2 + \dots + a^{n-1})} \quad (8-31)$$

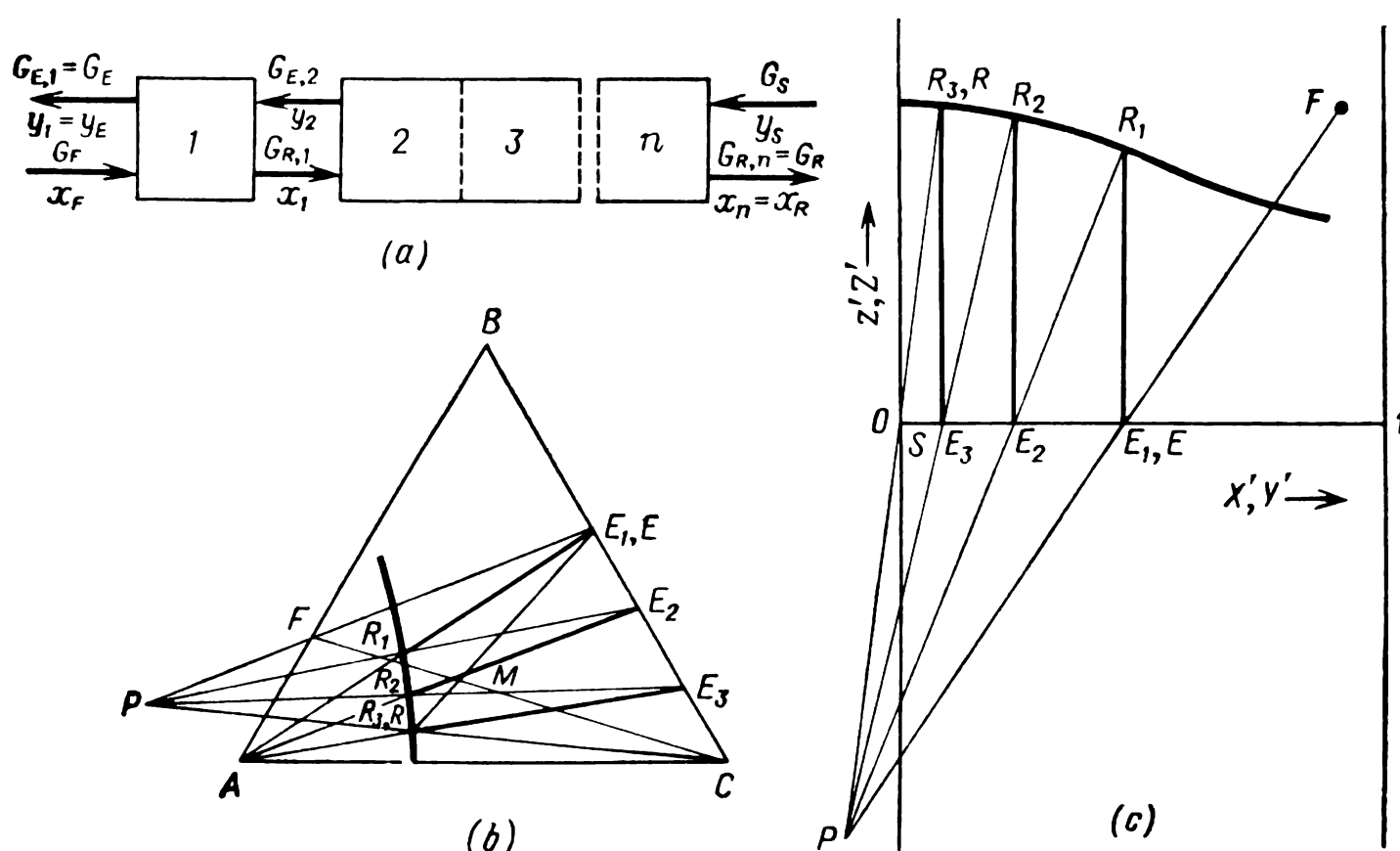


Fig. 8-11. Countercurrent leaching

If, in addition, the feed already contains the same amount of solution as there is between the stages, i. e. $a_1 = a$, we get:

$$\Psi = \frac{1}{1 + a + a^2 + \dots + a^n} \quad (8-32)$$

The required number of theoretical leaching stages* n_s with constant flow ratios $a_2 = a_3 = \dots = a = \text{const}$ can be determined by the formula:

$$n_s - 1 = \frac{\log \frac{x_R - y_S}{x_1 - y_2}}{\log \frac{y_2 - y_S}{x_1 - x_R}} = \frac{\log \frac{x_1 - y_2}{x_R - y_S}}{\log \frac{x_1 - x_R}{y_2 - y_S}} \quad (8-33)$$

In the general case, the calculations can be carried out with the aid of a triangular diagram (Fig. 8-11b) or a rectangular one in the coordinates X', Y' against z', Z' (Fig. 8-11c). The calculation procedure is similar to that for countercurrent liquid extraction.

* This number can be found graphically, in the same way as for a binary system, using a rectangular diagram in the coordinates y against x' , where $x' = x_B / (1 - x_A)$, i. e. the mass of the insoluble solid does not participate in the calculations. The equation of the equilibrium line in this case is $y^* = x'$; the equations of the operating lines are easily derived from the material balance equations.

EXAMPLES

Example 8-1. Construct a triangular diagram of phase equilibrium for the system water-acetone-chlorobenzene. Take the data needed from Table 8-1. Use the diagram to determine (a) the content of water and chlorobenzene in an aqueous layer with an acetone concentration of 45 mass per cent; (b) the composition of a chlorobenzene layer in equilibrium with it; and (c) the amount of acetone whose addition will cause a mixture of 0.11 kg of chlorobenzene and 0.09 kg of water to stop stratifying.

Solution. We draw an equilateral triangle (Fig. 8-12). We find points 1 and 1' corresponding to the first line of Table 8-1 on side AC. We find points 2 and 2' corresponding to the second line of the table inside the triangle and join them with straight line 2-2'. When finding points inside the triangle, we first lay off the acetone content in the solution along side AB and then the chlorobenzene content parallel to side AC. After finding all the points, we draw a smooth curve through them.

TABLE 8-1.

EQUILIBRIUM COMPOSITIONS
OF COEXISTING PHASES
(IN MASS PER CENT)

Aqueous layer			Chlorobenzene layer		
water	acetone	chloro- benzene	water	acetone	chloro- benzene
99.89	0	0.11	0.18	0	99.82
89.79	10	0.21	0.49	10.79	88.72
79.69	20	0.31	0.79	22.23	76.98
69.42	30	0.58	1.72	37.48	60.80
58.64	40	1.36	3.05	49.44	47.51
46.28	50	3.72	7.24	59.19	33.57
27.41	60	12.59	22.85	61.07	15.08
25.66	60.58	13.76	25.66	60.58	13.76

(a) We draw straight line *ab* through point *a* on side *AB* parallel to side *AC*. Line *ab* is the locus of points characterizing mixtures containing 45 mass per cent of acetone. At the intersection of line *ab* with the left branch of the binodal curve, we find point *c* characterizing the following composition of the aqueous layer: 52.8% (mass) of water and 2.2% (mass) of chlorobenzene.

(b) To determine the composition of the coexisting phase, we draw straight lines parallel to the sides of the triangle through

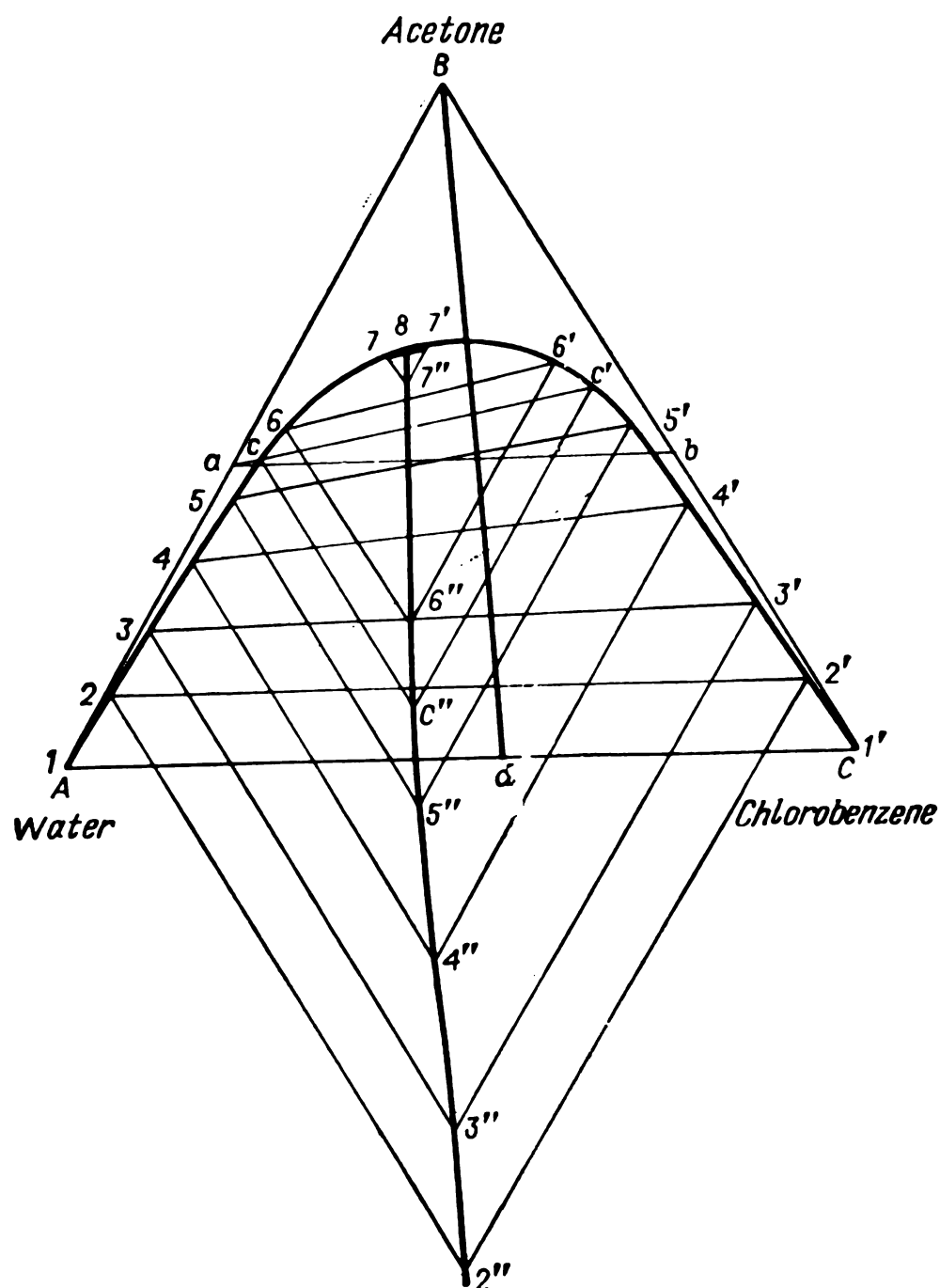


Fig. 8-12. To Example 8-1

points 2, 2', 3, 3', ..., 7, 7' and connect the points of their intersection 2'', 3'', ..., 7'' with a smooth curve. Next through point *c*, we draw a straight line parallel to side *BC* up to its intersection with the connecting curve at point *c''*, and from point *c''* a straight line parallel to side *AB* up to its intersection with the binodal curve. Point *c'* is the required one. The composition of the layer is (by mass) 54.9% of acetone, 4.3% of water, and 40.8% of chlorobenzene.

(c) On side *AC*, we find point *d* characterizing the total composition of the given mixture: $[0.09/(0.09 + 0.11)] 100 = 45\%$ by mass of water and 55% by mass of chlorobenzene. When acetone is added to this mixture, its total composition will change along line *dB* whose intersection with the binodal curve gives us the required point *e*. We find the amount of acetone needed from the

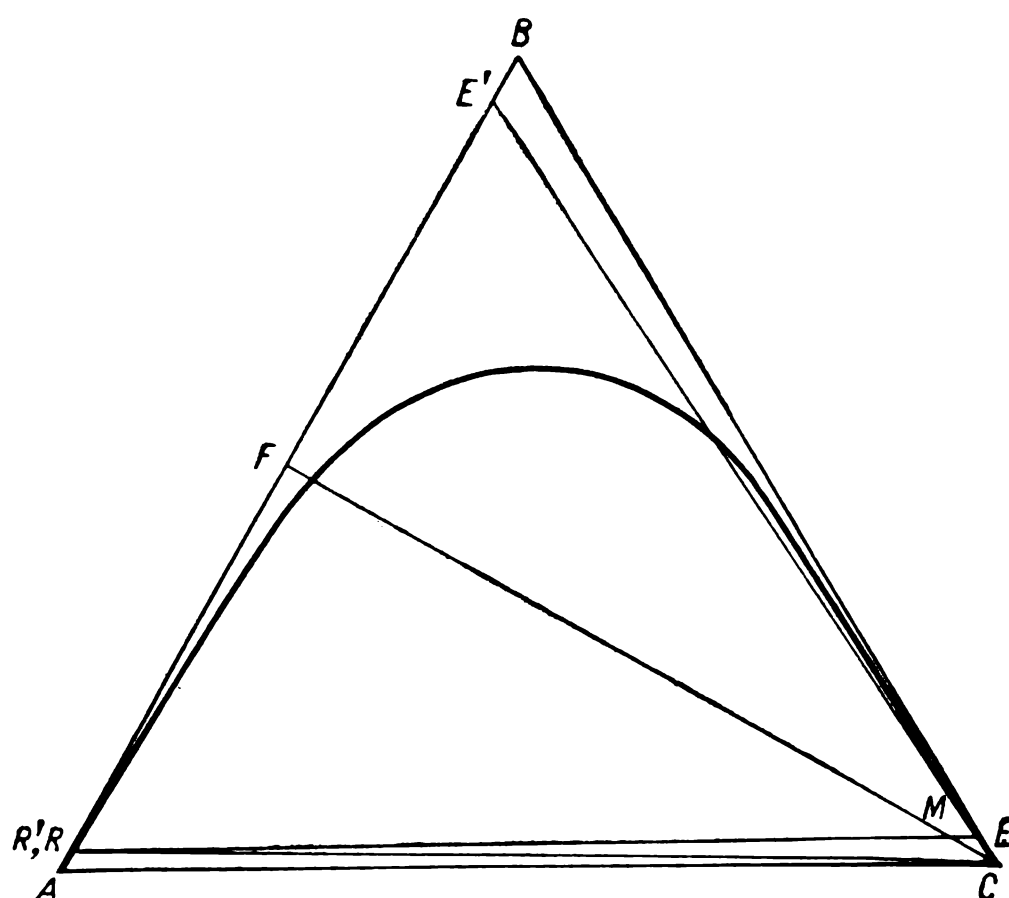


Fig. 8-13. To Example 8-2

ratio of segments Be and ed :

$$\frac{G_d}{G_B} = \frac{Be}{ed} \text{ and } G_B = \frac{(0.09 + 0.11) 53.5}{33} = 0.325 \text{ kg}$$

Example 8-2. Acetone is extracted with chlorobenzene from a 50% aqueous solution. The raffinate should contain not over 2 mass per cent of acetone. Using the diagram constructed in the preceding example, determine the amount of solvent needed for processing 100 kg of the feed if extraction is conducted in one stage. Also determine the yield of the raffinate, and the yield and composition of the extract-product.

Solution. We draw tie line RE through point R (Fig. 8-13) characterizing the composition of the raffinate. We connect point F characterizing the composition of the feed with vertex C of the triangle. Point M of intersection of lines FC and RE determines the composition of a mixture of the feed and the solvent needed to obtain a raffinate of the given composition. We find the required amount of solvent from the ratio

$$\frac{G_S}{G_F} = \frac{FM}{MC} \text{ and } G_S = \frac{100 \times 81.5}{5} = 1630 \text{ kg}$$

The mass of the mixture obtained is:

$$G_M = 1630 + 100 = 1730 \text{ kg}$$

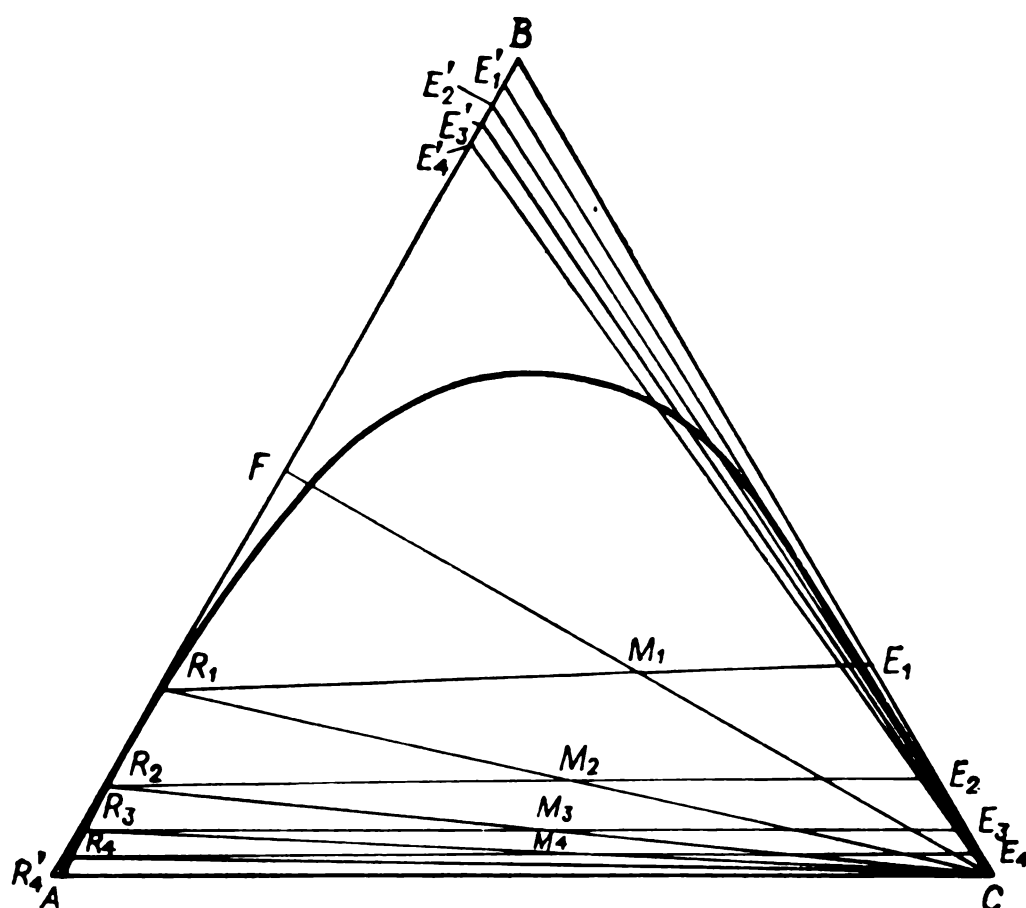


Fig. 8-14. To Example 8-3

We find the amount of extract from the ratio

$$\frac{G_E}{G_M} = \frac{RM}{RE} \quad \text{and} \quad G_E = \frac{1730 \times 94.4}{97} = 1682 \text{ kg}$$

The mass of the raffinate is:

$$G_R = G_M - G_E = 1730 - 1682 = 48 \text{ kg}$$

The amount of the extract-product is:

$$G'_E \approx G_E - G_S = 1682 - 1630 = 52 \text{ kg}$$

because the solubility of the chlorobenzene in the raffinate in the given case may be ignored.

The composition of the extract-product is determined by the point of intersection E' of side AB with the ray drawn from vertex C through point E : the acetone content is 95.5 mass per cent, and the water content is 4.5 mass per cent.

Example 8-3. Determine the required amount of solvent, the compositions and yields of the products, and the number of extraction stages for the conditions of the preceding example if fresh solvent in an amount equal to the mass of the mixture being processed is used for extraction in each stage.

Solution. When equal amounts of the feed solution and chlorobenzene are mixed, the position of point M_1 (Fig. 8-14) characterizing the total composition of the mixture is determined from

the ratio

$$\frac{FM_1}{M_1C} = \frac{100}{100} = 1 \text{ and } FM_1 = M_1C$$

We draw tie line R_1E_1 through point M_1 . Points R_1 and E_1 characterize the compositions and the amounts of raffinate and extract of the first stage. The raffinate from the first stage is separated and again mixed with an equal (in mass) amount of the solvent. The position of point M_2 determining the composition of the mixture in the second stage is found from the ratio

$$\frac{G_{R,1}}{G_S} = \frac{M_2C}{R_1M_2} = 1 \text{ and } R_1M_2 = M_2C$$

We again draw a tie line through point M_2 , and so on. We continue the construction until we reach the required raffinate composition. Four extraction stages are needed. We determine the amounts of raffinate and solvent by stages:

$$\begin{aligned} G_{S,i} &= G_{R,i-1} \\ G_{R,i} &= \frac{2G_{R,i-1}M_iE_i}{R_iE_i} \\ G_{S,1} &= G_{R,0} = G_F = 100 \text{ kg} \\ G_{S,2} &= G_{R,1} = \frac{2 \times 100 \times 23.5}{74} = 63.5 \text{ kg} \\ G_{S,3} &= G_{R,2} = \frac{2 \times 63.5 \times 37.5}{86.5} = 55.1 \text{ kg} \\ G_{S,4} &= G_{R,3} = \frac{2 \times 55.1 \times 44}{93} = 52.1 \text{ kg} \\ G_{R,4} &= \frac{2 \times 52.1 \times 45}{96} = 49.0 \text{ kg} \end{aligned}$$

The total amount of the solvent is:

$$G_S = \sum G_{S,i} = 100 + 63.5 + 55.1 + 52.1 = 270.8 \text{ kg}$$

The total amount of the extract is:

$$G_E = G_F + G_S - G_{R,4} = 100 + 270.8 - 49.0 = 321.8 \text{ kg}$$

After separation of the solvent, there remains:

$$G'_E \approx G_E - G_S = 321.8 - 270.8 = 51 \text{ kg}$$

The extract contains approximately 96% by mass of acetone.

Example 8-4. Determine the composition and yield of the products, and also the number of theoretical extraction stages for the conditions of Example 8-2 if countercurrent extraction is used with a flow ratio of 1:1.

Solution. We draw a straight line through point R (Fig. 8-15) characterizing the composition of the raffinate and point M

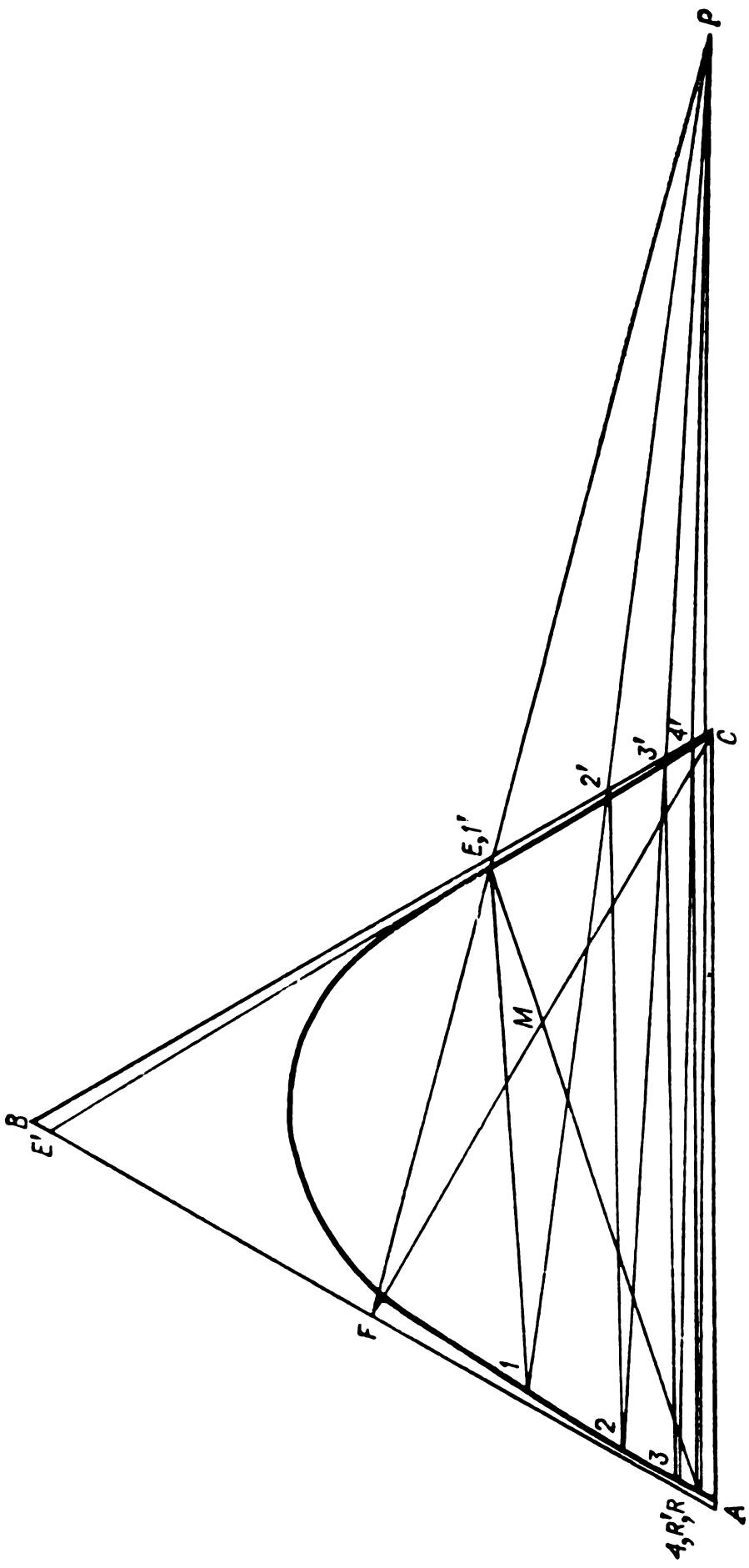


Fig. 8-15. To Example 8-4

determining the total fictitious composition of the feed solution with all of the solvent (since $G_F:G_S=1$, we have $FM=MC$) up to its intersection with the right branch of the binodal curve at point E corresponding to the composition of the extract. When segments FE and RC are continued, they intersect at point P (the pole). The latter is the common point of intersection of all the rays passing through points characterizing the composition of the raffinate at any stage and the composition of the extract at the next stage. Lines $1-1'$, $2-2'$, ..., $4-4'$ are tie lines. Their number determines the number of theoretical extraction stages. Thus, the number of stages is determined graphically by consecutively drawing the lines FC , RME ($1'$), FEP , RCP , $1'-1$, $1-P$, $2'-2$, $2-P$, $3'-3$, $3-P$, $4'-4$ (R). In the given case, we have $n_s=4$.

We find the amount of extract from the ratio

$$\frac{G_E}{G_M} = \frac{G_E}{G_F + G_S} = \frac{RM}{RE} \text{ and } G_E = \frac{(100 + 100) 64}{85} = 150.5 \text{ kg}$$

The amount of the extract-product is:

$$G'_E = 150.5 - 100 = 50.5 \text{ kg}$$

The composition of the extract-product is characterized by point E' : 97.5 mass per cent of acetone and 2.5 per cent of water.

The amount of raffinate is:

$$G'_R \approx G_R = G_M - G_E = 200 - 150.5 = 49.5 \text{ kg}$$

The results of Examples 8-2, 8-3, and 8-4 are compared in Table 8-2.

TABLE 8-2

Characteristic of process	Crosscurrent		Countercurrent
Number of stages	1	4	4
Consumption of solvent, kg	1630	270.8	100
Yield of raffinate, kg	48	49	49.5
Yield of extract, kg	1682	321.8	150.5
Yield of extract-product, kg	52	51	50.5
Acetone content in extract-product, mass per cent	95.5	96	97.5

Initial data: the mixture consists of water (A), acetone (B), and chlorobenzene (C); $x_F=50\%$ (mass); $x_R=2\%$ (mass); $y_S=0$; $G_F=100$ kg.

Examination of the table shows that countercurrent extraction has great advantages in the given case (a smaller consumption of solvent, a higher purity of the extract, etc.).

Example 8-5. Assess approximately the expediency of using extraction in Example 8-4 if the solvent is separated from the extract by continuous rectification, the tolerated content of chlorobenzene in the overhead product is 10% by mass, and that of acetone in the bottom product is 1% by mass. Take the surplus reflux factor in both cases equal to 2.

Solution. For purposes of simplification, we assume that the extract fed for rectification is a binary mixture. We determine the minimum reflux ratio and the number of theoretical plates according to equilibrium data by conventional procedures (the calculations are omitted). We enter the results of our calculations and those for rectification of an acetone-water mixture in Table 8-3.

TABLE 8-3

Characteristic of process	Rectification of acetone-water mixture	Extraction of acetone with subsequent rectification of acetone-chlorobenzene mixture
Acetone content in feed, %:		
mass	50	32.5
mole	23.7	47.8
Acetone content in overhead product, %:		
mass	97.5	90
mole	92.4	94.5
Acetone content in bottom product, %:		
mass	2	1
mole	0.63	1.9
Reflux ratio:		
R_{\min}	0.38	0.16
R	0.76	0.32
Number of theoretical plates	10	7
Ratio of consumptions of heat for rectification $\frac{q_1}{q_2} = \frac{L(R_1 + 1)}{L(R_2 + 1)}$	$\frac{0.76 + 1}{0.32 + 1} = 1.33$	

A comparison of the results allows us to arrive at the conclusion that it is expedient to use extraction, but the economy, if the equipment and operating costs are taken into account, will be negligible. A more detailed analysis is needed for final solution of the problem.

Example 8-6. A binary mixture of 1,4-dioxan and water cannot be separated by rectification at atmospheric pressure owing to the formation of an inseparably boiling mixture. The dioxan is extracted from an aqueous solution by benzene with the subsequent rectification of the dioxan-benzene mixture. Determine the final content of the dioxan in the water if 150 kg of a 20% solution of the dioxan is consecutively processed with five 100-kg

portions of benzene. The fresh solvent contains 2% by mass of the dioxan. Equilibrium is reached at each extraction stage. Disregard the miscibility of water and benzene. The solubility of dioxan in water and benzene at 25°C is given below:

Content of dioxan in water, % (mass)	5.1	18.9	25.2
Equilibrium content of dioxan in benzene, % (mass)	5.2	22.5	32.0

Solution. We convert the mass fractions (in per cent) to mass ratios.

The content of the dioxan in water is:

$$x' = \frac{x}{100-x} \frac{\text{kg of dioxan}}{\text{kg of water}}$$

The equilibrium content of dioxan in benzene is:

$$y' = \frac{y}{100-y} \frac{\text{kg of dioxan}}{\text{kg of benzene}}$$

Conversion of the above data yields:

x' , kg of dioxan/kg of water	0.0537	0.233	0.337
y' , kg of dioxan/kg of benzene	0.0548	0.291	0.471

The content of dioxan in the feed is:

$$x'_F = \frac{20}{100-20} = 0.25 \frac{\text{kg of dioxan}}{\text{kg of water}}$$

The content of dioxan in the fresh solvent is:

$$y'_S = \frac{2}{100-20} = 0.022 \frac{\text{kg of dioxan}}{\text{kg of benzene}}$$

The amount of water G_A in the feed G_F can be determined by the formula:

$$G_A = G_F \frac{100-x_F}{100} = \frac{150(100-20)}{100} = 120 \text{ kg}$$

The amount of benzene G_C per $G_S=100$ kg of fresh solvent can be found as follows:

$$G_C = G_S \frac{100-y_S}{100} = \frac{100(100-2)}{100} = 98 \text{ kg}$$

The slope of the operating lines [Eq. (8-17)] is:

$$\tan \alpha = \frac{G_A}{G_C} = \frac{120}{98} = 1.225$$

In Fig. 8-16, we draw a straight line having a slope of 1.225 : 1 through point 1 with the coordinates x'_F and y'_S up to its intersection with the equilibrium line. The coordinates of the point of intersection characterize the compositions of the extract y'_1 and the raffinate x'_1 of the first stage. We again draw a straight line

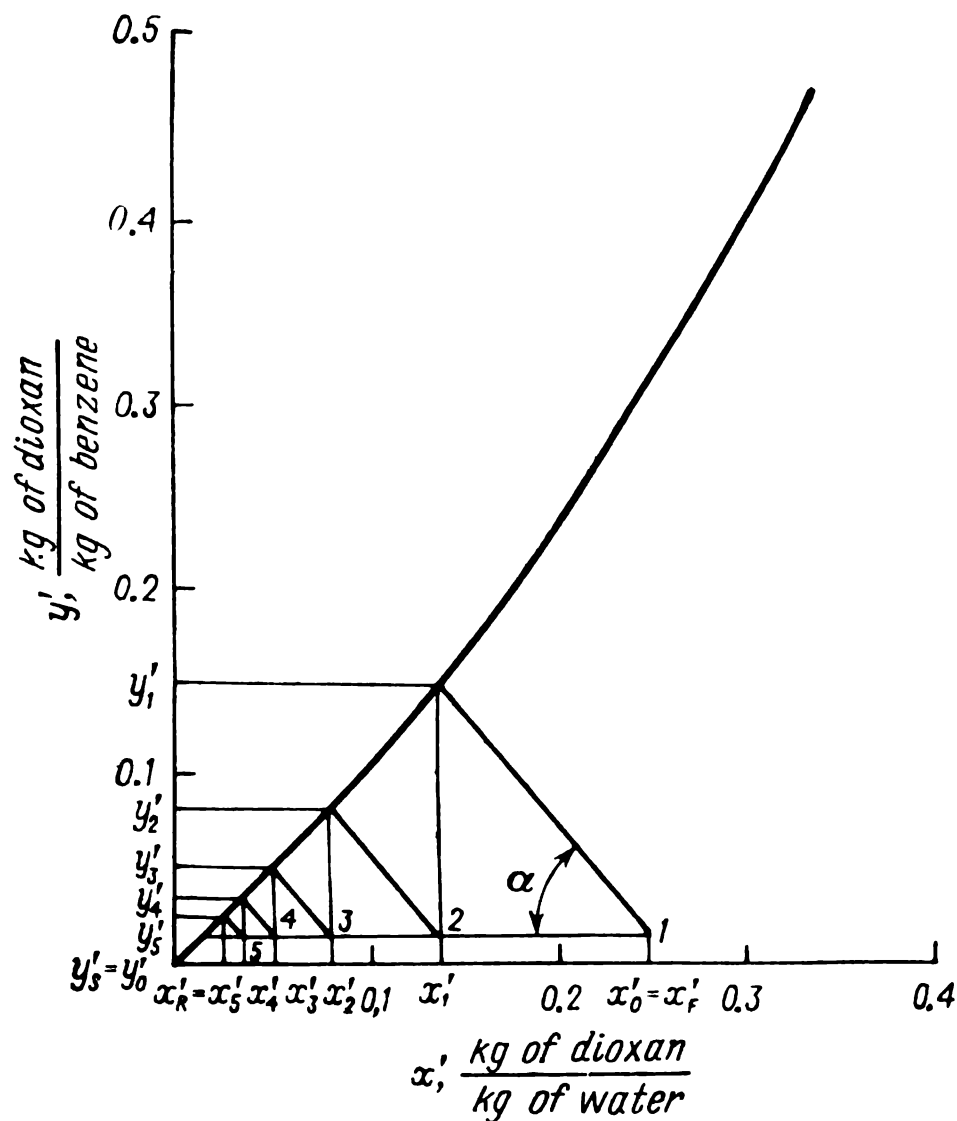


Fig. 8-16. To Example 8-6

having the slope of 1.225 : 1 through point 2 with the coordinates x'_1 and y'_5 up to its intersection with the equilibrium curve, and so on. The raffinate content in the last stage is:

$$x'_R = x'_5 = 0.022 \frac{\text{kg of dioxan}}{\text{kg of water}}$$

or

$$x_R = x_5 = \frac{0.022 \times 100}{1 + 0.022} = 2.15 \text{ mass per cent of dioxan}$$

Example 8-7. A continuous countercurrent extractor is used to treat phenol sewage water with pure benzene for purification of the water and extraction of the phenol. Determine the required amount of the solvent and the number of theoretical extraction stages if 10 m³ of sewage water is processed an hour. The initial and final contents of the phenol in the water are 8 and 0.5 kg/m³, respectively, the final phenol content in the benzene is 25 kg/m³, and the temperature of the liquids is 25 °C.

Solution. We take the data on equilibrium from the tables in [0-30]:

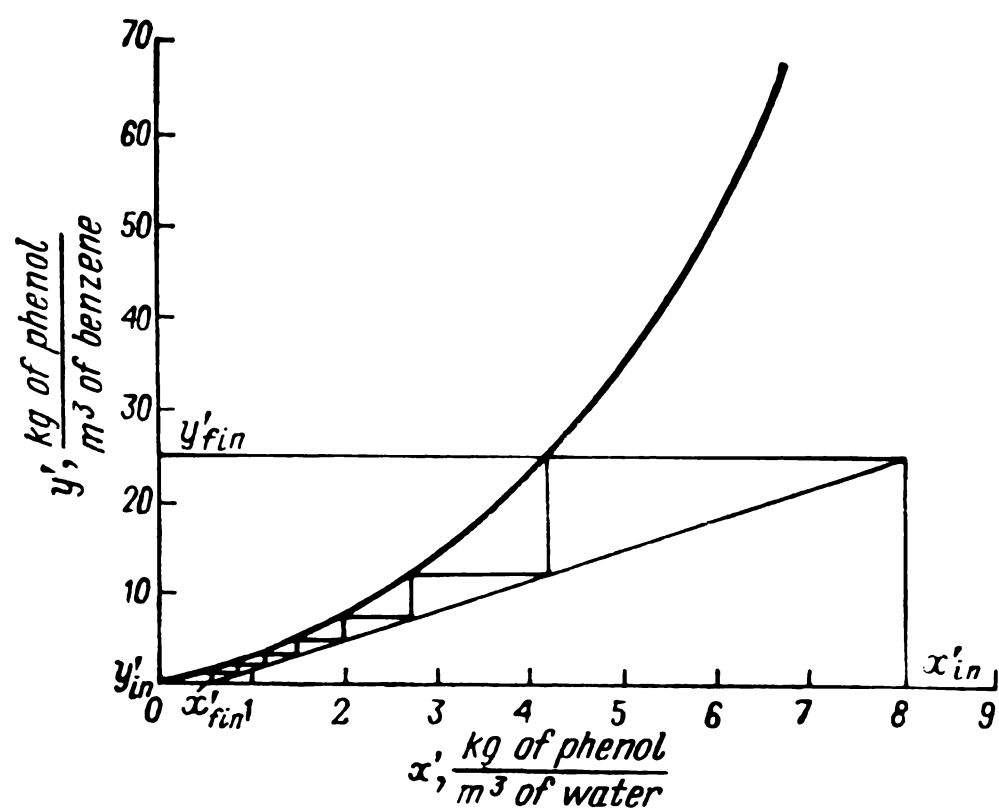


Fig. 8-17. To Example 8-7

Content of phenol in water, c_1 , g-eq/dm ³ . . .	0.0272	0.1013	0.3660
Equilibrium content of phenol in benzene, c_2 , g-eq/dm ³	0.062	0.279	2.978

We convert the concentrations to kg/m³: $x' \approx 15.686c_1$ and $y' \approx 15.686c_2$. The results of our calculations are:

Content of phenol in water, x' , kg/m ³	0.426	1.59	5.74
Equilibrium content of phenol in benzene, y' , kg/m ³	0.974	4.37	46.7

We find the required amount of benzene from the material balance equation:

$$V_F (x_{in}' - x_{fn}') = V_S (y_{fn}' - y_{in}')$$
$$V_S = \frac{10}{3600} \times \frac{8 - 0.5}{25 - 0} = 0.000\,83 \text{ m}^3/\text{s}$$
$$G_S = 0.000\,83 \times 879 = 0.73 \text{ kg/s}$$

We find the number of theoretical stages graphically: we construct the operating line passing through points having the coordinates x_{in}' , y_{fn}' and x_{fn}' , y_{in}' on an equilibrium diagram plotted according to the above data (Fig. 8-17) and inscribe the stages of the change in concentration between the operating line and the equilibrium curve. We find that seven stages are needed.

Example 8-8. Use the equilibrium compositions of coexisting phases (in per cent by mass) given in Table 8-4 to construct the equilibrium phase diagrams for the system water (A)-acetic acid (B)-diethyl ether (C) at 25 °C in the coordinates: (a) X , Y against z , Z , and (b) X against Y .

TABLE 8-4

Water layer			Ether layer		
water	acetic acid	diethyl ether	water	acetic acid	diethyl ether
93.3	0	6.7	2.3	0.	97.7
88.0	5.1	6.9	3.6	3.8	92.6
84.0	8.8	7.2	5.0	7.3	87.7
78.2	13.8	8.0	7.2	12.5	80.3
72.1	18.4	9.5	10.4	18.1	71.5
65.0	23.1	11.9	15.1	23.6	61.3
55.7	27.9	16.4	23.6	28.7	47.7

Solution. We convert the concentrations by means of Eq. (8-4):

$$X = \frac{x_B}{x_A + x_B} \frac{\text{kg of acetic acid}}{\text{kg of water} + \text{acetic acid}}$$
$$Y = \frac{y_B}{y_A + y_B} \frac{\text{kg of acetic acid}}{\text{kg of water} + \text{acetic acid}}$$
$$z = \frac{x_C}{x_A + x_B} \frac{\text{kg of diethyl ether}}{\text{kg of water} + \text{acetic acid}}$$
$$Z = \frac{y_C}{y_A + y_B} \frac{\text{kg of diethyl ether}}{\text{kg of water} + \text{acetic acid}}$$

We enter the data obtained in Table 8-5.

TABLE 8-5

Water layer		Ether layer	
X	z	Y	Z
0	0.072	0.	42.55
0.055	0.074	0.514	12.53
0.095	0.078	0.593	7.14
0.150	0.087	0.635	4.07
0.204	0.105	0.635	2.51
0.263	0.135	0.610	1.58
0.333	0.196	0.549	0.913

We construct the diagrams themselves in the usual way (Fig. 8-18). We do not plot the tie lines in the diagram of X, Y against z, Z; the auxiliary X-Y diagram is used to find them in the calculations.

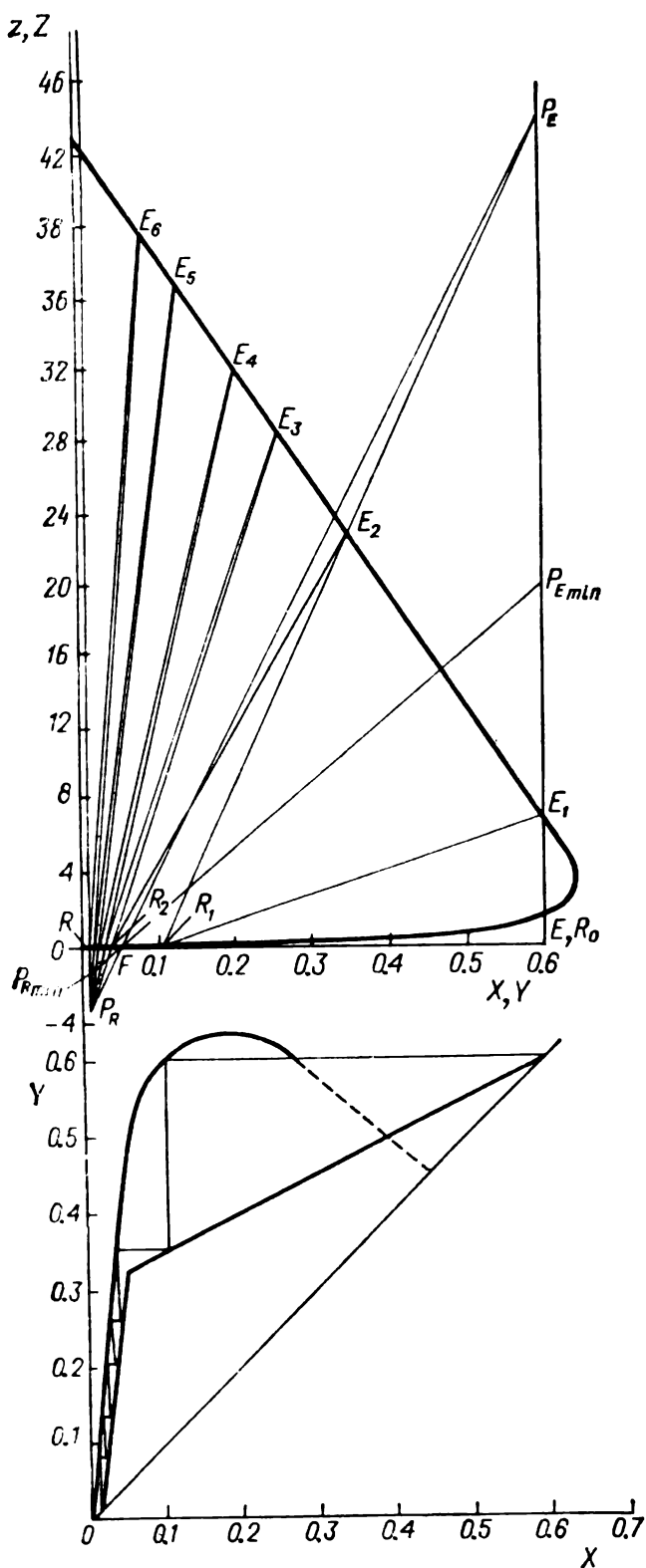


Fig. 8-18. To Examples 8-8, 8-9, and 8-10

Solution. Since it is impossible to achieve the required concentration of the extract by using conventional countercurrent extraction (see Example 8-9), we use a process with partial extract reflux (Fig. 8-19).

Example 8-9. Determine the greatest achievable concentration of the extract for the system water-acetic acid-diethyl ether at 25°C if countercurrent extraction is used with an acid content in the feed of (a) 15%, and (b) 5%*.

Solution. We find from the X - Y diagram in Fig. 8-18 that an extract with an acetic acid content of $Y = 0.635$ is in equilibrium with the feed for which $X = 0.15$; an extract with $Y = 0.46$ is in equilibrium with the feed for which $X = 0.05$. Thus, in the first case, the maximum concentration of the acetic acid in the extract (after separation of the solvent) is 63.5% (mass), and in the second case 46% (mass); in real conditions the greatest achievable concentrations will be somewhat smaller.

Example 8-10. Calculate the required number of stages and the amount of solvent for the extraction of acetic acid from an aqueous solution using diethyl ether ($t = 25^\circ\text{C}$) if the concentration of the feed is 5% (mass), and that of the extract-product is 60% (mass). The amount of feed processed an hour is 1000 kg; the ether is completely separated from the raffinate and the extract; the content of the acid in the raffinate is not over 1% (mass).

* It is good to separate acetic acid from water by extraction at moderate concentrations because its boiling point is 118.1°C , while to separate a small amount of acid by rectification it is necessary to evaporate a considerable amount of water equal to $(G_F - G_W)(R + 1)$, where G_W is the mass of the concentrated acid, and R is the reflux ratio.

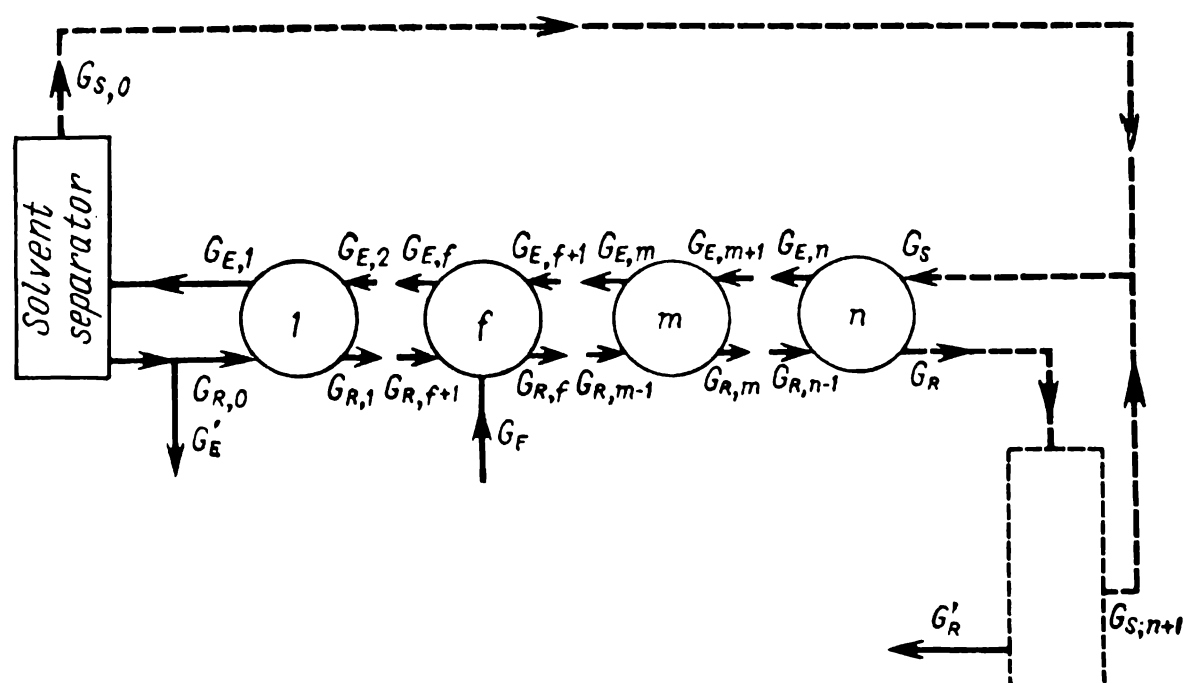


Fig. 8-19. To Example 8-10

We determine the minimum extract reflux ratio. We draw a tie line through point F (Fig. 8-18) characterizing the feed ($X_F = 5/100 = 0.05$; $z_F = 0$) up to its intersection with the vertical line drawn through point E ($X_E = 60/100 = 0.6$; $Z_E = 0$) corresponding to the final extract.

Since in our case the solvent is completely separated, we have, by Eq. (8-21):

$$R_{E\min} = \frac{G_{R,0}}{G_E} = \frac{\overline{P_{E\min}E_1}}{\overline{E_1E}} = \frac{Z_{P,E\min} - Z_{E,1}}{Z_{E,1} - z_{R,0}} = \frac{19 - 6.7}{6.7 - 0} = 1.835$$

We assume that the surplus reflux factor $\beta = 3$; hence, $R_E = 1.835 \times 3 = 5.51$.

We determine the ordinate of the pole of the extract-enriching section of the plant $Z_{P,E}$:

$$5.51 = \frac{Z_{P,E} - 6.7}{6.7 - 0} \quad \text{and} \quad Z_{P,E} = 43.6$$

We draw a straight line through points P_E and F up to its intersection with the vertical line drawn through point R ($X_R = 1/93.3 \approx 0.01$ and $z_R = 6.7/93.3 = 0.072$) characterizing the composition of the final raffinate (before separation of the solvent). The point obtained is the pole of the raffinate-stripping section of the plant.

Next we determine the number of extraction stages by consecutively drawing tie lines and rays and counting the number of tie lines. The sequence of construction is E_1R_1 , R_1P_E , E_2R_2 , $P_ER_2E_3$, and so on until the preset raffinate concentration is obtained.

We use an auxiliary X - Y diagram to construct the tie lines. It allows us to determine the equilibrium concentration Y accord-

ing to a given value of X (or vice versa). In our case, six extraction stages are needed. The feed is supplied in the second stage. Figure 8-18 shows how to find the number of stages according to an X - Y diagram (it must be taken into consideration that the operating lines in this case are not straight ones).

We calculate the values of the flows from the material balance equations.

The balance of components A and B is:

$$G_F = G'_R + G'_E$$

The balance of component B is:

$$X_F G_F = X_R G'_R + X_E G'_E$$

We solve the simultaneous equations:

$$1000 = G'_R + G'_E$$

$$0.05 \times 1000 = 0.01 G'_R + 0.6 G'_E$$

The mass of the extract is $G'_E = G_E = 68$ kg, and that of the raffinate is $G'_R = 932$ kg. Hence,

$$G_R = G'_R (1 + z_R) = 932 (1 + 0.072) = 1000 \text{ kg}$$

The mass of the ether in the raffinate is $1000 - 932 = 68$ kg.

The mass of the extract reflux is:

$$G_{R,0} = G_E R_E = 68 \times 5.51 = 375 \text{ kg}$$

The mass of the solvent removed in the separator is:

$$G_{S,0} = (G_E + G_{R,0}) Z_{E,1} = (68 + 375) 6.7 = 2970 \text{ kg}$$

The overall balance of the plant is:

$$G_F + G_S = G_E + G_{S,0} + G_R$$

$$1000 + G_S = 68 + 2970 + 1000$$

whence the required amount of the solvent is $G_S = 3038$ kg.

Example 8-11. A continuous countercurrent extractor is used to extract styrene from its 38% solution in ethylbenzene with diethyleneglycol. The capacity of the extractor with respect to the feed is 100 kg/h. Since conventional countercurrent extraction cannot provide the required purity of separation, extraction with partial extract and raffinate refluxes is employed. The extract and raffinate products contain 95 and 3% (mass) of styrene, respectively. Determine the amount of solvent needed, the compositions and amounts of the extract, raffinate, and refluxes, and also the required number of theoretical extraction stages assuming that the extract reflux is 1.5 times greater than the minimum amount.

Solution. After working out the scheme of the process* and constructing the phase diagrams X, Y against z, Z and X against Y (the equilibrium data are not given here), we determine the compositions of the products and plot the points corresponding to them on a diagram (Fig. 8-20). We enter the data as we calculate them in Table 8-6. Since the raffinate on any portion of the raffinate curve contains very small amounts of the solvent, the installation of a raffinate and a second extract distillation column is not expedient in the given case.

We determine the coordinates of the poles of the extract-enriching and raffinate-stripping sections of the extraction column corresponding to the minimum refluxes by drawing through point F a straight line coinciding with a tie line up to its intersection with the vertical lines passing through points E and R . The number of extraction stages will be infinitely great.

The minimum reflux is

$$\frac{G_{R,0}}{G_E} = \frac{Z_{P,Emin} - Z_{E,1}}{Z_{E,1} - z_{E,1}} = \frac{33 - 2.95}{2.95 - 0} = 10.35$$

$$\frac{G_{R,n+1}}{G_R} = \frac{z_R - Z_{P,Rmin}}{Z_{E,n+2} - z_R} = \frac{0.007 + 20}{8.65 - 0.007} = 2.31$$

Upon the complete recycling of the extract and raffinate to the extractor, we have:

$$\frac{G_{R,0}}{G_E} = \infty; \quad \frac{G_{R,n+1}}{G_R} = \infty$$

$$Z_{P,Emax} = \infty; \quad Z_{P,Rmax} = \infty$$

i.e. the rays transform into parallel vertical lines. The minimum number of extraction stages corresponds to this case.

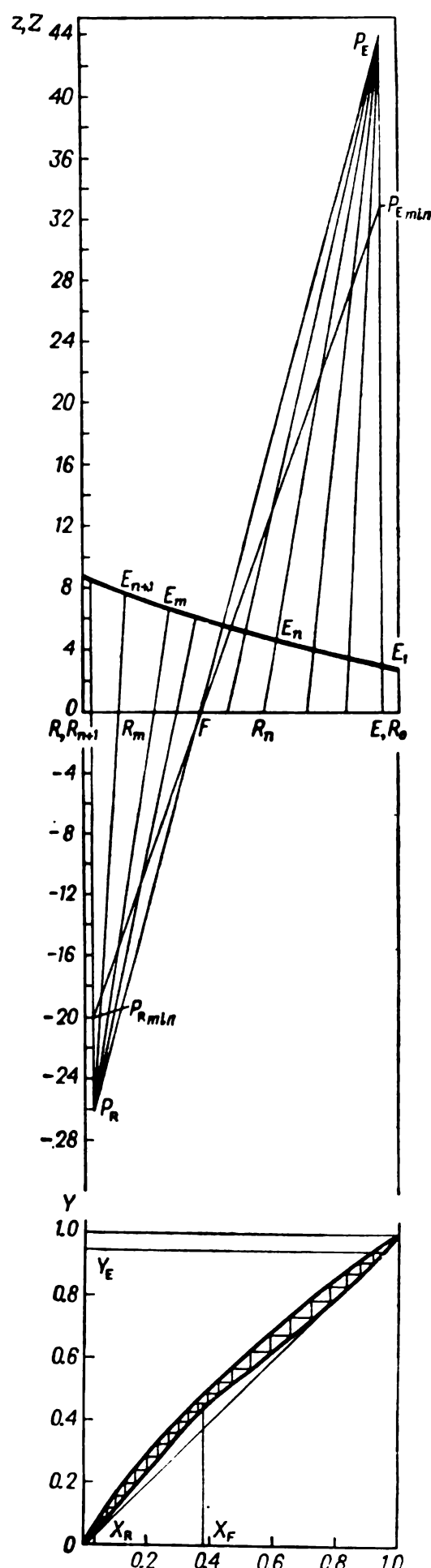


Fig. 8-20. To Example 8-11

* See Fig. 8-9a. The apparatus for the distillation of the solvent marked with an asterisk in the figure is not included in the given plant.

TABLE 8-6

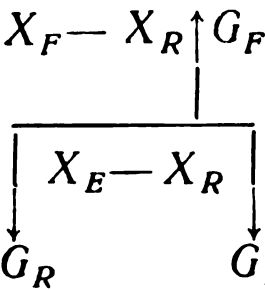
	Symbol of figurative point	Symbol of stream	Composition (concentration)		Flow rate, kg/h
			$X (Y),$ $\frac{\text{kg of B}}{\text{kg of A + B}}$	$z, (Z),$ $\frac{\text{kg of C}}{\text{kg of A + B}}$	
Feed	F	G_F	0.38	0	100
Solvent to mixer	S^*	G_S	—	∞	1805.5
Crude extract	E_1	$G_{E, 1}$	0.95	2.95	2418
Extract-product	E	G_E	0.95	0**	38
Extract reflux	R_0	$G_{R, 0}$	0.95	0**	575
Raffinate-product	R	G_R	0.03	0.007 ^{3*}	62.5
Raffinate reflux	R_{n+1}	$G_{R, n+1}$	0.03	0.007 ^{3*}	190
Solution from mixer	E_{n+1}	$G_{E, n+1}$	0.03	8.65	1995.5
Added solvent	—	ΔG_S	—	∞	0.5 ^{4*}
Solvent after extract separator	S_0^*	$G_{S, 0}$	—	∞	1805
Pole of extract-enriching section:					
minimum	$P_{E\min}$	—	0.95	3.3	—
operating	P_E	—	0.95	43.5	—
Pole of raffinate-stripping section:					
minimum	$P_{R\min}$	—	0.03	—20	—
operating	P_R	—	0.03	—26.2	—

- * The figurative point is absent in the diagram.
- ** Purification is conducted entirely in one column.
- 3* Purification is not expedient.
- 4* Some solvent is carried off with the raffinate-product.

We find the coordinates of the poles when $\beta = 1.5$:

$$\frac{Z_{P, E} - 2.95}{2.95 - 0} = 10.35 \times 1.5 \text{ and } Z_{P, E} = 43.53$$

The amount of extract according to the lever rule is:



$$G_E = \frac{G_F (X_F - X_R)}{X_E - X_R} = \frac{100 (0.38 - 0.03)}{0.95 - 0.03} = 38.0 \text{ kg/h}$$

The extract reflux is:

$$G_{R, 0} = 10.35 \times 2 \times 38.0 = 575 \text{ kg/h}$$

The amount of raffinate (together with the solvent carried off) is:

$$\Delta G_S + G_R = (G_F - G_E)(1 + z_R) = (100 - 38.0)(1 + 0.007) = 62.5 \text{ kg/h}$$

The raffinate reflux is:

$$G_{R, n+1} = G_R \frac{Z_{P, E_{\min}} - Z_{P, E}}{Z_{E, n+2} - z_R} = 62.5 \times \frac{33 + 26.2}{8.65 - 0.007} \approx 190 \text{ kg/h}$$

The amount of solvent circulating during one hour is:

$$G_{S, 0} = (G_E + G_{R, 0}) z_{E, 1} = (38.0 + 575) 2.95 = 1805 \text{ kg/h}$$

The amount of solvent added (equal to the losses with the raffinate product) is:

$$\Delta G_S = G_R z_R = 62 \times 0.007 \approx 0.5 \text{ kg/h}$$

The capacity with respect to the crude extract (processed in the distillation column) is:

$$G_{E, 1} = G_{R, 0} + G_E + G_{S, 0} = 575 + 38 + 1805 = 2418 \text{ kg/h}$$

The rate of flow of the solution leaving the mixer is:

$$G_{E, n+1} = G_S + G_{R, n+1} = 1805.5 + 190 = 1995.5 \text{ kg/h}$$

The rate of flow of the solvent fed to the mixer is:

$$G_S = G_{S, 0} + \Delta G_S \approx 1805.5 \text{ kg/h}$$

The merging of the rays and tie lines makes it simpler to determine the required number of extraction stages according to the X - Y diagram. We draw a number of rays from the operating poles up to their intersection with the boundary curves (points $E_n, R_n, \dots, E_m, R_m, \dots$) and find points having the coordinates $X_R^n, Y_E^n, \dots, X_E^m, Y_R^m, \dots$ in the X - Y diagram. After drawing through these points a smooth curve (which is the operating line), we inscribe stages between the operating and the equilibrium lines and count their number. We need 23 extraction stages; the feed is supplied to the 12th stage from the top.

Example 8-12. A vertical settling tank with a conical bottom contains a sediment and 7 m³ of a solution in which two tons of NaOH are dissolved. After settling, the transparent part is poured out in an amount of 6 m³, pure water is added to the settling tank, and the suspension is agitated. After repeated settling, 6 m³ of the clean solution are again poured out. Three solutions poured out from the settling tank are mixed and fed for evaporation. Determine: (a) the amount of NaOH remaining in the sediment (slurry), (b) the per cent of extraction of NaOH, and (c) the content in per cent of the NaOH in the solution fed for evaporation.

Solution. (a) The sediment in the settling tank is washed three times with the ratio of the volumes of the poured out and remaining solutions equal to $a = 6 : 1 = 6$. According to Eq. (8-29), after three-fold washing, the slurry will contain

$$\frac{1}{(1+a)^3} = \frac{1}{7^3} = \frac{1}{343}$$

of the original amount of NaOH, or

$$G_{\text{NaOH}} = 2000 \times \frac{1}{343} = 5.8 \text{ kg}$$

(b) the per cent of extraction of the NaOH is:

$$\frac{2000 - 5.8}{2000} \times 100 = 99.7\%$$

According to Table A-54, the amount of extracted substance upon three-fold washing with a six-fold amount of the solvent is 99.71%.

(c) The amount of the solution is $V_s = 6 \times 3 = 18 \text{ m}^3$. The content of NaOH in it is:

$$G_{\text{NaOH}} = 2000 - 5.8 = 1994.2 \text{ kg or } \frac{1994.2}{18000 + 1994.2} \times 100 \approx 10\%$$

Example 8-13. To extract copper, pyrite cinders are subjected to chlorinating roasting with sodium chloride. The copper is contained in the roasted material in the form of cupric chloride (CuCl_2) whose content is 11%. The roasted product is leached in a countercurrent battery using acidified water obtained as a result of washing the flue gases. The inert solid material retains 2 kg of water per kg of solid. Equilibrium is reached in each stage. How many stages are needed in the battery to produce a solution containing 12% (mass) of CuCl_2 , and extract 98% of the copper from the roasted product?

Solution. Since the solid phase upon its movement from stage to stage retains a constant amount of water (except for the first stage because the latter is supplied with the dry roasted product that leaves it retaining 2 kg of water per kg of solid phase), we can use Eq. (8-33) to determine the number of stages in the plant.

We adopt as the basis of our calculations 100 kg of dry solid residue free of copper and calculate the amounts of the products and the concentrations.

The amount of CuCl_2 supplied with the roasted product is:

$$100 \times \frac{11}{89} = 12.36 \text{ kg}$$

The amount withdrawn with the residue is:

$$\frac{12.36(100-98)}{100} = 0.25 \text{ kg}$$

The amount withdrawn with the extract is:

$$\frac{12.36 \times 98}{100} = 12.11 \text{ kg}$$

We find the amount of acidified water G_S fed into the plant, taking into account that 200 kg of the water is carried off by the solid residue, and the remainder is withdrawn with the extract:

$$\frac{(G_S - 200) 12}{88} = 12.11$$

whence $G_S = 288.8 \text{ kg}$.

The content of CuCl_2 in the extract (in kg per 100 kg of water) is:

$$y_E = \frac{12}{88} \times 100 = 13.64$$

The solution retained by the solid matter leaving the first stage will have the same content, i. e. $x'_1 = y_E = 13.64$.

The content of CuCl_2 in the solution withdrawn with the solid residue is:

$$x'_R = \frac{0.25}{200} \times 100 = 0.125$$

The acidified water fed to the battery contains no copper salts, and $y_S = 0$.

The content of copper chloride y_2 in the overflow from the second stage to the first one is determined according to the balance of the CuCl_2 in the first stage. The amount of solvent in the overflow is 288.8 kg. The first stage receives 12.36 kg and A kg of CuCl_2 together with 288.8 kg of the solvent from the second stage per 100 kg of inert dry matter, a total of $12.36 + A$ kg of CuCl_2 . The amount leaving the first stage is 12.11 kg with the extract, and $\frac{13.64}{100} \times 200 = 27.28$ kg with the underflow solution, a total of $12.11 + 27.28 = 39.39$ kg.

The CuCl_2 balance in the first stage is $12.36 + A = 39.39$ kg, whence $A = 39.39 - 12.36 = 27.03$ kg.

The CuCl_2 content in the overflow (in kg per 100 kg of water) is:

$$y_2 = \frac{27.03}{288.8} \times 100 = 9.36$$

The number of stages (without the first one) is:

$$n_s - 1 = \frac{\log \frac{x_1 - y_2}{x_R - y_S}}{\log \frac{x_1 - x_R}{y_2 - y_S}} = \frac{\log \frac{13.64 - 9.36}{0.125 - 0}}{\log \frac{13.64 - 0.125}{9.36 - 0}} = 10 \text{ stages}$$

and altogether $n_s = 10 + 1 = 11$ stages.

Example 8-14. Sodium hydroxide is produced by the reaction
 $\text{Na}_2\text{CO}_3 + \text{CaO} + \text{H}_2\text{O} = \text{CaCO}_3 + 2\text{NaOH}$

The products are fed into the first stage of a continuous countercurrent three-stage battery with a water content of 50% of the mass of the slurry (CaCO_3). Subsequently, upon passing from stage to stage and leaving the battery, the slurry retains water in an amount equal to 1.5 times of its own mass. It is desirable to achieve extraction of 98% of the NaOH. Determine the consumption of water (per 100 kg of dry slurry) and the concentrations of the solutions at each stage.

Solution. Sufficient data are not available for drawing up material balances for each stage.

According to Table A-55, to extract 97.5% of the product using three-stage continuous leaching, the ratio of the amounts of the solvent and the solution retained is 3. Let us use these figures and draw up consecutively balances beginning with the third stage. We shall calculate for 100 kg of dry CaCO_3 .

The amount of the NaOH fed into the first stage per 100 kg of CaCO_3 is 80 kg. Of this amount, 97.5% passes over into the extract, or

$$\frac{80 \times 97.5}{100} = 78 \text{ kg}$$

The amount lost with the residual solution is $80 - 78 = 2$ kg. The amount of water retained by 100 kg of slurry is $100 \times 1.5 = 150$ kg. The content of the NaOH in the residual solution (in kg per kg of pure solution) is:

$$x'_R = \frac{2}{150} = 0.0133$$

Water is fed into the system as a solvent through the third stage in an amount of $150 \times 3 = 450$ kg. Thus, the overflow contains 450 kg of water, and the underflow contains 150 kg together with the solid phase.

The NaOH content in the solvent is $y_S = 0$, and that in the overflow from the third stage to the second one is $y_3 = x'_R = 0.0133$ kg/kg.

The unknown content of the NaOH in the underflow is x_2 . Let us determine it according to the NaOH balance for the third

stage:

$$450y_s + 150x_2' = 150x_R' + 450y_3$$
$$450 \times 0 + 150x_2' = 150 \times 0.0133 + 450 \times 0.0133$$
$$150x_2' = 8$$
$$x_2' = \frac{8}{150} = 0.0533 \text{ kg/kg}$$

The NaOH content in the overflow from the second stage to the first one is $y_2 = x_2' = 0.0533$ kg/kg. Now it remains to find the content of the NaOH in the underflow from the first stage to the second one. This content x_1' is determined from the balance for the second stage:

$$450y_3 + 150x_1' = 450y_2 + 150x_2'$$
$$450 \times 0.0133 + 150x_1' = 450 \times 0.0533 + 150 \times 0.0533$$
$$150x_1' + 6 = 24 + 8$$
$$x_1' = \frac{32 - 6}{150} = \frac{26}{150} = 0.1733 \text{ kg/kg}$$

To determine the NaOH content in the extract in the first stage, we must draw up a water balance for this stage, denoting by A the amount of water withdrawn with the extract: $A + 150 = 450 + 50$, whence $A = 350$ kg. The extract contains 78 kg of NaOH which, by mass, is:

$$y_E = \frac{78}{350 + 78} \times 100 = 18.2\%$$

Example 8-15. An oil-extracting plant processes one ton an hour of meal (crushed and partly degreased sunflower seeds) containing 28% of oil and 2.5% of benzene. The solvent is the regenerated benzene containing 1.5% of oil and fed in an amount of 50% of the mass of the meal. Experiments show that the amount of solution retained by the solid phase depends on the oil content in it (Table 8-7) *.

TABLE 8-7

Oil content, kg/kg of solution	Solution retained, kg/kg of solid	Oil content, kg/kg of solution	Solution retained, kg/kg of solid
0.0	0.500	0.4	0.550
0.1	0.505	0.5	0.571
0.2	0.515	0.6	0.595
0.3	0.530	0.7	0.620

* McCabe, W. L. and Smith, J. C. *Unit Operations of Chemical Engineering*, 2nd ed. New York, McGraw-Hill (1967), p. 717.

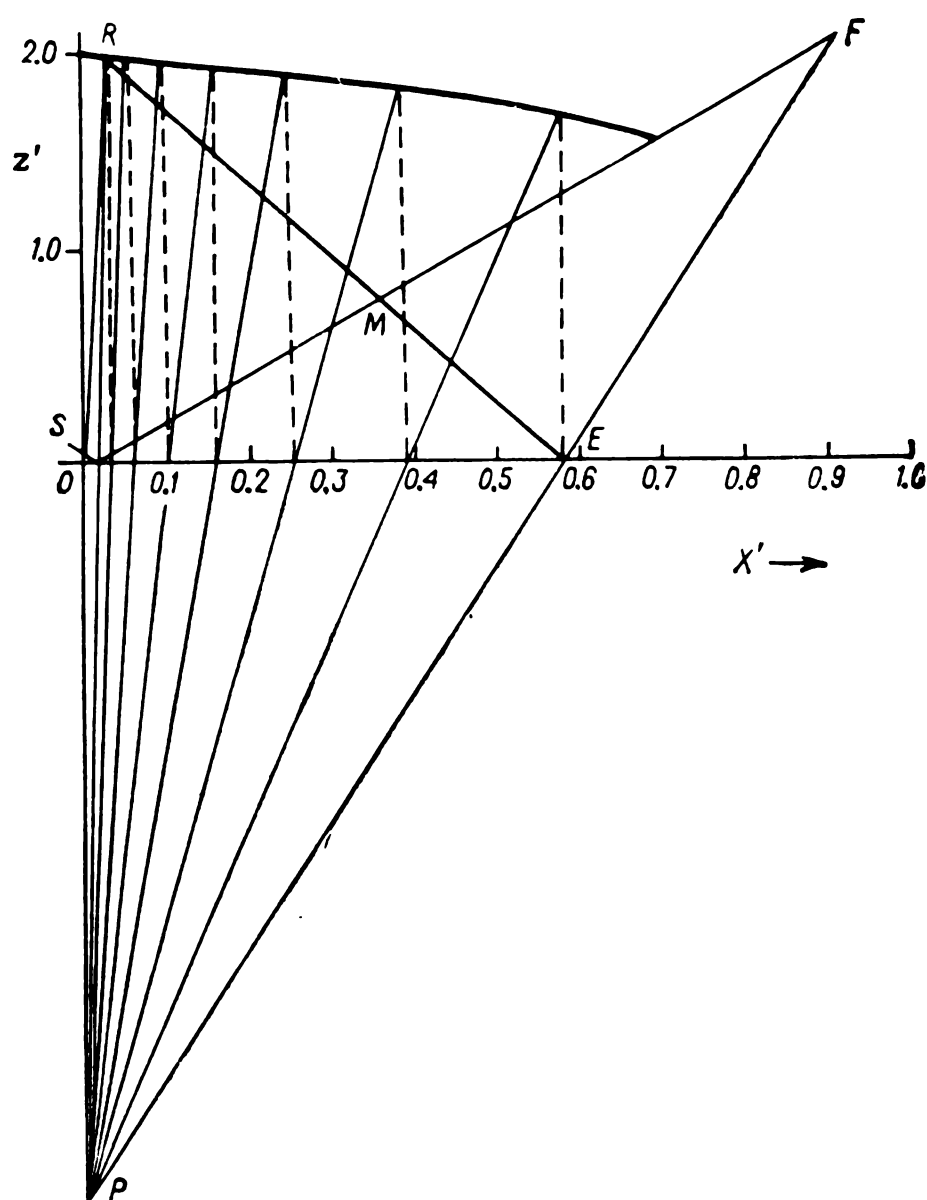


Fig. 8-21. To Example 8-15

The solid residue after extraction contains 5% of oil.

Determine: (1) the amount of extract and its oil content; (2) the amount of the solution retained by the solid phase (meal cake) and the oil content in it; and (3) the number of leaching stages.

Solution. The problem is solved graphically in the rectangular system of coordinates X' against z' (Fig. 8-21).

The data of the first column of Table 8-7 equal the ratio $X' = x_B/(x_B + x_C)$.

The data of the second column—the mass of the solution per unit mass of the solid—have to be recalculated to their reciprocals, when they express the ratios $z' = x_A/(x_B + x_C)$, i. e. the amounts of the solid phase per unit mass of the solution (see Table 8-8).

The data of Table 8-8 are the coordinates of points on the underflow curve (Fig. 8-21). There is no solid matter in the overflow ($z' = 0$), therefore the overflow line merges with the axis of abscissas. After plotting the curve, we draw the material balance lines in the diagram.

TABLE 8-8

$x' = \frac{x_B}{x_B + x_C}$	$z' = \frac{x_A}{x_B + x_C}$	$x' = \frac{x_B}{x_B + x_C}$	$z' = \frac{x_A}{x_B + x_C}$
0.0	2.0	0.4	1.818
0.1	1.980	0.5	1.751
0.2	1.942	0.6	1.681
0.3	1.887	0.7	1.613

We determine the coordinates of point *F* corresponding to the composition of the meal fed in for extraction. According to the initial conditions, we have

$$X_F' = \frac{28}{28 + 2.5} = 0.92 \text{ and } z_F' = \frac{100 - 28 - 2.5}{28 + 2.5} = 2.12$$

The coordinates of point *S* corresponding to the composition of the solvent are $x_S = 1.5/100 = 0.015$ and $z_S = 0$.

Points *F* and *S* are plotted on the diagram and are connected by a straight line which in accordance with the lever rule is divided by point *M* into parts proportional to the amounts of the delivered products excluding the insoluble solid substance (otherwise the lever rule cannot be used in such diagrams). The solvent is fed into the plant in an amount equal to 50% of the mass of the meal. If we take this mass as a unit, then the liquid part in it will be $0.28 + 0.025 = 0.305$ mass unit, and the amount of the solvent will be 0.5 mass unit. Hence, *M* is at a distance of $0.305/(0.5 + 0.305) = 0.38$ of segment *SF* counting from point *S*.

We have no numerical data at our disposal for constructing the line of the rate of flow *RE*. We only know that line *RE* intersects *SF* at point *M* and that point *R* is on the underflow curve and point *E* is on the axis of abscissas because its ordinate $z_E' = 0$. Without having the coordinates of point *R*, we can nevertheless determine their ratio because we know from the initial conditions that the solid residue contains 5% of oil:

$$\frac{z_R'}{X_R'} = \frac{\frac{x_{AR}}{x_{BR} + x_{CR}}}{\frac{x_{BR}}{x_{BR} + x_{CR}}} = \frac{x_{AR}}{x_{BR}} = \frac{95}{5} = 19$$

The straight line passing through the origin of coordinates and having a slope of 19 will intersect the curve at point *R*. We draw this line, and its intersection with the curve gives us point *R*. We join points *R* and *M* and by continuing the straight line to its intersection with the axis of abscissas, we find point *E*.

We use the diagram to determine the abscissas of points R , M , and E : we have $X'_R = 0.03$, $X'_M = 0.36$, and $X'_E = 0.58$.

The amount of the liquid phase fed into the plant with the meal and solvent is $1000 \times 0.305 + 1000 \times 0.5 = 805$ kg/h. This amount is divided between the residue and the extract proportionally to the segments EM and MR . We can use the abscissas of the ends of these segments for calculations:

$$G_R = \frac{X'_E - X'_M}{X'_E - X'_R} \times 805 = \frac{0.58 - 0.36}{0.58 - 0.03} \times 805 = 322 \text{ kg/h}$$

The amount of the residual solution is 322 kg/h, and its composition is determined from the assumption that the oil lost with the residue is contained in this solution. The content of oil in the residual solution is:

$$\frac{1000 \times 0.695 \times 5}{95 \times 322} \times 100 = 11.7\%$$

The amount of extract is $G_E = 805 - 322 = 483$ kg/h.

The content of the oil in the extract is:

$$\frac{100 \times 0.28 - \frac{1000 \times 0.695 \times 5}{95}}{483} \times 100 = 50.4\%$$

To find the number of stages, we draw two rays through points F , E and R , S up to their intersection at pole P . Since in leaching there is only one solution with a single concentration in each stage of a plant, all the tie lines will be vertical ones. We erect a perpendicular from point E —the tie line for the first stage—up to its intersection with the boundary curve. We connect the point of intersection of the tie line and the boundary curve with point P . From the point of intersection of this ray with the axis of abscissas, we again erect a perpendicular to its intersection with the boundary curve, and so on, until we reach point R or are in its direct proximity.

The number of tie lines indicates the number of stages. In the given case we need seven leaching stages.

PROBLEMS

8-1. Construct a triangular equilibrium diagram for the system water-acetic acid-ethyl ether at 25°C using the data of Table 8-4. Compare the diagram obtained with a diagram of X , Y against z , Z (see Example 8-8).

8-2. Determine the compositions and the amounts of coexisting phases into which a mixture of 10 kg of water, 5 kg of ethyl ether, and 5 kg of acetic acid stratifies. What amount of the ethyl ether should be removed for the mixture to stop stratifying? Use the triangular diagram constructed in Problem 8-1 for solving this problem.

8-3. Acetic acid is extracted from its 15% (mass) aqueous solution at 25 °C. The mass of the initial mixture is 1200 kg. Determine the compositions and amounts of the final products after distillation of the solvent if crosscurrent extraction with pure ether is used. The process is carried out in two stages with a ratio of the mass of the solvent to that of the feed equal to 1.5. Use the triangular diagram constructed in Problem 8-1 for solving this problem.

8-4. Acetic acid is extracted from its 20% (mass) aqueous solution in a countercurrent with ethyl ether. Determine the amount of solvent needed for 1000 kg/h of the feed and the number of theoretical extraction stages if the extract-product is to contain 60% (mass) and the raffinate-product not over 2% (mass) of the acid.

8-5. Benzoic acid is extracted from an aqueous solution containing 1.5 kg per m³ of water by consecutive washing with benzene containing 0.2 kg of benzoic acid per m³ of benzene with a ratio of the volumes of the water and benzene of $V_F : V_S = 4$. Determine how many stages are needed if the benzene content in the raffinate-product is 0.2 kg/m³. Also determine the compositions of the extracts obtained. The equilibrium data at the operating temperature are:

Concentration of benzoic acid in water, kg/m ³	0.104	0.456	0.707	1.32	1.56
Concentration of benzoic acid in benzene, kg/m ³	0.182	2.45	6.12	18.2	24.5

8-6. A countercurrent extractor is used to extract 1,4-dioxan from a 25% aqueous solution with benzene containing 0.5% (mass) of dioxan. The dioxan content in the raffinate-product is 2% (mass). Find (a) the minimum amount of the solvent needed per 100 kg of the feed; (b) the required number of theoretical extraction stages; and (c) the composition of the extract assuming that the amount of solvent used is 1.5 times greater than the minimum one. See Example 8-6 for the equilibrium data.

8-7. Construct equilibrium phase diagrams in the coordinates X, Y against z, Z and X against Y for the system water-acetic acid-isopropyl ether at 20 °C using the data on the equilibrium compositions of the coexisting phases (in per cent by mass) given in Table 8-9. Do not draw tie lines in the diagram of X, Y against z, Z . Determine the maximum concentrations of the extract in countercurrent operation for feed compositions of 5 and 10% (mass).

8-8. Determine the minimum amount of extract reflux and the corresponding minimum amount of solvent per 100 kg of the feed—a 10% (mass)

TABLE 8-9

Water layer			Ether layer		
water	acetic acid	isopropyl ether	water	acetic acid	isopropyl ether
98.1	0.69	1.2	0.5	0.18	99.3
97.1	1.41	1.5	0.7	0.37	98.9
95.5	2.89	1.6	0.8	0.79	98.4
91.7	6.42	1.9	1.0	1.93	97.1
84.4	13.30	2.3	1.9	4.82	93.3
71.1	25.50	3.4	3.9	11.40	84.7
58.9	36.70	4.4	6.9	21.60	71.5
45.1	44.30	10.6	10.8	31.10	58.1
37.1	46.40	16.5	15.1	36.20	48.7

aqueous solution of acetic acid, if extraction is conducted with diethyl ether at 25 °C. The extract-product should contain 75% (mass) of the acetic acid, and the raffinate-product 1% (mass); the solvent is separated completely. Use the equilibrium diagram constructed according to the data of Example 8-8 (see Fig. 8-18).

8-9. Solve Problem 8-8 assuming that the extract reflux is double the minimum amount. Also determine the number of theoretical extraction stages. Use the equilibrium diagram constructed according to the data of Example 8-8 (see Fig. 8-18).

8-10. Construct the equilibrium phase diagrams in the coordinates X, Y against z, Z for the system heptane-methylcyclohexane-aniline at 25 °C. Take the data on the equilibrium compositions of the coexisting phases (in per cent by mass) from Table 8-10. Determine the concentrations of the products obtained in conventional countercurrent extraction of a 40% solution of methylcyclohexane in heptane with pure aniline. Also determine the minimum number of extraction stages (with complete reflux of the extract and raffinate) if the extract contains 98% (mass) and the raffinate 1% (mass) of the methylcyclohexane (after separation of the solvent).

8-11. Methylcyclohexane is extracted with aniline from its 40% solution in heptane at 25 °C in an extractor with reflux of part of the extract and raffi-

TABLE 8-10

Raffinate layer		Extract layer		Raffinate layer		Extract layer	
X	z	Y	Z	X	z	Y	Z
0	0.064	0	15.7	0.610	0.087	0.770	6.58
0.085	0.064	0.150	13.8	0.730	0.099	0.870	5.67
0.216	0.070	0.365	11.2	0.810	0.105	0.920	5.10
0.445	0.078	0.623	8.2	0.885	0.117	0.960	4.80
0.525	0.079	0.700	7.33	1.0	0.124	1.0	4.70

nate. The extract contains 98% (mass) and the raffinate 1% (mass) of methylcyclohexane (excluding the solvent). The ratio of the amounts of reflux of the extract and the extract-product is 1.615 times greater than the minimum value. Determine the number of extraction stages, the compositions and the amounts of the raffinate, extract, refluxes, and solvent per 100 kg/h of the feed.

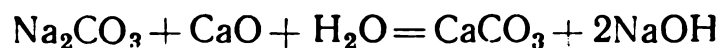
8-12. A battery of three countercurrent settling tanks each having a volume of 7 m³ is fed with 2 tons of a solution of NaOH in 1 m³ of water together with a sediment of CaCO₃, and 6 m³ of a transparent concentrated solution are withdrawn for evaporation. Pure water is fed into the other side of the battery as a solvent in an amount of 6 m³ per 2000 kg of NaOH. The CaCO₃ sediment when transferred from stage to stage and when removed from the battery retains 1 m³ of the solution. Find (a) the amount of NaOH in the slurry; (b) the extraction factor of the NaOH; (c) the content of NaOH in the extract delivered for evaporation, in per cent.

8-13. Determine the number of extraction stages in the conditions of Example 8-12 if the extraction factor of the NaOH equals 0.98.

8-14. Determine the number of extraction stages in the conditions of Example 8-13 if the content of CuCl₂ in the extract equals 9% (mass), and the extraction factor of the copper is 92%.

8-15. A plant processes 10 tons of barium sulphide a day with the corresponding amount of soda and 35 tons of water to produce barium carbonate and a solution of sodium sulphide. A five-stage countercurrent battery is used. The barium carbonate sediment during the process retains a double (by mass) amount of water. The processing results in a 10% solution of sodium sulphide. It is desired to achieve 98% of sodium sulphide extraction. Determine (a) the loss of sodium sulphide in the residue; (b) the amount of water that must be added as the solvent; and (c) the concentrations in each thickener.

8-16. A countercurrent extraction battery is used to extract sodium hydroxide from the products of the reaction



The feed contains water in an amount half that of the mass of the sediment (CaCO_3). The battery extracts 95% of the NaOH, and a 15% solution is obtained. How much water should be fed to the battery as a solvent and how many stages should the battery have? It is known from experimental data that the sediment retains the solution in the following amounts depending on the content of NaOH in it:

Content of NaOH, % (mass)	0	5	10	15	20
Solution retained per kg of sediment,					
kg	1.39	1.72	2.04	2.70	3.85

SYMBOLS

a	ratio of flows
G	amount of substance
k	distribution coefficient
n	number of stages
R	reflux ratio
V	volumetric rate of flow
X	mass ratio, raffinate phase
x	mass fraction, raffinate phase
x'	mass ratio, raffinate phase
Y	mass ratio, extract phase
y	mass fraction, extract phase
y'	mass ratio, extract phase
Z	mass ratio, extract phase
z	mass ratio, raffinate phase

Greek Letters

α	angle
β	surplus reflux factor
φ	fraction of solute remaining unextracted

**FUNDAMENTAL RELATIONSHIPS
AND FORMULAS FOR CALCULATIONS**

1. The limiting state for a substance being adsorbed (adsorbate) in adsorption is the state of equilibrium between its content in the adsorbent (the so-called static activity) a_0^* and its partial pressure p (or concentration \bar{c}_y) in a vapour-gas or liquid mixture.

The curve of equilibrium at constant temperature $a_0^* = f(p)$ is called an adsorption isotherm and is the main characteristic of the statics of a process. At relatively high partial pressures p , the content of the adsorbate a_0^* approaches a constant value a_∞ characterizing the state of the maximum possible saturation of the adsorbent at the given temperature.

According to the Clapeyron equation, there is direct proportionality between the concentration \bar{c}_y (in kg/m³) of the adsorbate in a gas mixture and its partial pressure p :

$$\bar{c}_y = \frac{p}{RT} \quad (9-1)$$

where R is the gas constant, J/kg·K.

2. The most complete equations of adsorption isotherms having no points of inflection in the coordinates a_0 against p and also taking into account the structural features of the sorbents are M. Dubinin's equations obtained on the basis of the theory of the dimensional filling of micropores*.

For sorbents having a homogeneous microporous structure (synthetic zeolites) this equation has the form:

$$a_0^* = \frac{w_0}{V} \exp \left[-B \frac{T^2}{\beta^2} \left(\log \frac{p_s}{p} \right)^2 \right] \quad (9-2)$$

* Dubinin, M. M. *et al.* In: *Sinteticheskie tseolity* (Synthetic Zeolites). Moscow, Izd. AN SSSR (1962), p. 7. Dubinin, M. M. In: *Tseolity, ikh sintez, svoistva i primeneniye* (Zeolites, Their Synthesis, Properties and Application). Moscow, Nauka (1965), p. 5.

For adsorbents having complex microporous structures (finely porous silica gels, active carbons), we have:

$$a_0^* = \frac{w_{01}}{V} \exp \left[-B_1 \frac{T^2}{\beta^2} \left(\log \frac{p_s}{p} \right)^2 \right] + \frac{w_{02}}{V} \exp \left[-B_2 \frac{T^2}{\beta^2} \left(\log \frac{p_s}{p} \right)^2 \right] \quad (9-3)$$

Here a_0^* = amount of the adsorbate, mmol/g
 w_0 , w_{01} , and w_{02} = constants characterizing the volumes of the micropores

B , B_1 , and B_2 = constants depending on the pore size

T = temperature, K

β = affinity coefficient of the characteristic curves

p_s/p = ratio of the saturated vapour pressure of the adsorbate to its partial pressure.

3. According to the theory of A. Eucken and M. Polanyi, if we have at our disposal an adsorption isotherm for the vapour of a standard substance at the temperature T_1 , we can calculate the adsorption isotherm for another vapour at the temperature T_2 .

The following formula is used to calculate the amount of adsorbate:

$$a_2^* = \frac{a_1^* V_1}{V_2} \quad (9-4)$$

where a_1^* = ordinate of the isotherm of the standard substance (generally benzene), kg/kg or mmol/g

a_2^* = ordinate of the isotherm being determined, kg/kg or mmol/g

V_1 and V_2 = molar volumes of the standard substance and the one being studied (in the liquid state), m³/kmol.

The molar volumes are determined by the equation:

$$V = \frac{M}{\rho} \quad (9-5)$$

where M = molar mass, kg/kmol

ρ = density of the liquid, kg/m³.

The pressures are found by the equation:

$$\log p_2 = \log p_{s,2} - \beta \frac{T_1}{T_2} \log \frac{p_{s,1}}{p_1} \quad (9-6)$$

where p_1 and p_2 = partial pressures of the standard substance and the one being studied, mm Hg

$p_{s,1}$ = saturated vapour pressure of the standard substance at the temperature T_1 (in K), mm Hg

$p_{s,2}$ = saturated vapour pressure of the substance being studied at the temperature T_2 (in K), mm Hg

β = affinity coefficient equal to the ratio of the molar volumes:

$$\beta = \frac{V_2}{V_1}$$

(9-7)

Table 9-1 gives the affinity coefficients β for selected substances. Benzene has been taken as the standard substance.

TABLE 9-1

Substance	β	Substance	β
Methyl alcohol	0.40	Acetic acid	0.97
Methyl bromide	0.57	Benzene	1.00
Ethyl alcohol	0.61	Cyclohexane	1.03
Formic acid	0.61	Carbon tetrachloride	1.05
Carbon disulphide	0.70	Diethyl ether	1.09
Ethyl chloride	0.76	Pentane	1.12
Propane	0.78	Toluene	1.25
Chloroform	0.86	Chloropicrin	1.28
Acetone	0.88	Hexane	1.35
Butane	0.90	Heptane	1.59

When calculating points on the isotherm of a vapour being studied, the coordinates a_1^* and p_1 are taken from a curve for the standard substance, the values of $p_{s,1}$ and $p_{s,2}$ from tables of saturated vapour pressure, and p_2 is calculated by Eq. (9-6).

4. The heat of adsorption consists of the heat of condensation and the heat of wetting. In practice, we can assume that the value of the heat of adsorption of organic substances is independent of the temperature. The dependence of the specific heat of adsorption q (in J/kg of carbon) on the amount of adsorbed vapour (for the substances designated with an asterisk in Table A-53) is determined by the equation [9-4]:

$$q = ma^n$$

(9-8)

where a = amount of adsorbed vapour, dm³/kg of carbon
 m and n = constants whose values are given in Table 9-2.

When water vapour is adsorbed by carbon, the heat of adsorption depends on the temperature as follows:

Temperature, °C	− 15	10	40	80	128	187
Heat of adsorption $q \times 10^{-3}$, J/kmol . . .	46 500	41 900	39 000	34 800	30 900	21 800

In the absence of experimental data, the heat of adsorption (related to one kmole of a gas) is determined by means of an

TABLE 9-2

Substance	Formula	<i>n</i>	<i>m</i> × 10 ⁻³
Benzene	C ₆ H ₆	0.959	3.24
Carbon disulphide	CS ₂	0.9205	3.15
Carbon tetrachloride	CCl ₄	0.930	3.74
Chloroform	CHCl ₃	0.935	3.47
Diethyl ether	(C ₂ H ₅) ₂ O	0.9215	3.84
Ethyl alcohol	C ₂ H ₅ OH	0.928	3.65
Ethyl bromide	C ₂ H ₅ Br	0.900	3.77
Ethyl chloride	C ₂ H ₅ Cl	0.915	3.06
Ethyl formate	HCOOC ₂ H ₅	0.9075	3.96
Ethyl iodide	C ₂ H ₅ I	0.956	3.10
Methyl alcohol	CH ₃ OH	0.938	3.11

approximate formula similar to Trouton's rule:

$$\frac{q}{\sqrt{T_b}} = \text{const}$$

(9-9)

where *q* = heat of adsorption, J/kmol of gas
T_b = boiling point of the adsorbate at atmospheric pressure, K.
The value of the constant depends on the nature of the adsorbent. For instance for activated carbon, it equals 2180.
The heat of adsorption (in J/kmol) can also be calculated by the formula:

$$q = \frac{44 \times 10^3 \log \frac{p_2}{p_1}}{\frac{1}{T_1} - \frac{1}{T_2}}$$

(9-10)

where *p₁* and *p₂* are the equilibrium pressures of the adsorbate over the adsorbent at the temperatures *T₁* and *T₂* (in K).
5. In practice, gases and vapours are absorbed, as a rule, in dynamic conditions—from a stream of a carrier gas. In this case, the sorbent bed is characterized by the value of the dynamic adsorption capacity (activity) *a_d*—the amount of sorbate adsorbed by the sorbent bed up to the breakpoint—the moment when the sorbate appears in the effluent:

$$a_d = \bar{c}_0 v \tau$$

(9-11)

where *a_d* = dynamic adsorption capacity of the sorbent bed, kg/m³
c₀ = initial concentration of the sorbate in the gas stream, kg/m³
v = velocity of the vapour and gas mixture related to the total cross section of the apparatus, m/s
τ = breakpoint time — the time elapsed up to breakthrough, s.

The degree of utilization of the equilibrium adsorption capacity $\eta = a_d/a_0^*$ (here a_0^* is the equilibrium static adsorption capacity of the sorbent bed, kg/m³) usually ranges from 0.8 to 0.9 for microporous sorbents in dynamic conditions with a bed from 30 to 50 cm deep, velocities of the vapour and air stream from 0.3 to 0.5 m/s, and initial sorbate concentrations of from 10 to 20 g/m³.

6. The process of adsorption in dynamic conditions can be characterized by a kinetic equation according to which the rate of adsorption (or the amount of substance adsorbed in a unit time by a unit volume of the adsorbent) is directly proportional to the mass transfer coefficient and the driving force of the process:

$$\frac{da}{d\tau} = \beta_y (\bar{c} - \bar{c}^*) \quad (9-12)$$

where \bar{c} = concentration of the adsorbate in the vapour and gas mixture, kg/m³ of inert gas

\bar{c}^* = concentration of the adsorbate in the vapour and gas mixture in equilibrium with the amount of adsorbate in a unit volume of the adsorbent, kg/m³ of inert gas

β_y = kinetic coefficient (coefficient of mass transfer), s⁻¹.

The following equation is used for approximate calculations of the coefficient of mass transfer β_y in adsorption on activated carbon ($d = 1.7$ to 2.2 mm, the stream velocity v is from 0.3 to 2 m/s), provided that the adsorption isotherm is described by the Langmuir equation:

$$Nu' = 1.6 Re^{0.54} \quad (9-13)$$

where

$$Nu' = \frac{\beta_y d^2}{D} \quad (9-14)$$

$$Re = \frac{vd}{\nu} \text{ [Eq. (1-27)]}$$

d = mean diameter of adsorbent particles, m

D = coefficient of diffusion of the adsorbate in the gas at the temperature of the process, m²/s

v = velocity of the stream of the vapour and gas mixture calculated for the free section of the apparatus, m/s

ν = kinematic viscosity of the vapour and gas mixture, m²/s.

7. The process of adsorption in dynamic conditions can also be characterized by the time elapsing from the beginning of passage of the vapour and gas mixture through the adsorbent bed to the breakpoint when the sorbate appears in the effluent (the indicated breakpoint concentration). This time τ is called the breakpoint

time and is determined by N. Shilov's equation:

$$\tau = K(H - h) \quad (9-15)$$

in which $Kh = \tau_0$. Hence,

$$\tau = KH - \tau_0 \quad (9-16)$$

where K = breakpoint coefficient of the sorbent bed, s/m

H = depth of the sorbent bed, m

h = depth of the unutilized sorbent bed in dynamic conditions, m

τ_0 = kinetic coefficient, or the loss in the breakpoint time of the sorbent bed, equal to the time spent for the formation of the adsorption zone, s.

The breakpoint coefficient of a bed can be found by the equation:

$$K = \frac{a_0^*}{v\bar{c}_0} \quad (9-17)$$

where a_0^* = equilibrium absorption capacity, kg/m³

v = velocity of the vapour and gas mixture related to the total cross section of the apparatus, m/s

\bar{c}_0 = initial concentration of the adsorbate in the vapour and gas mixture, kg/m³.

8. The following relationships called the dynamic characteristics B_1 and B_2 are observed for the same adsorbent and adsorbate at a constant concentration and temperature of the vapour and gas stream:

$$B_1 = K_1 v_1 = K_2 v_2 = \frac{a_0^*}{\bar{c}_0} = \text{const} \quad (9-18)$$

$$B_2 = \frac{\tau_{01} \sqrt{v_1}}{d_{g1}} = \frac{\tau_{02} \sqrt{v_2}}{d_{g2}} = \text{const} \quad (9-19)$$

where K = breakpoint coefficient of the bed, s/m

v = velocity of the vapour and gas stream, m/s

τ_0 = loss in the breakpoint time of the bed, s

d_g = mean diameter of the sorbent grains, m.

9. The duration of adsorption in a batch process is determined by solving a system of equations that consists of the equation of the adsorbate balance, the equation of the adsorption kinetics, and the equation of the adsorption isotherm.

The adsorption isotherm is divided into three regions: the first is characterized by the ratio $p/p_s < 0.17$ (relative to benzene), for the second $p/p_s \approx 0.17$ to 0.5, and for the third region $p/p_s > 0.5$.

When determining the duration of a process, the adsorption isotherm and the preset concentration of the initial vapour and air mixture \bar{c}_0 are used to find a_0^* , and the region of the isotherm which the quantity \bar{c}_0^* relates to is established. Next, the coefficient of mass transfer β_y is determined by Eq. (9-13).

Depending on the position of the quantity \bar{c}_0 on the isotherm, one of the following three methods of determining the duration of adsorption is employed [9-4]:

(1) for the first region, where the adsorption isotherm is considered to be a straight line and approximately corresponds to Henry's law, the following equation is used:

$$\sqrt{\tau} = \sqrt{\frac{a_0^*}{v\bar{c}_0}} \sqrt{H} - b \sqrt{\frac{a_0^*}{\beta_y \bar{c}_0}} \tag{9-20}$$

where τ = duration of adsorption, s

v = velocity of the vapour and gas stream related to the total cross section of the apparatus, m/s

H = depth of the bed of activated carbon, m

\bar{c}_0 = initial concentration of the adsorbate in the vapour and gas stream, kg/m³

a_0^* = amount of the adsorbate corresponding to equilibrium with the concentration of the stream \bar{c}_0 , kg/m³ (it is taken from the adsorption isotherm in kg/kg and is multiplied by the bulk density of the carbon in kg/m³)

β_y = coefficient of mass transfer, s⁻¹

b = coefficient whose value is taken for a number of values of \bar{c}/\bar{c}_0 from Table 9-3 (\bar{c} is the content of the adsorbate in the gas stream leaving the adsorber, kg/m³).

TABLE 9-3

\bar{c}/\bar{c}_0	b	\bar{c}/\bar{c}_0	b	\bar{c}/\bar{c}_0	b
0.005	1.84	0.2	0.63	0.7	−0.27
0.01	1.67	0.3	0.42	0.8	−0.46
0.03	1.35	0.4	0.23	0.9	−0.68
0.05	1.19	0.5	0.07		
0.1	0.94	0.6	−0.10		

(2) for the second region of the adsorption isotherm, we use the equation

$$\tau = \frac{a_0^*}{v\bar{c}_0} \left\{ H - \frac{v}{\beta_y} \left[\frac{1}{P} \ln \left(\frac{\bar{c}_0}{\bar{c}} - 1 \right) + \ln \frac{\bar{c}_0}{\bar{c}} - 1 \right] \right\} \tag{9-21}$$

where $P = \bar{c}_0/\bar{c}^*$

\bar{c}^* = content of the substance in the gas stream in equilibrium with half the maximum amount adsorbed by the given adsorbent, i. e. in equilibrium with $a_\infty/2$, kg/m³.

(3) for the third region of the adsorption isotherm, we have:

$$\tau = \frac{a_0^*}{v\bar{c}_0} \left[H - \frac{v}{\beta_y} \left(\ln \frac{\bar{c}_0}{\bar{c}} - 1 \right) \right] \quad (9-22)$$

10. A very important kinetic characteristic of the adsorption process is the depth of the mass transfer zone (the depth of the operating bed) h_0 which is calculated on the basis of the output curves (response curves) by the equation:*

$$h_0 = H \frac{\tau_s - \tau_b}{\tau_s - (1-f)(\tau_s - \tau_b)} \quad (9-23)$$

where H = depth of the sorbent bed

τ_s = time elapsing up to equilibrium saturation

τ_b = breakpoint time of the bed at the minimum indicated breakpoint concentration

f = unutilized equilibrium adsorption capacity of the sorbent in dynamic conditions in the zone of mass transfer (for microporous sorbents $f \approx 0.5$).

11. Continuous adsorption processes are conducted in adsorbers having an adsorbent bed moving downward**, while the vapour and gas mixture flows in the opposite direction (in countercurrent). The minimum velocity u of the sorbent (the velocity at which 95 to 98 per cent of the equilibrium adsorption capacity of a sorbent is utilized in dynamic conditions, and the preset degree of purification or drying of the vapour and gas streams is ensured) and the operating depth of the bed in a hypersorption apparatus H_0 are found by calculations.

The velocity of the bed u is calculated by the formula:

$$u = \frac{1}{K} = \frac{(\bar{c}_0 - \bar{c}_b) v}{a_0^*} \quad (9-24)$$

where v = velocity of the gas stream related to the total cross section of the apparatus

K = breakpoint coefficient of the bed

\bar{c}_0 = initial concentration of the adsorbate in the vapour and gas mixture

\bar{c}_b = constant indicated breakpoint concentration of the adsorbate in the vapour and gas stream.

* Michaels, A. *Ind. Eng. Chem.*, 44: 1922 (1952).

** In hypersorption or a moving bed.

In processes of fine drying or purification of gas streams, \bar{c}_p varies from 0.025 to 0.008% depending on the value of \bar{c}_0 (when \bar{c}_0 varies from 20.0 to 10.0 g/m³). Therefore in the given case, the constant indicated breakpoint concentration may be disregarded.

The operating depth of the bed in a hypersorption apparatus is calculated by the equation:

$$H_v = \gamma h_0 \quad (9-25)$$

where γ = factor characterizing the ratio of the bulk density with vibrational compacting of the bed to the density without compacting (on an average $\gamma = 1.4$)

h_0 = depth of mass transfer zone of the fixed bed.

12. The depth of the adsorbent bed can be calculated according to the general method through the number of transfer units:

$$H = \frac{Q_{v,g}}{A\beta_y} \int_{\bar{c}_1}^{\bar{c}_0} \frac{d\bar{c}}{\bar{c} - \bar{c}^*} = h_{eq} m \quad (9-26)$$

where $Q_{v,g}$ = rate of flow of the vapour and gas mixture, m³/s

A = cross-sectional area of the bed, m²

β_y = coefficient of mass transfer, s⁻¹

\bar{c}_0 and \bar{c}_1 = concentrations of the vapour and gas mixture at the inlet and the outlet of the adsorber, respectively, g/m³

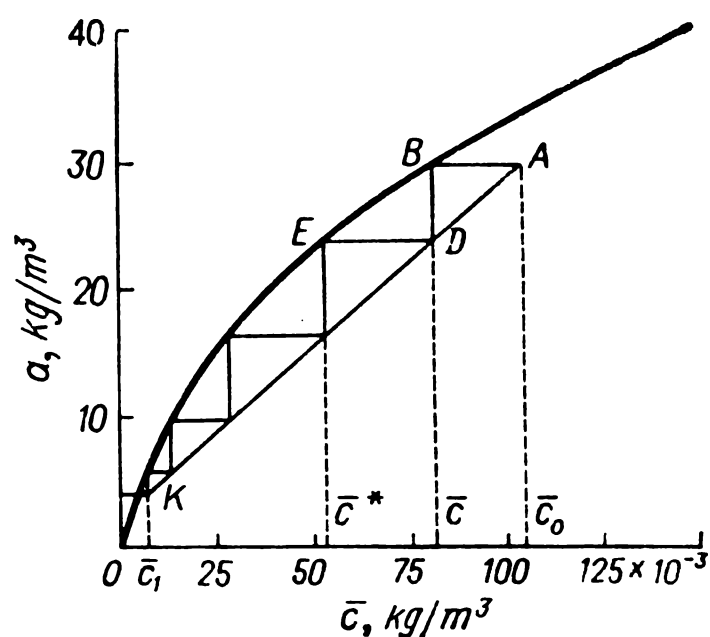
\bar{c}^* = equilibrium concentration of the adsorbate in the vapour and gas mixture according to the isotherm, g/m³.

The concentration difference $\bar{c} - \bar{c}^*$ determines the driving force of the process. An example of such a difference is the segment DE in Fig. 9-1 for which the abscissa of point D is \bar{c} and of point E is \bar{c}^* . The ratio $Q_{v,g}/A\beta_y$ is the depth of the bed of adsorbent equivalent to one transfer unit and is denoted by h_{eq} , while the integral in Eq. (9-26) is the total number of transfer units and is denoted by m . To determine H , we must find the value of the integral. This can be done graphically.

Knowing the values of \bar{c}_0 , a_{d0} and \bar{c}_1 , a_{d1} , we plot both these points on the diagram of the adsorption isotherm, and straight line AK drawn between them is the operating line. Next we construct a curve in the coordinates \bar{c} against $1/(\bar{c} - \bar{c}^*)$ (for similar calculations see Chaps. 6 and 7).

Instead of finding the integral in Eq. (9-26) in this way, we can use a simpler graphical method. Beginning from point A (Fig. 9-1) on the operating line (the point of admission of the vapour and gas mixture into the adsorber), we construct stages

Fig. 9-1. Determination of the number of stages of the change in concentration



of the change in the concentration between the equilibrium curve and the operating line up to point *K* (the outlet of the vapour and gas mixture from the adsorber).

Let m_0 stand for the number of transfer units corresponding to one stage. If the number of stages equals n and they are all the same, then $m = m_0 n$, and the depth of the bed is:

$$H = h_{eq} m = h_{eq} m_0 n \quad (9-27)$$

Such a case is possible only on the straight portion of the isotherm. The general form of the equation for calculating the depth of the bed is:

$$H = h_{eq} (m_0^I + m_0^{II} + \dots + m_0^n) = h_{eq} \sum_1^n m_0^i \quad (9-28)$$

The number of transfer units m_0 is calculated separately for each stage of the change in the concentration by the formula:

$$m_0 = \frac{\bar{c}_{in} - \bar{c}_{fn}}{\frac{(\bar{c}_{in} - \bar{c}_{in}^*) + (\bar{c}_{fn} - \bar{c}_{fn}^*)}{2}} = \frac{2(\bar{c}_{in} - \bar{c}_{fn})}{\bar{c}_{in} - \bar{c}_{fn}^*} \quad (9-29)$$

where \bar{c}_{in} = concentration of the vapour and gas mixture entering a stage, for example the abscissa of point *A*

\bar{c}_{fn} = concentration of the vapour and gas mixture leaving the stage, for example the abscissa of point *D*

\bar{c}_{in}^* = equilibrium concentration, for example the abscissa of point *B*; it must be noted that $\bar{c}_{fn} = \bar{c}_{in}^*$

\bar{c}_{fn}^* = equilibrium concentration, for example the abscissa of point *E*.

We may also use the following equation to determine the depth of the bed for the straight portion of the isotherm:

$$H = \frac{G}{A\beta_y\Delta\bar{c}_m} \quad (9-30)$$

Here G = amount of substance adsorbed in a unit time

$\Delta\bar{c}_m$ = mean driving force of adsorption, which can be computed as the logarithmic mean:

$$\Delta\bar{c}_m = \frac{\Delta\bar{c}_0 - \Delta\bar{c}_1}{\ln \frac{\Delta\bar{c}_0}{\Delta\bar{c}_1}} \quad (9-31)$$

where $\Delta\bar{c}_0$ = greater driving force at one end of the bed ($\bar{c}_0 - \bar{c}_0^*$)
 $\Delta\bar{c}_1$ = smaller driving force at the other end of the bed ($\bar{c}_1 - \bar{c}_1^*$).

EXAMPLES

Example 9-1. Determine the required amount of activated carbon, the depth of the adsorbent bed and the diameter of a batch adsorber for absorbing petrol vapour from its mixture with air. The rate of flow of the vapour and air mixture is 3450 m³/h. The initial concentration of the petrol is $\bar{c}_0 = 0.02$ kg/m³. The velocity of the vapour and air mixture is $v = 0.23$ m/s reduced to the total cross section of the apparatus, the dynamic activity of the carbon relative to petrol is 7% (mass), the residual activity after desorption is 0.8% (mass), the bulk density of the carbon is $\rho_b = 500$ kg/m³. The duration of desorption, drying, and cooling of the adsorbent is 1.45 h.

Solution. The amount of adsorbent needed to adsorb the petrol during 1.45 h is:

$$G = \frac{3450 \times 1.45 \times 0.02}{0.07 - 0.008} = 1612 \text{ kg}$$

At the given velocity of the vapour and air mixture equal to 0.23 m/s and a rate of flow of 3450 m³/h, the diameter of the adsorber should be

$$D = \sqrt{\frac{3450}{3600 \times 0.785 \times 0.23}} = 2.3 \text{ m}$$

The depth of the adsorbent bed should be:

$$H = \frac{1612}{500 \times 0.785 \times 2.3^2} \approx 0.8 \text{ m}$$

Example 9-2. According to experimental data, the duration of adsorbing chloropicrin vapour ($\bar{c}_0 = 6.6$ g/m³) using a bed of ac-

tivated carbon with a depth of $H = 0.05$ m and a cross-sectional area of $A = 0.01$ m² at a volumetric rate of flow of $Q_v = 0.03$ m³/min is $\tau = 336$ min. According to the isotherm for chloropicrin, the activity of the carbon is $a_0^* = 222$ kg/m³. The diameter of the carbon particles is $d_g = 1.5$ mm. Find (a) the breakpoint coefficient of the bed K ; (b) the loss in the breakpoint time (kinetic coefficient τ_0); and (c) the value of the dynamic characteristics B_1 and B_2 .

Solution. By Eq. (9-17), we have

$$K = \frac{a_0^*}{v\bar{c}_0}$$

The quantities a_0^* and \bar{c}_0 are known, and the linear velocity of the vapour is calculated according to the volumetric rate of flow and the cross-sectional area of the adsorber:

$$v = \frac{Q_v}{A} = \frac{0.03}{0.01} = 3 \text{ m/min}$$

The introduction of values yields:

$$K = \frac{222}{3 \times 0.0066} = 11\,200 \text{ min/m} \approx 187 \text{ h/m}$$

The loss in the breakpoint time is found by Shilov's equation (9-16):

$$\tau_0 = KH - \tau = 11\,200 \times 0.05 - 336 = 224 \text{ min}$$

The dynamic characteristics B_1 and B_2 are determined by Eqs. (9-18) and (9-19):

$$B_1 = Kv = 11\,200 \times 3 = 33\,600$$

$$B_2 = \frac{\tau_0 \sqrt{v}}{d_g} = \frac{224 \sqrt{3}}{0.0015} = 259\,000$$

Example 9-3. In the conditions of the preceding example, the velocity of the vapour and air mixture is $v' = 6$ m/min. Find (a) the breakpoint coefficient of the bed; (b) the loss in the breakpoint time; and (c) the duration of adsorption for a bed having a depth of $H' = 0.1$ m.

Solution. Since the characteristic B_1 is a constant quantity, then

$$K' = \frac{Kv}{v'} = \frac{B_1}{v'} = \frac{33\,600}{6} = 5600 \text{ min/m}$$

From Eq. (9-19) for the characteristic B_2 , we get:

$$\tau'_0 = \frac{B_2 d_g}{\sqrt{v'}} = \frac{259\,000 \times 0.0015}{\sqrt{6}} \approx 159 \text{ min}$$

We find the duration of adsorption by Eq. (9-16):

$$\tau' = K'H' - \tau_0' = 5600 \times 0.1 - 159 = 401 \text{ min}$$

Example 9-4. A batch adsorber processes 2000 m³ of a vapour and air mixture having a diethyl ether concentration of $\bar{c}_0 = 0.006 \text{ kg/m}^3$ during one cycle. The temperature of the process is 20 °C, the pressure is atmospheric, the velocity of the stream of vapour and gas mixture $v = 13 \text{ m/min}$, the concentration of the mixture after it leaves the adsorber is $\bar{c} = 3 \times 10^{-5} \text{ kg/m}^3$. The adsorbent is activated carbon of Grade АГ-5 with a grain diameter $d_g = 0.004 \text{ m}$ and a bulk density of 500 kg/m³. The depth of the carbon bed is $H = 0.7 \text{ m}$.

According to the benzene isotherm for 20 °C using the same grade of carbon, construct an isotherm of the adsorption of diethyl ether from air at 20 °C. Using this isotherm, find the amount of activated carbon needed for one batch, the diameter of the adsorber, and the breakpoint time.

Solution. We calculate the ordinates and abscissas of the points on the isotherm for diethyl ether by Eqs. (9-4) and (9-6):

$$a_2^* = a_1^* \frac{V_1}{V_2}$$

$$\log p_2 = \log p_{s,2} - \beta \frac{T_1}{T_2} \log \frac{p_{s,1}}{p_1}$$

where a_1 and a_2 = concentrations of the adsorbed benzene and ether, kg/kg

V_1 and V_2 = molar volumes of the benzene and ether in the liquid state, m³/kmol

p_1 and p_2 = partial pressures of the benzene and ether vapours, mm Hg

$p_{s,1}$ and $p_{s,2}$ = saturated vapour pressures of the benzene and ether at 20 °C, mm Hg

T_1 and T_2 = temperatures of the benzene and ether upon adsorption (in the given case $T_1 = T_2 = 293 \text{ K}$)

β = affinity coefficient.

The molar volumes of the benzene and ether are:

$$V_1 = \frac{M_1}{\rho_1} = \frac{78}{879} = 0.0887 \text{ m}^3/\text{kmol}$$

$$V_2 = \frac{M_2}{\rho_2} = \frac{74}{714} = 0.1036 \text{ m}^3/\text{kmol}$$

The affinity coefficient is:

$$\beta = \frac{V_2}{V_1} = \frac{0.1036}{0.0887} = 1.17$$

We take a number of points on the benzene isotherm (Fig. 9-2).

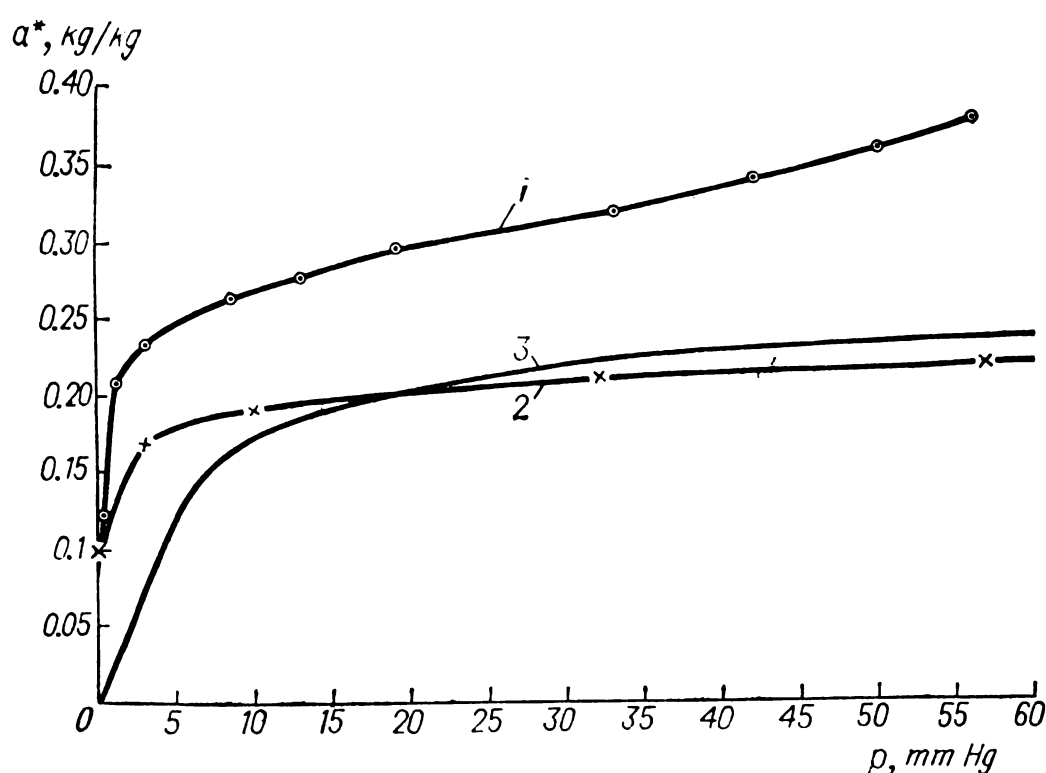


Fig. 9-2. Adsorption isotherms for 20°C:
1—benzene; 2—diethyl ether; 3—ethyl alcohol (70%)+diethyl ether (30%)

The first point is $a_1^* = 0.262$ kg/kg and $p_1 = 8$ mm Hg. We calculate the coordinates of the corresponding point on the isotherm for diethyl ether:

$$a_1^* = \frac{0.262}{78} \text{ kmol/kg}$$

$$a_2^* = a_1^* \frac{V_1}{V_2} = \frac{0.262}{78} \frac{0.0887}{0.1036} = 0.00287 \text{ kmol/kg} = 0.00287 \times 74 \text{ kg/kg} = 0.212 \text{ kg/kg}$$

$$\log p_2 = \log 442 - 1.17 \times \frac{293}{293} \log \frac{75}{8} = 1.50822$$

$$p_2 = 32.2 \text{ mm Hg}$$

We compute the ordinates and abscissas for other points in the same way and enter the results obtained in Table 9-4.

According to the found points, we construct an isotherm for the diethyl ether at 20°C (curve 2 in Fig. 9-2).

We use the isotherm to find the static activity of carbon with respect to the diethyl ether for a concentration of the vapour and air mixture $\bar{c}_0 = 0.006$ kg/m³.

We must first calculate the partial pressure corresponding to \bar{c}_0 by Eq. (9-1):

$$p_0 = \bar{c}_0 RT = 0.006 \times \frac{848}{74} \times 293 \times \frac{760}{10330} = 1.4 \text{ mm Hg}$$

According to the diagram, the abscissa $p_0 = 1.4$ mm Hg gives us the ordinate $a_0^* = 0.132$ kg/kg.

TABLE 9-4

Benzene isotherm		Diethyl ether isotherm	
a_1^* , kg/kg	p_1 , mm Hg	a_2^* , kg/kg	p_2 , mm Hg
0.103	0.105	0.0837	0.0202
0.122	0.223	0.0992	0.0501
0.208	1	0.169	2.82
0.233	3	0.190	10.1
0.262	8	0.212	32.2
0.276	13	0.224	56.9
0.294	19	0.24	88.2
0.318	33	0.258	169.2
0.338	42	0.273	224.5
0.359	50	0.292	275.0

The amount of activated carbon per batch is:

$$G_1 = \frac{2000 \times 0.006}{0.132} = 91 \text{ kg or } \frac{91}{500} = 0.182 \text{ m}^3$$

We find the diameter of the adsorber from the equation

$$\frac{\pi D_a^2 H}{4} = 0.182 \text{ m}^3$$

whence

$$D_a = \sqrt{\frac{0.182 \times 4}{\pi H}} = \sqrt{\frac{0.182 \times 4}{3.14 \times 0.7}} = 0.69 \text{ m}$$

Since the point on the isotherm corresponding to the initial concentration of the vapour and air mixture $\bar{c}_0 = 0.006 \text{ kg/m}^3$ is on the first (linear) portion, we calculate the duration of the process by Eq. (9-20):

$$\sqrt{\tau} = \sqrt{\frac{\Gamma}{v}} \sqrt{H} - b \sqrt{\frac{\Gamma}{\beta_y}}$$

where $\Gamma = a_0^* / \bar{c}_0 = (0.132 \times 500) / 0.006 = 11\,000$
 $v = 13 \text{ m/min} = \text{velocity of the gas stream}$
 $H = 0.7 \text{ m} = \text{depth of the carbon bed}$
 $b = \text{function determined from Table 9-3 (for } \bar{c} / \bar{c}_0 = 0.000\,03 / 0.006 = 0.005, \text{ the value of } b \text{ is } 1.84)$
 $\beta_y = \text{coefficient of mass transfer that is calculated by Eq. (9-13) after the latter is transformed into a form convenient for calculations:}$

$$\beta_y = 1.6 \frac{Dv^{0.54}}{v^{0.54} d_g^{1.46}} \tag{a}$$

We find the kinematic viscosity of air. Since according to Fig. A-6 we have $\mu = 0.018 \times 10^{-3} \text{ Pa}\cdot\text{s}$, then

$$\nu = \frac{\mu}{\rho} = \frac{0.018 \times 10^{-3}}{1.2} = 0.15 \times 10^{-4} \text{ m}^2/\text{s}$$

Hence,

$$\nu^{0.54} = (0.15 \times 10^{-4})^{0.54} = 0.248 \times 10^{-2} \text{ m}^2/\text{s}$$

The diameter of the carbon particles is $d_g = 0.004 \text{ m}$. Therefore,
 $d_g^{1.46} = 0.004^{1.46} = (0.40 \times 10^{-2})^{1.46} = 0.3154 \times 10^{-3} \text{ m}$

The velocity $v = 13 \text{ m/min} = 13/60 = 0.217 \text{ m/s}$. Hence, $v^{0.54} = 0.217^{0.54} = 0.438 \text{ m/s}$.

The coefficient of diffusion at 0°C for the system diethyl ether-air is $D_0 = 0.028 \text{ m}^2/\text{h} = 0.0778 \times 10^{-4} \text{ m}^2/\text{s}$.

We compute the coefficient of diffusion for a temperature of 20°C by the equation:

$$\begin{aligned} D &= D_0 \frac{p_0}{p} \left(\frac{T}{T_0} \right)^{3/2} = 0.0778 \times 10^{-4} \times \frac{1}{1} \left(\frac{293}{273} \right)^{3/2} = \\ &= 0.0778 \times 10^{-4} \times 1.11 = 0.0864 \times 10^{-4} \text{ m}^2/\text{s} \end{aligned}$$

After inserting values into Eq. (a), we get the volume coefficient of mass transfer:

$$\beta_y = \frac{1.6 \times 0.0864 \times 10^{-4} \times 0.438}{0.248 \times 10^{-2} \times 0.3154 \times 10^{-3}} = 7.75 \text{ s}^{-1}$$

We find the duration of the process:

$$\sqrt{\tau} = \sqrt{\frac{11\,000}{0.217}} \sqrt{0.7} - 1.84 \sqrt{\frac{11\,000}{7.75}} = 188.1 - 69.4 = 118.7$$

and $\tau = 118.7^2 = 14\,090 \text{ s} = 234.9 \text{ min} = 3.9 \text{ h}$.

We determine the amount of the vapour and air mixture passing through the adsorber during this time:

$$V = \frac{\pi D_a^2}{4} v \tau = \frac{3.14 \times 0.69^2}{4} \times 13 \times 234.9 = 1142 \text{ m}^3$$

According to the initial conditions, 2000 m^3 of the mixture should pass through the adsorber during one cycle. Consequently, the diameter of the adsorber should be increased:

$$D_a = \sqrt{\frac{2000 \times 4}{\pi v \tau}} = \sqrt{\frac{2000 \times 4}{3.14 \times 13 \times 234.9}} = 0.83 \text{ m}$$

It is also necessary to increase the amount of activated carbon in a batch:

$$\frac{\pi D_a^2}{4} H \times 500 = 0.785 \times 0.83^2 \times 500 \times 0.7 = 190 \text{ kg}$$

Example 9-5. Using an adsorption isotherm for a mixture of vapours of ethyl alcohol and diethyl ether (curve 3 in Fig. 9-2), determine the duration of adsorption of this mixture by a bed of activated carbon having a depth of $H = 1.0$ m. The initial concentration of the mixture is $\bar{c}_0 = 0.072$ kg/m³; the mean concentration at the outlet from the adsorber is $\bar{c} = 0.0001$ kg/m³; the velocity of the vapour and gas mixture related to the total section of the adsorber is $v = 12$ m/min; the diameter of the activated carbon particles is $d_g = 0.004$ m, their bulk density is $\rho_b = 500$ kg/m³; the adsorption temperature is 20 °C; and the pressure is atmospheric.

Solution. We use the adsorption isotherm (Fig. 9-2) to find a_0^* corresponding to the concentration $\bar{c}_0 = 0.072$ kg/m³. The molar mass of the mixture is $M_{\text{mix}} = 0.3 \times 46 + 0.7 \times 74 = 65.6$ kg/kmol.

The pressure corresponding to \bar{c}_0 is:

$$p_0 = \bar{c}_0 RT = 0.072 \times \frac{848}{65.6} \times 293 \times \frac{760}{10\,330} = 20 \text{ mm Hg}$$

On the isotherm, we find $a_0^* = 0.20$ kg/kg = $0.2 \times 500 = 100$ kg/m³. This point is on the third portion of the isotherm; therefore the duration of adsorption is determined by Eq. (9-22):

$$\tau = \frac{a_0^*}{v \bar{c}_0} \left[H - \frac{v}{\beta_y} \left(\ln \frac{\bar{c}_0}{\bar{c}} - 1 \right) \right]$$

For calculations by this equation, all we need is the coefficient of mass transfer β_y for the mixture.

The coefficient of diffusion of ether in air at 0 °C is:

$$D'_0 = 0.028 \text{ m}^2/\text{h} = 0.0778 \times 10^{-4} \text{ m}^2/\text{s}$$

The coefficient of diffusion of ethyl alcohol in air at 0 °C is:

$$D''_0 = 0.0367 \text{ m}^2/\text{h} = 0.1019 \times 10^{-4} \text{ m}^2/\text{s}$$

We adopt the smaller diffusion coefficient (for the ether) and recalculate it for a temperature of 20 °C:

$$\begin{aligned} D &= D'_0 \frac{p_0}{p} \left(\frac{T}{T_0} \right)^{3/2} = 0.028 \times \frac{1}{1} \left(\frac{293}{273} \right)^{3/2} = 0.0312 \text{ m}^2/\text{h} = \\ &= 0.0866 \times 10^{-4} \text{ m}^2/\text{s} \end{aligned}$$

We calculate the kinematic viscosity for air (see the preceding example):

$$\nu = 0.15 \times 10^{-4} \text{ m}^2/\text{s}$$

We compute the values of $v^{0.54}$, $v^{0.54}$, and $d_g^{1.46}$:

$$v^{0.54} = (0.15 \times 10^{-4})^{0.54} = 0.248 \times 10^{-2}$$

$$\left(\frac{v}{60}\right)^{0.54} = \left(\frac{12}{60}\right)^{0.54} = 0.42$$

$$d_g^{1.46} = 0.004^{1.46} = 0.3154 \times 10^{-3}$$

Introduction of the found values into Eq. (a) gives the coefficient of mass transfer:

$$\beta_y = \frac{1.6 \times 0.0866 \times 10^{-4} \times 0.42}{0.248 \times 10^{-2} \times 0.3154 \times 10^{-3}} = 7.4 \text{ s}^{-1}$$

The duration of adsorption is:

$$\tau = \frac{100 \times 60}{12 \times 0.072} \left[1.0 - \frac{12}{60 \times 7.4} \left(2.3 \log \frac{0.072}{0.0001} - 1 \right) \right] = 5900 \text{ s} = 1 \text{ h } 38 \text{ min}$$

Example 9-6. A continuous adsorber with a diameter of $D = 0.32 \text{ m}$ processes 120 m^3 of a vapour and gas mixture an hour. The activated carbon supplied to the adsorption zone contains $a_1 = 4 \text{ kg/m}^3$ of the adsorbate; at the outlet from this zone its adsorbate content reaches $a_d = 30 \text{ kg/m}^3$. The concentration of the vapour and gas mixture entering the adsorber is $\bar{c}_0 = 0.105 \text{ kg/m}^3$, and that of the effluent mixture is $\bar{c}_1 = 0.0065 \text{ kg/m}^3$. The coefficient of mass transfer of the adsorbate in the conditions of adsorber operation is $\beta_y = 5 \text{ s}^{-1}$. The adsorption isotherm is known (Fig. 9-1). Determine the velocity of the activated carbon and the depth of its bed.

Solution. We find the velocity of the carbon by Eq. (9-24):

$$u = \frac{1}{K} = \frac{\bar{c}_0 v}{a_0}$$

We find the linear velocity of the vapour and mixture:

$$v = \frac{4V}{3600\pi D^2} = \frac{4 \times 120}{3600 \times 3.14 \times 0.32^2} = 0.415 \text{ m/s}$$

Using the found value of v in Eq. (9-24), we determine the velocity of the carbon:

$$u = \frac{0.105 \times 0.415}{34} = 0.00128 \text{ m/s}$$

To calculate the depth of the bed, we plot the operating line in the isotherm diagram (Fig. 9-1) according to the coordinates of its initial point A ($\bar{c}_0 = 0.105 \text{ kg/m}^3$ and $a_d = 30 \text{ kg/m}^3$) and terminal point K ($\bar{c}_1 = 0.0065 \text{ kg/m}^3$ and $a_1 = 4 \text{ kg/m}^3$).

We construct the stages of the change in concentration from the initial point of the operating line to its terminal point. We

get five stages. For each stage, we find the number of transfer units m_0 by Eq. (9-28):

$$\begin{aligned} m_0^I &= \frac{2(\bar{c}_0 - \bar{c}_1)}{\bar{c}_0 - \bar{c}_2^*} = \frac{2(0.105 - 0.0815)}{0.105 - 0.053} = 0.905 \\ m_0^{II} &= \frac{2(0.0815 - 0.053)}{0.0815 - 0.028} = 1.065 \\ m_0^{III} &= \frac{2(0.053 - 0.028)}{0.053 - 0.0115} = 1.205 \\ m_0^{IV} &= \frac{2(0.028 - 0.0115)}{0.028 - 0.0065} = 1.535 \\ m_0^V &= \frac{2(0.0115 - 0.0065)}{0.0115 - 0.004} = 1.335 \end{aligned}$$

We find the depth of the activated carbon bed by Eqs. (9-26) and (9-28):

$$\begin{aligned} H &= \frac{v}{A\beta_y} \sum_1^v m_0^t = \frac{v}{\beta_y} \sum_1^v m_0^t = \\ &= \frac{0.415}{5} (0.905 + 1.065 + 1.205 + 1.535 + 1.335) = \\ &= 0.083 \times 6.045 \approx 0.5 \text{ m} \end{aligned}$$

Example 9-7. Determine the amount of heat liberated during one cycle ($\tau = 133$ min) in the adsorption of ethyl alcohol vapour with activated carbon. The diameter of the adsorber is 2 m, the depth of the carbon bed is $H = 1.0$ m.

The velocity of the vapour and air mixture is $v = 25$ m/min; the initial concentration is $\bar{c}_0 = 0.029$ kg/m³; the concentration of the mixture leaving the adsorber is $\bar{c}_1 = 0.0002$ kg/m³; the bulk density of the bed is $\rho_b = 500$ kg/m³.

Solution. The cross-sectional area of the apparatus is:

$$A = \frac{\pi D^2}{4} = \frac{3.14 \times 2^2}{4} = 3.14 \text{ m}^2$$

During one cycle, the amount of vapour and gas mixture passing through the adsorber is $V = vA\tau = 25 \times 3.14 \times 133 = 10\,400$ m³.

The amount of ethyl alcohol vapour adsorbed is:

$$G_{alc} = \frac{10\,400(29 - 0.2)}{1000} = 300 \text{ kg} = \frac{300}{46} \approx 6.52 \text{ kmol}$$

The amount of activated carbon charged into the adsorber is: $AH\rho_b = 3.14 \times 1.0 \times 500 = 1570$ kg = $1570/6.52 \approx 240.8$ kg of carbon per kmol of the alcohol.

We cannot use the data of Table A-53 here because it has been compiled on the basis of adsorption of 1 kmol per 500 kg of carbon.

We can calculate the heat of adsorption by Eq. (9-8):

$$q = ma^n$$

The amount of adsorbed vapour a per kilogram of carbon is:

$$a = \frac{6.52 \times 22.4 \times 1000}{1570} \approx 93 \text{ dm}^3/\text{kg}$$

We take the numerical values of m and n from Table 9-2: $m = 3.65 \times 10^3$ and $n = 0.928$.

By Eq. (9-8), the amount of heat liberated per kilogram of carbon is:

$$q = 3.65 \times 93^{0.928} = 245 \text{ kJ/kg}$$

The total amount of heat liberated during one cycle is:

$$q_1 = 245 \times 1570 = 385\,000 \text{ kJ}$$

This heat goes to heat the carbon and the apparatus, to compensate for heat losses, and mainly to heat the vapour and gas mixture. If we assume that all the liberated heat is used only for heating the vapour and gas mixture and that its specific heat capacity and density are the same as for air ($c = 1.01 \times 10^3 \text{ J/kg} \cdot \text{K}$ and $\rho = 1.2 \text{ kg/m}^3$), then the temperature of the mixture will grow by

$$\Delta t = \frac{385\,000 \times 10^3}{10\,400 \times 1.2 \times 1.01 \times 10^3} = 30.5 \text{ K}$$

Example 9-8. Determine the operating depth of a fixed bed of zeolite of type NaA ($d_g = 0.002 \text{ m}$) and the operating height of a column apparatus for the process of deep drying of gases ($\bar{c}_b = 2.94 \times 10^{-6} \text{ kg/m}^3$) in the following conditions: the depth of the fixed bed is 0.26 m , $\bar{c}_0 = 0.01 \text{ kg/m}^3$, the velocity of the vapour and air stream related to the total section of the apparatus is 0.5 m/s , $\tau_s = 190 \text{ min}$, $\tau_b = 110 \text{ min}$.*

Solution. The depth of the mass transfer zone is calculated by Eq. (9-23):

$$\begin{aligned} h_0 &= H \frac{\tau_s - \tau_b}{\tau_s - (1-f)(\tau_s - \tau_b)} = 0.26 \times \frac{190 - 110}{190 - (1-0.5)(190 - 110)} = \\ &= \frac{26 \times 80}{150} = 13.8 \text{ cm} \end{aligned}$$

where H = depth of the sorbent bed, cm

τ_s = time elapsing up to equilibrium saturation, min

* Shiryayev, A. N. "The Kinetics of Adsorption and Desorption on Zeolites." Synopsis of Candidate's Thesis. Leningrad Technological Institute named after the Leningrad Soviet (1967).

τ_b = breakpoint time of the bed at the minimum indicated breakpoint concentration, min

f = unutilized equilibrium adsorption capacity of the adsorbent in the mass transfer zone.

The operating depth of the bed is calculated by Eq. (9-25):

$$H_o = \gamma h_o = 1.4 \times 13.8 = 19.3 \text{ cm}$$

where γ is a factor characterizing the ratio of the bulk density with vibrational compacting of the layer to the density without compacting ($\gamma = 1.4$).

PROBLEMS

9-1. Determine the charge of activated carbon, the diameter of the adsorber, and the duration of the adsorption of 100 kg of octane vapour from its mixture with air for the following data: the initial concentration of the octane vapour in the mixture $\bar{c}_0 = 0.012 \text{ kg/m}^3$, the velocity of the mixture $v = 20 \text{ m/min}$, the activity of the carbon with respect to benzene is 7%, the bulk density of the carbon $\rho_b = 350 \text{ kg/m}^3$, and the depth of the carbon bed $H = 0.8 \text{ m}$.

9-2. Determine the breakpoint time τ and the loss in the breakpoint time τ_0 for the adsorption of carbon tetrachloride vapour by a bed of activated carbon having a depth of $H = 0.10 \text{ m}$. The velocity of the vapour and gas mixture is $v = 5 \text{ m/min}$, the diameter of the carbon particles is $d_g = 2.75 \text{ mm}$, the dynamic characteristics are $B_1 = 14\,500$ and $B_2 = 52\,945$.

9-3. According to the benzene adsorption isotherm for 20°C (see Fig. 9-2), construct an adsorption isotherm for ethyl alcohol vapour at 25°C .

9-4. Using the benzene adsorption isotherm in Fig. 9-2, determine the velocity and the operating depth of the bed of activated carbon in the continuous adsorption of a vapour and gas mixture having an initial concentration of $\bar{c}_0 = 0.11 \text{ kg/m}^3$, a velocity of $v = 20 \text{ m/min}$, and a mass transfer coefficient of $\beta_y = 4 \text{ s}^{-1}$. The carbon in the process of adsorption is saturated up to 80% of its static activity. The residual activity of the carbon after desorption is 14.5% of the initial static activity. The vapour and gas mixture must be purified to a concentration of not over $\bar{c}_1 = 0.01 \text{ kg/m}^3$.

9-5. A vertical adsorber 3 m in diameter with a steel tube 0.35 m in diameter is fed with $170 \text{ m}^3/\text{min}$ of a vapour and gas mixture. The initial concentration of ethyl alcohol vapour in the mixture is $\bar{c}_0 = 0.02 \text{ kg/m}^3$. The concentration of the ethyl alcohol in the discharged gas is $\bar{c}_1 = 0.0002 \text{ kg/m}^3$, the depth of the activated carbon bed is $H = 1.5 \text{ m}$, the bulk density of the carbon is $\rho_b = 500 \text{ kg/m}^3$, and the duration of one adsorption cycle is 4 h 37 min. Find the amount of heat liberated in the adsorber during the first cycle.

9-6. Determine the minimum velocity of type NaA zeolite in a column apparatus in deep drying of air with the following data: $\bar{c}_0 = 0.01 \text{ kg/m}^3$, $\bar{c}_b = 2.94 \times 10^{-6} \text{ kg/m}^3$, $d_g = 0.002 \text{ m}$, and $a_0^* = 170 \text{ kg/m}^3$. The velocity of the gas stream related to the total cross section of the apparatus is 0.5 m/s .

SYMBOLS

A area

a amount of adsorbed substance

a^* amount of adsorbate; static activity

B constant depending on pore size; dynamic characteristic

b coefficient

\bar{c}	mass-volume concentration
D	diffusion coefficient
d	diameter
f	unutilized equilibrium adsorption capacity of sorbent
G	amount of substance
g	specific consumption of heating steam
H	depth of sorbent layer
h	depth
K	breakpoint coefficient of sorbent bed
M	molar mass
m	constant; number of transfer units
Nu	Nusselt dimensionless number
n	constant
P	ratio of mass-volume concentrations
p	pressure
Q_v	volumetric rate of flow
q	specific heat of adsorption
R	gas constant
Re	Reynolds dimensionless number
T	absolute temperature
u	sorbent velocity
V	volume
v	velocity
w	volume of micropores

Greek Letters

β	affinity coefficient of characteristic curves; individual coefficient of mass transfer
Γ	ratio
γ	factor characterizing density ratio
η	degree of utilization of equilibrium adsorption capacity
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
τ	breakpoint time
τ_0	kinetic coefficient (loss in breakpoint time, equal to the time spent for the formation of the adsorption zone)

**FUNDAMENTAL RELATIONSHIPS
AND FORMULAS FOR CALCULATIONS**

1. The moisture content of a material can be expressed as a percentage either of the total mass of the wet substance (u) or of the mass of the dry substance (u'). The quantities u and u' are related by the equations:

$$u' = \frac{100u}{100-u} \text{ and } u = \frac{100u'}{100+u'} \quad (10-1)$$

2. The amount of moisture W removed from a material in drying when its moisture content changes from u_{in} to u_{fn} is:

$$W = G_{in} \frac{u_{in} - u_{fn}}{100 - u_{fn}} \text{ or } W = G_{fn} \frac{u_{in} - u_{fn}}{100 - u_{in}} \quad (10-2)$$

where G_{in} and u_{in} = initial mass and moisture content of the material fed for drying, respectively

G_{fn} and u_{fn} = final mass and moisture content of the dried material, respectively.

If the moisture content of the material is given as a percentage of the mass of the dry substance u' , then

$$W = G_{dry} \frac{u'_{in} - u'_{fn}}{100} \quad (10-3)$$

where G_{dry} is the capacity of the dryer expressed in the amount of bone-dry product (dry basis).

3. The vapour content of a vapour and gas mixture x (in kg of vapour per kg of dry gas) is:

$$x = \frac{M_v}{M_g} \frac{p_v}{p_{tot} - p_v} \quad (10-4)$$

where M_v and M_g = molar masses of the vapour and gas, respectively

p_{tot} = total pressure of the vapour and gas mixture

p_v = partial pressure of the vapour.

The moisture content of a water vapour and air mixture x (in kg of water vapour per kg of dry air) is:

$$x = 0.622 \frac{\varphi p_s}{p_{\text{tot}} - \varphi p_s} \quad (10-5)$$

where $0.622 =$ ratio of the molar masses of water vapour and air
 $\varphi =$ relative humidity of the air

$$\varphi = \frac{p_v}{p_s} \quad (10-6)$$

$p_v =$ partial pressure of the water vapour in the air (at the dry bulb temperature)

$p_s =$ pressure of the saturated water vapour at the same temperature (Tables A-38 and A-56)*.

4. The enthalpy of moist or humid air H (in kJ per kg of dry air) is:

$$H = (c_a + c_v x) t + L_0 x = (1.01 + 1.97x) t + 2493x \quad (10-7)$$

where $c_a = 1.01$ kJ/kg·K = mean specific heat capacity of the dry air (at constant pressure)

$c_v = 1.97$ kJ/kg·K = mean specific heat capacity of water vapour

$x =$ moisture content of the air, kg of vapour per kg of dry air

$t =$ dry bulb temperature, °C

$L_0 = 2493$ kJ/kg = specific heat of vaporization of water at 0 °C.

5. The relationship between the parameters of humid air x , t , φ , and H are easily determined according to a Ramzin H - x diagram (Fig. 10-1), which is exactly what is mainly used to solve problems in the statics of convective air drying.

6. The density of humid air $\rho_{h.a}$ (in kg/m³) at the pressure p_{tot} and the temperature T expressed in kelvins is determined by the equation:

$$\rho_{h.a} = \rho_a + \rho_v \quad (10-8)$$

in which the densities of the dry air ρ_a and of the water vapour ρ_v are each taken at the relevant partial pressure:

$$\rho_a = \frac{M_a T_0 (p_{\text{tot}} - \varphi p_s)}{22.4 T p_{\text{tot}}^\circ} \quad (10-9)$$

$$\rho_v = \frac{M_v T_0 \varphi p_s}{22.4 T p_{\text{tot}}^\circ} \quad (10-10)$$

* If the temperature of humid air is higher than the temperature of saturation of water vapour at the pressure p_{tot} , then $p_s = p_{\text{tot}}$ and $x = 0.622 \frac{\varphi}{1 - \varphi}$.

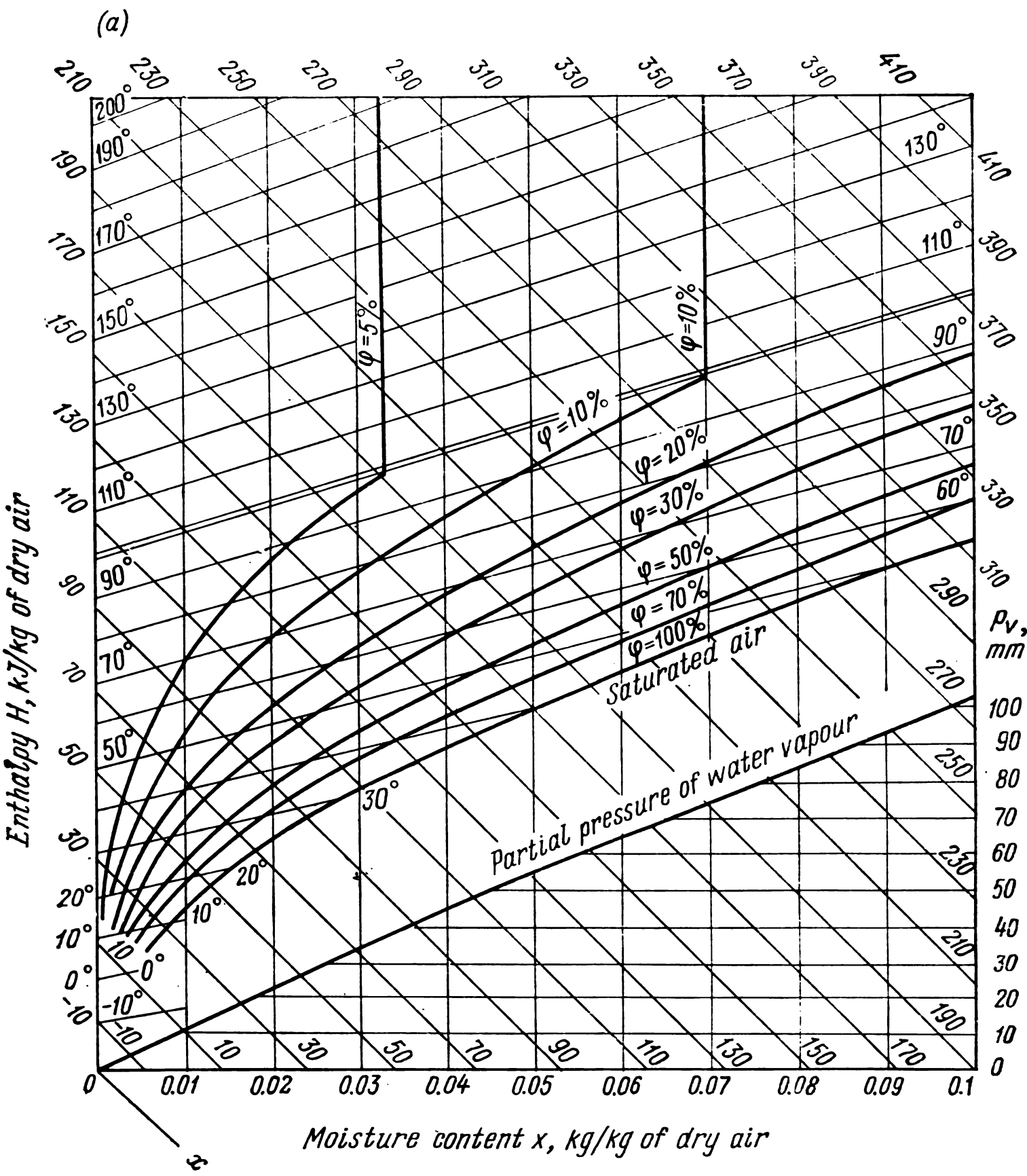


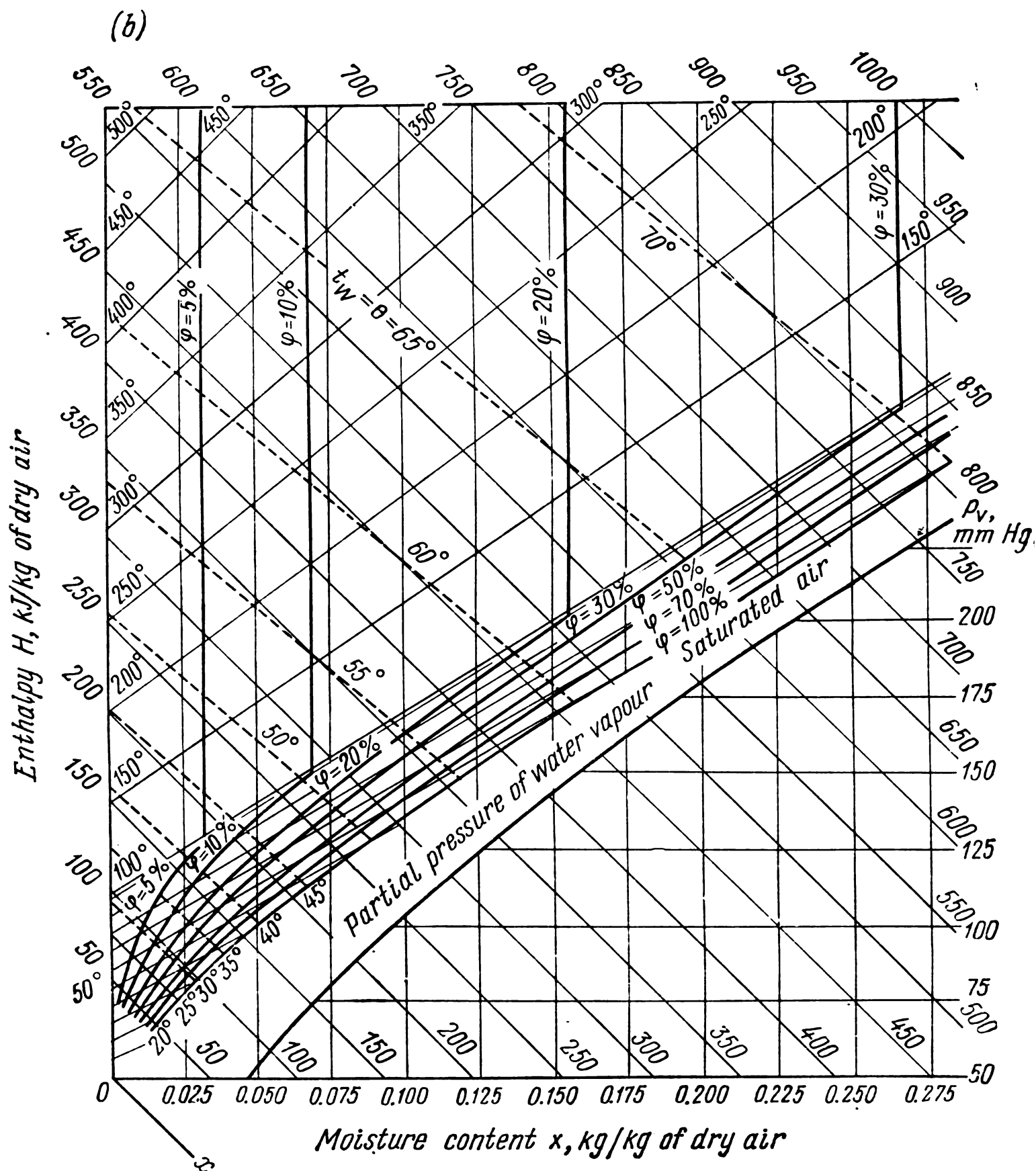
Fig. 10-1. Ramzin diagram for humid air:
a— $x=0$ to 0.1 ; b— $x=0$ to 0.275

where p_{tot} = total pressure of the water vapour and air mixture
 p_{tot}° = standard pressure (0.1013 MPa or 1 atm).

From Eqs. (10-8) to (10-10), we get:

$$\rho_{\text{h.a}} = \frac{M_a T_0 p_{\text{tot}}}{22.4 T p_{\text{tot}}^{\circ}} \left[1 - \left(1 - \frac{M_v}{M_a} \right) \frac{\varphi p_s}{p_{\text{tot}}} \right] = 1.293 \frac{273 p_{\text{tot}}}{T \times 101300} \times$$

$$\times \left(1 - 0.378 \frac{\varphi p_s}{p_{\text{tot}}} \right) = \frac{3.48 \times 10^{-3}}{T} (p_{\text{tot}} - 0.378 \varphi p_s) \quad (10-11)$$



7. The specific volume of humid air (per kilogram of dry air) (in m^3/kg) is calculated by the equation:

$$v = \frac{R_a T}{p_{\text{tot}} - \phi p_s} \quad (10-12)$$

where R_a = gas constant for air equal to $287 \text{ J/kg} \cdot \text{K}$

T = temperature of the air, K

p_{tot} = total pressure of the vapour and air mixture, Pa

$\phi p_s = p_v$ = partial pressure of the water vapour, Pa.

8. The rate of flow of dry air in a dryer G (in kg/s) is:

$$G = Wg \quad (10-13)$$

where W = capacity of the dryer with respect to the moisture being evaporated, kg/s

g = specific rate of flow of the dry air, kg/kg of evaporated moisture:

$$g = \frac{1}{x_2 - x_0} \quad (10-14)$$

x_0 and x_2 = initial and final moisture content of the air.

9. The consumption of heat in an air heater Q (in W) in the normal (basic) variant of a drying process is:

$$Q = G(H_1 - H_0) \quad (10-15)$$

where H_0 and H_1 are the enthalpies of the air at the inlet and outlet of the air heater, respectively, J/kg of dry air.

It follows from the heat balance of a dryer for the normal drying variant that

$$Q = G(H_2 - H_0) + \sum Q \quad (10-16)$$

where H_2 = enthalpy of the air at the outlet from the dryer

$\sum Q$ = sum of the heat consumptions for heating the material, heating the conveying means, and for losses to the surroundings (see below).

Ignoring the quantity $\sum Q$ in comparison with $G(H_2 - H_0)$ — the main consumption of heat for evaporation of the moisture and heating of the air and vapour — we get an equation for a theoretical dryer:

$$Q_{th} = G(H_2 - H_0) \quad (10-17)$$

10. The specific consumption of heat q (in J/kg of moisture being evaporated) in an actual dryer is:

$$q = \frac{Q}{W} = \frac{H_1 - H_0}{x_2 - x_0} = g(H_1 - H_0) \quad (10-18)$$

and in a theoretical dryer with the same final state of the air it is:

$$q_{th} = \frac{H_2 - H_0}{x_2 - x_0} \quad (10-19)$$

The difference between the specific consumptions of heat in an actual and a theoretical dryer is:

$$q - q_{th} = \frac{H_1 - H_2}{x_2 - x_0} = \Delta \quad (10-20)$$

In the absence of additional heating in the drying chamber, it equals:

$$\Delta = \frac{\sum Q}{W} = q_{\text{mat}} + q_{\text{con}} + q_1 - c\theta_{\text{in}} \quad (10-21)$$

where

$$q_{\text{mat}} = \frac{G_{\text{fin}}}{W} c_{\text{fin}} (\theta_{\text{fin}} - \theta_{\text{in}})^* \quad (10-22)$$

$$q_{\text{con}} = \frac{G_{\text{con}}}{W} c_{\text{con}} (\theta_{\text{fin}} - \theta_{\text{in}}) \quad (10-23)$$

$$q_1 = \frac{Q_1}{W} \quad (10-24)$$

$G_{\text{fin}}, G_{\text{con}}$ = final mass of the dried material and mass of the conveying means, respectively, kg

$c_{\text{fin}}, c_{\text{con}}, c$ = specific heat capacities of the dried material, the conveying means, and the water, J/kg·K

$\theta_{\text{in}}, \theta_{\text{fin}}$ = initial (of the moist material fed into the dryer) and final (of the dried material discharged from the dryer) temperatures, °C.

11. The thermal efficiency of a dryer is:

$$\eta = \frac{L}{q} \quad (10-25)$$

where L = specific heat of vaporization of the water, determined according to the temperature of the material in drying (the wet bulb temperature), J/kg.

q = specific consumption of heat in the dryer, J/kg.

12. When a psychrometer is used to measure the relative humidity of the moving air, the partial pressure of the water vapour in it can be calculated according to the psychrometric formula:

$$p_v = p'_s - C(t - t_w)p_{\text{tot}} \quad (10-26)$$

where p'_s = saturated water vapour pressure at the wet bulb temperature

$t - t_w$ = difference between the dry bulb and wet bulb temperatures

p_{tot} = barometric pressure

C = coefficient depending on a number of factors, the main one of which is the velocity of the air.

When $v > 0.5$ m/s, we have:

$$C = 0.000\,01 \left(65 + \frac{6.75}{v} \right) \quad (10-27)$$

* $Q_{\text{mat}} = G_{\text{fin}}c_{\text{fin}}\theta_{\text{fin}} - G_{\text{in}}c_{\text{in}}\theta_{\text{in}} = G_{\text{fin}}c_{\text{fin}}(\theta_{\text{fin}} - \theta_{\text{in}}) - Wc\theta_{\text{in}}$.

For the approximate determination of the relative humidity of air according to the readings of a psychrometer using a Ramzin H - x diagram, see Example 10-12, Fig. 10-6.

13. The rate of evaporation of water from the moist surface of a material (during the first period of drying) G (in $\text{kg}/\text{m}^2 \cdot \text{h}$) can be calculated by the equation:

$$G = 0.04075 v^{0.8} \Delta p \quad (10-28)$$

where v = velocity of the air over the material, m/s

$\Delta p = (p_s - p_v)$ = difference between the pressure of the saturated vapour in the boundary layer of the air at the surface of the moist material and the partial pressure of the vapour in the passing air, mm Hg .

We take the value of p_s from tables of vapour properties for the wet bulb temperature; we find the value of p_v by Eq. (10-26) or from the Ramzin diagram (Fig. 10-1).

14. The time of drying in constant conditions (with respect to the air) can be determined according to the following approximate equations:

(a) for the period of constant rate:

$$\tau_1 = \frac{1}{R} (u'_{in} - u'_{cr}) \quad (10-29)$$

(b) for the period of falling rate:

$$\tau_2 = \frac{u'_{cr} - u'_{eq}}{R} 2.3 \log \frac{u'_{cr} - u'_{eq}}{u'_{fin} - u'_{eq}} \quad (10-30)$$

Here R = rate of drying during the first period expressed by the number of kilograms of moisture (per kg of dry substance) evaporated in one second

u'_{in} , u'_{cr} , u'_{fin} , u'_{eq} = initial, critical, final, and equilibrium moisture content of the material (dry basis).

The total time of drying is:

$$\tau = \tau_1 + \tau_2 \quad (10-31)$$

Owing to the non-uniform flow of air around the material, the presence of "dead" zones and to other reasons, a correction factor equal to from 1.5 to 2 has to be introduced into the theoretical equations (10-29) and (10-30).

An approximate equation for determining the time of drying (without account taken of the equilibrium moisture content) has the form:

$$\tau = \frac{u'_{in} - u'_{cr}}{R} + \frac{u'_{cr}}{R} 2.3 \log \frac{u'_{cr}}{u'_{fin}} \quad (10-32)$$

15. The driving force of a drying process (during the first period) can be expressed as follows:

(a) As the difference between the temperatures of the air t and the surface of the moist material taken equal to the wet bulb temperature t_w :

$$\kappa = t - t_w \quad (10-33)$$

The quantity κ is also called the drying potential.

(b) As the difference between the moisture contents of saturated air x_s (in the surface layer) and unsaturated air x (in the nucleus of the air stream):

$$\Delta x = x_s - x \quad (10-34)$$

The mean driving force is determined by the equations:

$$\kappa_m = \frac{\kappa_1 - \kappa_2}{2.3 \log \frac{\kappa_1}{\kappa_2}} \quad (10-35)$$

and

$$\Delta x_m = \frac{\Delta x_1 - \Delta x_2}{2.3 \log \frac{\Delta x_1}{\Delta x_2}} \quad (10-36)$$

where $\kappa_1 = t_1 - t_w$ and $\kappa_2 = t_2 - t_w$

$\Delta x_1 = x_s - x_1$ and $\Delta x_2 = x_s - x_2$.

For the designations of the temperatures and moisture contents see Example 10-14 and Fig. 10-8.

16. The rate of drying R during the first period can be determined either experimentally or through the coefficient of mass transfer.

Since the amount of evaporated moisture (in kg/s) is:

$$W = \beta A \Delta x_m \quad (10-37)$$

then

$$R = \frac{W}{G_{dry}} = \frac{\beta A \Delta x_m}{G_{dry}} = \beta a \Delta x_m \quad (10-38)$$

Here β = coefficient of mass transfer in the gaseous phase,
kg/m²·s (kg/kg)

A = area of evaporation surface, m²

Δx_m = mean driving force, kg of vapour/kg of dry air

$a = A/G_{dry}$ = specific surface area (per kg of dry substance),
m²/kg.

17. The coefficient of mass transfer β can be determined by the dimensionless equation:

$$Nu'_g = C Re_g^n (Pr'_g)^{0.33} Gu^{0.135}$$

(10-39)

where $Nu'_g = \frac{\beta l}{D}$; $Re_g = \frac{v l}{\nu}$; and $Pr'_g = \frac{\nu}{D}$.

The defining dimension in calculating the dimensionless numbers Nu'_g and Re_g is the length of the evaporation surface l in the direction of motion of the drying agent.

The parametric Gukhman dimensionless number Gu is determined by the equation:

$$Gu = \frac{T_d - T_w}{T_d}$$

(10-40)

where T_d and T_w are the dry bulb and wet bulb temperatures, respectively, K.

The quantities C and n in Eq. (10-39) depend on the number Re_g (Table 10-1).

TABLE 10-1

Re_g	C	n
1-200	0.9	0.5
200-6 000	0.87	0.54
6 000-70 000	0.347	0.65

It must be noted that for an intensive drying process the value of the coefficient of mass transfer may be higher than that computed by Eq. (10-39).

18. The time of drying and the dimensions of a countercurrent dryer in variable conditions (with respect to the air and the material) can be determined with the aid of Eqs. (10-41) and (10-43). For the first period of drying in the dryer, it is necessary to ensure a surface area of the material (in m^2) equal to:

$$A_1 = \frac{G}{\beta} \ln \frac{x_s - x_1}{x_s - x_2}$$

(10-41)

For the second period of drying, it is necessary to develop a surface area of the material (in m^2) equal to:

$$A_2 = \frac{Gu_{cr}^*}{\beta} \int_{x_0}^{x_1} \frac{dx}{(x_s - x) \left[u^* - \frac{G}{V_m} (x_1 - x_0) \right]}$$

(10-42)

The right-hand side of Eq. (10-42) can be integrated because only x is a variable.

Integration yields the following expression:

$$A_2 = \frac{u_{cr}^*}{\beta} \frac{V_m}{x_s + \frac{u_{cr}^* V_m}{G} - x_1} \ln \frac{(x_s - x_0) \frac{u_{cr}^* V_m}{G}}{(x_s - x_1) \left(x_0 + \frac{u_{cr}^* V_m}{G} - x_1 \right)} \quad (10-43)$$

The total surface area of the material that must be provided in the dryer is:

$$A = A_1 + A_2$$

The following notation is used in Eqs. (10-41) to (10-43):

G = rate of flow of dry air passing through the dryer, kg/h

V_m = rate of flow of the material passing through the dryer (dry basis), m³/h

β = coefficient of mass transfer determined experimentally, kg/m²·h (kg/kg)

u^* = content of surplus (free) moisture in the material ($u^* = u' - u'_{eq}$) at the moment of time τ after the beginning of the period of the falling rate, kg of moisture per m³ of dry material

u'_{cr} = content of surplus (free) moisture in the material at the critical point, kg of moisture per m³ of dry material

x_0 = moisture content of the air entering the second zone of a countercurrent dryer, kg/kg of air (see Fig. 10-13)

x_1 = moisture content of the air leaving the second zone of the dryer and entering the first zone, kg/kg of air

x_2 = moisture content of the air leaving the dryer, kg/kg of air

x_s = moisture content of the saturated air, kg/kg of air.

EXAMPLES

Example 10-1. Use the H - x Ramzin diagram in Fig. 10-1 to determine the enthalpy and moisture content of air at 60 °C and $\varphi = 0.3$.

Solution. As shown in Fig. 10-2, we find $H = 163$ kJ/kg of dry air and $x = 0.04$ kg/kg of dry air.

Example 10-2. Find the partial pressure of water vapour in a vapour and air mixture at 80 °C and $H = 150$ kJ/kg of dry air.

Solution. According to the Ramzin diagram (Fig. 10-1), we find the point of intersection of the isotherm for 80 °C with the line $H = 150$ kJ/kg of dry air. We project this point onto the line of the partial pressure of the water vapour at the bottom of the diagram. We further project the point obtained to the right onto the axis of ordinates along which the partial pressures of the water vapour are laid off (in mm Hg).

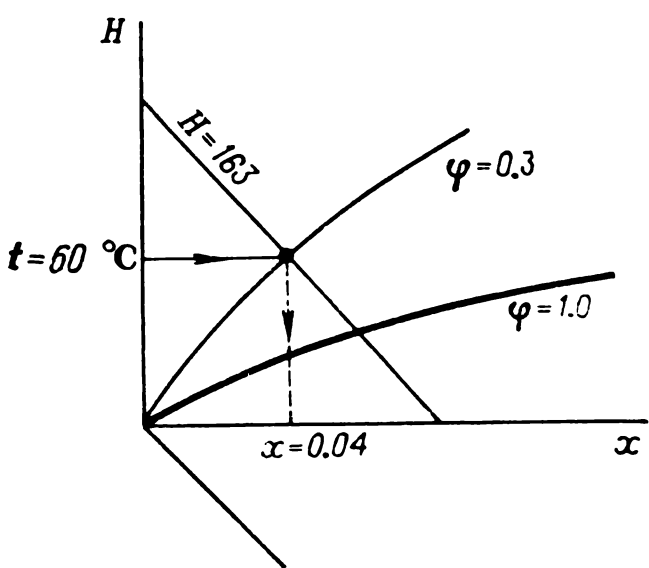


Fig. 10-2. To Example 10-1

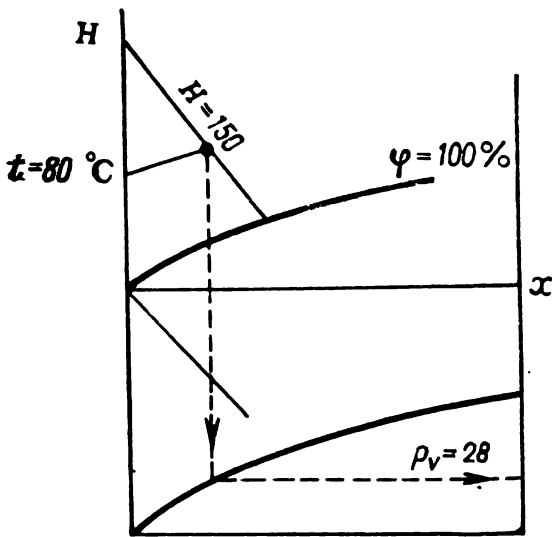


Fig. 10-3. To Example 10-2

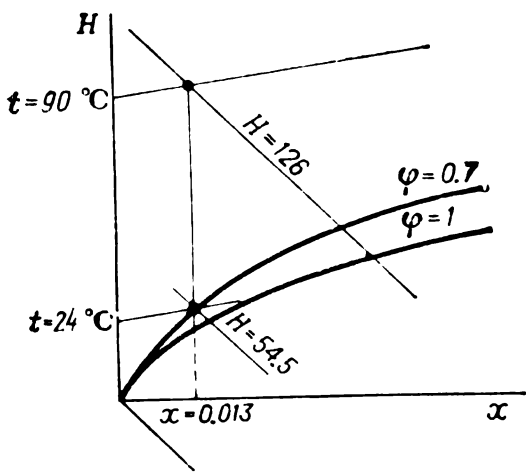


Fig. 10-4. To Example 10-3

For $t = 80\text{ }^{\circ}\text{C}$ and $H = 150\text{ kJ/kg}$ of dry air, we find $p_v = 28\text{ mm Hg}$. For a graphical scheme of the solution, see Fig. 10-3.

Example 10-3. Air having a temperature of $24\text{ }^{\circ}\text{C}$ and a relative humidity of $\varphi = 0.7$ is heated in an air heater to $90\text{ }^{\circ}\text{C}$. Find the enthalpy and moisture content of the air leaving the heater.

Solution (Fig. 10-4). The initial state of the air is determined in an H - x diagram by the point of intersection of the isotherm

for $t = 24\text{ }^{\circ}\text{C}$ with the line for $\phi = 0.7$. For this point, we get $x = 0.013\text{ kg/kg}$ of dry air, and $H = 54.5\text{ kJ/kg}$ of dry air. The change in the state of the air when it is heated in the air heater occurs at a constant moisture content, i. e. when $x = \text{const.}$ Hence, the final state of the air is determined by the point of intersection of the line $x = 0.013$ with the isotherm for $t = 90\text{ }^{\circ}\text{C}$.

The enthalpy $H = 126\text{ kJ/kg}$ of dry air corresponds to this point.

Example 10-4. Determine analytically the moisture content and enthalpy of air at $30\text{ }^{\circ}\text{C}$ and $\phi = 0.75$. The barometric pressure is $p_{\text{tot}} = 0.098\text{ MPa}$ (1 at).

Solution. We use Eq. (10-5) to determine the moisture content of the air analytically.

In our case at $t = 30\text{ }^{\circ}\text{C}$, the saturated vapour pressure is $p_s = 0.0433$ at (Table A-56). Inserting values, we get:

$$x = 0.622 \frac{\phi p_s}{p_{\text{tot}} - \phi p_s} = 0.622 \times \frac{0.75 \times 0.0433}{1 - 0.75 \times 0.0433} = 0.021\text{ kg/kg of dry air}$$

We calculate the enthalpy of air by Eq. (10-7). Introducing the relevant values, we have:

$$H = (1.01 + 1.97x)t + 2493x = (1.01 + 1.97 \times 0.021)30 + 2493 \times 0.021 = 83\text{ kJ/kg of dry air}$$

Example 10-5. According to the data of the preceding example, find the specific volume of humid air per kg of dry air, i. e. per $(1 + x)$ kg of the air and vapour mixture.

Solution. We find the specific volume of the humid air by Eq. (10-12):

$$v = \frac{R_a T}{p_{\text{tot}} - \phi p_s} = \frac{287 \times 303}{9.81 \times 10^4 - 0.75 \times 0.0433 \times 9.81 \times 10^4} = 0.92\text{ m}^3/\text{kg of dry air}$$

Example 10-6. Find the partial pressure of water vapour and the density and moisture content of air if its temperature is $60\text{ }^{\circ}\text{C}$, its absolute pressure is $p_{\text{tot}} = 380\text{ mm Hg}$, and the relative humidity $\phi = 0.4$.

Solution. In Table A-38, we find $p_s = 149.4\text{ mm Hg}$. According to Eq. (10-6), the partial pressure of the water vapour is:

$$p_v = p_s \phi = 149.4 \times 0.4 = 59.8\text{ mm Hg}$$

The density of the humid air, by Eq. (10-11), is:

$$\begin{aligned} \rho_{h.a} &= 1.293 \frac{T_0 p_{\text{tot}}}{T p_{\text{tot}}^{\circ}} \left(1 - \frac{0.378 \phi p_s}{p_{\text{tot}}} \right) = \\ &= 1.293 \times \frac{273 \times 380}{333 \times 760} \left(1 - \frac{0.378 \times 59.8}{380} \right) = 0.5\text{ kg/m}^3 \end{aligned}$$

The moisture content of the air, by Eq. (10-5), is:

$$x = 0.622 \frac{p_v}{p_{\text{tot}} - p_v} = 0.622 \times \frac{59.8}{380 - 59.8} = 0.116 \text{ kg/kg of dry air}$$

Example 10-7. Determine the relative humidity of air at $t = 150^\circ\text{C}$ and $p_{\text{tot}} = 760 \text{ mm Hg}$ if its moisture content is $x = 0.07 \text{ kg/kg}$ of dry air.

Solution. Saturated water vapour under atmospheric pressure cannot have a temperature above 100°C . Consequently, at a temperature of an air and vapour mixture above 100°C , we have $p_s = p_{\text{tot}}$. Hence, by Eq. (10-5):

$$x = 0.622 \frac{\varphi p_{\text{tot}}}{p_{\text{tot}} - \varphi p_{\text{tot}}} = 0.622 \frac{\varphi}{1 - \varphi}$$

i.e. at a given moisture content x , the relative humidity φ is a constant quantity independent of the temperature.

In our case:

$$0.07 = 0.622 \frac{\varphi}{1 - \varphi}$$

whence $\varphi \approx 0.1 = 10\%$.

Example 10-8. Find the moisture content of humid air at $t = 60^\circ\text{C}$ and $\varphi = 50\%$ if the barometric pressure $p_{\text{tot}} = 765 \text{ mm Hg}$ and determine the error in calculating this moisture content with the aid of a Ramzin diagram constructed for $p_{\text{tot}} = 745 \text{ mm Hg}$.

Solution. In Table A-38 for $t = 60^\circ\text{C}$, we find $p_s = 149.4 \text{ mm Hg}$. Hence,

$$x = 0.622 \frac{\varphi p_s}{p_{\text{tot}} - \varphi p_s} = 0.622 \times \frac{0.5 \times 149.4}{765 - 0.5 \times 149.4} = 0.0672 \frac{\text{kg of vapour}}{\text{kg of dry air}}$$

According to a Ramzin diagram (Fig. 10-1) for $t = 60^\circ\text{C}$ and $\varphi = 50\%$, we find $x = 0.0695 \text{ kg/kg}$.

The error is:

$$\frac{0.0695 - 0.0672}{0.0672} \times 100 = 3.4\%$$

Example 10-9. Determine the rate of flow of dry air and the demand for heat in a theoretical dryer for removing 100 kg/h of moisture from a wet material if the initial state of the air (before the air heater) is $t_0 = 15^\circ\text{C}$ and $\varphi_0 = 0.8$, and the final state (after the dryer) $t_2 = 44^\circ\text{C}$ and $\varphi_2 = 0.5$.

Solution. According to an H - x diagram, we find $x_0 = 0.009 \text{ kg/kg}$, and $x_2 = 0.03 \text{ kg/kg}$. By Eq. (10-14), we determine the specific rate of flow of the dry air:

$$g = \frac{1}{x_2 - x_0} = \frac{1}{0.03 - 0.009} = 47.6 \frac{\text{kg}}{\text{kg of evaporated moisture}}$$

We find the specific consumption of heat by Eq. (10-19). The values of the enthalpies are determined from an H - x diagram: $H_2 = 121.5$ kJ/kg and $H_0 = 40$ kJ/kg, and the specific consumption of heat is:

$$q = \frac{H_2 - H_0}{x_2 - x_0} = \frac{121.5 - 40}{0.03 - 0.009} = 3880 \frac{\text{kJ}}{\text{kg of evaporated moisture}}$$

The rate of flow of the dry air, by Eq. (10-13), is:

$$G = Wg = 100 \times 47.6 = 4760 \text{ kg/h}$$

The consumption of heat is:

$$Q = Wq = \frac{100 \times 3880}{3600} = 108 \text{ kW}$$

Example 10-10. An air heater is supplied with a mixture of fresh air ($t_0 = 25^\circ\text{C}$, $\varphi_0 = 0.5$) and spent air ($t_2 = 50^\circ\text{C}$, $\varphi_2 = 0.8$) in a mass ratio of 1 : 3, dry basis. Find the parameters of the mixture entering the air heater and after being heated in it to 80°C .

Solution. According to an H - x diagram, we determine the moisture contents and enthalpies of the fresh air: for $t_0 = 25^\circ\text{C}$ and $\varphi_0 = 0.5$, we get $x_0 = 0.01$ and $H_0 = 50$ kJ/kg, and of the spent air: for $t_2 = 50^\circ\text{C}$ and $\varphi_2 = 0.8$, we get $x_2 = 0.069$ and $H_2 = 228$ kJ/kg. Hence, the moisture content of the mixture is $x_{\text{mix}} = 0.25 \times 0.01 + 0.75 \times 0.069 = 0.0542$ kg/kg, and the enthalpy is $H_{\text{mix}} = 0.25 \times 50 + 0.75 \times 228 = 183.5$ kJ/kg. Next, we find in the diagram a point with the coordinates $x = 0.0542$ and $H = 183.5$. This point gives us $t_{\text{mix}} = 45^\circ\text{C}$ and $\varphi_{\text{mix}} = 0.85$.

Heating of the mixture in the air heater occurs at $x = \text{const}$. Therefore, the point of intersection of the line $x = 0.0542$ kg/kg with the isotherm for 80°C gives us the state of the mixture leaving the air heater. This point corresponds to $H_{\text{mix}} = 147$ kJ/kg and $\varphi_{\text{mix}} = 0.018$.

Example 10-11. Find the dew point for air having a temperature of $t = 40^\circ\text{C}$ and a relative humidity of $\varphi = 0.8$.

Solution. The dew point corresponds to the temperature at which a vapour and air mixture having a given moisture content becomes saturated with water vapour. When humid air is cooled to below this temperature, the water vapour condenses. To determine the dew point, we must find a point in an H - x diagram that corresponds to the given state of the air, and then drop along the line $x = \text{const}$ to its intersection with the curve $\varphi = 1$, i.e. to the saturation line. In our case, $x = 0.039$ kg/kg, and the dew point corresponds to a temperature of $t = 36^\circ\text{C}$ (see the scheme of the solution in Fig. 10-5).

Example 10-12. The dry bulb thermometer of a psychrometer shows $t = 40^\circ\text{C}$, and the wet bulb thermometer shows $t_w = 35^\circ\text{C}$.

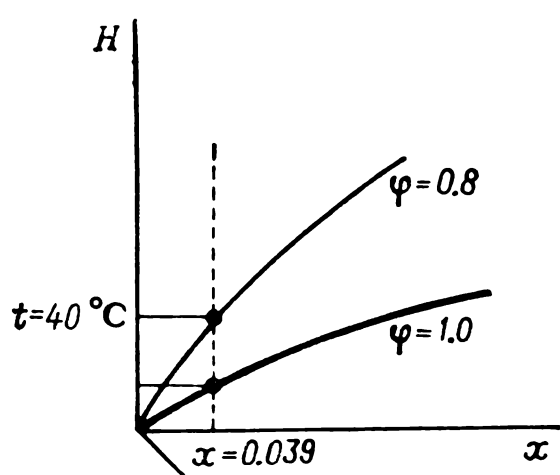


Fig. 10-5. To Example 10-11

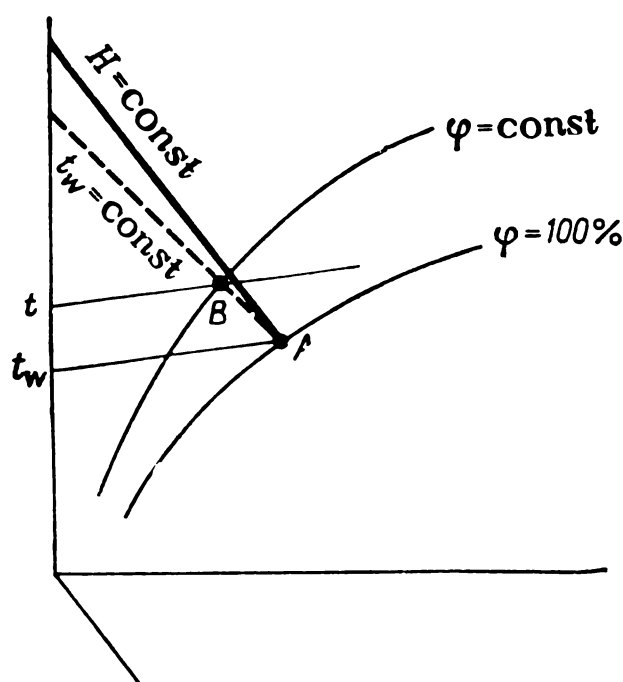


Fig. 10-6. To Example 10-12

Determine approximately the relative humidity of the air according to a Ramzin H - x diagram.

Solution. We find in the diagram the point of intersection of the isotherm for t_w with the line $\varphi = 100\%$ (point A in Fig. 10-6). From this point, we follow the isotherm of the wet bulb thermometer ($t_w = \text{const}$)* up to its intersection with the isotherm for t . At the point of intersection B we find the required value of φ .

If the Ramzin diagram being used has no lines $t_w = \text{const}$ for purposes of simplification, as in Fig. 10-1a, then the required relative humidity of the air can be approximately found by travelling from point A along a line $H = \text{const}$ to its intersection with the isotherm for t . The lines $t_w = \text{const}$ and $H = \text{const}$ are close to each other, and with low values of the difference $t - t_w$ the error in such an approximate determination will not be great.

* In Fig. 10-1b, the lines $t_w = \text{const}$ are the dash lines.

Fig. 10-7. To Example 10-13

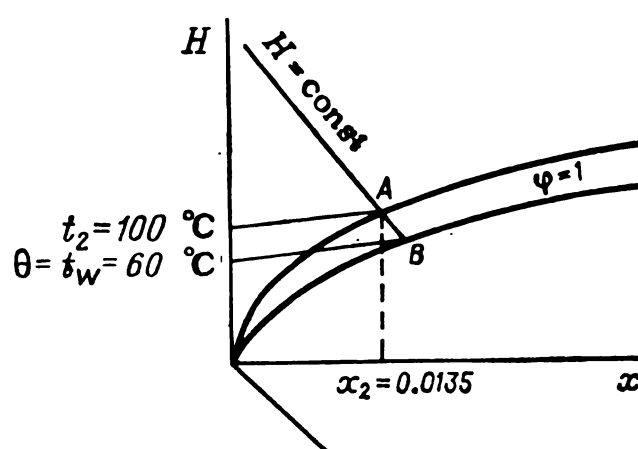
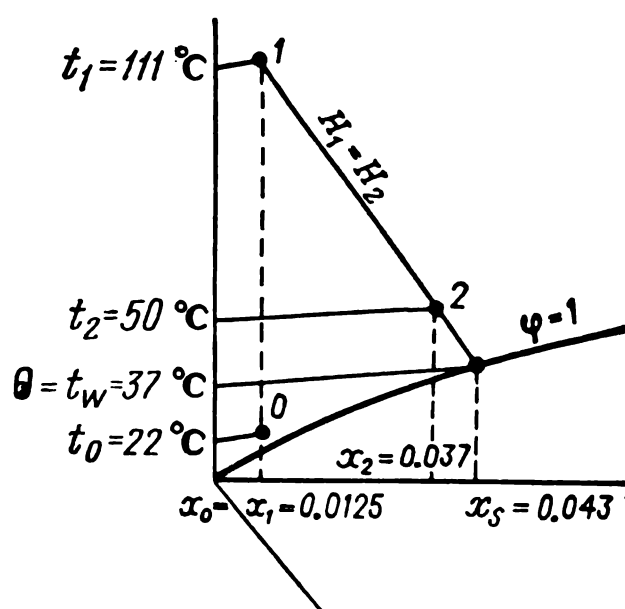


Fig. 10-8. To Example 10-14



In our case for $t = 40^\circ\text{C}$ and $t_w = 35^\circ\text{C}$, we find that $\phi = 70\%$ when travelling along the line $H = \text{const}$.

Example 10-13. Find the temperature of the material discharged from a dryer if its moisture content is above the critical value, and the air leaving the dryer has a temperature of $t_2 = 100^\circ\text{C}$ and a moisture content of $x_2 = 0.0135$ kg/kg.

Solution. During the first period of drying, the temperature of the wet material equals the temperature of the wet bulb thermometer t_w . We find it (Fig. 10-7) by travelling from point A along the line $H = \text{const}$ up to its intersection with the line $\phi = 1$ at point B through which the isotherm for $t_w = 60^\circ\text{C}$ passes.

Example 10-14. Find the driving force of a drying process Δx_m and κ_m for a theoretical dryer in the following conditions: $t_0 = 22^\circ\text{C}$, $t_2 = 50^\circ\text{C}$, $\phi_0 = 0.75$, and $\phi_2 = 0.45$.

Solution. In the H - x diagram (Fig. 10-8), we find $x_1 = 0.0125$ kg/kg, $x_2 = 0.037$ kg/kg, $x_s = 0.043$ kg/kg, and $\theta \approx 37^\circ\text{C}$. Hence,

$$\Delta x_m = \frac{\Delta x_1 - \Delta x_2}{2.3 \log \frac{\Delta x_1}{\Delta x_2}} = \frac{(0.043 - 0.0125) - (0.043 - 0.037)}{2.3 \log \frac{0.043 - 0.0125}{0.043 - 0.037}} = 0.0152 \text{ kg/kg}$$

$$\kappa_m = \frac{\kappa_1 - \kappa_2}{2.3 \log \frac{\kappa_1}{\kappa_2}} = \frac{(111 - 37) - (50 - 37)}{2.3 \log \frac{111 - 37}{50 - 37}} = 35 \text{ K}$$

Example 10-15. Use the readings of a psychrometer to determine the relative humidity of air leaving a dryer if the dry bulb temperature is $t=85^{\circ}\text{C}$, the wet bulb temperature is $t_w=68^{\circ}\text{C}$, the barometric pressure $p_{\text{tot}}=750\text{ mm Hg}$, and the velocity of the air is $v=1\text{ m/s}$.

Solution. The relative humidity of the air is $\varphi=p_v/p_s$. We find from Table A-56 that at $t=85^{\circ}\text{C}$ the saturated vapour pressure is $p_s=0.59\text{ at}$. Next we determine the partial pressure of water vapour p_v by Eq. (10-26);

$$p_v=p'_s-C\left(t-t_w\right)p_{\text{tot}}$$

In Table A-56, we find the saturated vapour pressure at the wet bulb temperature t_w : we get $p'_s=0.2912\text{ at}$.

We determine the coefficient C by Eq. (10-27):

$$C=0.000\,01\left(65+\frac{6.75}{1}\right)=0.000\,72$$

Next, we find the relative humidity:

$$\varphi=\frac{0.2912-0.000\,72\left(85-68\right)\frac{750}{735}}{0.59}=0.47$$

Example 10-16. Determine the rate of flow of the air and also the rate of flow and the required pressure of the heating steam for a continuous countercurrent dryer operating according to the normal drying variant.

Capacity of dryer, wet basis	$G_{\text{in}}=350\text{ kg/h}$
Initial moisture content of material	$u_{\text{in}}=42\%$
Final moisture content of material	$u_{\text{fn}}=11\%$
Temperature of the material fed in for drying . . .	$\theta_1=18^{\circ}\text{C}$
Temperature of the material discharged from the dryer	$\theta_2=47^{\circ}\text{C}$
Characteristics of the state of the air:	
entering the air heater	$t_0=15^{\circ}\text{C}, \varphi_0=70\%$
leaving the dryer	$t_2=45^{\circ}\text{C}, \varphi_2=60\%$
Specific heat capacity of the dried ($u_{\text{fn}}=11\%$) material	$c_{\text{fn}}=2.35\times10^3\text{ J/kg}\cdot\text{K}$
Mass of the conveying means (a steel conveyor) . .	$G_{\text{con}}=600\text{ kg}$
Heat losses of the dryer and air heater to the surroundings	$Q_1=12\%$ of sum of all the other addends of the heat balance
Moisture content of heating steam	6%

Solution. The amount of moisture evaporated in the dryer is determined by the equation:

$$W=G_{\text{in}}\frac{u_{\text{in}}-u_{\text{fn}}}{100-u_{\text{fn}}}=350\times\frac{42-11}{100-11}=122\text{ kg/h}$$

We use an H - x diagram to find the moisture content and the enthalpy of the air entering the air heater and of the air leaving the dryer. We get $x_0 = 0.0077$, $x_2 = 0.038$, $H_0 = 35$ kJ/kg, and $H_2 = 145$ kJ/kg.

The rate of flow of the dry air in the dryer for evaporating W kg/h of moisture is:

$$G_{d.a} = \frac{W}{x_2 - x_0} = \frac{122}{0.038 - 0.0077} = 4030 \text{ kg/h}$$

The consumption of heat in a theoretical dryer is:

$$Q_{th} = G_{d.a} (H_2 - H_0) = \frac{4030 (145 \times 10^3 - 35 \times 10^3)}{3600} = 123\,000 \text{ W}$$

In a real dryer, heat is also used to heat the material:

$$G_{fn} c_{fn} (\theta_2 - \theta_1) = \frac{(350 - 122) 2.35 \times 10^3 (47 - 18)}{3600} = 4300 \text{ W}$$

and also for heating the conveying means:

$$G_{con} c_{con} (\theta_2 - \theta_1) = \frac{600 \times 0.5 \times 10^3 (47 - 18)}{3600} = 2420 \text{ W}$$

where 0.5×10^3 is the specific heat capacity of steel, J/kg·K, taken from Table A-25.

From the total amount of heat that must be supplied to the dryer, we have to subtract the amount of heat introduced by the moisture that is in the wet material:

$$W \theta_1 c_w = \frac{122 \times 18 \times 4.19 \times 10^3}{3600} = 2560 \text{ W}$$

Hence, the total amount of heat that must be supplied to the air heater with account taken of the losses to the surroundings is:

$$Q = (123\,000 + 4300 + 2420 - 2560) 1.12 = 142\,500 \text{ W}$$

Comparing the consumptions of heat in a theoretical and a real dryer, we can see that the consumption of heat in the latter is higher by 15%.

Since $Q = G_{d.a} (H_1 - H_0) = 142\,500$ W, then

$$H_1 - H_0 = \frac{Q}{G_{d.a}} = \frac{142\,500 \times 3600}{4030} = 127.5 \times 10^3 \frac{\text{J}}{\text{kg of dry air}}$$

Consequently,

$$H_1 = 127.5 + H_0 = 127.5 + 35 = 162.5 \frac{\text{kJ}}{\text{kg of dry air}}$$

A temperature of the air leaving the air heater of $t_1 \approx 138^\circ\text{C}$ corresponds to this value of H_1 (according to an H - x diagram).

We assume that the difference between the temperature of the heating steam and that of the air leaving the air heater is

$\Delta t = t_{st} - t_1 = 10$ K. Hence, $t_{st} = 138 + 10 = 148^\circ\text{C}$. From Table A-56, we find that the pressure of the heating steam corresponding to this temperature is $p_{abs} \approx 0.461$ MPa = 4.7 at.

The rate of flow of the heating steam is:

$$G_{st} = \frac{Q}{Lx'} = \frac{142\,500}{2122 \times 10^3 \times 0.94} = 0.0715 \text{ kg/s} = 257 \text{ kg/h}$$

where $L = 2122$ kJ/kg is the specific heat of condensation of the heating steam at 148°C (Table A-56)

x' = steam content (dryness fraction) of the heating steam.

The specific consumption of the heating steam is:

$$g_{st} = \frac{G_{st}}{W} = \frac{257}{122} = 2.1 \frac{\text{kg of heating steam}}{\text{kg of moisture being evaporated}}$$

Example 10-17. Determine the mean overall coefficient of heat transfer (relating it to the difference between the temperature of the heating steam and that of the material being dried) in a pilot element of a steam tubular dryer for peat according to the following data:

Capacity of dryer element (bone-dry peat basis) . . .	$G_{dry} = 11.2$ kg/h
Initial moisture content of peat (dry peat basis) . . .	$u'_{in} = 0.57$ kg/kg
Final moisture content of peat	$u'_{fn} = 0.148$ kg/kg
Temperature of peat charged into dryer	$\theta_1 = 20^\circ\text{C}$
Temperature of peat discharged from dryer	$\theta_2 = 52^\circ\text{C}$
Specific heat capacity of bone-dry peat	$c = 1.26 \times 10^3$ J/kg·K
Characteristics of state of air:	
entering the dryer	$t_0 = 22^\circ\text{C}, \varphi_0 = 0.34$
leaving the dryer	$t_2 = 82^\circ\text{C}, \varphi_2 = 0.37$
Barometric pressure	$p_{tot} = 773$ mm Hg
Temperature of heating steam	$t = 100^\circ\text{C}$
Heating surface area of tube	$A = 2.18$ m ²

Solution. We determine the mean overall coefficient of heat transfer from Eq. (4-66):

$$K = \frac{Q}{A\Delta t_m}$$

where Q = amount of heat transferred through the heating surface
 Δt_m = mean temperature drop.

The amount of heat transferred through the heating surface is:

$$Q = Q_1 + Q_2 + Q_3$$

where Q_1 = heat spent for evaporation of the moisture and for heating the air

Q_2 = heat spent for heating the peat

Q_3 = heat losses to the surroundings.

The rate of flow of the evaporated moisture, by Eq. (10-3), is:

$$W = G_{dry} (u'_{in} - u'_{fn}) = \frac{11.2 (0.57 - 0.148)}{3600} = 0.0013 \text{ kg/s}$$

We find the initial and final moisture content of the air by Eq. (10-5):

$$x_0 = 0.622 \times \frac{0.34 \times 0.0270}{1.017 - 0.34 \times 0.0270} = 0.0057 \text{ kg/kg}$$

$$x_2 = 0.622 \times \frac{0.37 \times 0.5233}{1.017 - 0.37 \times 0.5233} = 0.146 \text{ kg/kg}$$

The enthalpy of the air entering the dryer, by Eq. (10-7), is:

$$H_0 = (1.01 \times 10^3 + 1.97 \times 10^3 \times 0.0057) 22 + 2493 \times 10^3 \times 0.0057 = 36.5 \times 10^3 \text{ J/kg}$$

and that of the air leaving the dryer is:

$$H_2 = (1.01 \times 10^3 + 1.97 \times 10^3 \times 0.146) 82 + 2493 \times 10^3 \times 0.146 = 470 \times 10^3 \text{ J/kg}$$

Hence,

$$Q_1 = W \frac{H_2 - H_0}{x_2 - x_0} = 0.0013 \times \frac{470 \times 10^3 - 36.5 \times 10^3}{0.146 - 0.0057} = 4050 \text{ W}$$

$$Q_2 = G_{\text{dry}} (H'_2 - H'_0)$$

where H'_2 and H'_0 are the enthalpies of the peat discharged from and fed into the dryer (per kg of dry peat) equal to:

$$H'_2 = (1.26 \times 10^3 \times 1 + 4.19 \times 10^3 \times 0.148) 52 = 97.8 \times 10^3 \text{ J/kg}$$

$$H'_0 = (1.26 \times 10^3 \times 1 + 4.19 \times 10^3 \times 0.57) 20 = 73 \times 10^3 \text{ J/kg}$$

Consequently,

$$Q_2 = \frac{11.2}{3600} (97.8 \times 10^3 - 73 \times 10^3) = 77 \text{ W}$$

We take the losses of heat to the surroundings Q_3 equal to 10 per cent of Q_1 . Hence, the total amount of heat is:

$$Q = 4050 + 77 + 405 = 4532 \text{ W}$$

The mean temperature drop in the dryer is

$$\Delta t_m = \frac{(100 - 20) + (100 - 52)}{2} = 64 \text{ K}$$

The overall coefficient of heat transfer is:

$$K = \frac{Q}{A \Delta t_m} = \frac{4532}{2.18 \times 64} = 32.4 \text{ W/m}^2 \cdot \text{K}$$

Example 10-18. Determine the efficiency of a theoretical air dryer for the following conditions: the drying process is conducted at $H = 115 \text{ kJ/kg}$, the state of the air changes from $\varphi_0 = 0.8$ and $t_0 = 20^\circ\text{C}$ to $\varphi_2 = 0.6$ and $t_2 = 40^\circ\text{C}$.

Solution. We use the Ramzin H - x diagram (Fig. 10-1) to determine by Eq. (10-19) the specific consumption of heat in the dryer for evaporating 1 kg of moisture $q=3820$ kJ/kg. According to Table A-56 for $t_w=33^\circ\text{C}$ at $H=115$ kJ/kg, the specific heat of vaporization $L=2420$ kJ/kg.

Consequently, by Eq. (10-25), the efficiency of the dryer is:

$$\eta = \frac{L}{q} = \frac{2420}{3820} \times 100 = 63.3\%$$

Example 10-19. A dye paste was dried in a chamber dryer with recirculation of the air. An analysis of samples for their moisture content gave the results shown in Table 10-2.

TABLE 10-2

Time from beginning of drying, h	0	2	2.5	4	5	6	8	10	12	14	16	18	20
Moisture content of material, % (dry basis)	104.0	84.0	79.1	63.9	53.9	43.9	32.0	21.9	14.0	8.0	5.0	3.0	1.5

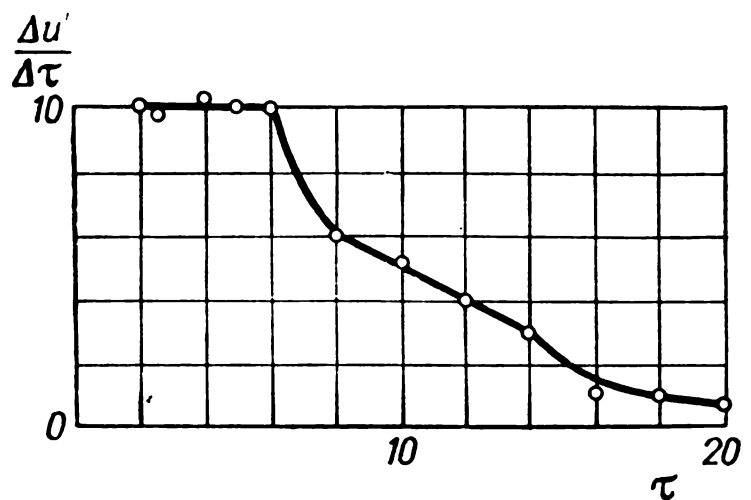
Determine the rate of drying depending on the time. Use the data obtained to construct a curve and find the critical moisture content of the material.

Solution. We compile Table 10-3.

TABLE 10-3

Time from beginning of drying τ , h	Rate of drying $\Delta u'/\Delta \tau$ (dry basis), %/h	Time from beginning of drying τ , h	Rate of drying $\Delta u'/\Delta \tau$ (dry basis), %/h
2	$\frac{104-84}{2}=10$	10	$\frac{32-21.9}{2}=5.05$
2.5	$\frac{84-79.1}{0.5}=9.8$	12	$\frac{21.9-14}{2}=3.95$
4	$\frac{79.1-63.9}{1.5}=10.14$	14	$\frac{14-8}{2}=3.0$
5	$63.9-53.9=10$	16	$\frac{8-5}{2}=1.5$
6	$53.9-43.9=10$	18	$\frac{5-3}{2}=1.0$
8	$\frac{43.9-32}{2}=5.95$	20	$\frac{3.0-1.5}{2}=0.75$

Fig. 10-9. To Example 10-19



According to the data of this table, we construct a curve in the coordinates rate of drying against drying time (Fig. 10-9) and find that the critical moisture content of the material is reached in six hours after the beginning of drying. The moisture content corresponding to this point is 43.9% (bone-dry basis) or $43.9 \times 100 / (100 + 43.9) = 30.6\%$ of moisture per total mass of the product (wet basis).

Example 10-20. A commercial dryer needed seven hours to dry a moist material from a moisture content of 33% to one of 9% (bone-dry basis). The critical moisture content of the material was 16%, and the equilibrium one 5%. Determine the time needed to dry this material from a moisture content of 37% to one of 7% if the drying conditions remain unchanged. Disregard the initial starting period.

Solution. We determine R —the rate of drying in the first (constant-rate) period. Its duration, by Eq. (10-29), is:

$$\tau_1 = \frac{u'_{in} - u'_{cr}}{R} = \frac{0.33 - 0.16}{R} = \frac{0.17}{R}$$

The duration of the second (falling-rate) period, by Eq. (10-30), is:

$$\tau_2 = \frac{u'_{cr} - u'_{eq}}{R} 2.3 \log \frac{u'_{cr} - u'_{eq}}{u'_{fin} - u'_{eq}} = \frac{0.16 - 0.05}{R} 2.3 \log \frac{0.16 - 0.05}{0.09 - 0.05} = \frac{0.111}{R}$$

The total time of drying was seven hours. Hence,

$$\tau_1 + \tau_2 = 7 = \frac{0.17}{R} + \frac{0.111}{R} = \frac{0.281}{R}$$

whence $R = 0.0402 \text{ kg/kg} \cdot \text{s}$.

For the new values of the initial and final moisture contents of the material, we have:

$$\tau_1 = \frac{0.37 - 0.16}{0.0402} = 5.22 \text{ h}$$

$$\tau_2 = \frac{0.16 - 0.05}{0.0402} 2.3 \log \frac{0.16 - 0.05}{0.07 - 0.05} = 4.66 \text{ h}$$

The total drying time is $\tau = 5.22 + 4.66 = 9.9$ h.

Example 10-21. Determine the time needed to dry crystals of salicylic acid (angular particles) in a pneumatic dryer and the required dryer length for the following conditions: the capacity is $G_{\text{fn}} = 250$ kg/h of dry product, the equivalent particle diameter is $d_{\text{eq}} = 1$ mm, the density of the material is $\rho_{\text{mat}} = 1480$ kg/m³.

Characteristics of the state of the air:

entering the air heater	$t_0 = 15^\circ\text{C}, \quad \varphi_0 = 0.7$
leaving the air heater	$t_1 = 90^\circ\text{C}$
leaving the dryer	$t_2 = 50^\circ\text{C}$
Temperature of the crystals fed to the dryer	$\theta_1 = 15^\circ\text{C}$
Temperature of the crystals discharged from the dryer	$\theta_2 = 40^\circ\text{C}$
Specific heat capacity of dry crystals	$c_{\text{fn}} = 1.16 \times 10^3$ J/kg·K
Moisture content of crystals (bone-dry basis):	
initial	$u'_{\text{in}} = 15\%$
final	$u'_{\text{fn}} = 1\%$

Solution. We can determine the time of drying from the heat transfer equation:

$$\tau = \frac{Q}{\alpha A_d \Delta t_m}$$

To determine the rate of flow of the air and the consumption of heat for drying, we plot the drying process in an H - x diagram (Fig. 10-10).

In a theoretical dryer with $H_1 = 111$ kJ/kg, the drying process would follow the constant enthalpy line BC' , and the specific consumption of heat q'_{th} would equal:

$$q'_{\text{th}} = \frac{H_1 - H_0}{x'_2 - x_0} = \frac{111 - 33.5}{0.023 - 0.0075} = 5000 \text{ kJ/kg of evaporated moisture}$$

where $x'_2 = 0.023$ is the moisture content of the air at point C' .

In a real dryer, the final moisture content of the air x_2 (at point C) is less than x'_2 . We find its value as follows.

The equation of the line of a real drying process (line BC) is:

$$H = H_1 - \Delta(x - x_0)$$

We calculate the rate of flow of the moisture being evaporated W , then the correction Δ for the real drying process, and adopt an arbitrary value of x . This gives us all the quantities needed to solve the above equation.

By Eq. (10-3), we have:

$$W = G_{\text{dry}} \frac{u'_{\text{in}} - u'_{\text{fn}}}{100} = 250 \times 0.99 (0.15 - 0.01) = 34.6 \text{ kg/h}$$

By Eq. (10-21) with $q_{\text{con}} = 0$, we get:

$$\Delta = q_{\text{mat}} + q_1 - c\theta_1$$

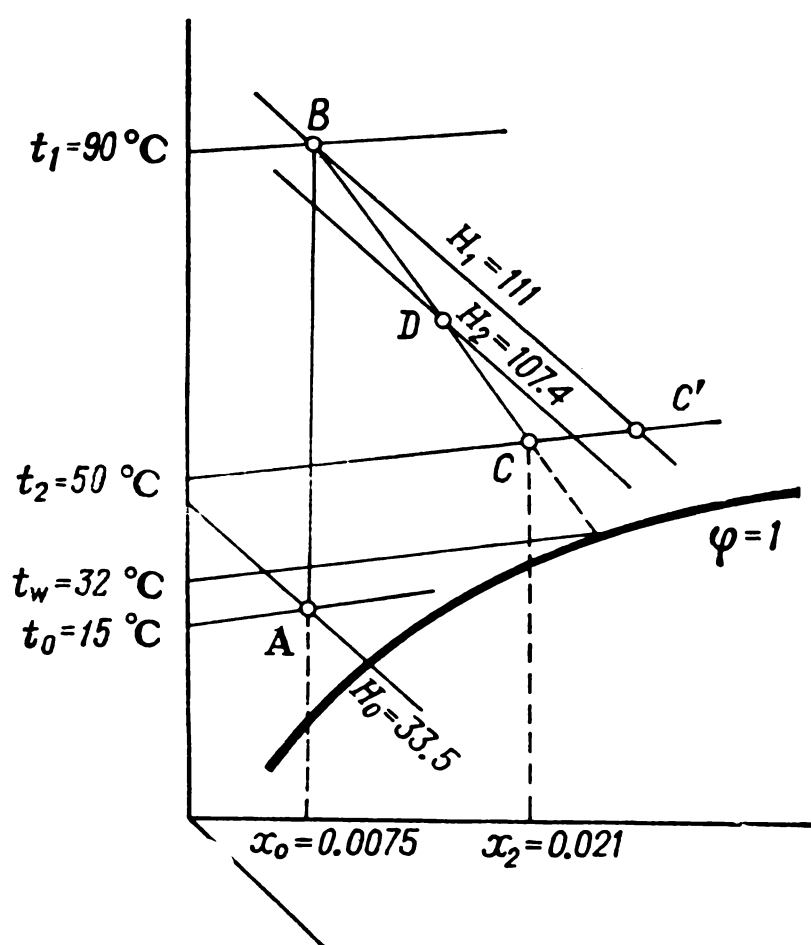


Fig. 10-10. To Example 10-21

By Eq. (10-22), we have:

$$q_{\text{mat}} = \frac{G_{\text{fin}} c_{\text{fin}} (\theta_2 - \theta_1)}{W} = \frac{250 \times 1.16 \times 10^3 (40 - 15)}{34.6} = 209.5 \times 10^3 \text{ J/kg of evaporated moisture}$$

We assume that the specific heat losses q_1 equal 5.5% of q'_{th} , i. e. $q_1 = 0.055 \times 5000 = 275 \text{ kJ/kg of evaporated moisture}$. Hence, Eq. (10-21) yields:

$$\Delta = 209.5 + 275 - 4.19 \times 15 = 421.7 \text{ kJ/kg of evaporated moisture}$$

We adopt $x = 0.016$ and find:

$$H = H_1 - \Delta (x - x_0) = 111 - 421.7 (0.016 - 0.0075) = 107.4 \text{ kJ/kg}$$

By drawing a straight line through points B and D ($x_D = 0.016$ and $H_D = 107.4$) up to its intersection with the isotherm for $t_2 = 50^\circ\text{C}$, we get point C , which $x_2 = 0.021 \text{ kg/kg}$ corresponds to.

The rate of flow of the dry air is:

$$G_{\text{d.a}} = \frac{W}{x_2 - x_0} = \frac{34.6}{0.021 - 0.0075} = 2560 \text{ kg/h}$$

The amount of heat transferred to the air in the air heater is:

$$Q = G_{\text{d.a}} (H_1 - H_0) = \frac{2560 (111 \times 10^3 - 33.5 \times 10^3)}{3600} = 55\,200 \text{ W}$$

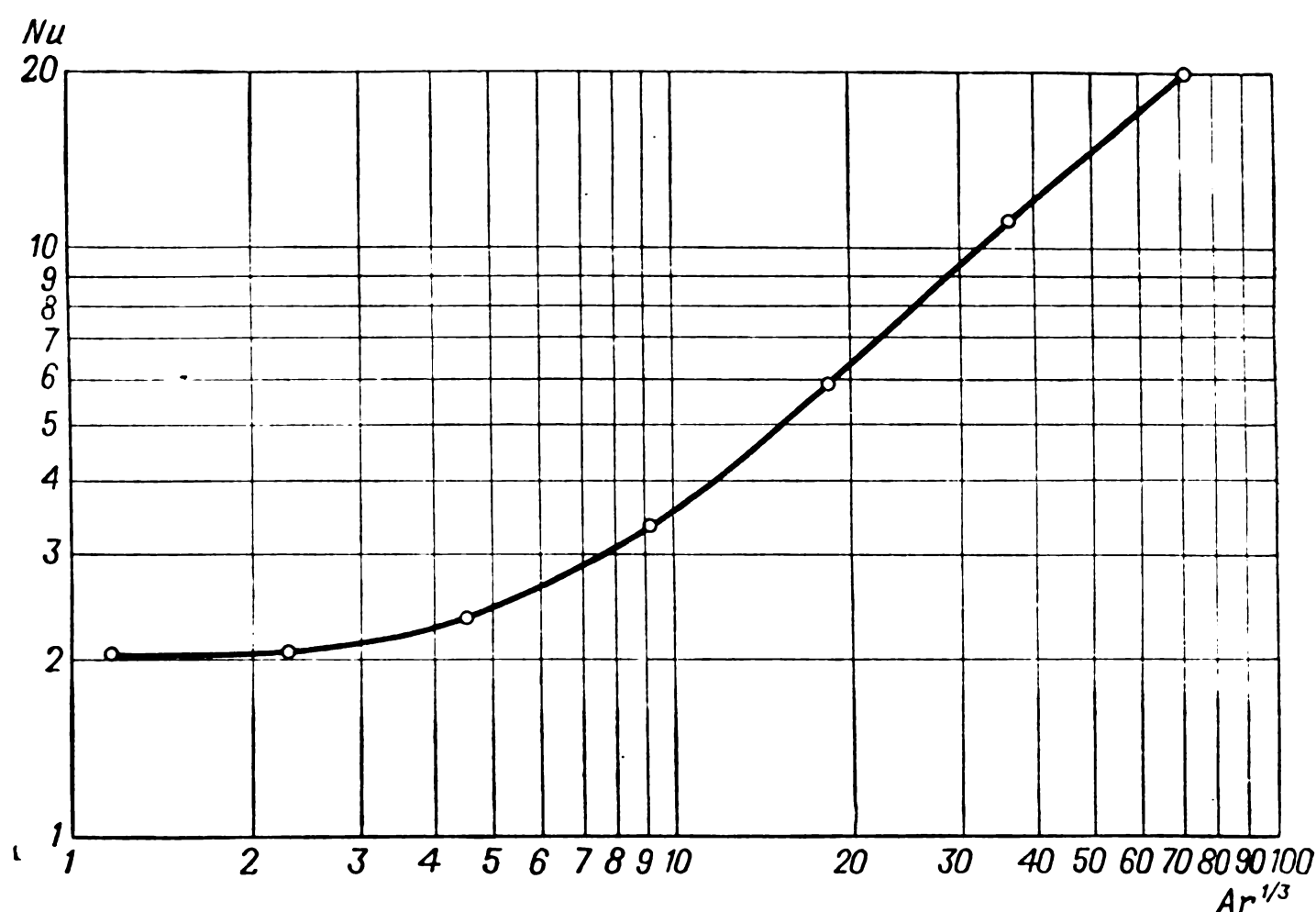


Fig. 10-11. Dependence of number Nu on number Ar (to Example 10-21)

The coefficient of heat transfer α from the hot air to the particles of the material being dried in a tubular pneumatic dryer can be determined approximately according to Fig. 10-11 which shows the relationship $Nu = f(Ar)$ obtained from data of I. Fedorov *.

The Nusselt dimensionless number, by Eq. (4-11), is:

$$Nu = \frac{\alpha d_{eq}}{\lambda}$$

The Archimedes number, according to Eq. (3-4), is:

$$Ar = \frac{d_{eq}^3 \rho_{mat} g}{\nu_{\eta}^2 \rho_{\eta}}$$

where $d_{eq} = 10^{-3} \text{ m}$ = diameter of a particle

$\lambda = 0.0285 \text{ W/m} \cdot \text{K}$ = coefficient of thermal conductivity of air at its mean temperature of $(90 + 50)/2 = 70^\circ \text{C}$

$\rho_{mat} = 1480 \text{ kg/m}^3$ = density of the material

$\rho_{\eta} = 1.03 \text{ kg/m}^3$ = density of the air at 70°C

$\nu_{\eta} = 2 \times 10^{-5} \text{ m}^2/\text{s}$ = kinematic viscosity of air.

* Fedorov, I. M. *Teoriya i raschet protsessa sushki vo vzveshennom sostoyanii* (Theory and Design of Drying Process in Fluidized State). Moscow, Gosenergoizdat (1955).

Inserting these values, we get:

$$Ar = \frac{10^{-9} \times 1480 \times 9.81}{2^2 \times 10^{-10} \times 1.03} = 3.52 \times 10^4$$

and $Ar^{1/3} = 32.8$.

From Fig. 10-11, we find $Nu = 10$, whence

$$\alpha = \frac{Nu \lambda}{d_{eq}} = 10 \times \frac{0.0285}{0.001} = 285 \text{ W/m}^2 \cdot \text{K}$$

The number of particles of the material passing through the dryer in one second is:

$$n = \frac{G}{\frac{\pi d_{eq}^3}{6} \rho_{mat} \times 3600}$$

They give the total surface area:

$$A_d = n \pi d_{eq}^2 = \frac{6G}{d_{eq} \rho_{mat} \times 3600} = \frac{6 \times 250}{0.001 \times 1480 \times 3600} = 0.281 \text{ m}^2/\text{s}$$

We determine the mean temperature difference approximately* as follows:

$$\begin{array}{ccc} 90 & \xrightarrow{\text{Air}} & 50 \\ 32 & \xrightarrow{\text{Material}} & 40 \\ \hline \Delta t_{gr} = 58 & & \Delta t_{sm} = 10 \end{array}$$

and by Eq. (4-70) we have:

$$\Delta t_m = \frac{58 - 10}{2.3 \log \frac{58}{10}} = 27.3 \text{ K}$$

The amount of heat received by the material in the dryer from the hot air consists of the following addends (assuming approximately that the entire moisture evaporates at $t_w = 32^\circ\text{C}$).

1. Heating of the moist material:

$$\begin{aligned} Q_1 &= (G_{fn} c_{fn} + Wc) (t_w - \theta_1) = \\ &= \left(\frac{250}{3600} \times 1.16 + \frac{34.6}{3600} \times 4.19 \right) (32 - 15) = 2.05 \text{ kW} \end{aligned}$$

2. Evaporation of the moisture:

$$Q_2 = WL = \frac{34.6}{3600} \times 2420 = 23.25 \text{ kW}$$

* Disregarding the brief initial period of heating the material, its initial temperature here can be assumed equal to the wet bulb temperature, i. e. 32°C (see Fig. 10-10).

3. Heating of the dried material:

$$Q_3 = G_{\text{fin}} c_{\text{fin}} (t_2 - t_w) = \frac{250}{3600} \times 1.16 (40 - 32) = 0.65 \text{ kW}$$

The total amount of heat is:

$$Q = Q_1 + Q_2 + Q_3 = 2.05 + 23.25 + 0.65 = 25.95 \text{ kW}$$

The time of drying is:

$$\tau = \frac{Q}{\alpha A_0 \Delta t_m} = \frac{25.95 \times 10^3}{285 \times 0.281 \times 27.3} = 11.9 \text{ s}$$

The length of the dryer can be found from the following equation:

$$\tau = \frac{l}{v - v_{\text{dep}}}$$

whence

$$l = \tau (v - v_{\text{dep}})$$

where l = length of the dryer tube, m

v = velocity of the air in the dryer, m/s

v_{dep} = velocity of deposition (soaring), m/s; we find it from Fig. 3-1 in which the relationship $Ly = f(Ar)$ is given for sharp-edged particles.

For $Ar = 3.52 \times 10^4$, we find $Ly = 205$, whence

$$v_{\text{dep}} = \sqrt[3]{\frac{Ly v_{\text{fl}} \rho_{\text{mat}} g}{\rho_{\text{fl}}}} = \sqrt[3]{\frac{205 \times 2 \times 10^{-5} \times 1.48 \times 10^3 \times 9.81}{1.03}} = 3.86 \text{ m/s}$$

It is generally assumed that $v = (1.1 \text{ to } 1.25) v_{\text{dep}}$. We adopt $v = 3.86 \times 1.2 = 4.64 \text{ m/s}$. Hence, $l = 11.9 (4.64 - 3.86) = 9.28 \text{ m}$.

The actual time spent by the particles in a dryer tube is greater than that obtained by calculations because no account is taken of the time needed to accelerate the particles, while only after this do the flow conditions set in; this is confirmed by experimental data.

The additional length l_{acc} of the tube for 0.2 to 1 mm particles can be found by the empirical relationship*:

$$l_{\text{acc}} = vd$$

where v = velocity of the air, m/s

d = diameter of a particle, mm.

For our example, $l_{\text{acc}} = 4.64 \times 1 = 4.64 \text{ m}$. Consequently, the required length of the dryer tube is $L = 9.28 + 4.64 \approx 14 \text{ m}$.

The diameter of the dryer tube is determined from the equation

* The authors established this relationship in processing the experimental data of I. Fedorov for an air velocity from 15 to 50 m/s.

The parameters of the atmospheric air are $t_0 = 25^\circ\text{C}$ and $x_0 = 0.0095 \text{ kg/kg}$ of dry air.

The parameters of the spent air are $t_2 = 60^\circ\text{C}$ and $x_2 = 0.041 \text{ kg/kg}$ of dry air.

Solution. We determine the amount of moisture evaporated in drying by Eq. (10-2):

$$W = G_{\text{in}} \frac{u_{\text{in}} - u_{\text{fn}}}{100 - u_{\text{fn}}} = 1000 \times \frac{50 - 6}{100 - 6} = 468 \text{ kg/h}$$

(a) Normal drying variant (ABC in Fig. 10-12).

The specific rate of flow of the dry air, by Eq. (10-14), is:

$$g = \frac{1}{x_2 - x_0} = \frac{1}{0.041 - 0.0095} = 31.8 \text{ kg/kg of evaporated moisture}$$

The total rate of flow of the dry air, by Eq. (10-13), is:

$$G = Wg = 468 \times 31.8 = 14\,900 \text{ kg/h}$$

The specific heat consumption is:

$$q = g(H_2 - H_0)$$

We find the values of the enthalpies using a Ramzin H - x diagram (see Fig. 10-1), and

$$q = 31.8(167 - 46) = 3860 \text{ kJ/kg of evaporated moisture}$$

The total consumption of heat is:

$$Q = Wq = \frac{468}{3600} \times 3860 = 502 \text{ kW}$$

(b) When the drying process is conducted with intermediate heating ($AB'C'C''C$ in Fig. 10-12), two air heaters are needed for heating the air, in each of them to 100°C . In the first zone of the dryer, the moisture content of the air grows to 0.02525 kg/kg of dry air, and the specific rate of flow of the dry air is:

$$g' = \frac{1}{0.02525 - 0.0095} = 63.6 \text{ kg/kg of evaporated moisture}$$

But since only half of all the moisture evaporates in the first zone of the dryer, i. e. $468/2 = 234 \text{ kg/h}$, then the rate of flow of the dry air is:

$$G = g' \frac{W}{2} = 63.6 \times 234 = 14\,900 \text{ kg/h}$$

The remaining moisture is evaporated in the second zone of the dryer by the same air, but heated to 100°C in the intermediate air heater.

The total amount of heat spent in two air heaters is:

$$Q = g' (H'_2 - H_0) \frac{W}{2} + g' (H_2 - H'_2) \frac{W}{2} = \frac{W}{2} g' (H_2 - H_0)$$

but since $g'/2 = g$, then

$$Q = W g (H_2 - H_0) = \frac{468}{3600} \times 31.8 (167 - 46) = 500 \text{ kW}$$

(c) For the drying process with recirculation of 80% of the spent air ($AMB''C$ in Fig. 10-12), we determine the characteristics of the mixture fed to the air heater:

$$\begin{aligned} x_{\text{mix}} &= 0.2x_0 + 0.8x_2 = 0.2 \times 0.0095 + 0.8 \times 0.41 = \\ &= 0.0347 \text{ kg of moisture/kg of dry air} \end{aligned}$$

$$H_{\text{mix}} = 0.2H_0 + 0.8H_2 = 0.2 \times 46 + 0.8 \times 167 = 143 \text{ kJ/kg of dry air}$$

Therefore, the specific rate of flow of the dry air is:

$$g'' = \frac{1}{x_2 - x_{\text{mix}}} = \frac{1}{0.041 - 0.0347} = 159 \text{ kg/kg of moisture}$$

and the rate of flow of the air (mixture) fed to the dryer is:

$$G'' = W g'' = 468 \times 159 = 74\,500 \text{ kg/h}$$

The rate of flow of the atmospheric air (20%) is:

$$G = 74\,500 \times 0.2 = 14\,900 \text{ kg/h}$$

The specific consumption of heat is:

$$q = \frac{H_2 - H_{\text{mix}}}{x_2 - x_{\text{mix}}} = \frac{467 - 143}{0.041 - 0.0347} = 3820 \text{ kJ/kg of moisture}$$

The consumption of heat is:

$$Q = W q = \frac{468}{3600} \times 3820 = 495 \text{ kW}$$

A comparison of the rates of flow of the air and the consumption of heat for drying for the three considered variants shows that with the same initial and final parameters of the air (points A and C in Fig. 10-12), these quantities are the same.

We calculate the mean drying potentials:

$$(a) \quad \kappa_m = \frac{(t_1 - t_w) - (t_2 - t_w)}{2.3 \log \frac{t_1 - t_w}{t_2 - t_w}} = \frac{(140 - 40) - (60 - 40)}{2.3 \log \frac{140 - 40}{60 - 40}} = 49.8 \text{ K}$$

$$(b) \quad \kappa_{m_1} = \frac{(100 - 35) - (60 - 35)}{2.3 \log \frac{100 - 35}{60 - 35}} = 41.8 \text{ K}$$

$$\kappa_{m_2} = \frac{(100 - 40) - (60 - 40)}{2.3 \log \frac{100 - 40}{60 - 40}} = 36.5 \text{ K}$$

$$x_m = \frac{41.8 + 36.5}{2} = 39.15 \text{ K}$$

$$(c) \quad x_m = \frac{(75 - 40) - (60 - 40)}{2.3 \log \frac{75 - 40}{60 - 40}} = 26.7 \text{ K}$$

A comparison of the drying potentials shows that the lowest value of the potential (the mildest drying conditions) has been obtained in the variant with recirculation of part of the spent air, and the highest value in the normal drying variant.

Example 10-23. A material is dried in a continuous countercurrent dryer from a moisture content of 50% to one of 3.5%, wet basis. The capacity of the dryer, wet basis, is 2260 kg/h. The density of the dry material is 640 kg/m³. The evaporation surface area per kilogram of dry material is 0.0615 m².

It was found in preliminary experimental drying that the critical moisture content of the material is 20%, and the equilibrium moisture content is 1.5%, wet basis. During the first drying period, when the surface of the material is saturated with moisture, the drying rate was 2.44 kg of moisture per hour from 1 m². The air used had a moisture content of 0.0306 kg/kg. The moisture content of the saturated air at the temperature of the material was $x_s = 0.0495$ kg/kg. The coefficient of mass transfer has been determined from these data:

$$\beta = \frac{2.44}{0.0495 - 0.0306} = 129 \text{ kg/m}^2 \cdot \text{h} (\Delta x = 1)$$

Determine the drying time.

Solution (Fig. 10-13). We use Eq. (10-41) to determine the duration of the first drying period in which the entire surface of the material is saturated with moisture. According to the given parameters, namely, $t_0 = 20^\circ\text{C}$, $\varphi_0 = 0.5$, $t_1 = 140^\circ\text{C}$, and $t_2 = 63^\circ\text{C}$, we find $x_0 = 0.0075$ and $x_2 = 0.0294$. Next, by conventional calculations not given here, we find that the rate of flow of the dry air is $G = 49\,700$ kg/h; $x_1 = 0.0124$ kg/kg, and $x_s = 0.0495$ kg/kg.

Hence, by Eq. (10-41), we have:

$$A_1 = \frac{2.3 \times 49\,700}{129} \log \frac{0.0495 - 0.0124}{0.0495 - 0.0294} = 237 \text{ m}^2$$

At the given capacity of the dryer, the feed of the material per hour corresponds to the evaporation surface:

$$2260 \times 0.5 \times 0.0615 = 69.5 \text{ m}^2/\text{h}$$

Thus, the first drying zone corresponding to the first drying period should contain the amount of the material charged during $237/69.5 = 3.41$ h, i.e. the duration of the first drying period is 3 h 25 min.

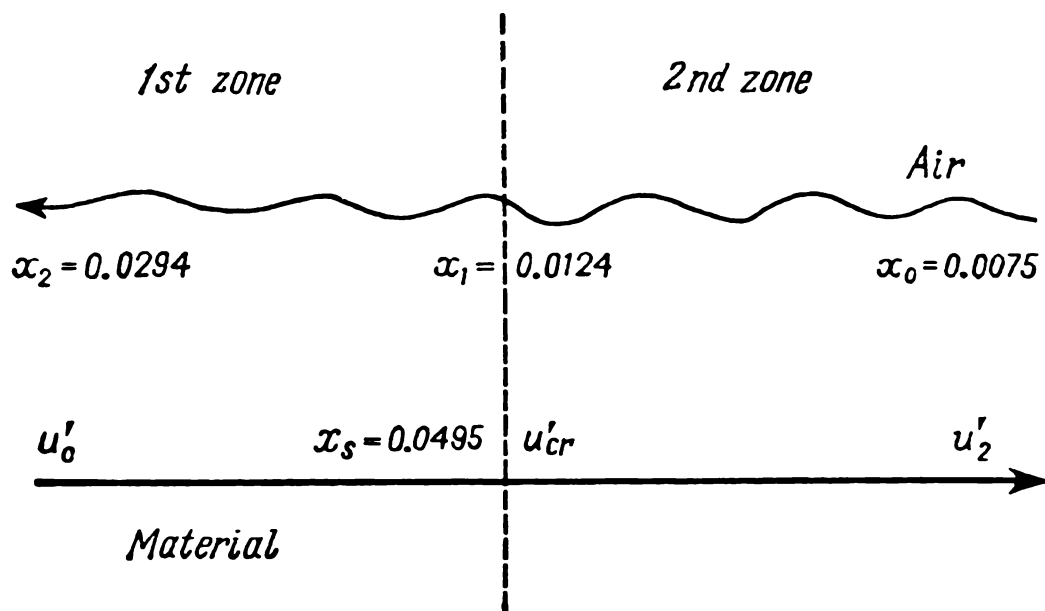


Fig. 10-13. To Example 10-23

Passing over to determining the duration of the second drying period, we find:

(a) the volume of the dry material is:

$$V = \frac{2260 \times 0.5}{640} = 1.77 \text{ m}^3/\text{h}$$

(b) the critical moisture content is:

$$u'_{\text{cr}} = \frac{20 \times 640}{80} = 160 \text{ kg/m}^3 \text{ of dry material}$$

(c) the equilibrium moisture content is:

$$u'_{\text{eq}} = \frac{1.5 \times 640}{98.5} = 9.75 \text{ kg/m}^3 \text{ of dry material}$$

The content of free moisture at the critical point is:

$$u'_{\text{cr}}^* = u'_{\text{cr}} - u'_{\text{eq}} = 160 - 9.75 = 150.25 \text{ kg/m}^3 \text{ of dry material}$$

We find the required evaporation surface area for the second zone of the dryer A_2 by Eq. (10-43) after first calculating the term $u'_{\text{cr}}^* V_m / G$ in it:

$$\frac{u'_{\text{cr}}^* V_m}{G} = \frac{150.25 \times 1.77}{49\,700} = 0.0054$$

$$A_2 = \frac{150.25}{129} \times \frac{1.77}{0.0495 + 0.0054 - 0.0124} \times \ln \frac{(0.0495 - 0.0075) 0.0054}{(0.0495 - 0.0124) (0.0075 + 0.0054 - 0.0124)} = 120 \text{ m}^2$$

i.e. the second zone of the dryer corresponding to the second drying period must contain the amount of material charged during $120/69.5 = 1.73$ h. Hence, the duration of the second drying period is 1 h 44 min.

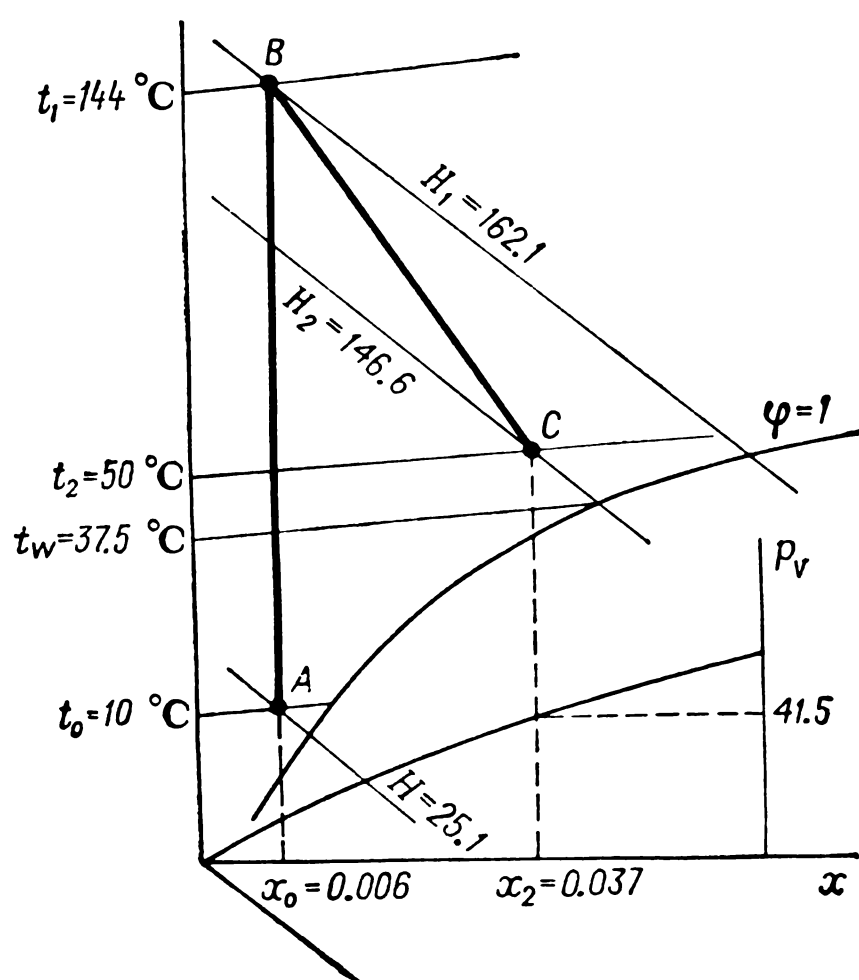


Fig. 10-14. To Example 10-24

The total time of drying is $\tau = 3 \text{ h } 25 \text{ min} + 1 \text{ h } 44 \text{ min} = 5 \text{ h } 9 \text{ min}$.

Example 10-24. A dryer operating according to the normal drying variant withdraws 1000 kg/h of moisture from a material. The atmospheric air ($t_0 = 10^\circ\text{C}$, $\phi_0 = 78\%$) is heated in a steam air heater in which the gauge pressure of the heating steam is 4 at. The psychrometer on the air duct after the dryer shows $t_2 = 50^\circ\text{C}$ and $t_w = 37.5^\circ\text{C}$. Assuming that the specific consumption of heat is 13% greater than in a theoretical dryer, determine the capacity of the exhaust fan, the rate of flow of the heating steam having a moisture content of 5%, and the heating surface area of the air heater if the overall coefficient of heat transfer in it is $30 \text{ W/m}^2 \cdot \text{K}$.

Solution. In a Ramzin diagram, we find $x_0 = 0.006 \text{ kg/kg}$ of dry air, $H_0 = 25.1 \text{ kJ/kg}$, $x_2 = 0.037 \text{ kg/kg}$ of dry air, $H_2 = 146.6 \text{ kJ/kg}$, and $p_v = 41.5 \text{ mm Hg}$ (Fig. 10-14).

The specific rate of flow of the dry air, by Eq. (10-14), is:

$$g = \frac{1}{x_2 - x_0} = \frac{1}{0.037 - 0.006} = 32.3 \text{ kg/kg of evaporated moisture}$$

The rate of flow of the dry air, by Eq. (10-13), is:

$$G = Wg = 1000 \times 32.3 = 32\,300 \text{ kg/h}$$

The specific volume of the humid air, by Eq. (10-12), is:

$$v = \frac{R_a T}{p_{\text{tot}} - p_v} = \frac{287 (273 + 50)}{(745 - 41.5) 133.3} = 0.988 \text{ m}^3/\text{kg of dry air}$$

The capacity of the exhaust fan is

$$V = Gv = 32\,300 \times 0.988 = 31\,900 \text{ m}^3/\text{h}$$

The specific consumption of heat in a theoretical dryer, by Eq. (10-19), is:

$$q_{\text{th}} = \frac{H_2 - H_0}{x_2 - x_0} = g (H_2 - H_0) = 32.3 (146.6 - 25.1) = 3920 \text{ kJ/kg of evaporated moisture}$$

For a real dryer, we have:

$$q = 1.13 q_{\text{th}} = 1.13 \times 3920 = 4430 \text{ kJ/kg of evaporated moisture}$$

From Eq. (10-18), we have:

$$q = g (H_1 - H_0)$$

whence

$$H_1 = H_0 + \frac{q}{g} = 25.1 + \frac{4430}{32.3} = 162.1 \text{ kJ/kg}$$

The temperature $t_1 = 144^\circ\text{C}$ corresponds to this value of H_1 at point B .

The consumption of heat in the air heater is:

$$Q = Wq = \frac{1000}{3600} \times 4430 = 1230 \text{ kW}$$

The consumption of heating steam is:

$$G_{\text{st}} = \frac{Q}{Lx} = \frac{1230}{2117 \times 0.95} = 0.613 \text{ kg/s} = 2.2 \text{ ton/h}$$

We determine the mean temperature drop in the air heater as follows:

$$\begin{array}{ccc} 151.1 & \longrightarrow & 151.1 \\ 10 & \longrightarrow & 144 \\ \hline \Delta t_{\text{gr}} = 141.1 & & \Delta t_{\text{sm}} = 7.1 \end{array}$$

By Eq. (4-70), we have:

$$\Delta t_m = \frac{\Delta t_{\text{gr}} - \Delta t_{\text{sm}}}{2.3 \log \frac{\Delta t_{\text{gr}}}{\Delta t_{\text{sm}}}} = \frac{141.1 - 7.1}{2.3 \log \frac{141.1}{7.1}} = 45 \text{ K}$$

The heating surface area of the air heater, by Eq. (4-65), is:

$$A = \frac{Q}{K \Delta t_m} = \frac{1230 \times 10^3}{30 \times 45} = 910 \text{ m}^2$$

PROBLEMS

- 10-1. How many times more moisture will have to be removed from 1 kg of a wet material when it is dried from 50 to 25% moisture content than when it is dried from 2 to 1% moisture content (wet basis)? In both cases 1 kg of the moist material is fed in for drying.
- 10-2. Find the moisture content, enthalpy, wet bulb temperature and dew point for the air leaving a dryer at $t=50^{\circ}\text{C}$ and $\varphi=0.7$.
- 10-3. The dry bulb and wet bulb temperatures of air are 50 and 30°C , respectively. Find all the characteristics of the air.
- 10-4. Determine the moisture content and relative humidity of a vapour and air mixture at 50°C if the partial water vapour pressure in the mixture is known to be 0.1 at.
- 10-5. Find the content of water vapour in its mixture (a) with air, (b) with hydrogen, and (c) with ethane (per kg of dry gas) at $t=35^{\circ}\text{C}$ and $\varphi=0.45$. The total pressure (absolute) is $p_{\text{tot}}=1.033$ at.
- 10-6. Compare the specific rates of flow of air and the specific consumption of heat in a dryer for summer and winter conditions (in Leningrad) if in both cases the air leaving the dryer has a temperature $t_2=40^{\circ}\text{C}$ and a relative humidity $\varphi_2=0.6$. The dryer is a theoretical one of the normal drying variant. For the characteristics of the state of the air in different localities in different seasons of the year see Table A-40.
- 10-7. The total pressure (absolute) of a vapour and air mixture at 150°C and a relative humidity of $\varphi=0.5$ is 745 mm Hg. Find the partial pressure of the water vapour and the air, and the moisture content of the air.
- 10-8. Humid air having a temperature of 130°C and $\varphi=0.3$ is under a pressure of $p_{\text{abs}}=7$ at. Determine the partial pressure of the air, its density, and moisture content.
- 10-9. What amount of moisture is removed from a material in a dryer if air is fed into the dryer at a rate of 200 kg/h (bone-dry basis) with $t_1=95^{\circ}\text{C}$ and $\varphi_1=5\%$, and leaves the dryer with $t_2=50^{\circ}\text{C}$ and $\varphi_2=60\%$? Also find the specific rate of flow of the air.
- 10-10. Humid air with a temperature of 130°C and a relative humidity of $\varphi=1$ is under an absolute pressure of $p_{\text{tot}}=7$ at. Find the partial pressure of the water vapour, the density of the humid air, and its moisture content.
- Compare the results obtained in problems 10-10 and 10-8.
- 10-11. Determine the capacity of an exhaust fan for a dryer in which 100 kg/h of moisture is removed from the material being dried in the following conditions: $t_0=15^{\circ}\text{C}$, $\varphi_0=0.8$, $t_2=45^{\circ}\text{C}$, $\varphi_2=0.6$, $p_{\text{tot}}=750$ mm Hg.
- 10-12. Air before being fed into a dryer is heated in an air heater to 113°C . The temperature of the air leaving the dryer is 60°C and its relative humidity is $\varphi_2=0.3$. Determine the dew point of the air entering the air heater. The drying process follows a line $H=\text{const}$.
- 10-13. Determine the rate of flow of atmospheric air and the consumption of heat an hour, and also the temperature of the vapour and air mixture entering the air heater, for a dryer with recirculation of part of the spent air in the following conditions:

Characteristics of air (dry basis):	
atmospheric	$H_0=50$ kJ/kg,
	$\varphi_0=0.7$
spent	$H_2=260$ kJ/kg,
	$\varphi_2=0.8$
Amount of recycled air	80% (of amount
	leaving the
	dryer)

Moisture content of material
(wet basis):
 initial $u_{in} = 47\%$
 final $u_{fin} = 5\%$
 Capacity of dryer (wet basis) $G_{in} = 1.5$ ton/h

10-14. Find the required rate of flow of air in a dryer and the consumption of heat for the air heater in the following conditions:

Characteristics of air:

atmospheric $x_0 = 0.01$, $t_0 = 20^\circ\text{C}$
 spent $x_2 = 0.028$, $t_2 = 34^\circ\text{C}$

Moisture content of material (wet basis):

initial $u_{in} = 50\%$
 final $u_{fin} = 13\%$

Capacity of dryer (bone-dry basis) $G_d = 1$ ton/h

Losses of heat $\sum Q$ with the material, conveying means and to the surroundings (minus the heat introduced by the moisture) 15% of the total amount of heat

10-15. Determine the efficiency of a theoretical dryer if the state of the air in it changes from $\varphi_0 = 0.7$ and $t_0 = 20^\circ\text{C}$ to $\varphi_2 = 0.6$ and $t_2 = 50^\circ\text{C}$. The moisture evaporates at the wet bulb temperature.

10-16. Find the mean drying potential in a theoretical dryer at $t_0 = 20^\circ\text{C}$, $\varphi_0 = 0.7$, and $t_2 = 50^\circ\text{C}$, $\varphi_2 = 0.4$. Evaporation proceeds at the wet bulb temperature.

10-17. A theoretical dryer is supplied with air from an air heater with a temperature of 85°C , the drying potential being 43°C . The drying potential of the air leaving the dryer is 8°C . Find the partial pressure of the water vapour in the air leaving the dryer and the per cent by volume of the water vapour in it if the absolute pressure in the dryer is $p_{tot} = 750$ mm Hg.

10-18. A moist material with an initial moisture content of 33%, a critical one of 17%, and an equilibrium moisture content of 2% is dried in constant conditions to a moisture content of 9% during 8 hours. Determine the time of drying to a moisture content of 3% in the same conditions. The moisture content is indicated on a bone-dry basis, in mass per cent.

10-19. Determine the heating surface area of a drum vacuum dryer having a capacity of 200 kg/h (dry basis). The initial moisture content is 50%, the final moisture content is 5% (wet basis). The overall coefficient of heat transfer is $350 \text{ W/m}^2\cdot\text{K}$, the drying temperature is 60°C , the specific heat capacity of the dry material is $1.26 \times 10^3 \text{ J/kg}\cdot\text{K}$, the initial temperature of the material is 20°C , and the pressure of the heating steam is $p_{abs} = 1.5$ at. The heat losses are 10% of the total amount of heat given up by the heating steam.

10-20. Find the dew point and the relative humidity of the air leaving a dryer according to the readings of a psychrometer: $t_d = 50^\circ\text{C}$ and $t_w = 35^\circ\text{C}$.

10-21. Find the temperature of moist material in a theoretical dryer (during the first drying period) if atmospheric air is fed into the air heater at $t_0 = 15^\circ\text{C}$ and $\varphi_0 = 0.8$ and is heated in it to $t_1 = 123^\circ\text{C}$.

10-22. A dryer with a capacity of 1 ton/h (wet basis) is used to dry a material from a moisture content of 55% to one of 8% (wet basis). The atmospheric air has the parameters $t_0 = 20^\circ\text{C}$, $\varphi_0 = 0.75$, and is heated in an air heater to $t_1 = 110^\circ\text{C}$. The drying potential at the outlet from the dryer is $x_2 = 10^\circ\text{C}$. Determine the rates of flow of the air and the heating steam if the steam pressure is $p_{abs} = 2.5$ at, and its dryness fraction is 95%.

10-23. A dryer with a capacity of 500 kg/h (bone-dry basis) is used to dry a material from a moisture content of 42% to one of 9% (bone-dry basis). The temperature of the air fed to the air heater is $t_0 = 20^\circ\text{C}$, and its dew point is

$t_{\text{dew}} = 8^\circ\text{C}$. The drying process would proceed in a theoretical dryer at $H = 125 \text{ kJ/kg}$. The temperature of the air leaving the dryer is $t_2 = 45^\circ\text{C}$. A normal drying variant is used. Determine the rate of flow of the heating steam and the heating surface area of the air heater if the absolute pressure of the heating steam is 2 at and its moisture content is 5%. The overall coefficient of heat transfer $K = 32 \text{ W/m}^2\cdot\text{K}$. The sum of all the heat losses is 15% of the consumption of heat in the theoretical dryer.

10-24. Air at $t = 60^\circ\text{C}$ and $\phi = 0.2$ is cooled with cold water in a tubular counterflow heat exchanger to its dew point. The cooling water gets heated from 15 to 25°C . Determine the rate of flow of the cooled air, the partial pressure of the water vapour and its per cent by volume in the air, and also the rate of flow of the cooling water if the surface area of the exchanger is 15 m^2 , and the overall coefficient of heat transfer $K = 46 \text{ W/m}^2\cdot\text{K}$.

10-25. Find the temperature and moisture content of the air leaving a theoretical dryer if the mean drying potential is $\kappa_m = 41^\circ\text{C}$. Air is fed into the air heater at $t_0 = 15^\circ\text{C}$ and $\phi_0 = 70\%$. The enthalpy of the air fed into the dryer from the air heater is $H = 144.2 \text{ kJ/kg}$. Also find the temperature of the wet material (during the first period of drying).

10-26. Determine the temperature of the air fed to a theoretical dryer if the mean driving force of the drying process $\Delta x_m = 0.0136 \text{ kg/kg}$, the temperature and the relative humidity of the air leaving the dryer are $t_2 = 45^\circ\text{C}$ and $\phi_2 = 60\%$, respectively.

10-27. Determine the rate of flow of the air, the rate of flow of the heating steam, and its required pressure for a countercurrent dryer operating according to the normal drying variant. The capacity of the dryer is 600 kg/h of moist material whose initial moisture content is 50% (wet basis) and whose final moisture content is 9%. The air fed into the air heater is at $t_0 = 10^\circ\text{C}$ and $\phi_0 = 80\%$, and that leaving the dryer is at $t_2 = 50^\circ\text{C}$ and $\phi_2 = 50\%$. Choose the temperature of the heating steam. Its moisture content is 6%.

Perform the calculations (a) for a theoretical dryer, and (b) for a real dryer, assuming that the temperature of the material at its inlet is 16°C and at its outlet is 55°C . The specific heat capacity of the dried material is $1.68 \text{ kJ/kg}\cdot\text{K}$. The mass of the conveying means (steel conveyor) carrying the hourly batch of the moist material is 450 kg . The losses of heat by the dryer and air heater to the surroundings are 10% of the amount of heat transferred to the air in the air heater.

10-28. A theoretical dryer operating with intermediate (by stages) heating of the air is supplied with 1800 kg/h of a moist material having an initial moisture content of 39%. The final moisture content is 8% (wet basis). The air leaves the dryer at a temperature of 45°C . The temperature of the atmospheric air is 20°C . The drying plant has a total of three air heaters, in each of which the air is heated to 70°C . After each heater, the air in the dryer is saturated with water vapour to $\phi = 0.7$. Find the rates of flow of the dry air and the heating steam. The pressure of the heating steam is $p_{\text{abs}} = 3 \text{ at}$, its moisture content is 5%. Give a scheme of the process in a Ramzin diagram.

10-29. The rate of flow of steam in the air heater of a dryer at a gauge pressure of $p_{\text{gauge}} = 2 \text{ at}$ and a moisture content of 10% is 200 kg/h . The consumption of heat is 10% greater than that in a theoretical dryer. The heating surface area of the air heater is 41 m^2 . The atmospheric air has a temperature of $t_0 = 25^\circ\text{C}$ and a dew point of $t_{\text{dew}} = 10^\circ\text{C}$. The enthalpy of the drying process is $H_2 = 100 \text{ kJ/kg}$. The partial pressure of the water vapour in the air leaving the dryer is 25 mm Hg .

Determine the overall coefficient of heat transfer in the air heater and the capacity of the dryer (wet basis) if the initial moisture content of the material is 60%, and the final moisture content is 10% (wet basis).

10-30. A dryer having a capacity of 500 kg/h (dry basis) dries a material from $\phi = 70$ to 10% (wet basis). The readings of the psychrometer measuring the atmospheric air are 15 and 20°C . The air leaves the dryer at a tem-

perature of 45 °C and a relative humidity of 50%. The losses of heat in the dryer and the air heater are 8% of the loss of heat in a theoretical dryer.

Determine the heating surface area of the air heater and the rate of flow of the heating steam if it has a pressure of $p_{\text{abs}}=2$ at and a moisture content of 5%. The overall coefficient of heat transfer in the air heater is 35 W/m²·K.

10-31. A wet material having an initial moisture content of 50% is dried at a rate of 1000 kg/h to a final moisture content of 8% (wet basis). Drying is performed (a) in a vacuum dryer with the temperature of the material during drying equal to 40 °C, and (b) in an atmospheric air dryer at the same temperature of the material (during the first period). The atmospheric air has a temperature of $t_0=20$ °C and a relative humidity of $\varphi_0=0.7$; the air leaving the dryer has a temperature of $t_2=55$ °C. In both cases, the moist material enters the dryer at 15 °C and leaves it at 40 °C. The specific heat capacity of the dried material is 1.26×10^3 J/kg·K.

Disregarding the losses of heat to the surroundings and for heating the conveying means, determine the specific consumptions of heat in both dryers.

10-32. A theoretical dryer having a capacity of 600 kg/h of bone-dry material is used to dry a material from a moisture content of 35 to 8% (wet basis). The readings of a psychrometer installed in the premises from which air is supplied to the air heater are $t_d=18$ °C and $t_w=15$ °C. The air leaving the dryer has a temperature of $t_2=40$ °C and a relative humidity of $\varphi_2=0.65$.

Determine the rate of flow of the heating steam in the air heater and the heating surface area if the vapour pressure $p_{\text{abs}}=2$ at and the overall coefficient of heat transfer $K=33$ W/m²·K.

10-33. Find the rate of flow of the air, the rate of flow and the required pressure of the heating steam, and the surface area of the air heater for a dryer whose capacity is 600 kg/h of moist material with an initial moisture content of 50% and a final moisture content of 9% (wet basis). The readings of the psychrometer for the air entering the air heater are 10 and 5 °C. The air leaves the dryer at $t_2=50$ °C and $\varphi_2=50\%$. Assume that the temperature of the heating steam is 15 °C above the temperature of the air at the outlet from the heater. The moisture content of the heating steam is 6%. The consumption of heat is 10% greater than that in a theoretical dryer. The overall coefficient of heat transfer in the air heater is 35 W/m²·K.

10-34. Determine the capacity (dry basis), the heating surface area of the air heater, and the fraction of recycled spent air in a theoretical dryer with recirculation. The rate of flow of the fresh atmospheric air is 6000 kg/h, its enthalpy is 50 kJ/kg, and the partial water vapour pressure in it is 12 mm Hg. The initial moisture content of the material is 40%, and its final moisture content is 7% (wet basis).

The parameters of the air mixture entering the air heater are $x=0.034$, and $t=40$ °C. The air is heated up to 88 °C in the heater. The overall coefficient of heat transfer in the air heater is 47 W/m²·K. The pressure of the heating steam is $p_{\text{gauge}}=2$ at.

EXAMPLE OF CALCULATING
AND SELECTING A FLUIDIZED-BED
DRYER FOR DRYING POTASSIUM CHLORIDE

<i>Data for Calculations</i>	
Capacity (dry basis)	$G_{\text{fin}}=20$ ton/h
Moisture content of salt (wet basis):	
initial	$u_{\text{in}}=10\%$
final	$u_{\text{fin}}=0.5\%$
Mean diameter of particles	$d=0.25$ mm ($d_{\text{max}}=$ $=0.5$ mm, $d_{\text{min}}=$ $=0.1$ mm)

Temperature of salt supplied for drying $\theta_1 = 20\text{ }^\circ\text{C}$
Specific heat capacity of dry salt $c_{\text{mat}} = 0.712 \times 10^3\text{ J/kg}\cdot\text{K}$

Density of salt $\rho_{\text{mat}} = 2 \times 10^3\text{ kg/m}^3$

Use flue gases for drying. Take the heat losses equal to 15% of the amount of heat used to heat the material and evaporate the moisture.

Selection of Design of Dryer and the Conditions of Its Operation.

The factors affecting the choice of the design of the dryer are:

1. The state of the material being dried—loose, forming lumps in the moist state.
2. The material withstands heating to a high temperature ($t_{\text{melt}} = 770\text{ }^\circ\text{C}$).
3. The ratio of the maximum particle size to the minimum one $d_{\text{max}}/d_{\text{min}} = 0.5/0.1 = 5$.
4. The surface moisture is to be mainly removed.
5. We select a turbogrid (recommended when drying salts) with an opening diameter of $d_0 = 5\text{ mm}$.
6. A certain non-uniformity of the dried material with respect to the final moisture content is tolerated because in storage all the salt particles acquire the same moisture content.

The third factor makes it possible to preliminarily choose a single-chamber drying apparatus with vertical walls.

To ensure better hydrodynamic conditions, we select an apparatus having a round cross section.

We adopt the depth of the fluidized bed in the apparatus four times greater than the height of the zone of action of the jets—the zone of hydrodynamic stabilization [10-11]. The latter is determined from the relationship $h_{\text{jet}} = 20d_0 = 20 \times 5 = 100\text{ mm}$, whence $h = 4h_{\text{jet}} = 4 \times 100 = 400\text{ mm}$.

We adopt the temperature of the flue gases diluted with air and fed under the grid equal to $t_1 = 800\text{ }^\circ\text{C}$, and the temperature of the exhaust gases $t_2 = 125\text{ }^\circ\text{C}$, which makes it possible to prevent condensation of vapour in the dust-separating apparatus (cyclones and filters). The temperature of the discharged salt can approximately be taken equal to that of the exhaust gases, i.e. $\theta_g = 125\text{ }^\circ\text{C}$.

To ensure uniform fluidization, the moist salt, which may form lumps, should be fed in with the aid of scattering devices that distribute the material uniformly over the surface of the bed.

It is good to discharge the salt from the apparatus directly at the grid in order to remove the lumps from it.

Calculations. 1. The amount of moist material, from Eqs. (10-2), is:

$$G_{\text{in}} = G_{\text{fn}} \frac{100 - u_{\text{fn}}}{100 - u_{\text{in}}} = 20\,000 \times \frac{0.995}{0.9} = 22\,100\text{ kg/h} = 6.15\text{ kg/s}$$

2. The amount of evaporated moisture is:

$$W = G_{\text{in}} - G_{\text{fn}} = 22\,100 - 20\,000 = 2\,100\text{ kg/h} = 0.584\text{ kg/s}$$

3. The consumption of heat is:

$$\begin{aligned} Q &= Q_e + Q_h + Q_1 = 1.15 \{ W [L + c_v (t_2 - \theta_1)] + G_{\text{fn}} c_{\text{mat}} (\theta_2 - \theta_1) \} = \\ &= 1.15 \{ 0.584 [2490 \times 10^3 + 1.97 \times 10^3 (125 - 20)] + \\ &\quad + 5.56 \times 0.712 \times 10^3 (125 - 20) \} = 2280 \times 10^3\text{ W} \end{aligned}$$

The specific consumption of heat is

$$q = \frac{Q}{W} = \frac{2280 \times 10^3}{0.584} = 3900\text{ kW/kg of moisture}$$

4. The rate of flow of the drying gases is:

$$G_g = \frac{Q}{c_g (t_1 - t_2)} = \frac{2280 \times 10^3}{1.05 \times 10^3 (800 - 125)} = 3.22 \text{ kg/s}$$

The specific heat capacity of the gases c_g has been taken for a mean temperature of $0.5 (800 + 125) \approx 463^\circ\text{C}$.

The specific rate of flow of the gases, according to Eq. (10-13), is:

$$g_g = \frac{G_g}{W} = \frac{3.22}{0.584} = 5.5 \text{ kg/kg of moisture}$$

5. Velocity of the gases. We preliminarily calculate the critical velocity of fluidization for particles of the mean size using a graph of Ly against Ar for a temperature in the bed that can be considered equal to the temperature of the exhausted gases, i.e. 125°C . We determine the Archimedes number Ar by Eq. (3-63):

$$Ar = \frac{d^3 \rho_{\text{mat}} g}{v^2 \rho_g}$$

The values of d , ρ_{mat} and g are known ($d = 0.25 \text{ mm} = 2.5 \times 10^{-4} \text{ m}$, $\rho_{\text{mat}} = 2 \times 10^3 \text{ kg/m}^3$, and $g = 9.81 \text{ m/s}^2$). We calculate the value of ρ_g as for air. From Eq. (1-5), with $p = p_0$, we get

$$\rho_g = \rho_0 \frac{T_0}{T}$$

From Table A-5, we get $\rho_0 = 1.293 \text{ kg/m}^3$, and

$$\rho_g = 1.293 \times \frac{273}{273 + 125} = 0.885 \text{ kg/m}^3$$

In Fig. A-6 for air and $t = 125^\circ\text{C}$, we find $\mu = 0.218 \times 10^{-3} \text{ Pa}\cdot\text{s}$ and by Eq. (1-10), we get

$$v = \frac{\mu}{\rho_g} = \frac{0.218 \times 10^{-3}}{0.885} = 2.47 \times 10^{-5} \text{ m}^2/\text{s}$$

The critical value of the Lyashchenko number, from Fig. 3-8, is $Ly_{\text{cr}} = 10^{-4}$. We find the critical fluidization velocity from Eq. (3-6), substituting $v\rho_g$ for μ in it according to Eq. (1-10):

$$v_{\text{cr}} = \sqrt[3]{\frac{Ly_{\text{cr}} v g \rho_{\text{mat}}}{\rho_g}} = \sqrt[3]{\frac{10^{-4} \times 2.47 \times 10^{-5} \times 9.81 \times 2 \times 10^3}{0.885}} = 0.038 \text{ m/s}$$

We choose the operating value of the number Ly for a porosity of the fluidized bed of $\varepsilon = 0.75$ because for drying processes taking place in the first period the intensity of a process is the higher, the greater is the velocity of the gases.

For $\varepsilon = 0.75$, we find $Ly = 3.4 \times 10^{-1}$ (Fig. 10-15). By Eq. (3-70) the fluidization number is $K_v = v/v_{\text{cr}}$. From Eq. (3-6), other quantities being equal, we find that $v/v_{\text{cr}} = Ly^3/Ly_{\text{cr}}^3$, whence

$$K_v = \sqrt[3]{\frac{Ly}{Ly_{\text{cr}}}} = \sqrt[3]{\frac{3.4 \times 10^{-1}}{10^{-4}}} = 15$$

Now we find the velocity of the gases (for the total cross section of the grid).

$$v = K_v v_{\text{cr}} = 0.038 \times 15 = 0.57 \text{ m/s}$$

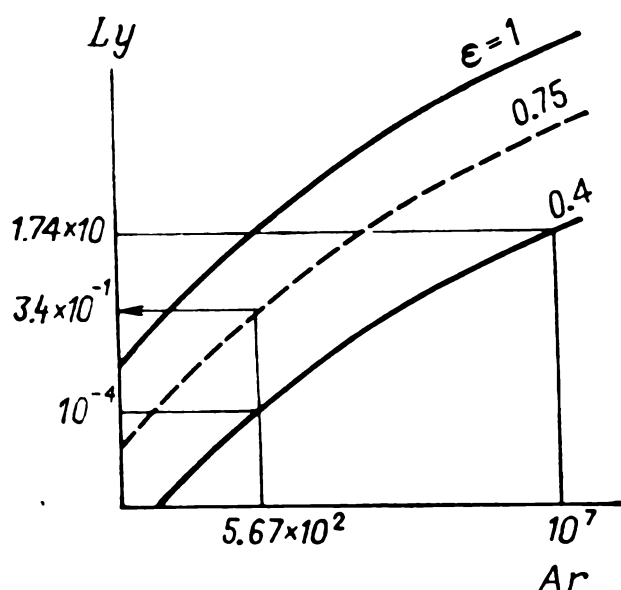


Fig. 10-15. Determination of the dimensionless number Ly

The velocity of the gases directly at the grid is higher owing to their higher temperature. This velocity is:

$$v_{gr} = v \frac{273 + t_1}{273 + t_2} = 0.57 \times \frac{273 + 800}{273 + 125} = 1.54 \text{ m/s}$$

If the cross-sectional area of the grid openings is taken equal to 10% of its entire area, then the velocity of the gas in the grid openings will be:

$$v_0 = 10 \times 1.54 = 15.4 \text{ m/s}$$

This velocity is sufficient for fluidization of the bed not only with particles of the maximum diameter ($d_{\max} = 0.5 \text{ mm}$), but also with larger formations (lumps), which can be seen from the following calculations.

If we adopt a small value of the fluidization number for the consolidated particles, only sufficient for their movement (for instance $K_v = 3$), then the critical fluidization velocity for these particles will be:

$$v_{cr (o)} = \frac{v_0}{K_v} = \frac{15.4}{3} = 5.13 \text{ m/s}$$

The density of the gases at the grid openings will be:

$$\rho_g = 1.293 \times \frac{273}{273 + 800} = 0.33 \text{ kg/m}^3$$

the kinematic viscosity is $\nu = 1.31 \times 10^{-4} \text{ m}^2/\text{s}$, and by Eq. (3-6), we have:

$$Ly_{cr (o)} = \frac{v_{cr (o)}^3 \rho_g}{\nu g \rho_{mat}} = \frac{5.13^3 \times 0.33}{1.31 \times 10^{-4} \times 9.81 \times 2 \times 10^3} = 1.74 \times 10^1$$

For $Ly_{cr} = 1.74 \times 10^1$, we get $Ar = 10^7$. Therefore, the diameter of the consolidated particles (lumps), by Eq. (3-63), is:

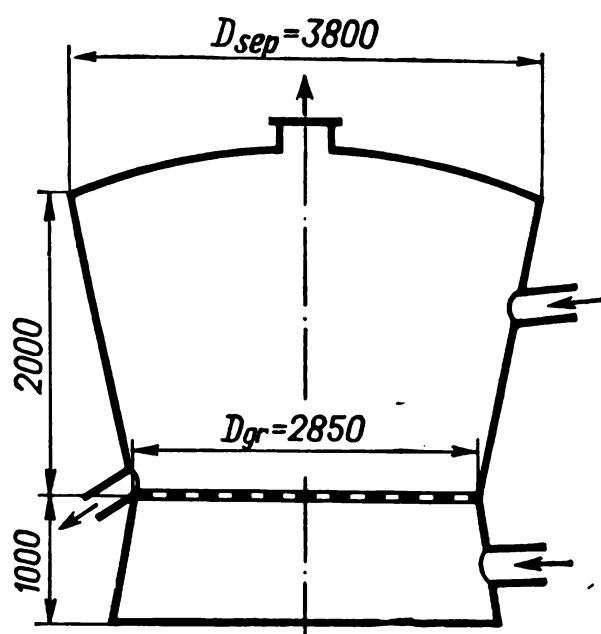
$$d_{lump} = \sqrt[3]{\frac{Ar \nu^2 \rho_g}{\rho_{mat} g}} = \sqrt[3]{\frac{10^7 \times 1.31^2 \times 10^{-8} \times 0.33}{2 \times 10^3 \times 9.81}} = 0.0143 \text{ m}$$

Thus, even lumps having a diameter of about 14 mm are capable of movement at the grid openings.

6. Grid dimensions. We determine the total grid area by the expression:

$$A_{gr} = \frac{G_g}{\rho_g v} = \frac{3.22}{0.885 \times 0.57} = 6.39 \text{ m}^2$$

Fig. 10-16. Sketch of the dryer



The diameter of the grid is:

$$D_{gr} = \sqrt{\frac{A}{0.785}} = \sqrt{\frac{6.39}{0.785}} = 2.85 \text{ m}$$

7. Separation space. We take the height of the separation space four times greater than the depth of the fluidized bed: $h_{sep} = 4 \times 400 = 1600 \text{ mm}$.

The total height of the apparatus (above the grid) is $h + h_{sep} = 400 + 1600 = 2000 \text{ mm}$.

Let us see whether the smallest particles of the salt (with a diameter of about 0.1 mm) will be carried out of the apparatus.

The Archimedes number for these particles is:

$$Ar = \frac{d^3 \rho_{mat} g}{\nu^2 \rho_g} = \frac{1 \times 10^{-12} \times 2 \times 10^3 \times 9.81}{2.47^2 \times 10^{-10} \times 0.885} = 3.64 \times 10^1$$

The Lyashchenko number corresponding to the carrying out of the particles is $Ly_s = 0.15$, and the soaring velocity for particles with a diameter of 0.1 mm will be:

$$v_s = \sqrt[3]{\frac{Ly_s \nu \rho_{mat} g}{\rho_g}} = \sqrt[3]{\frac{0.15 \times 2.47 \times 10^{-5} \times 2 \times 10^3 \times 9.81}{0.885}} = 0.435 \text{ m/s}$$

The actual velocity of the gas in the separation space for an apparatus with vertical walls will be:

$$v = \frac{G_g + W}{\rho_g A_{gr}} = \frac{(3.22 + 0.584)(273 + 125)}{1.293 \times 273 \times 6.39} = 0.672 \text{ m/s}$$

(with a certain error, the density of the flue gases has been calculated here as for air).

Thus, an apparatus with vertical walls does not ensure the settling in the separation space of salt particles with a diameter less than 0.1 mm. To ensure their settling, the cross section of the separation space should be increased to the value:

$$A_{sep} = 1.1 A_{gr} \frac{v}{v_s} = 1.1 \times 6.39 \times \frac{0.672}{0.435} = 11 \text{ m}^2$$

Here the factor 1.1 is introduced to take into account a certain lowering of the stream velocity in comparison with the soaring velocity needed to ensure settling of the particles.

The diameter of the separation space will be:

$$D_{\text{sep}} = \sqrt{\frac{11}{0.785}} \approx 3.8 \text{ m}$$

A sketch of the dryer is given in Fig. 10-16.

EXAMPLE OF CALCULATING AND SELECTING A DRUM DRYER

Determine the main dimensions of a double-drum dryer for drying a nickel carbonate paste with a capacity of 90 kg/h of the paste. The initial and final moisture contents are 75% and 10% (wet basis), respectively. The dryer is heated with indirect steam ($p_{\text{abs}} = 1 \text{ at}$). The thickness of the layer of material is about 1 mm. The thickness of an iron drum wall is 10 mm. Air is blown over the surface of the material with a velocity of 1.5 m/s. The temperature of the air is 40 °C, $\varphi = 40\%$. The temperature of the paste is 15 °C.

Solution. The dryer can be calculated through the overall coefficient of heat transfer from the steam to the air. Heat is transferred in a drum dryer as follows: heat is transferred from the condensing steam inside the drum to the drum wall and thence to the material being dried. The moisture evaporating from the material on the outer surface of the drum diffuses into the air carrying off with it the corresponding amount of heat. It is possible to compute the amount of diffusing moisture and on the basis of this amount determine the equivalent coefficient of heat transfer.

We adopt the coefficient of heat transfer from the condensing steam to the drum wall equal to $\alpha = 9280 \text{ W/m}^2 \cdot \text{K}$. The thermal conductivity coefficient of iron is $\lambda_i = 46.4 \text{ W/m} \cdot \text{K}$, the mean thermal conductivity coefficient of the material being dried is $\lambda_{\text{mat}} = 0.8 \text{ W/m} \cdot \text{K}$, and its heat capacity is $3.46 \text{ kJ/kg} \cdot \text{K}$.

The equivalent coefficient of heat transfer upon evaporation of the moisture is determined from the following equation:

$$\alpha_e = \frac{q_e}{\Delta t} = \frac{GL}{\theta_{\text{mat}} - t_{\text{air}}}$$

where L is the specific heat of vaporization, J/kg.

Since by Eq. (10-28) the specific rate of flow of the moisture being evaporated G (in $\text{kg/m}^2 \cdot \text{h}$) is

$$G = 0.04075 v^{0.8} \Delta p$$

then the coefficient of heat transfer α_e can be calculated by the equation

$$\alpha_e = \frac{0.04075 v^{0.8} \Delta p L}{\Delta t \times 3600}$$

We adopt (with subsequent verification) a temperature of the outer surface of the material $\theta_{\text{mat}} = 80 \text{ }^\circ\text{C}$ (the permissible temperature for nickel carbonate is not above $85 \text{ }^\circ\text{C}$). The pressure of saturated water vapour at $80 \text{ }^\circ\text{C}$ is $p_s = 355 \text{ mm Hg}$; the partial pressure of the water vapour in air at $t = 40 \text{ }^\circ\text{C}$ and $\varphi = 0.4$ is 22.4 mm Hg . The specific heat of vaporization of water at atmospheric pressure is $L = 2264 \times 10^3 \text{ J/kg}$.

Consequently, the equivalent coefficient of heat transfer in evaporation is:

$$\alpha_e = \frac{0.04075 \times 1.5^{0.8} (355 - 22.4) 2264 \times 10^3}{(80 - 40) 3600} = 294 \text{ W/m}^2 \cdot \text{K}$$

The overall coefficient of heat transfer from the condensing steam to the air, by Eq (4-67), is:

$$K = \frac{1}{\frac{1}{9280} + \frac{0.01}{46.4} + \frac{0.001}{0.8} + \frac{1}{294}} = 203 \text{ W/m}^2 \cdot \text{K}$$

The unit heat load, by Eq. (4-66), is:

$$q = K (t_{\text{st}} - t_{\text{air}}) = 203 (100 - 40) = 12\,180 \text{ W/m}^2$$

We verify the temperature of the surface of the material θ_{mat} adopted above by the equation

$$\Delta t = \frac{q}{\alpha_p} = \frac{12\,180}{294} = 41.4 \text{ K}$$

The temperature of the surface of the material is:

$$\theta_{\text{mat}} = t_{\text{air}} + \Delta t = 40 + 41.4 = 81.4 \text{ }^\circ\text{C}$$

which is close to the adopted temperature.

The rate of flow of the water evaporated in the dryer is:

$$W = G_{\text{in}} \frac{u_{\text{in}} - u_{\text{fin}}}{100 - u_{\text{fin}}} = 90 \times \frac{75 - 10}{100 - 10} = 65 \text{ kg/h}$$

The consumption of heat for heating the material and evaporating the moisture is:

$$Q = \frac{90 \times 3.46 \times 10^3 (81.4 - 15) + 65 \times 2264 \times 10^3}{3600} = 46\,500 \text{ W}$$

The required heating surface area of the drum dryer is:

$$A = \frac{Q}{q} = \frac{46\,500}{12\,180 \times 0.75} = 5.38 \text{ m}^2$$

where 0.75 is a factor taking into account the actual surface of contact of the material with the heating surfaces of the drums.

According to the relevant catalogues, the closest double-drum dryer has a surface area of $A = 5.2 \text{ m}^2$ (the diameter of the drums is 600 mm and their length is 1400 mm). We choose this dryer although the area of its heating surface is slightly smaller than the calculated one. To ensure the required capacity, it will be necessary to increase somewhat the pressure of the heating steam. This is simple to adjust in practice.

The heat losses of the dryer must be taken into account when determining the rate of flow of the heating steam.

SYMBOLS

A	area
Ar	Archimedes dimensionless number
a	unit area
C	coefficient
D	diameter
d	diameter
G	amount of substance; rate of flow of gas
Gu	Gukhman dimensionless number
g	acceleration due to gravity; specific rate of flow

H	enthalpy
L	latent heat of vaporization; length
Ly	Lyashchenko dimensionless number
l	characteristic linear dimension; length
M	molar mass
Nu'	Nusselt dimensionless number for mass transfer
n	constant
Pr'	Prandtl dimensionless number for mass transfer
p	pressure
Q	amount of heat
R	gas constant; rate of drying
Re	Reynolds dimensionless number
T	absolute temperature
t	temperature, °C
u	moisture content
V	volumetric rate of flow
v	specific volume
W	amount of moisture; rate of flow of moisture or water
x	vapour content of vapour and gas mixture

Greek Letters

α	individual coefficient of heat transfer
β	individual coefficient of mass transfer
θ	temperature
κ	drying potential
λ	thermal conductivity
ν	kinematic viscosity
ρ	density
τ	time
φ	relative humidity of air

FUNDAMENTAL RELATIONSHIPS
AND FORMULAS FOR CALCULATIONS

1. For a refrigeration (reverse) Carnot cycle 1-2-3-4 consisting of two isothermal and two isentropic processes (Fig. 11-1), the refrigeration coefficient (coefficient of performance) is:

$$\varepsilon_C = \frac{Q_0}{P} = \frac{Q}{P_{\text{com}} - P_{\text{exp}}} = \frac{Q_0}{Q - Q_0} = \frac{T_0}{T - T_0} \quad (11-1)$$

Here Q_0 = refrigerating capacity—the amount of heat received by the refrigerant (working fluid) from the medium being cooled at the temperature T_0 , W

Q = amount of heat given up by the refrigerant to water at the temperature T , W

P_{com} = power spent in the compressor upon the isentropic compression of the refrigerant vapour, W

P_{exp} = power received upon the isentropic expansion of the refrigerant in the expansion device, W

$P = P_{\text{com}} - P_{\text{exp}} = Q - Q_0$ = theoretical power spent in the cycle, W.

Examination of Eq. (11-1) shows that the quantity ε_C theoretically depends only on the values of the temperatures T and T_0 and does not depend on the nature of the refrigerant.

2. For an actual wet cycle 1-2-3-4' (Fig. 11-2) of a vapour compression refrigerating installation, the refrigeration coefficient is:

$$\varepsilon = \frac{Q_0}{P} = \frac{Q_0}{Q - Q_0} = \frac{H_1 - H_4'}{H_2 - H_1} = \frac{H_1 - H_3}{H_2 - H_1} \quad (11-2)$$

where P = power used by the compressor in compressing the refrigerant vapour, W

H_1, H_2, H_3, H_4' = specific enthalpies of the refrigerant at the corresponding points of the cycle (Fig. 11-2), J/kg.

For the other symbols see Eq. (11-1).

3. For a dry cycle of a single-stage vapour compression refrigerating installation (Fig. 11-3) the refrigeration coefficient is:

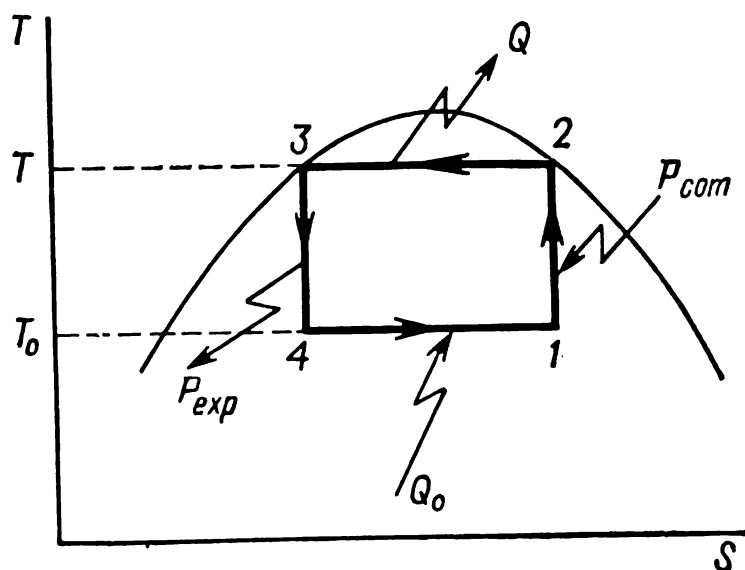


Fig. 11-1. Reverse Carnot cycle

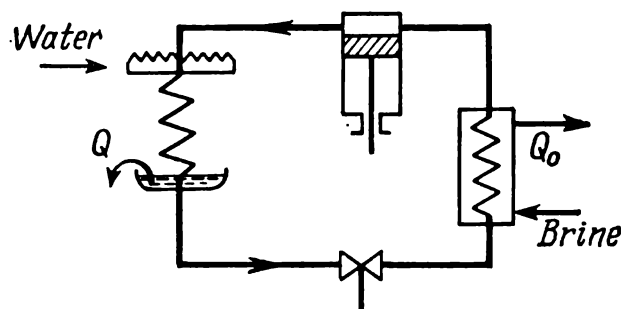
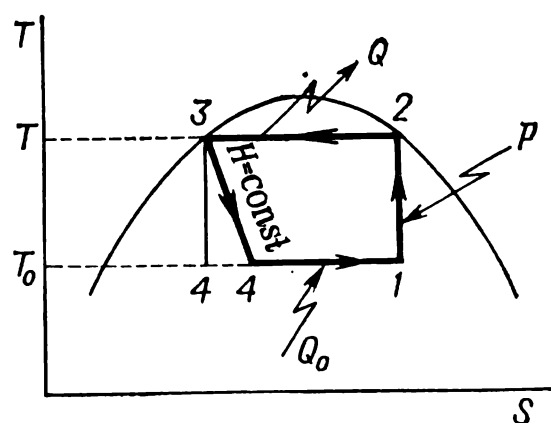


Fig. 11-2. Wet cycle



(a) without supercooling the liquid refrigerant (process 1-2-3-4-4'):

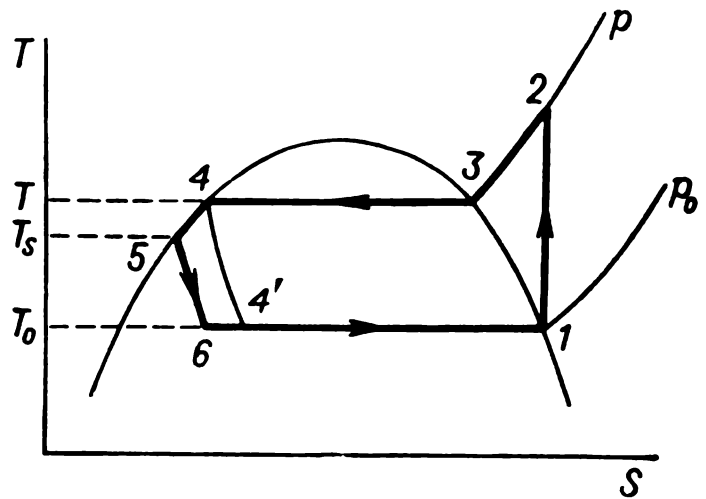
$$\epsilon = \frac{Q_0}{P} = \frac{H_1 - H_{4'}}{H_2 - H_1} = \frac{H_1 - H_4}{H_2 - H_1} \quad (11-3)$$

(b) with supercooling of the liquid refrigerant (process 1-2-3-4-5-6):

$$\epsilon = \frac{Q_0}{P} = \frac{H_1 - H_6}{H_2 - H_1} = \frac{H_1 - H_5}{H_2 - H_1} \quad (11-4)$$

where $Q_0 = G (H_1 - H_5)$ = refrigerating capacity of the installation, W
 $P = Q - Q_0 = G (H_2 - H_1)$ = theoretical power used by the compressor, W
 $Q = G (H_2 - H_5)$ = amount of heat given up by the refrigerant to the water in the condenser (including supercooling of the liquid refrigerant), W
 G = rate of flow of the refrigerant in the cycle, kg/s
 H_1, H_2, \dots = specific enthalpies of the refrigerant at the relevant points of the cycle, J/kg.

Fig. 11-3. Dry cycle



In Fig. 11-4, a dry cycle of a single-stage compression refrigerating installation is shown in the coordinates p against H .

4. The actual power P_a (in kW) used by the compressor of a refrigerating installation is:

$$P_a = \frac{P}{1000\eta} \quad (11-5)$$

Here η is the overall efficiency equal to

$$\eta = \eta_i \eta_{\text{mech}} \eta_{\text{tr}} \eta_{\text{mot}} \quad (11-6)$$

where η_i = indicated efficiency of the compressor that takes into account the difference of an actual working process from the theoretical (isentropic) one; the value of η_i depends on the compression ratio of the refrigerant, i. e. on the ratio of the condensation pressure p to the evaporation pressure p_0 ; approximate values of η_i for ammonia compressors are given in Fig. 11-5

η_{mech} = mechanical efficiency of the compressor taking into account the losses due to friction

η_{tr} = efficiency of the transmission

η_{mot} = efficiency of the compressor motor.

In approximate calculations, it is usually assumed that $\eta_{\text{mech}} = 0.8$ to 0.9 , and $\eta_{\text{tr}} = \eta_{\text{mot}} = 0.95$.

5. The refrigerating capacity of a compressor Q_0 (in W) is:

$$Q_0 = \lambda V_g q_v \quad (11-7)$$

where λ = volumetric efficiency of the compressor—the ratio of the actual volume of the vapour sucked in by the compressor a second to the geometrical volume V_g (in m^3/s) displaced by the piston; for ammonia compressors the values of the volumetric efficiencies given in Fig. 11-5 and depending on the ratio of the pressure p in the condenser to that in the evaporator p_0 may be used

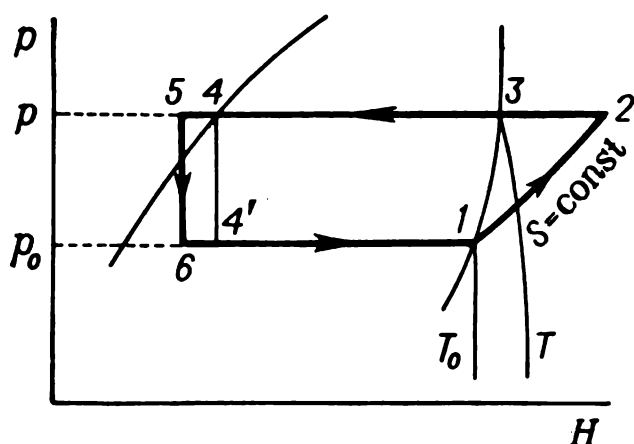


Fig. 11-4. Dry cycle in the coordinates p against H

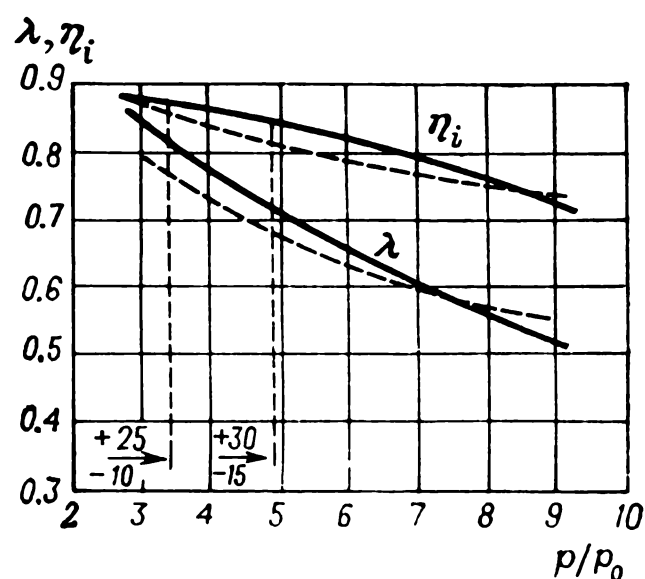


Fig. 11-5. Values of efficiencies η_i and λ for vertical in-line ammonia compressors (solid lines) and for double-action horizontal ammonia compressors (dash lines)

q_v = volumetric refrigerating capacity (in J/m^3) of the refrigerant equal to:

$$q_v = \rho_1 (H_1 - H_5) \quad (11-8)$$

Here H_1 and H_5 are the specific enthalpies of the refrigerant entering the evaporator and leaving it, respectively (see Fig. 11-3 or 11-4), J/kg , and ρ_1 is the density of the vapour sucked in by the compressor, kg/m^3 .

6. To recalculate the refrigerating capacity of a compressor Q_0 for other conditions at a constant speed (Q'_0), the following formula can be used:

$$\frac{Q_0}{Q'_0} = \frac{q_v \lambda}{q'_v \lambda'} \quad (11-9)$$

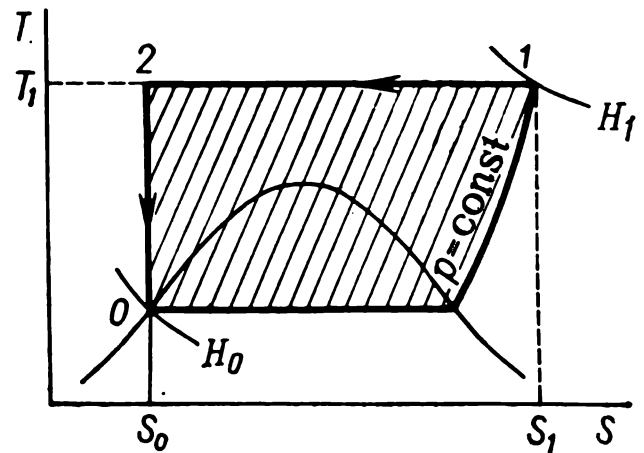
An evaporation temperature of -10°C , a condensation temperature of 25°C , and a temperature of supercooling the liquid refrigerant of 15°C are considered to be the standard conditions of operation of a vapour compression refrigerating installation with single-stage compression.

7. The minimum work needed to liquefy 1 kg of a gas in an ideal liquefaction process (Fig. 11-6) is:

$$W_{\min} = T_i (S_i - S_0) - (H_i - H_0) \quad (11-10)$$

where T_i , S_i , and H_i = temperature, specific entropy and enthalpy of the gas in the initial state (point 1)
 S_0 and H_0 = specific entropy and enthalpy of the liquid (point 0).

Fig 11-6. Ideal process of liquefaction



Although an ideal liquefaction process cannot be achieved in practice, W_{\min} is important as a reference which real cycles are compared with.

8. Liquefaction of air with its expansion without giving up external work—throttling (the Linde cycle).

(a) Simple regenerative cycle—see Example 11-12.

The specific refrigerating capacity of the cycle q (in J/kg) is:

$$q = H_1 - H_3 \quad (11-11)$$

where H_1 and H_3 are the specific enthalpies of the expanded and compressed air, respectively, reduced to the heat exchanger inlet temperature, J/kg.

The fraction of air being liquefied is:

$$y = \frac{q - q_1}{H_1 - H_0} = \frac{H_1 - H_3 - q_1}{H_1 - H_0} \quad (11-12)$$

where H_0 = specific enthalpy of liquid air (at the pressure of the expanded air), J/kg

q_1 = total losses of cold related to 1 kg of air being liquefied.

(b) Cycle with preliminary (ammonia) cooling.

The specific refrigerating capacity of the cycle q' (in J/kg):

$$q' = H'_1 - H'_3 \quad (11-13)$$

where H'_1 and H'_3 are the specific enthalpies of the expanded and compressed air, respectively, reduced to the temperature of admission into the main heat exchanger after the ammonia cooler, J/kg.

The fraction of air being liquefied is:

$$y = \frac{q' - q_1}{H'_1 - H_0} = \frac{H'_1 - H'_3 - q_1}{H'_1 - H_0} \quad (11-14)$$

The specific amount of heat q_a (in J/kg) transferred in the ammonia heat exchanger (per kilogram of compressed air) is:

$$q_a = q' - q + y(H_1 - H'_1) \quad (11-15)$$

where q' = specific refrigerating capacity of the cycle [Eq. (11-13)]
 q = specific refrigerating capacity of the cycle if there is no preliminary ammonia cooling [Eq. (11-11)]

H_1 and H'_1 = the same as in Eqs. (11-11) and (11-13).

(c) Cycle with air circulation under pressure—see Example 11-14.

The specific refrigerating capacity of the cycle q (in J/kg) is:

$$q = (H_2 - H_3) + z(H_1 - H_2) \quad (11-16)$$

where H_1 , H_2 , H_3 = specific enthalpies of the expanded air, the compressed air with a medium pressure, and the compressed air with a high pressure, respectively, reduced to the temperature of admission to the main heat exchanger, J/kg
 z = fraction of the air throttled to a low pressure (it usually ranges from 0.2 to 0.4).

The liquefied fraction of the air is:

$$y = \frac{(H_2 - H_3) + z(H_1 - H_2) - q_1}{H_1 - H_0} \quad (11-17)$$

The notation is the same as in Eq. (11-12).

9. Liquefaction of air by its expansion with giving up of external work in an expander.

(a) Medium-pressure (Claude) cycle—see Example 11-15.

The specific refrigerating capacity of the cycle q (in J/kg) is:

$$q = (H_1 - H_2) + z(H_3 - H_4) \quad (11-18)$$

where H_1 , H_2 = specific enthalpies of the expanded and compressed air, respectively, reduced to the temperature of admission into the main heat exchanger, J/kg

H_3 , H_4 = specific enthalpies of the compressed air at the inlet to the expander and of the air at the outlet from it, J/kg

z = fraction of air directed into the expander (it is generally assumed that $z = 0.8$).

For the air expanding in the expander, it is usually assumed in this cycle that

$$H_3 - H_4 = 0.65\Delta H_{ch} = 0.65(H_3 - H_5) \quad (11-19)$$

where ΔH_{ch} = change in the specific enthalpy of the air in an isentropic process

H_5 = specific enthalpy of the air at the pressure after the expander and at the same entropy as H_3 , J/kg.

The liquefied fraction of the air is:

$$y = \frac{(H_1 - H_2) + z(H_3 - H_4) - q_1}{H_1 - H_0} \quad (11-20)$$

The notation is the same as in Eq. (11-18).

(b) High-pressure (Heylandt) cycle — see Example 11-16.

The specific refrigerating capacity of the cycle q (in J/kg) is:

$$q = (H_1 - H_3) + z(H_3 - H_4) \quad (11-21)$$

where H_1, H_3 = specific enthalpies of the expanded and compressed air, respectively, at the temperature of admission into the main heat exchanger and into the expander, J/kg

H_4 = specific enthalpy of the air at the outlet from the expander, J/kg

z = fraction of air directed into the expander (it is generally assumed that z ranges from 0.45 to 0.5).

In this cycle, we can assume for the air expanding in the expander that:

$$H_3 - H_4 = 0.75\Delta H_{ch} = 0.75(H_3 - H_5) \quad (11-22)$$

The liquefied fraction of the air is:

$$y = \frac{(H_1 - H_3) + z(H_3 - H_4) - q_1}{H_1 - H_0} \quad (11-23)$$

(c) Low-pressure (Kapitza) cycle with expansion turbine — see Example 11-18.

The liquefied fraction of the air is:

$$y = \frac{(H_1 - H_2) + (1 - ay)(H_3 - H_4) - q_1}{H_1 - H_0} \quad (11-24)$$

where H_1, H_2 = specific enthalpies of the expanded and compressed air, respectively, reduced to the temperature of admission into the main heat exchanger, J/kg

H_3, H_4 = specific enthalpies of the compressed air entering the expansion turbine and leaving it, J/kg

a = factor taking into account evaporation in throttling the liquid from the condensation pressure to atmospheric pressure (at a condensation pressure of $p_{abs} = 5.89 \times 10^5 \text{ Pa} = 6 \text{ at}$, the factor a is 1.25).

We can assume for the air expanding in the expansion turbine that:

$$H_3 - H_4 = 0.8\Delta H_{ch} = 0.8(H_3 - H_5) \quad (11-25)$$

where H_5 is the specific enthalpy of the air at the pressure after the expansion turbine and at the same entropy as H_3 , J/kg.

10. The losses of cold q_1 consist of two addends:

$$q_1 = q_{inc} + q_{sur} \quad (11-26)$$

The losses of cold due to incomplete recuperation (in J/kg) are:

$$q_{inc} = c_p \Delta t \quad (11-27)$$

where c_p = specific heat capacity of the gas (at constant pressure) at the temperature at the outlet from the heat exchanger, J/kg·K

Δt = difference between the temperature of the compressed air entering the heat exchanger and that of the expanded air leaving the exchanger, K.

The losses of cold to the surroundings (through the insulation) q_{sur} are usually from 4 to 12 kJ per m³ (in standard conditions) of the air being processed.

EXAMPLES

Example 11-1. Determine the refrigeration coefficient of a compressor refrigerating installation operating according to the Carnot cycle if the temperature in the evaporator is -23°C , and in the condenser is 27°C .

Solution. By Eq. (11-1), we get:

$$\epsilon_C = \frac{T_0}{T - T_0} = \frac{273 - 23}{(273 + 27) - (273 - 23)} = 5$$

Example 11-2. Calculate the theoretical power used by a refrigerating installation operating according to the Carnot cycle and removing 17 400 J/s at -19°C (the evaporation temperature). The condensation temperature is 15°C .

Solution. The refrigeration coefficient is:

$$\epsilon_C = \frac{T_0}{T - T_0} = \frac{254}{268 - 254} = 7.5$$

According to Eq. (11-1), the theoretical power is:

$$P = \frac{Q_0}{\epsilon_C} = \frac{17\,400}{7.5} = 2320 \text{ W} = 2.32 \text{ kW}$$

Example 11-3. Find the minimum (for the Carnot cycle) theoretical power of the compressor in an ammonia refrigerating installation and the rate of flow of the water in the condenser when 500 kg of ice are produced an hour from water having a temperature of 0°C . Ammonia boils at -7°C and condenses at 20°C . The water in the condenser is heated from 10 to 15°C .

Solution. The heat liberated in freezing of the water is:

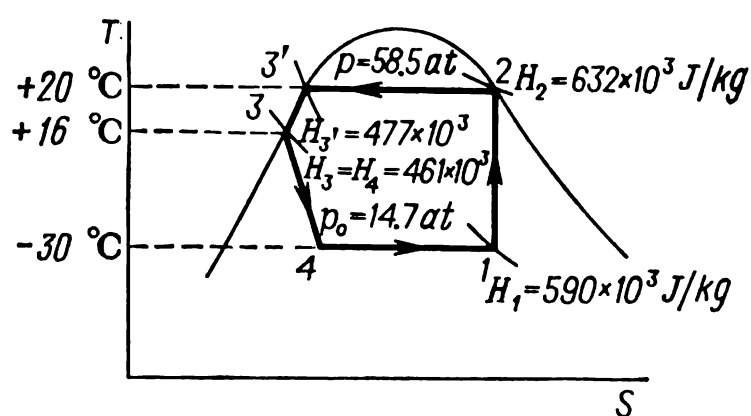
$$Q_0 = \frac{500 \times 339.1 \times 1000}{3600} = 47\,100 \text{ W}$$

where 339.1×10^3 J/kg is the specific heat of freezing of water.

The power of the compressor (minus the work of isentropic expansion) is:

$$P = \frac{Q_0}{\epsilon_C} = \frac{T - T_0}{T_0} Q_0 = \frac{293 - 266}{266} \times 47\,100 = 4780 \text{ W}$$

Fig. 11-7. To Example 11-4



The amount of heat removed by the water in the condenser is:

$$Q = Q_0 + P = 47\,100 + 4780 = 51\,880 \text{ W} = 51.9 \text{ kW}$$

The consumption of water is:

$$\frac{51\,880}{(15 - 10) 4.187 \times 1000} = 2.478 \text{ kg/s}$$

Example 11-4. Determine the following quantities for a carbon-dioxide refrigerating installation operating according to the wet cycle: the specific refrigerating capacity of the refrigerant, the refrigeration coefficient, the amount of heat removed in the condenser, the amount of circulating refrigerant, and the theoretical power used. The temperature of evaporation is -30°C , that of condensation is 20°C , and that of supercooling is 16°C . The required refrigerating capacity of the installation is $58\,150 \text{ W}$.

Figure 11-7 shows the cycle in the coordinates T - S . The values of the specific enthalpies have been determined with the aid of a T - S diagram for carbon dioxide (Fig. A-27). Line 1-2 shows compression, 2-3'—condensation, 3'-3—supercooling of the liquid refrigerant, 3-4—throttling, and 4-1—evaporation.

Solution. The specific refrigerating capacity of the refrigerant is:

$$q_0 = H_1 - H_4 = 590 \times 10^3 - 461 \times 10^3 = 129 \times 10^3 \text{ J/kg}$$

The refrigeration coefficient, by Eq. (11-2), is:

$$\varepsilon = \frac{H_1 - H_4}{H_2 - H_1} = \frac{(590 - 461) 10^3}{(632 - 590) 10^3} = 3.1$$

The amount of heat removed in the condenser per kg of circulating carbon dioxide is:

$$q = H_2 - H_3 = 632 \times 10^3 - 461 \times 10^3 = 171 \times 10^3 \text{ J/kg}$$

The rate of flow of the refrigerant circulating in the cycle is:

$$G = \frac{Q_0}{q_0} = \frac{58\,150}{129 \times 10^3} = 0.448 \text{ kg/s}$$

The amount of heat removed in the condenser is:

$$Q = 171 \times 10^3 \times 0.448 = 76\,900 \text{ W} = 76.9 \text{ kW}$$

The required theoretical power is:

$$P_{th} = G (H_2 - H_1) = 0.448 (632 \times 10^3 - 590 \times 10^3) = \\ = 0.448 \times 42 \times 10^3 = 18\,700 \text{ W} = 18.7 \text{ kW}$$

Example 11-5. Determine the volumetric efficiency of a simple-action ammonia compressor having a piston stroke of $s = 0.32 \text{ m}$, a cylinder diameter of $D = 0.25 \text{ m}$, and a speed of $n = 180 \text{ rpm}$. The cycle is dry without supercooling. The evaporation temperature is -10°C , the condensation temperature is 30°C , the refrigerating capacity is $93\,000 \text{ W}$.

Solution. The specific refrigerating capacity of ammonia is:

$$q_0 = H_1 - H_4 = (1430 - 324) \times 10^3 = 1106 \times 10^3 \text{ J/kg}$$

The specific enthalpies have been determined from a T - S diagram for ammonia (Fig. A-26).

The volumetric refrigerating capacity of ammonia is:

$$q_v = q_0 \rho_1 = 1106 \times 2.39 \times 10^3 = 2640 \times 10^3 \text{ J/m}^3$$

where $\rho_1 = 2.39 \text{ kg/m}^3$ is the density of the vapour sucked in by the compressor (Table A-48).

The volume displaced by the piston is:

$$V_g = \frac{\pi D^2}{4} s \frac{n}{60} = \frac{3.14 \times 0.25^2}{4} \times 0.32 \times \frac{180}{60} = 0.047 \text{ m}^3/\text{s}$$

The volumetric efficiency, by Eq. (11-7), is:

$$\lambda = \frac{Q_0}{V_g q_v} = \frac{93\,000}{0.047 \times 2640 \times 10^3} = 0.745$$

Example 11-6. A vertical ammonia compressor has a refrigerating capacity of $Q_0 = 174\,000 \text{ W}$ in standard conditions. What will the refrigerating capacity of this compressor be if it operates with an evaporation temperature of -25°C , a condensation temperature of 30°C , and a supercooling temperature of 25°C ?

Solution. The refrigerating capacity of the compressor in operating conditions can be determined by Eq. (11-9)

$$Q'_0 = Q_0 \frac{q'_v \lambda'}{q_v \lambda}$$

To find the values of q_v , q'_v , λ , and λ' we proceed as follows.

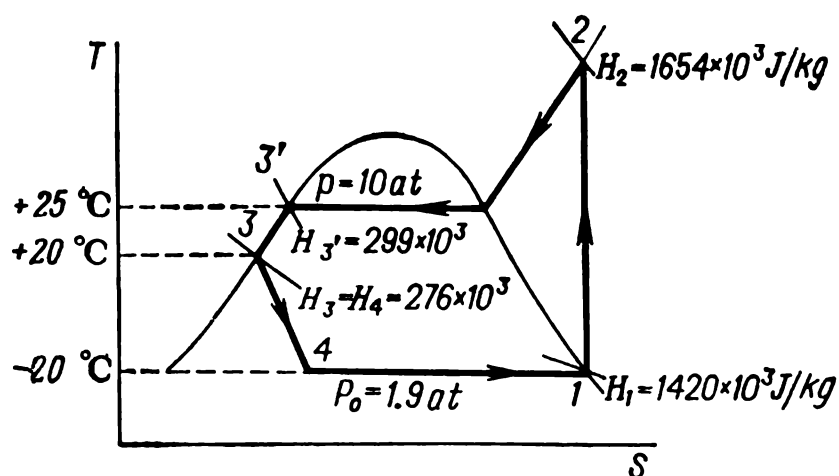
On the basis of the definition of the standard operating conditions ($t_e = -10^\circ\text{C}$, $t_{\text{cond}} = 25^\circ\text{C}$, $t_{\text{sup}} = 15^\circ\text{C}$), we find with the aid of a T - S diagram (Fig. A-26), Table A-48 and Eq. (11-8) that

$$q_v = q_0 \rho_1 = (H_1 - H_4) \rho_1 = (1430 \times 10^3 - 251 \times 10^3) 2.39 = 2830 \times 10^3 \text{ J/m}^3$$

Similarly, we find q'_v :

$$q'_v = (H'_1 - H'_4) \rho' = (1410 \times 10^3 - 299 \times 10^3) 1.297 = 1440 \times 10^3 \text{ J/m}^3$$

Fig. 11-8. To Example 11-7



To determine λ , we find the ratio of the condensation and evaporation pressures in standard conditions, i. e. the ratio of the pressure p corresponding to the condensation temperature of 25°C and the pressure p_0 corresponding to the evaporation temperature of -10°C (Table A-48):

$$\frac{p}{p_0} = \frac{1004}{281} = 3.45$$

Figure 11-5 gives us $\lambda = 0.82$ for this ratio of the pressures.

In a similar way, we determine λ' on the basis of the given operating conditions. We find the ratio of the pressure p corresponding to the condensation temperature of 30°C and the pressure p_0 corresponding to the evaporation temperature of -25°C :

$$\frac{p}{p_0} = \frac{1167}{152} = 7.7$$

For this pressure ratio, Fig. 11-5 gives us $\lambda' = 0.57$.

The operating refrigerating capacity of the compressor is:

$$Q'_0 = Q_0 \frac{q'_v \lambda'}{q_v \lambda} = 174\,000 \times \frac{1440 \times 10^3 \times 0.57}{2830 \times 10^3 \times 0.82} = 62\,000 \text{ W} = 62 \text{ kW}$$

Example 11-7. Determine the dimensions of the cylinder and also the actual power used for a single-stage single-action ammonia compressor operating according to the dry cycle. The required refrigerating capacity is $290\,000 \text{ W}$ at an evaporation temperature of -20°C . The condensation and supercooling temperatures are 25 and 20°C , respectively. Also find the refrigeration coefficient of the installation.

The cycle corresponding to the given conditions is shown in Fig. 11-8 in the coordinates T - S . Line 1-2 is compression, 2-3' is cooling of the superheated vapour and condensation, 3'-3 is supercooling of the liquid, 3-4 is throttling, and 4-1 is evaporation.

Solution. We find the specific enthalpies of the ammonia at the various points of the cycle in a T - S diagram (Fig. A-26).

The specific refrigerating capacity of the ammonia is:

$$q_0 = H_1 - H_4 = (1420 - 276) 10^3 = 1144 \times 10^3 \text{ J/kg}$$

The rate of flow of the circulating ammonia is:

$$G = \frac{Q_0}{q_0} = \frac{290\,000}{1144 \times 10^3} = 0.254 \text{ kg/s}$$

The volumetric refrigerating capacity of the ammonia is:

$$q_v = q_0 \rho_1 = 1144 \times 10^3 \times 1.6 = 1830 \times 10^3 \text{ J/m}^3$$

where $\rho_1 = 1.6 \text{ kg/m}^3$ is the density of the vapour sucked in by the compressor and determined from Table A-48.

The volume displaced by the piston a second is determined by the equation:

$$V_g = \frac{Q_0}{q_v \lambda}$$

The ratio $p/p_0 = 10/1.9 = 5.3$, and from Fig. 11-5 we find $\lambda = 0.7$. Hence,

$$V_g = \frac{290\,000}{1830 \times 10^3 \times 0.7} = 0.227 \text{ m}^3/\text{s}$$

If we assume a compressor speed of $n = 210 \text{ rpm}$, then the volume of the cylinder is:

$$V = \frac{0.227 \times 60}{210} = 0.065 \text{ m}^3$$

The ratio of the piston stroke s to the cylinder diameter D usually ranges from $s/D = 1.1$ to 1.3 . We adopt this ratio equal to 1.2 . Hence,

$$0.065 = \frac{\pi D^2}{4} \times 1.2 D$$

whence $D = 0.41 \text{ m} = 410 \text{ mm}$ and $s = 410 \times 1.2 = 490 \text{ mm}$.

The theoretical power of the compressor is:

$$P_{\text{th}} = G (H_2 - H_1) = 0.254 (1654 - 1420) 10^3 = 59.5 \times 10^3 \text{ W} = 59.5 \text{ kW}$$

The actual power used is:

$$P = \frac{P_{\text{th}}}{\eta}$$

By Eq. (11-16), the overall efficiency η is:

$$\eta = \eta_i \eta_{\text{mech}} \eta_{\text{tr}} \eta_{\text{mot}}$$

We find the value of the indicated efficiency η_i at $p/p_0 = 5.3$ from Fig. 11-5, $\eta_i = 0.84$.

We adopt $\eta_{\text{mech}} = 0.85$, $\eta_{\text{tr}} = 0.95$, and $\eta_{\text{mot}} = 0.95$. Hence,
 $\eta = 0.84 \times 0.85 \times 0.95 \times 0.95 = 0.643$

The actual power is thus:

$$P = \frac{59.5}{0.643} = 92.5 \text{ kW}$$

The refrigeration coefficient is:

$$\varepsilon = \frac{q_0}{H_2 - H_1} = \frac{1144 \times 10^3}{(1654 - 1420) 10^3} = 4.88$$

Example 11-8. Using the conditions of the preceding example, determine the amount of water supplied to the condenser and the heat-transfer surface area of the latter (including the supercooling of the liquid ammonia) if the initial temperature of the water is 15°C , the final temperature is 21°C , the overall coefficient of heat transfer for the condensation zone (including the cooling of the superheated vapour) is $K' = 2330 \text{ W/m}^2 \cdot \text{K}$, and for the liquid ammonia supercooling zone is $K'' = 465 \text{ W/m}^2 \cdot \text{K}$.

Solution. We determine the amount of heat transferred in the condenser by the equation:

$$Q = G (H_2 - H_3) = 0.254 (1654 - 276) 10^3 = 350 \times 10^3 \text{ W} = 350 \text{ kW}$$

The amount of heat transferred in the zone of cooling the superheated vapour and condensation is:

$$Q' = G (H_2 - H'_3) = 0.254 (1654 - 299) 10^3 = 344 \times 10^3 \text{ W} = 344 \text{ kW}$$

The amount of heat transferred in the zone of supercooling the liquid ammonia is:

$$Q'' = (350 - 344) 10^3 = 6 \times 10^3 \text{ W} = 6 \text{ kW}$$

The rate of flow of the water supplied to the condenser is:

$$G_w = \frac{Q}{(t_{\text{fin}} - t_{\text{in}}) c_w} = \frac{350 \times 10^3}{(21 - 15) 4.19 \times 10^3} = 13.9 \text{ kg/s}$$

The temperature of the water after the zone of cooling the liquid ammonia is:

$$t = 15 + \frac{6 \times 10^3}{13.9 \times 4.19 \times 10^3} = 15.1^\circ\text{C}$$

The mean temperature drop in the condensation zone without accounting for the temperature of ammonia superheating (Fig. 11-9) is:

$$\Delta t'_m = \frac{(25 - 15.1) - (25 - 21)}{2.3 \log \frac{25 - 15.1}{25 - 21}} = 6.5 \text{ K}$$

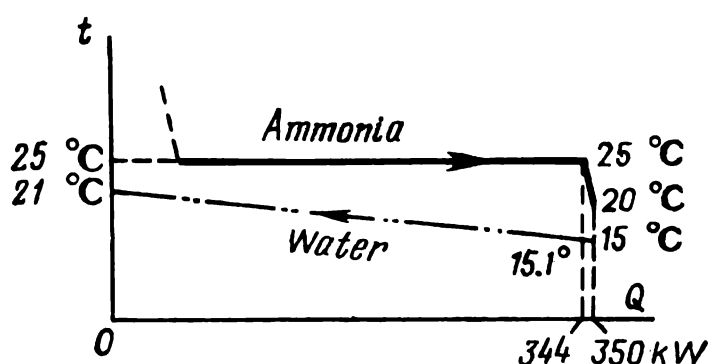


Fig. 11-9. To Example 11-8

The mean temperature drop in the zone of supercooling the liquid ammonia is:

$$\Delta t''_m = \frac{(25 - 15.1) + (20 - 15)}{2} = 7.4 \text{ K}$$

The area of the heat-transfer surface in the condensation zone is:

$$A' = \frac{Q'}{K' \Delta t'_m} = \frac{344 \times 10^3}{2330 \times 6.5} = 22.8 \text{ m}^2$$

The area of the heat-transfer surface in the supercooling zone is:

$$A'' = \frac{Q''}{K'' \Delta t''_m} = \frac{6 \times 10^3}{465 \times 7.45} = 1.68 \text{ m}^2$$

The total heat-transfer surface area of the condenser is:

$$A = A' + A'' = 22.8 + 1.68 = 24.5 \text{ m}^2$$

The unit heat load (the amount of heat removed from 1 m² of heat-transfer surface area) is:

$$q_A = \frac{350 \times 10^3}{24.5} = 14.3 \times 10^3 \text{ W/m}^2$$

Example 11-9. Air contains 5.8% (by volume) of carbon tetrachloride vapour. If this gas mixture is cooled under atmospheric pressure (760 mm Hg), then at what temperature will condensation of the carbon tetrachloride begin? Also determine the pressure which the gas mixture must be compressed to for 75% of the carbon tetrachloride contained in the air to condense upon its subsequent cooling at constant pressure to 30 °C.

Solution. Condensation of the carbon tetrachloride from the gas mixture will begin when its partial pressure in the mixture becomes equal to the saturated vapour pressure.

The partial pressure of the carbon tetrachloride in the gas mixture is $p_{c,t} = yp = 0.058 \times 760 = 44 \text{ mm Hg}$.

According to a diagram showing the temperature dependence of saturated carbon tetrachloride vapour (Fig. A-24), we find that a temperature of 8 °C corresponds to a pressure of 44 mm Hg. Consequently, the condensation of the carbon tetrachloride will begin at this temperature.

We find the pressure p up to which the initial gas mixture has to be compressed for 75% of the carbon tetrachloride to condense after cooling to 30°C.

Of the 5.8 moles of carbon tetrachloride contained in 100 moles of the initial mixture, according to the initial condition $5.8 \times 0.25 = 1.45$ moles should remain in the gaseous phase after condensation. The total number of moles in the gas mixture after condensation is $(100 - 5.8) + 1.45 = 95.65$.

The partial pressure of the carbon tetrachloride in the gaseous phase over the liquid equals its saturated vapour pressure at 30°C and is 145 mm Hg (Fig. A-24).

We find the total pressure p from the proportion $1.45/95.65 = 145/p$, whence

$$p = \frac{95.65 \times 145}{1.45} = 9565 \text{ mm Hg} = 13 \text{ at} = 1.275 \text{ MPa}$$

Example 11-10. Use a T - S diagram (Fig. A-29) to find the per cent of liquid air formed after the throttling of compressed air from 80 at and 140 K to 1 at.

Solution. We find the point of intersection of the isobar for 80 at with the isotherm for 140 K and from this point we follow a line $H = \text{const}$ up to the intersection with the isobar for 1 at. We take the ratio of the section on the 1-at isobar to the right of the found point up to the boundary line of the vapour to the entire length of the section between the line of the liquid and that of the vapour for $p = 1$ at and find that the vapour and liquid mixture formed contains 33% of liquid.

Example 11-11. Determine the amount of heat to be withdrawn from 1 kg of air to liquefy it at atmospheric pressure, and the theoretically minimum amount of work to be done for this purpose. The initial temperature of the air is 290 K.

Solution. From a T - S diagram for air (Fig. A-29), we find that at $p = 1$ at we have:

$$H_g - H_{lq} = 502 \times 10^3 - 92 \times 10^3 = 410 \times 10^3 \text{ J/kg}$$

We find the theoretically minimum amount of work to be done for liquefying 1 kg of air by Eq. (11-10):

$$\begin{aligned} W_{\min} &= T_1 (S_1 - S_0) - (H_1 - H_0) = 290 (3.75 - 0) 10^3 - 410 \times 10^3 = \\ &= 677 \times 10^3 \text{ J/kg} \end{aligned}$$

We have found the values of S_1 and S_0 from a T - S diagram for air.

Thus,

$$W_{\min} = \frac{677}{3600} = 0.188 \text{ kW} \cdot \text{h/kg}$$

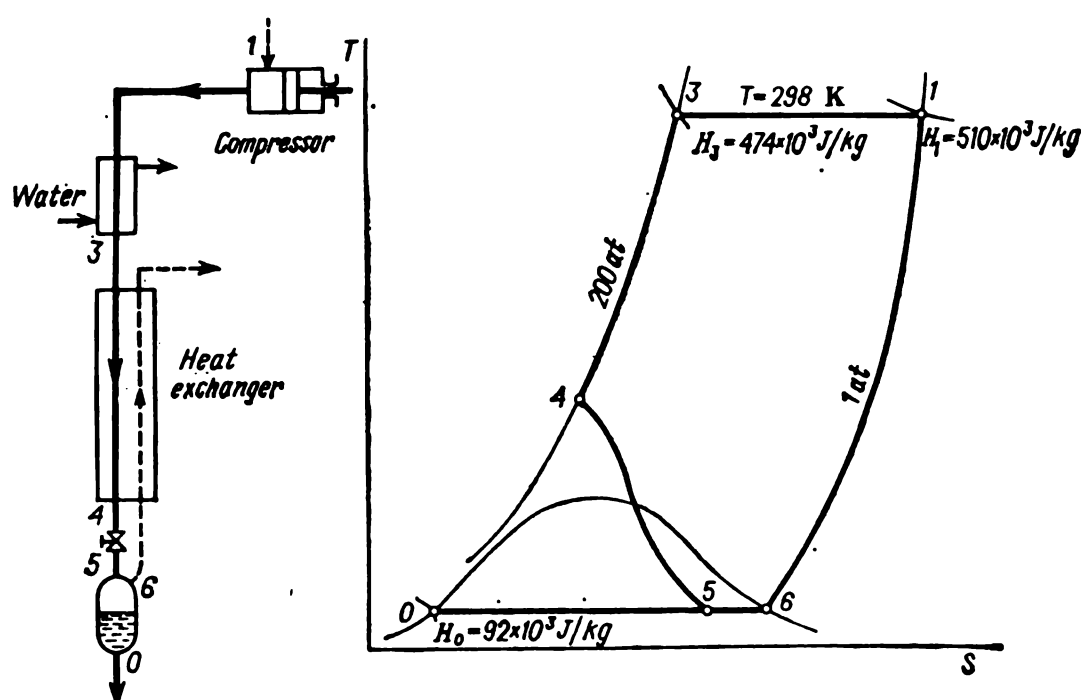


Fig. 11-10. To Example 11-12

Example 11-12. Determine the amount of air liquefied an hour and the power needed for this purpose when processing 200 kg/h of air compressed to 200 at. The installation operates according to a single-action regenerative Linde cycle. The temperature of the air entering and leaving the compressor (at the inlet to the heat exchanger) is 25°C. The air is throttled to 1 at. Assume that the losses of cold to the surroundings are 4.19×10^3 J per m^3 of air (in standard conditions). The deviation from complete recuperation is 5°C.

Solution. Figure 11-10 shows an elementary diagram of the installation and its cycle in T - S coordinates. We use a T - S diagram for air (Fig. A-29) to determine the specific enthalpies.

We determine the specific refrigerating capacity of the cycle by Eq. (11-11):

$$q = H_1 - H_3 = (510 - 474) 10^3 = 36 \times 10^3 \text{ J/kg}$$

We find the losses of cold per kg of air processed by Eqs. (11-26) and (11-27):

(a) the losses due to incomplete recuperation are*:

$$q_{\text{inc}} = c_p \Delta t = 1.01 \times 10^3 \times 5 = 5.05 \times 10^3 \text{ J/kg}$$

(b) the losses to the surroundings are:

$$q_{\text{sur}} = \frac{4.19 \times 10^3}{1.29} = 3.25 \times 10^3 \text{ J/kg}$$

The total losses are:

$$q_l = (5.05 + 3.25) 10^3 = 8.3 \times 10^3 \text{ J/kg}$$

* Here no account is taken of the reduction of the returning air stream by the amount of the liquefied air.

We find the fraction of the air that is liquefied by Eq. (11-12):

$$y = \frac{(H_1 - H_3) - q_1}{H_1 - H_0} = \frac{(36 - 8.3) 10^3}{(510 - 92.1) 10^3} = 0.067$$

The rate of flow of the liquefied air is $G_{\text{liq}} = 200 \times 0.067 = 13.4 \text{ kg/h}$.

We determine the power used on the compressor shaft by Eq. (2-25):

$$P = \frac{1.69 G_c R T \ln \frac{p_2}{p_1}}{3600 \times 1000} = \frac{1.69 \times 200 \times 287 \times 298 \times 2.3 \log \frac{200}{1}}{3600 \times 1000} = 42.5 \text{ kW}$$

The specific consumption of energy per kg of liquid air is $E_{\text{sp}} = 42.5/13.4 = 3.16 \text{ kW} \cdot \text{h/kg}$.

Example 11-13. Determine the rate of flow of the air processed to obtain 50 kg/h of liquefied air and the power used for this purpose if liquefaction follows a cycle with preliminary (ammonia) cooling of the air to -35°C . The compression pressure is 200 at, and throttling is to 1 at. The initial temperature of the air is 25°C . Assume that the total losses of cold to the surroundings and due to incomplete recuperation are 8.8 kJ/kg .

Solution. To determine the specific enthalpies, we use a T - S diagram for air (Fig. A-29).

We determine the specific refrigerating capacity of the cycle by Eq. (11-13):

$$q' = H'_1 - H'_3 = (449 - 386) 10^3 = 63 \times 10^3 \text{ J/kg}$$

where H'_1 = specific enthalpy of the air at $p = 1 \text{ at}$ and $t = -35^\circ\text{C}$
 H'_3 = ditto at $p = 200 \text{ at}$ and $t = -35^\circ\text{C}$.

We find the fraction of the air being liquefied by Eq. (11-14):

$$y = \frac{(H'_1 - H'_3) - q_1}{H'_1 - H'_0} = \frac{(63 - 8.8) 10^3}{(449 - 92) 10^3} = 0.152$$

The amount of air to be liquefied in order to obtain 50 kg/h of liquid air is:

$$G = \frac{50}{0.152} = 329 \text{ kg/h}$$

We find the amount of heat transferred in the ammonia heat exchanger by Eq. (11-15):

$$\begin{aligned} q_a &= q' - q + y(H_1 - H'_1) = \\ &= 63 \times 10^3 - 36.0 \times 10^3 + 0.152(510 - 449) 10^3 = 36.3 \times 10^3 \text{ J/kg} \end{aligned}$$

where $q = 36.0 \times 10^3 \text{ J/kg}$ and $H_1 = 510 \times 10^3 \text{ J/kg}$ (see Example 11-12).

We determine the power on the shaft of the air compressor:

$$P = \frac{1.69 G_c R T \ln \frac{p_2}{p_1}}{3600 \times 1000} = \frac{1.69 \times 329 \times 287 \times 298 \times 2.3 \log 200}{3600 \times 1000} = 69.5 \text{ kW}$$

The power used by the ammonia compressor is:

$$P_a = \frac{36.0 \times 329}{8540 \times 0.66} = 2.11 \text{ kW}$$

where 8540 kJ/kW·h is the theoretical specific refrigerating capacity of an ammonia refrigerating installation at the temperature of evaporation of the ammonia* of -40°C [11-7], and 0.66 is the adopted overall efficiency of the ammonia compressor.

The total power used is $P_{\text{tot}} = 69.5 + 2.11 = 71.61 \text{ kW}$.

The specific consumption of energy per kg of liquid air is $E_{\text{sp}} = 71.61/50 = 1.43 \text{ kW} \cdot \text{h/kg}$.

Example 11-14. Determine the liquefied fraction and the specific consumption of energy in liquefying air with circulation of the air under pressure. The high pressure of the air is 200 at, the medium pressure is 50 at. The temperature of the air entering the heat exchanger is 20°C . The fraction of the air throttled to a low pressure is $z = 0.3$. The total losses of cold due to incomplete recuperation and to the surroundings are 10.5 kJ per m^3 of high-pressure air (in standard conditions) entering the installation.

Solution. Figure 11-11 contains an elementary diagram of an installation with circulation of the air under pressure and shows the process in T - S coordinates. We use a T - S diagram for air (Fig. A-29) to determine the specific enthalpies. We find the specific refrigerating capacity of the cycle by Eq. (11-16):

$$q = (H_2 - H_3) + z(H_1 - H_2) = (494 - 467) 10^3 + 0.3(505 - 494) 10^3 = 30 \times 10^3 \text{ J/kg}$$

We determine the liquefied fraction y by Eq. (11-17):

$$y = \frac{(H_2 - H_3) + z(H_1 - H_2) - q_l}{H_1 - H_0} = \frac{30 \times 10^3 - \frac{10.5 \times 10^3}{1.29}}{(505 - 92) 10^3} = 0.053$$

We find the power on the compressor shaft by Eq. (2-25):

$$P' = 1.69 \times \frac{1}{3600} \times 287 \times 293 \times 2.3 \log \frac{200}{50} = 54.5 \text{ W}$$

$$P'' = 1.69 \times \frac{0.3}{3600} \times 287 \times 293 \times 2.3 \log \frac{50}{1} = 46.1 \text{ W}$$

* The temperature of -40°C has been taken with account of $\Delta t = 5 \text{ K}$ between the temperatures of the air and the ammonia.

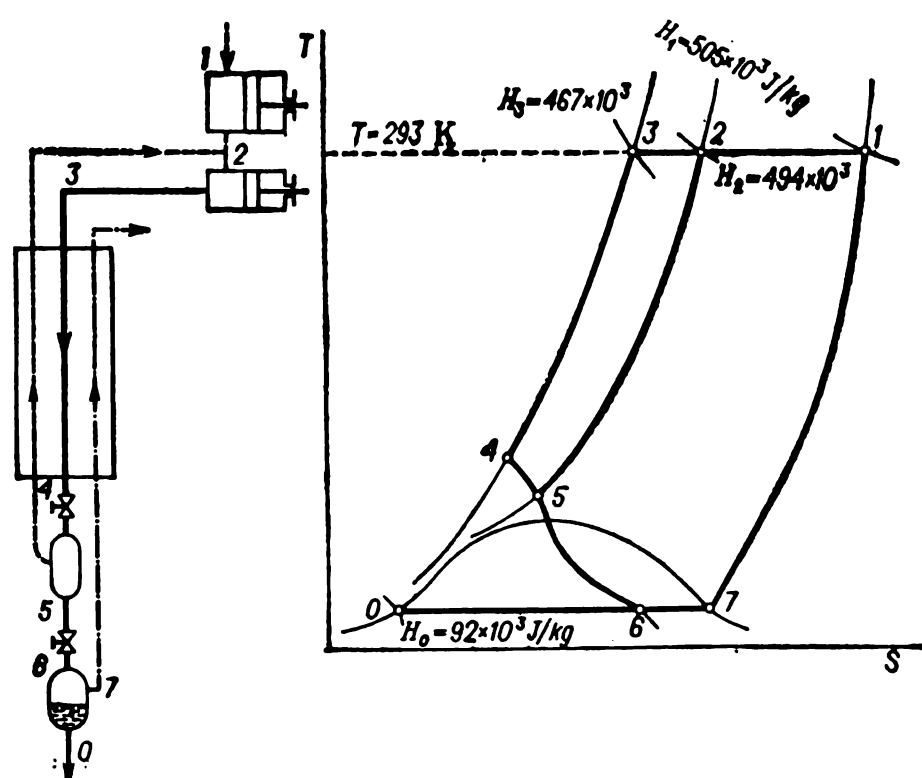


Fig. 11-11. To Example 11-14

The total power is:

$$P = \frac{54.5 + 46.1}{1000} = 0.1 \text{ kW}$$

The specific consumption of energy per kg of liquid air is:

$$E_{sp} = \frac{0.1}{0.053} = 1.89 \text{ kW} \cdot \text{h/kg}$$

Example 11-15. In liquefying air with the giving up of external work in an expander according to a medium-pressure cycle, a power of 110 kW (on the compressor shaft) is used. How many kilograms of liquid air an hour will be produced if the air is compressed to 35 at? The expander receives 80% of the entire amount of processed air; the temperature of the air entering the expander is -110°C , the deviation from complete recuperation is 5°C . The losses of cold to the surroundings (through the insulation) are 0.93 kW. Assume that the work returned by the expander to the compressor shaft is half the theoretical (isentropic) heat losses in the expander. The temperature of the air entering the heat exchanger after the compressor is 25°C . Also determine the specific consumption of energy per kg of liquid air.

Solution. Figure 11-12 contains an elementary diagram of the installation and also shows the process in the coordinates T - S . We use a T - S diagram for air (Fig. A-29) to determine the specific enthalpies.

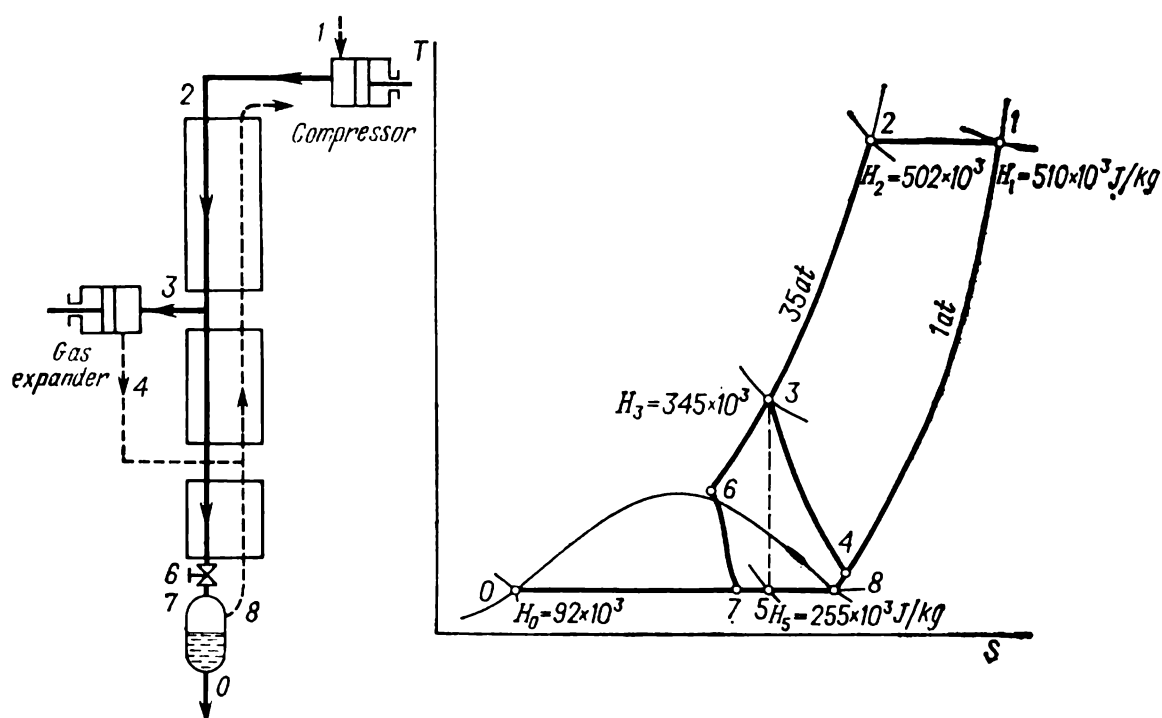


Fig. 11-12. To Example 11-15

We find the specific refrigerating capacity of the cycle by Eq. (11-18):

$$q = (H_1 - H_2) + z(H_3 - H_4) = (510 - 502) 10^3 + 0.8 \times 58 \times 10^3 = 54 \times 10^3 \text{ J/kg}$$

where $H_3 - H_4 = 0.65(H_3 - H_5) = 0.65(345 - 255) 10^3 = 58 \times 10^3 \text{ J/kg}$.

The work returned by the expander when 1 kg of air expands in it is:

$$w_{\text{exp}} = 0.5(345 - 255) 10^3 = 44.6 \times 10^3 \text{ J/kg}$$

We determine the amount of air G (in kg/s) compressed by the compressor from the equation

$$P = P_{\text{com}} - P_{\text{exp}} = 1.69GRT \times 2.3 \log 35 - 0.8G \times 44.6$$

According to the initial conditions, we have $P = 110 \text{ kW}$. Hence,

$$110 = 1.69G \times 287 \times 298 \times 2.3 \log 35 - 0.8 \times 44.6G$$

whence $G = 0.231 \text{ kg/s} = 0.231 \times 3600 = 835 \text{ kg/h}$.

We determine the losses of cold by equations * (11-26) and (11-27):

$$Q_{\text{inc}} = Gc_p \Delta t = 0.231 \times 1.01 \times 10^3 \times 5 = 1.17 \times 10^3 \text{ W}$$

According to the initial conditions, $Q_{\text{sur}} = 0.93 \times 10^3 \text{ W}$. Consequently, the total losses of cold are:

$$Q_1 = (1.17 + 0.93) 10^3 = 210 \times 10^3 \text{ W}$$

* Here no account is taken of the reduction of the returning air stream by the amount of the liquefied air.

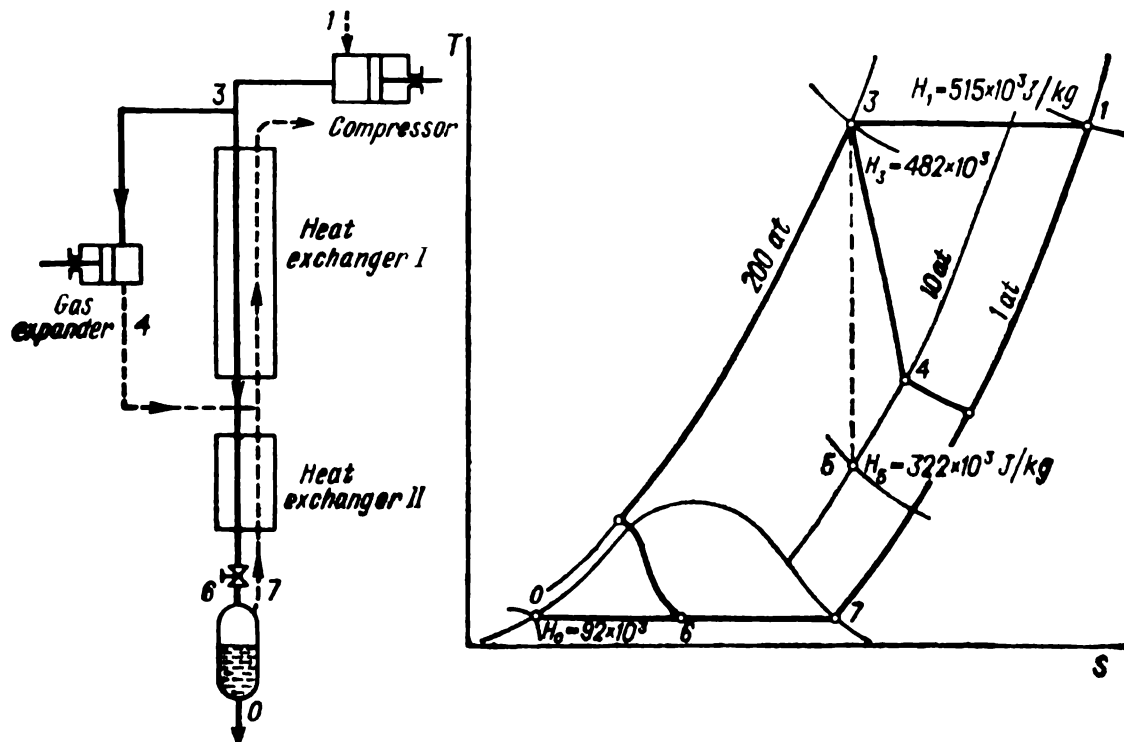


Fig. 11-13. To Example 11-16

or, per kilogram of processed air:

$$q_1 = \frac{2.10 \times 10^3}{0.231} = 9.1 \times 10^3 \text{ J/kg}$$

We determine the liquefied fraction of the air by Eq. (11-20):

$$y = \frac{(H_1 - H_2) + z(H_3 - H_4) - q_1}{H_1 - H_0} = \frac{(54 - 9.1) 10^3}{(510 - 92) 10^3} = 0.107$$

The amount of liquid air produced in the installation is $G_{lq} = 835 \times 0.107 = 89.3 \text{ kg/h}$. The specific consumption of energy per kg of liquid air is:

$$E_{sp} = \frac{110}{89.3} = 1.23 \text{ kW} \cdot \text{h/kg}$$

Example 11-16. Determine the efficiency of an air liquefaction process with giving up of external work according to a high-pressure cycle if the air is compressed by a compressor to 200 at. The initial temperature of the air is 30°C , and half the processed air is directed into a gas expander. Assume that the total losses of cold are 8.4 kJ per kg of air being processed. The air is expanded in the expander from 200 to 10 at. From 10 to 1 at, it is throttled. Take the work returned by the expander to the compressor shaft as in the preceding example.

Solution. Figure 11-13 contains an elementary diagram of the installation and also shows the process in the coordinates T - S . We use a T - S diagram for air (Fig. A-29) to determine the specific enthalpies.

We find the specific refrigerating capacity of the cycle by Eq. (11-21):

$$q = (H_1 - H_2) + z(H_3 - H_4) = (515 - 482) 10^3 + 0.5 \times 119 \times 10^3 = 93.2 \times 10^3 \text{ J/kg}$$

where, by Eq. (11-22), we have:

$$H_3 - H_4 = 0.75(H_3 - H_5) = 0.75(482 - 322) 10^3 = 119 \times 10^3 \text{ J/kg}$$

We find the liquefied fraction of the air by Eq. (11-23):

$$y = \frac{(H_1 - H_3) + z(H_3 - H_4) - q_1}{H_1 - H_0} = \frac{(93.2 - 8.4) 10^3}{(515 - 92) 10^3} = 0.2$$

The power used to compress the air is:

$$P_{\text{com}} = 1.69 G_c R T \ln \frac{p_2}{p_1} = 1.69 \times \frac{1}{3600} \times 287 \times 303 \times 2.3 \log 200 = 216 \text{ W}$$

The power returned to the compressor shaft when the air expands in the expander is:

$$P_{\text{exp}} = \frac{0.5}{3600} \times 0.5(482 - 322) 10^3 = 11.1 \text{ W}$$

The power actually spent on the compressor shaft is:

$$P = P_{\text{com}} - P_{\text{exp}} = 216 - 11.1 = 204.9 \text{ W} = 0.205 \text{ kW}$$

The specific consumption of energy per kg of liquid air is:

$$E_{\text{sp}} = \frac{0.205}{0.2} = 1.02 \text{ kW} \cdot \text{h/kg}$$

The efficiency of the liquefaction process is:

$$\eta = \frac{0.188}{1.02} \times 100 = 18.5\%$$

where 0.188 kW·h/kg is the theoretically minimum work of liquefaction of 1 kg of air at its initial temperature of 303 K (see Example 11-11).

Example 11-17. Determine the specific consumption of energy in the liquefaction of air following a low-pressure cycle with an expansion turbine (Fig. 11-14). The air enters the installation at 30 °C and an absolute pressure of 6 at. The deviation from complete recuperation is 5 °C, the losses to the surroundings are 8.4 kJ/m³ (at 0 °C and 760 mm Hg).

Solution. We adopt a temperature of the air entering the expansion turbine of 122 K. Hence, by Eq. (11-25), we have:

$$H_3 - H_4 = 0.8(H_3 - H_5) = 0.8(325 - 283) 10^3 = 33.5 \times 10^3 \text{ J/kg}$$

We use a *T-S* diagram for air (Fig. A-29) to determine the specific enthalpy.

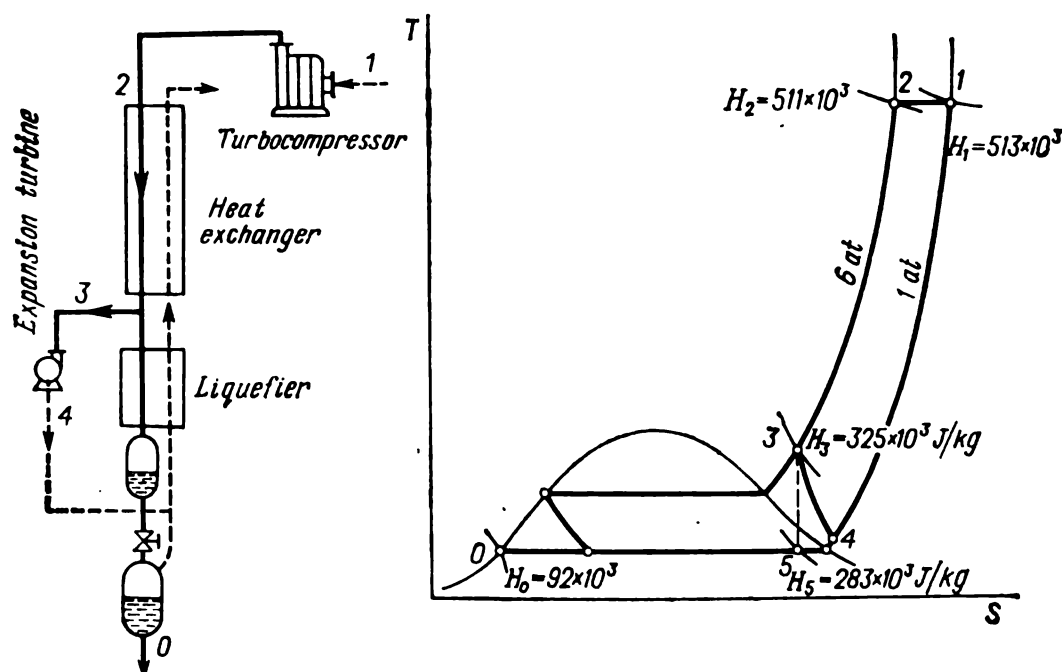


Fig. 11-14. To Example 11-17

We find the liquefied fraction of the air by Eq. (11-27):

$$y = \frac{\left[513 - 511 + 33.5 (1 - 1.25y) - \frac{8.4}{1.29} - 1.01 \times 5 (1 - y) \right] \times 10^3}{(513 - 92) \times 10^3}$$

whence $y = 0.052$.

We calculate the work returned by the expansion turbine per kg of processed air:

$$W_{\text{exp}} = \frac{(H_3 - H_4) (1 - ay) 0.85}{3600} = \frac{33.5 \times 10^3 (1 - 1.25 \times 0.052) 0.85}{3600} = 0.0074 \text{ kW} \cdot \text{h/kg}$$

where 0.85 is the power return factor of the expansion turbine, and $a = 1.25$ is a factor described in the explanations to Eq. (11-24).

The specific consumption of energy per kg of liquid air with an isothermal efficiency of the compressor of 0.6 is:

$$E_{\text{sp}} = \frac{\frac{287 \times 303 \times 2.3 \log 6.3}{1000 \times 0.6 \times 3600} - 0.0074}{0.052} = 1.26 \text{ kW} \cdot \text{kg/h}$$

where 6.3 is the compression ratio in the compressor (with account taken of the hydraulic losses of the installation).

Example 11-18. In an installation for the liquefaction of air operating with the giving up of external work according to a low-pressure cycle, the measured absolute pressure before the expansion turbine is 6 at at a temperature of 120 K. The pressure and temperature after the expansion turbine are 1.6 at and 89 K, respectively. The measured power given up by the expansion turbine is 100 kW. Determine the amount of air passing through the turbine and its thermodynamic efficiency.

Solution. We determine the actual ΔH_{act} and isentropic ΔH_{is} drop of heat in the expansion turbine using a T - S diagram for air (Fig. A-29).

Specific enthalpy of air at 6 at and 120 K	322×10^3 J/kg
Ditto, after isentropic expansion to 1.6 at	289×10^3 J/kg
Ditto, at 1.6 at and 89 K	296×10^3 J/kg

Hence,

$$\Delta H_{\text{act}} = (322 - 296) 10^3 = 26.6 \times 10^3 \text{ J/kg}$$

$$\Delta H_{\text{is}} = (322 - 289) 10^3 = 33.3 \times 10^3 \text{ J/kg}$$

The thermodynamic efficiency of the expansion turbine is:

$$\eta = \frac{26.6 \times 10^3}{33.3 \times 10^3} = 0.8$$

We approximately find the amount of air passing through the turbine (disregarding the energy losses):

$$G = \frac{3600P}{\Delta H_{\text{act}}} = \frac{3600 \times 100 \times 10^3}{26.6 \times 10^3} = 13\,500 \text{ kg/h}$$

Example 11-19. Find the required air pressure in steady-state conditions in an installation for the production of gaseous oxygen. The installation operates without preliminary cooling according to a single-action regenerative cycle. The deviation from complete recuperation is 5°C , the losses of cold through the insulation are 12.6 kJ per m^3 of processed air (at 0°C and 760 mm Hg). The temperature of the air entering the heat exchanger is 25°C .

Solution. We find the losses of cold by Eqs. (11-26) and (11-27):

$$q_{\text{inc}} = 1.01 \times 10^3 \times 5 = 5.05 \times 10^3 \text{ J/kg}$$

$$q_{\text{sur}} = \frac{12.6 \times 10^3}{1.29} = 9.74 \times 10^3 \text{ J/kg}$$

$$q_1 = 5.05 \times 10^3 + 9.74 \times 10^3 = 14.8 \times 10^3 \text{ J/kg}$$

To cover these losses, 1 kg of compressed air throttled in the installation should provide $\Delta H = 14.8 \text{ kJ}$.

We determine the throttling effect at 25°C (the temperature at which the compressed air enters the heat exchanger):

$$\Delta H = H_1 - H_3$$

where H_1 is the specific enthalpy of air at $p_{\text{abs}} = 1 \text{ at}$ and 25°C equal to $510 \times 10^3 \text{ J/kg}$ (according to a T - S diagram, Fig. A-29). Therefore, $14.8 \times 10^3 = 510 \times 10^3 - H_3$, whence $H_3 = 495.2 \times 10^3 \text{ J/kg}$.

We find from a T - S diagram that a pressure of $p_{\text{abs}} = 65 \text{ at}$ corresponds to this specific enthalpy of air at 25°C .

Example 11-20. An air-separating installation operating with the giving up of external work according to a medium-pressure cycle produces 100 m³/h (at 0 °C and 760 mm Hg) of gaseous oxygen and 25.8 kg of liquid oxygen; the purity of the oxygen is 99%. The air is compressed to 40 at. The temperature of the compressed air entering the expander is 160 K. Expansion proceeds in the expander to 6 at. The thermodynamic efficiency of the expander is 0.65. The discarded nitrogen contains 5% of oxygen. The deviation from complete recuperation is 5 °C. The temperature of the air entering the installation is 300 K. The losses to the surroundings are 6.3 kJ per m³ (at 0 °C and 760 mm Hg) of air handled. Determine the fraction of the air directed into the expander.

Solution. The total amount of oxygen produced is:

$$V = 100 + \frac{25.8}{1.43} = 118 \text{ m}^3/\text{h}$$

where 1.43 is the density of oxygen in standard conditions, kg/m³.

We find the amount (in standard conditions) of processed air (A) and discarded nitrogen (N) from the material balance equations:

$$A = 118 + N$$

$$0.21A = 0.99 \times 118 + 0.05N$$

whence $A = 695 \text{ m}^3/\text{h}$ and $N = 577 \text{ m}^3/\text{h}$.

We determine the losses of cold:
due to incomplete recuperation:

$$q_{\text{inc}} = \frac{(100 \times 1.43 \times 0.92 \times 10^3 + 577 \times 1.25 \times 1.05 \times 10^3)}{3600} = 1230 \text{ W} = 1.23 \text{ kW}$$

to the surroundings

$$q_{\text{sur}} = \frac{695 \times 6.3 \times 10^3}{3600} = 1210 \text{ W} = 1.21 \text{ kW}$$

carried off with the liquid oxygen

$$q_{\text{liq. oxyg}} = \frac{25.8 \times 406 \times 10^3}{3600} = 2930 \text{ W} = 2.93 \text{ kW}$$

Here 406×10^3 is the difference between the specific enthalpies of gaseous oxygen at 300 K and liquid oxygen (at 1 at), J/kg.

The total losses are:

$$Q_{\text{tot}} = 1230 + 1210 + 2930 = 5370 \text{ W} = 5.37 \text{ kW}$$

The refrigerating capacity of the installation obtained as a result of throttling is:

$$G\Delta H_{\text{thr}} = \frac{695 \times 1.29 \times 8.38 \times 10^3}{3600} = 2080 \text{ W} = 2.08 \text{ kW}$$

where 8.38×10^3 is the difference between the specific enthalpies of air at 1 and 40 at (at 300 K), J/kg.

The theoretical (isentropic) heat drop in the expander is:

$$\Delta H_{is} = H' - H'' = (333 - 283) 10^3 = 50 \times 10^3 \text{ J/kg}$$

where H' and H'' are the specific enthalpies for $p = 40$ at and $T = 160$ K, and for $p = 6$ at, respectively.

The actual heat drop is:

$$\Delta H_{act} = 50 \times 10^3 \times 0.65 = 32.5 \times 10^3 \text{ J/kg}$$

We find the amount of air entering the expander from the equation:

$$5370 - 2080 = 32.5 \times 10^3 x$$

whence $x \approx 0.1 \text{ kg/s} = 360 \text{ kg/h}$.

The fraction of the air directed to the expander is $360/695 \times 1.29 = 0.40$.

Example 11-21. Determine the amount of moisture liberated and the consumption of energy for cooling 3000 kg/h of air (dry basis) saturated with water vapour at $p_{abs} = 6$ at. The air is cooled (by boiling ammonia) from 0 to -40°C . The refrigerating capacity of the ammonia refrigerating installation is 5130 kJ/kW·h.

Solution. When cooling dry air, it is necessary to remove the following amount of heat:

$$Q_1 = G_{air} c_p (t_{in} - t_{fin}) = \frac{3000 \times 1.03 \times 10^3 [0 - (-40)]}{3600} = 34\,300 \text{ W}$$

where $c_p = 1.03 \times 10^3 \text{ J/kg}\cdot\text{K}$ is the specific heat capacity of dry air at $p_{abs} = 6$ at within the given temperature range.

We find the amount of moisture liberated from the air when it is cooled.

The amount of moisture in the air entering the refrigerator is:

$$G'_w = G_{air} \times 0.622 \times \frac{\varphi p_s}{p - \varphi p_s} = 3000 \times \frac{0.622 \times 1 \times 4.6}{6 \times 760 - 4.6} = 1.89 \text{ kg/h}$$

where $p_s = 4.6 \text{ mm Hg}$ is the saturated vapour pressure of water at 0°C (Table A-38).

The amount of moisture in the air leaving the refrigerator is:

$$G''_w = 3000 \times 0.622 \times \frac{0.09}{6 \times 760 - 0.09} = 0.038 \text{ kg/h}$$

where $p_s = 0.09 \text{ mm Hg}$ is the saturated vapour pressure of water at -40°C .

The amount of moisture liberated in the refrigerator is:

$$G_w = G'_w - G''_w = 1.89 - 0.038 = 1.852 \text{ kg/h}$$

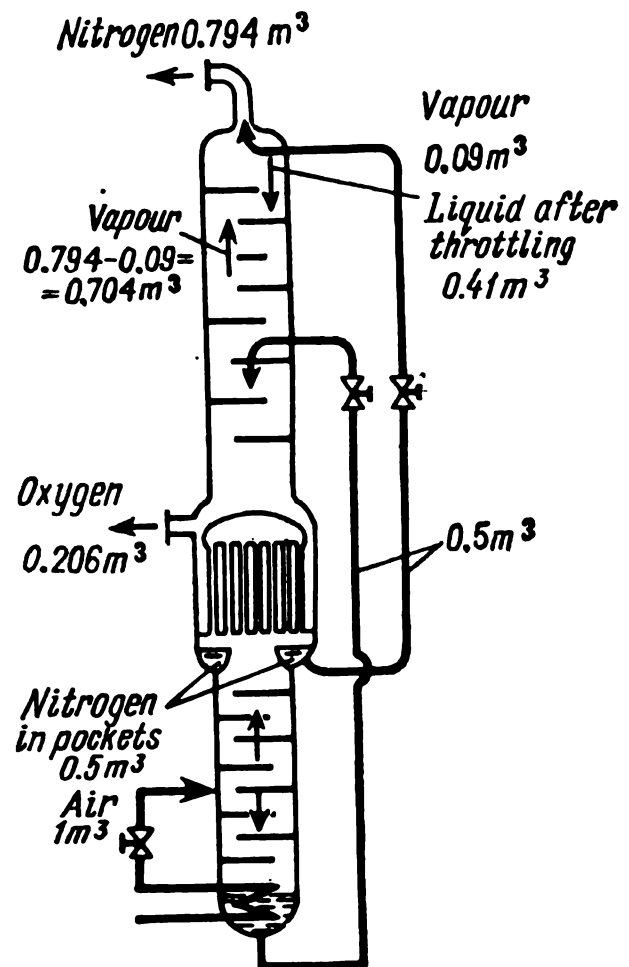


Fig. 11-15. To Examples 11-23 and 11-24

The amount of heat liberated upon condensation and freezing of the moisture is:

$$Q_2 = G_w [L_c + L_f + c(t_{in} - t_{fn})] = \frac{1.852 [2490 \times 10^3 + 335 \times 10^3 + 2.09 \times 10^3 (0 + 40)]}{3600} = 1490 \text{ W} = 1.49 \text{ kW}$$

where $L_c = 2490 \times 10^3 \text{ J/kg}$ is the specific heat of condensation at 0°C

$L_f = 335 \times 10^3 \text{ J/kg}$ is the specific heat of fusion

$c = 2.09 \times 10^3 \text{ J/kg} \cdot \text{K}$ is the specific heat capacity of ice.

The total amount of heat removed by the ammonia is:

$$Q_1 + Q_2 = 34\,300 + 1490 = 35\,790 \text{ W} \approx 35.8 \text{ kW}$$

The power used by the ammonia refrigerating installation is:

$$P = \frac{35.8 \times 3600}{5130} = 25 \text{ kW}$$

Example 11-22. Determine the rate of flow of the air used to produce $100 \text{ m}^3/\text{h}$ of oxygen having a purity of 99%. The discarded nitrogen contains 4% of oxygen.

Solution. We denote by x the required amount of air (in m^3). Hence,

$$0.21x = 100 \times 0.99 + (x - 100) 0.04$$

where 0.21 is the volume fraction of oxygen in air. Solution of the equation yields $x = 560 \text{ m}^3/\text{h}$.

Example 11-23. Determine for the bottom column of a two-column air-separating apparatus (Fig. 11-15) the amount of nitrogen formed in the pockets and the amount of still liquid enriched with oxygen per cubic metre of fed in air. The purity of the nitrogen in the pockets is 98%, the still liquid contains 40% of oxygen.

Solution. We denote by x the amount of nitrogen (in m^3) obtained in the pockets, and by y the amount of still liquid. Hence $x + y = 1$, and (with respect to oxygen) $0.02x + 0.4y = 0.21 \times 1$.

Solution of this material balance equation yields $x = 0.5 \text{ m}^3$ and $y = 0.5 \text{ m}^3$ (at 0°C and 760 mm Hg).

Example 11-24. Determine the reflux ratio and the amount of nitrogen and oxygen produced (per cubic metre of air) in the top column of a two-column air-separating apparatus (Fig. 11-15) taking into account the data of the preceding example and assuming that the purity of the nitrogen is 99% and that of the oxygen 98%.

Solution. We compile a material balance equation. Let x stand for the amount of withdrawn nitrogen and y for the amount of withdrawn oxygen. Hence,

$$\begin{aligned}x + y &= 1 \\0.01x + 0.98y &= 0.21\end{aligned}$$

whence $x = 0.794 \text{ m}^3$ and $y = 0.206 \text{ m}^3$.

We find the ratio of the amount of liquid L flowing down the column to the amount of rising vapour G .

The liquid nitrogen reflux is fed to the top of the column from the pockets of the bottom column, and as can be seen from a phase diagram for nitrogen, when the liquid nitrogen is throttled from 6 to 1 at about 18% of vapour is formed. It was found in the preceding example that 0.5 m^3 of nitrogen is obtained in the pockets of the bottom column. Hence, the amount of liquid nitrogen entering the top column is:

$$L = (1 - 0.18) 0.5 = 0.41 \text{ m}^3$$

The amount of vapour rising along the top column is:

$$G = 0.794 - 0.5 \times 0.18 = 0.704 \text{ m}^3$$

where 0.5×0.18 is the amount of vapour formed after throttling of the liquid nitrogen from the pockets of the bottom column into the top column, m^3 .

The ratio $L/G = 0.41/0.704 = 0.582$.

We determine the reflux ratio [see the explanations to Eq. (7-24)]:

$$\frac{R}{R+1} = \frac{L}{G} = 0.582$$

whence $R = 1.39$.

Example 11-25. Find the consumption of energy in the preparation of 1 m³ of gaseous oxygen having a purity of 99.5% in an installation with double rectification operating according to a single-action regenerative cycle without preliminary ammonia cooling. The waste nitrogen contains 5% of oxygen. The total losses of cold are 6.3 kJ per kg of air being processed. The temperature of the air delivered to the installation (after the compressor) is 25 °C.

Solution. We determine the amount of air x needed to produce 1 m³ of oxygen:

$$0.21x = 1 \times 0.995 + (x - 1) 0.05$$

whence $x = 5.9$ m³.

We determine the pressure to which the air is compressed from 1 at in steady-state conditions when the losses of cold are 6.3 kJ/kg:

$$\Delta H = H_1 - H_3$$

$$6.3 \times 10^3 = 510 \times 10^3 - H_3$$

where 510×10^3 is the specific enthalpy of air at 1 at and 25 °C, J/kg (from the T - S diagram in Fig. A-29).

Hence $H_3 = (510 - 6.3) 10^3 = 504.7 \times 10^3$ J/kg. We find from the T - S diagram that an absolute pressure of 60 at corresponds to this specific enthalpy of air at 25 °C.

We find the work needed to compress 1 kg of air from 1 to 60 at by the formula:

$$\begin{aligned} W &= 1.69 G R T \ln \frac{p_2}{p_1} = 1.69 \times 1 \times 287 \times 298 \times 2.3 \log 60 = \\ &= 590 \times 10^3 \text{ J/kg} = 590 \text{ kJ/kg} \end{aligned}$$

To obtain 1 m³ of oxygen, we need 5.9 m³ of air whose weight is 1.29 kg/m³. Hence, the consumption of energy for 1 m³ of oxygen is:

$$E_{sp} = \frac{590 \times 5.9 \times 1.29}{3600} = 1.25 \text{ kW} \cdot \text{h/m}^3$$

Example 11-26. Heat from the surroundings penetrates through the insulation of a liquid oxygen storage tank at a rate of 83.8 kJ/m²·h (with respect to the outer surface). The internal diameter of the spherical tank is 1200 mm. The external diameter of the outer housing having a cylindrical shape is 1900 mm,

and its height is 2300 mm. Determine how much oxygen evaporates an hour and how long it will take the entire oxygen to evaporate if the tank is filled up to 0.75 of its volume. The oxygen is stored under atmospheric pressure.

Solution. The area of the external surface of the cylindrical tank housing is:

$$A = 2 \frac{\pi D^2}{4} + \pi DH = 2 \times \frac{3.14 \times 1.9^2}{4} + 3.14 \times 1.9 \times 2.3 = 19.34 \text{ m}^2$$

The amount of heat that penetrates into the tank is:

$$Q = \frac{19.34 \times 83.8 \times 10^3}{3600} = 450 \text{ W} = 0.45 \text{ kW}$$

The rate of evaporation of the oxygen is:

$$G_e = \frac{450 \times 3600}{213.7 \times 10^3} = 7.6 \text{ kg/h}$$

where $213.7 \times 10^3 \text{ J/kg}$ is the specific heat of evaporation of oxygen under atmospheric pressure.

The amount of oxygen in the tank when it is filled up to 0.75 of its volume is:

$$G = 0.75 \frac{\pi D_{\text{int}}^2}{6} \times 1100 = 0.75 \times \frac{3.14 \times 1.2^3}{6} \times 1100 = 743 \text{ kg}$$

where 1100 kg/m^3 is the density of liquid oxygen.

The time needed for evaporation of the entire oxygen is:

$$\tau_e = \frac{743}{7.6} = 98 \text{ h}$$

PROBLEMS

11-1. Calculate the refrigeration coefficient and the power used by a refrigerating installation operating according to the Carnot cycle if its refrigerating capacity is 6400 W at an evaporation temperature of -10°C . The condensation temperature is 22°C .

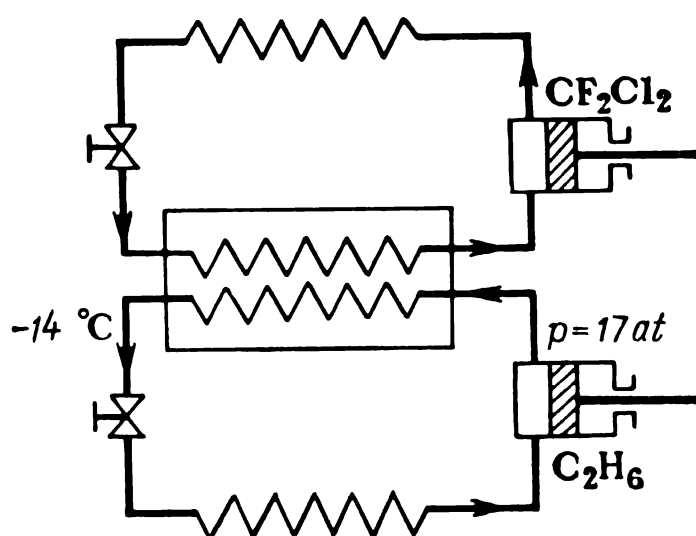
11-2. Find the minimum expenditure of work (according to the Carnot cycle) and the rate of flow of the water in a condenser when producing 100 kg/h of ice from water having a temperature of 0°C . The refrigerant evaporates at -5°C and condensates at 25°C . Water enters the condenser at 12°C and leaves it at 20°C . The specific heat of freezing of water is 335 kJ/kg .

11-3. Determine the specific refrigerating capacity of the refrigerant and the refrigeration coefficient of the cycle for (a) ammonia, (b) carbon dioxide, and (c) difluorodichloromethane CF_2Cl_2 (Freon-12). The evaporation temperature is -15°C , the condensation temperature is 30°C . A dry cycle is used, and there is no supercooling of the liquid before throttling.

11-4. Calculate the theoretical refrigeration coefficient for a carbon dioxide refrigerating installation if the condensation temperature is 20°C and the evaporation temperature is -40°C . A dry cycle is used, and there is no supercooling of the liquid before throttling.

11-5. Compare the theoretical refrigeration coefficients of an ammonia compression refrigerating installation operating at an evaporation temperature of

Fig. 11-16. To Problem 11-14



—20 °C and a condensation temperature of 30 °C (a) for a Carnot cycle, (b) for a real wet cycle, (c) for a dry cycle without supercooling of the liquid ammonia, and (d) for a dry cycle with supercooling of the liquid ammonia after condensation to 25 °C.

11-6. According to the conditions of the preceding problem, compare the theoretical refrigeration coefficients for a Freon refrigerating installation using a diagram of H against $\log p$ for Freon (Fig. A-28).

11-7. Water is heated by 6 K in the condenser of an ammonia refrigerating installation at a rate of 20 m³/h. The theoretical power used by the compressor is 23.5 kW. Determine the refrigerating capacity of the installation and the refrigeration coefficient.

11-8. Find the volumetric rate of flow of the ammonia entering a compressor an hour in the following conditions: the refrigerating capacity of the installation is 58 200 W, the condensation temperature is 25 °C, there is no supercooling, the evaporation temperature is —15 °C, a dry cycle is used.

11-9. Ethyl alcohol is to be cooled from 20 to —15 °C at a rate of 1000 kg/h. Ammonia boiling at —25 °C is used for this purpose. Find the theoretical power used by the compressor. The condensation temperature is 25 °C. A dry cycle is used, and there is no supercooling of the liquid before throttling.

11-10. A type ГД (horizontal) ammonia compressor has a refrigerating capacity of 697 800 W at an evaporation temperature of —15 °C and a condensation temperature of 25 °C. Determine its refrigerating capacity if the evaporation temperature is —5 °C and the condensation temperature is 30 °C.

11-11. A carbon dioxide refrigerating installation with a refrigerating capacity of 116 300 W operates at an evaporation temperature of —15 °C, an absolute pressure in the condenser of 75 at, and with supercooling to 25 °C. The compressor operates according to a dry cycle. Determine the refrigeration coefficient and the theoretically used power.

11-12. An ammonia refrigerating plant with a refrigerating capacity of 116 300 W having a vertical compressor operates at an evaporation temperature of —15 °C, a condensation temperature of 30 °C, and with supercooling to 25 °C. The compressor operates according to a dry cycle. Find the pressures in the condenser and the evaporator, the refrigeration coefficient, the volume of the vapour sucked in by the compressor an hour, the theoretical and actual power used, the temperature of the ammonia leaving the compressor, and the rate of flow of the water in the condenser when it is heated in it by 7 K.

11-13. An ammonia two-cylinder vertical compressor with single-stage compression has a cylinder diameter of 150 mm, a piston stroke of 150 mm, and a speed of 400 rpm. Calculate the refrigerating capacity of the compressor in standard conditions, and also in the operating conditions when evaporation occurs at $p_{\text{abs}} = 2$ at, and condensation at $p_{\text{abs}} = 12$ at. Before throttling, the ammonia is supercooled by 6 K. Also calculate the operating power.

11-14. In a cascade refrigerating installation operating according to Freon-ethane cycles (Fig. 11-16), the ethane condenses at -14°C under an absolute pressure of 17 at. The amount of heat transferred from the condensing ethane to the boiling Freon is 23 260 W. The evaporation temperature of the Freon is 5°C lower than the temperature of condensation of the ethane. The Freon condenses at 30°C , the liquid Freon is not supercooled, and the cycle is dry. Determine the compression ratio of the Freon in the compressor (the ratio of the condensation pressure to the evaporation one) and the rate of flow of water in the Freon condenser if it is heated by 8 K.

11-15. Air compressed to $p_{\text{abs}}=100$ at is cooled in the evaporator of a two-stage ammonia refrigerating installation to a temperature that is 5°C higher than the temperature of evaporation of ammonia. After cooling, the compressed air is throttled to a pressure of $p_{\text{abs}}=4$ at. Determine the temperature of the air after throttling if the ammonia evaporates under a pressure of $p_{\text{abs}}=0.42$ at.

11-16. Use a T - S diagram to determine the integral Joule-Thomson effect in the throttling of air to 1 at (a) with an initial temperature of the air of 15°C and an initial pressure of 50 at, and (b) at an initial temperature of the air of -50°C and an initial pressure of 200 at.

11-17. Determine the expenditure of energy per kilogram of liquid air produced according to a single-action regenerative cycle in the following conditions: (a) the initial temperature of the air is 15°C and it is compressed to 50 at, and (b) the initial temperature is 15°C and it is compressed to 200 at. Expansion in both cases is to 1 at. Disregard the losses of cold due to incomplete recuperation and to the surroundings.

11-18. Find the liquefied fraction of air and the consumption of energy per kg of liquid air in a single-action regenerative cycle with an initial air temperature of 30°C and a compression pressure of $p_{\text{abs}}=200$ at. The total losses of cold are 10.5 kJ per kilogram of processed air.

11-19. Determine the consumption of energy per kilogram of liquid air upon throttling air from 200 to 1 at in a cycle with preliminary ammonia cooling to -50°C . The specific refrigerating capacity of the ammonia refrigerating installation is 4820 kJ/kW·h. Disregard the losses of cold due to incomplete recuperation and to the surroundings. The initial temperature of the air is 15°C .

11-20. Determine the liquefied fraction of air and the consumption of energy per kilogram of liquid air in an installation operating with the circulation of air under pressure. The compression pressure is $p_{\text{abs}}=200$ at; the intermediate pressure is $p_{\text{abs}}=50$ at; the low pressure is 1 at; $z=0.2$; the initial air temperature is 25°C . Disregard the losses of cold.

11-21. Determine the power used and the amount of air liquefied in a medium pressure cycle with the giving up of external work when processing 300 m³/h of air (at 0°C and 760 mm Hg). The air is compressed to 40 at, the temperature of the air entering the expander is -80°C ; the temperature of the air leaving the compressor (entering the heat exchanger) is 30°C ; the fraction of the air directed into the expander is 0.8. Also find the consumption of energy per kilogram of liquid air. Assume that the total losses of cold are 11.5 kJ per kilogram of processed air.

11-22. Find the consumption of energy per kilogram of liquid air in a high pressure cycle with the giving up of external work when the air is compressed to 200 at and expanded to 8 at. The value of z is 0.5. The total losses of cold are 14.7 kJ per kilogram of processed air. The initial temperature of the air is 30°C .

11-23. When testing an expansion turbine, it was found that the air in it expands from 4 to 1.2 at, a power of 4 kW was obtained from the turbine, and 650 kg/h of air passed through it. Find the thermodynamic efficiency of the expansion turbine. Compressed air was fed into the turbine at 114 K.

11-24. Find the consumption of energy per kilogram of liquid air in a low pressure cycle with an expansion turbine if it is known that the compressor

compresses 6000 m³/h of air (in standard conditions) to $p_{\text{abs}} = 7$ at. The turbine gives up a power of 55 kW. The losses due to incomplete recuperation and to the surroundings are 6.3 kJ per m³ of compressed air (in standard conditions). Assume that the isothermal efficiency of the compressor is 0.7. Air is fed into the installation at 35 °C. The expansion turbine receives 80% of the air being processed. The evaporation factor is $a = 1.25$.

11-25. How many cubic metres of air must be processed to produce 200 m³ of oxygen with a purity of 99% if the waste nitrogen contains 10% of oxygen?

11-26. According to experimental data, the losses of cold are 335 kJ from each square metre of external surface of the housing of a thermally insulated cylindrical tank filled with liquid methane. The internal dimensions of the tank are $D = H = 1.1$ m. The insulation layer is 300 mm thick. How long will it take for all the liquid to evaporate if the tank was initially filled completely? The density of liquid methane is 415 kg/m³.

11-27. Determine the consumption of energy to liquefy 1 kg of methane according to a single-action regenerative cycle. The methane is compressed to a pressure of 150 at. The temperature of the methane leaving the compressor is 300 K. For a T - S diagram for methane see [11-9] or some other reference book.

11-28. Find the consumption of energy to liquefy one kilogram of methane in a cycle with preliminary ammonia cooling to -45 °C if the methane is compressed to 150 at. The specific refrigerating capacity of the ammonia refrigerating plant is 4820 kJ/kW·h.

11-29. Air is fed at a pressure of 20 at into an installation for producing gaseous oxygen operating according to a medium pressure cycle with the giving up of external work. The deviation from complete recuperation is 8 °C, and the losses of cold to the surroundings are 8.38 kJ per m³ of air being processed. The air expands in the expander from 20 at (at 140 K) to 6 at, the efficiency of the expander is 0.65. Determine the fraction of air directed into the expander ignoring the effect of throttling of the air from 6 to 1 at.

SYMBOLS

a	factor taking into account evaporation
c	specific heat capacity
D	diameter
E	energy
G	amount of gas; rate of flow of refrigerant
H	enthalpy; height
L	amount of liquid
n	speed (number of revolutions per unit of time)
P	power
Q	amount of heat
Q_0	refrigerating capacity
q	volumetric refrigerating capacity
S	entropy
s	piston stroke
T	absolute temperature
V	volume; volumetric rate of flow
W	work
w	specific work
y	liquefied fraction of air
z	fraction of air

Greek Letters

ε	refrigeration coefficient
η	efficiency
λ	volumetric efficiency of compressor
ρ	density

ANSWERS TO PROBLEMS

Chapter 1. Fundamentals of Applied Hydraulics

- 1-1. $M = 15.8$; $\rho = 0.616 \text{ kg/m}^3$.
- 1-2. 4.43 kg/m^3 .
- 1-3. 7.3% of CO_2 , 69.2% of N_2 , and 23.5% of H_2O .
- 1-4. $0.995 \times 10^5 \text{ Pa}$.
- 1-5. (a) 1.8 m; (b) 1.1 m.
- 1-6. 6 bolts, $0.716 \times 10^5 \text{ Pa}$ or 0.73 at.
- 1-7. $3.31 \times 10^4 \text{ N}$.
- 1-8. $0.33 \times 10^{-4} \text{ m}^2/\text{s}$.
- 1-9. $1.5 \times 10^{-5} \text{ Pa} \cdot \text{s}$.
- 1-10. 0.07 P = $0.007 \text{ Pa} \cdot \text{s}$.
- 1-11. 0.72 m/s.
- 1-12. $v_{\text{in}} = 13.1 \text{ m/s}$; $v_{\text{out}} = 10.1 \text{ m/s}$.
- 1-13. $v_{\text{lg}} = 2.0 \text{ m/s}$; $v_{\text{g}} = 10.4 \text{ m/s}$.
- 1-14. 73 mm.
- 1-16. 0.105 m.
- 1-17. Transition conditions.
- 1-18. (a) transition conditions; (b) laminar conditions.
- 1-19. 0.056 m/s.
- 1-20. 2840 kg/h.
- 1-21. $C_d = 0.632$; $\tau = 1 \text{ h } 13 \text{ min}$.
- 1-22. $0.06 \text{ m}^3/\text{s}$.
- 1-23. $v = 0.47 \text{ m/s}$; $Q_m = 47\,800 \text{ kg/h}$.
- 1-24. 280 kg/h.
- 1-25. $2.88 \times 10^4 \text{ Pa}$.
- 1-26. $1.37 \times 10^4 \text{ Pa}$ ($e = 0.01 \text{ mm}$).
- 1-27. 520 Pa ($e = 0.8 \text{ mm}$).
- 1-28. $1.15 \times 10^4 \text{ Pa}$ ($e = 0.2 \text{ mm}$).
- 1-29. (a) it will decrease to one-tenth of the original value;
(b) it will change $\frac{\lambda_1 T_1}{\lambda_0 T_0}$ times.
- 1-30. $56.5 \text{ m}^3/\text{h}$.
- 1-31. 50 mm.
- 1-32. (a) it will increase 16 times; (b) it will increase 32 times.
- 1-33. 7.1 m.
- 1-34. $5.9 \text{ dm}^3/\text{min}$.
- 1-35. 1.51 kW.
- 1-36. 821 Pa.
- 1-37. 15.6 Pa for the toluene, 513 Pa for the water.
- 1-39. The scale is 1 to 13.6; $v = 0.271 \text{ m/s}$.
- 1-40. 1.43 kW.

- 1-41. It will grow 8 times.
 1-42. 4.0 kW.
 1-43. 10.2 kW.
 1-44. 4.7 kW.
 1-45. $\Delta p = 3360$ Pa, $P = 0.35$ kW.
 1-46. $v = 1.7$ m/s; $\lambda = 0.0255$.
 1-47. 4.79×10^{-2} kg/s; laminar conditions.
 1-49. 21 m³/h.
 1-50. 1.71×10^4 kg/h in winter; 1.46×10^4 kg/h in summer.
 1-51. 776 Pa or 79 mm H₂O.
 1-52. 36.7×10^4 Pa or 3.74 at.
 1-53. 4.84 m.
 1-54. 354 Pa.

Chapter 2. Pumps, Fans, Compressors

- 2-1. 15.6 m.
 2-2. 467 m.
 2-3. 0.69.
 2-4. 18.3 kW.
 2-5. 43 °C.
 2-6. 0.037 m³/s; 0.0184 m³/s by the greater plunger and 0.0186 m³/s by the smaller one.
 2-7. 0.89.
 2-8. Not over 2.2 m.
 2-9. Yes, it can; 1.86 kW.
 2-10. $\eta = 0.59$; $Q = 71.2$ m³/h; $H = 68$ m; $P = 22.4$ kW.
 2-11. $Q_1 = 0.4$ m³/min; $Q_2 = 0.3$ m³/min.
 2-12. 0.257 m³/min.
 2-13. 18.2 m³/h.
 2-14. 4.2 kW.
 2-15. $Q = 4170$ m³/h; $\Delta p = 734$ Pa; $P = 1.77$ kW; $\eta = 0.48$.
 2-16. 940 m³/h.
 2-17. 1650 m³/h.
 2-18. 1800 rpm.
 2-19. 117 °C; 118 kJ/kg.
 2-20. 4.6 kW.
 2-21. 0.89.
 2-22. 3.25 m³/min; 13.0 kW.
 2-23. 5.46 m³/min; 13.2 kW.
 2-24. 20.3×10^5 Pa or 20.7 at.
 2-25. 3.62×10^5 Pa or 3.69 at for air; 9.20×10^5 Pa or 9.37 at for ethane.
 2-26. Single-stage compression—261 kJ/kg; double-stage compression—220 kJ/kg.
 2-27. 4 stages.
 2-28. Single-stage compression— 4.28×10^3 kJ/kg; double-stage compression— 3.54×10^3 kJ/kg.
 2-29. 10.7 m³/h (in standard conditions).
 2-30. Two-stage compression— $P = 83.2$ kW, the rate of flow of the water is 4.13 m³/h; three-stage compression— $P = 80$ kW, the rate of flow of the water is 3.96 m³/h.

Chapter 3. Hydromechanical Separation Methods. Hydrodynamics of a Fluidized Bed. Agitation in a Liquid Medium

- 3-1. (a) $d_2/d_1 = 1.73$; (b) $d_2/d_1 = 2.06$.
 3-2. (a) $v_{\text{set}} = 7.65 \times 10^{-5}$ m/s; (b) $v_{\text{set}} = 7.86 \times 10^{-3}$ m/s at 15 °C, and $v_{\text{set}} = 3.93 \times 10^{-3}$ m/s at 500 °C.
 3-3. 16.8 m/s.

- 3-4. 7.63 m/s.
- 3-5. 226 mm.
- 3-6. 11.4 μm (microns).
- 3-8. 5.14 m.
- 3-9. About two times.
- 3-10. UH-15; $\Delta p = 808 \text{ Pa} = 82.4 \text{ mm Hg}$.
- 3-12. 1070 kg/m^3 .
- 3-13. 0.08 m/s; 0.11 m/s.
- 3-14. 0.93 mm.
- 3-15. 2990 kg.
- 3-16. 4200 kg.
- 3-17. 1 h 36 min.
- 3-18. 2 h.
- 3-19. 1 h 20 min.
- 3-20. 58.5 min.
- 3-21. 110 cm^3/dm^3 .
- 3-23. r_2 increased by 8.5% in comparison with r_1 .
- 3-24. A suspended batch centrifuge.
- 3-25. A supercentrifuge.
- 3-26. $1.375 \times 10^5 \text{ Pa}$ or 1.40 at.
- 3-27. About 600 rpm.
- 3-28. (a) 1.04 mm; (b) 0.082 m.
- 3-30. 200 times.
- 3-31. 1 min 18 s.
- 3-32. 4.8 kW.
- 3-33. 3 centrifuges.
- 3-34. It will almost double.
- 3-35. 6.53 m^3/h .
- 3-36. 27 times.
- 3-37. 5.07 m^3/h .
- 3-38. 6.15 μm (microns).
- 3-39. 0.28 m/s; 2279 Pa.
- 3-40. 0.48; 462 mm.
- 3-41. 1.3 mm; 50%.
- 3-42. 137 W.
- 3-43. 240 rpm.
- 3-44. It will increase 1.3 times.
- 3-45. 0.58 m.

Chapter 4. Heat Transfer in a Chemical Apparatus

- 4-1. It will increase 10 times.
- 4-2. $1.74 \times 10^4 \text{ kJ}$.
- 4-3. 38 W.
- 4-4. It will grow by about 20%.
- 4-5. 217 $^\circ\text{C}$.
- 4-6. (a) 0.151 $\text{W/m}\cdot\text{K}$; (b) 0.0165 $\text{W/m}\cdot\text{K}$; (c) 0.471 $\text{W/m}\cdot\text{K}$.
- 4-7. (a) 285 kg/h ; (b) 259 kg/h ; (c) 2820 kg/h .
- 4-8. 94.6 $^\circ\text{C}$.
- 4-9. 92 000 W.
- 4-10. 548 kg/h .
- 4-11. 4.15 m^3/h .
- 4-12. Cocurrent — 36.7 $^\circ\text{C}$; countercurrent — 50.5 $^\circ\text{C}$.
- 4-13. 38.4 K.
- 4-14. No, its surface area is not sufficient.
- 4-15. 9.36 $\text{W/m}^2\cdot\text{K}$.
- 4-16. 2140 $\text{W/m}^2\cdot\text{K}$.

- 4-17. 63.8 W/m^2 .
 4-18. (a) It will not change; (b) it will grow by 9.1%.
 4-19. (a) from 52.6 to 50.3 $\text{W/m}^2\cdot\text{K}$; (b) from 1960 to 730 $\text{W/m}^2\cdot\text{K}$.
 4-20. 28 700 W/m^2 .
 4-21. 3.2 m^2 ; 582 kg/h.
 4-22. 124 $\text{W/m}^2\cdot\text{K}$.
 4-23. 664 $\text{W/m}^2\cdot\text{K}$.
 4-24. 1456 $\text{W/m}^2\cdot\text{K}$.
 4-25. (1) 41.8 $\text{W/m}^2\cdot\text{K}$; (2) 72.3 $\text{W/m}^2\cdot\text{K}$.
 4-26. 95 $\text{W/m}^2\cdot\text{K}$.
 4-27. It will grow 1.94 times.
 4-28. 4070 $\text{W/m}^2\cdot\text{K}$.
 4-29. 2110 $\text{W/m}^2\cdot\text{K}$.
 4-30. 284 $\text{W/m}^2\cdot\text{K}$.
 4-31. 1100 W/m^2 .
 4-32. 1675 $\text{W/m}^2\cdot\text{K}$.
 4-33. 5380 $\text{W/m}^2\cdot\text{K}$.
 4-34. 3900 $\text{W/m}^2\cdot\text{K}$.
 4-35. $L = 40.3 \text{ m}$; $Q_m = 1510 \text{ kg/h}$.
 4-36. $L = 3.38 \text{ m}$; $Q_m = 286 \text{ kg/h}$.
 4-37. (a) For water: $Q_v = 39.8 \text{ m}^3/\text{h}$, $A = 17.7 \text{ m}^2$;
 (b) For air: $Q_v = 111\,000 \text{ m}^3/\text{h}$ (in standard conditions), $A = 357 \text{ m}^2$.
 4-38. Two heat exchangers ($A = 56 \text{ m}^2$).
 4-39. $L = 3.36 \text{ m}$; $Q_m = 336 \text{ kg/h}$.
 4-40. (a) 258 $\text{W/m}^2\cdot\text{K}$; (b) 256 $\text{W/m}^2\cdot\text{K}$.
 4-41. $q = 128.5 \text{ W/m}^2$; $K = 1.92 \text{ W/m}^2\cdot\text{K}$.
 4-42. 214 W/m^2 .
 4-43. 771 kg/day.
 4-44. 1.8 times.
 4-45. 166 mm.

Chapter 5. Evaporation. Crystallization

- 5-1. (1a) 1.185 kg/kg; (1b) 1.023 kg/kg; (2a) 1.150 kg/kg; (2b) 1.068 kg/kg.
 5-2. 0.141 kg/s.
 5-3. It will decrease by 37.5%.
 5-4. $3.48 \times 10^5 \text{ Pa}$ or 3.55 at.
 5-5. 1100 kg.
 5-6. $W = 530 \text{ kg}$; $V = 0.56 \text{ m}^3$.
 5-7. 78 kW.
 5-8. 1993 J/kg·K.
 5-9. (a) $c = 2792 \text{ J/kg}\cdot\text{K}$; (b) $G_{st} = 196 \text{ kg}$.
 5-10. (a) $\bar{x}_{fn} = 44\%$; (b) $G_{st} = 2710 \text{ kg/h}$.
 5-11. 2.2 mm.
 5-12. (a) 178 kW; (b) 0.026 kW.
 5-13. (a) 1460 kg/h; (b) 980 kg/h; (c) 3.9 m^2 .
 5-14. $L = 305 \text{ kJ/kg}$; $c = 2080 \text{ J/kg}\cdot\text{K}$.
 5-15. $L = 2032 \text{ kJ/kg}$; $c = 2810 \text{ J/kg}\cdot\text{K}$.
 5-16. $t_b = 85^\circ\text{C}$; $L = 264 \text{ kJ/kg}$.
 5-17. 18.1 kPa or 136.1 mm Hg.
 5-18. 81.55 $^\circ\text{C}$.
 5-19. 152 m^2 .
 5-20. $2.92 \times 10^4 \text{ Pa}$ or 0.298 at.
 5-21. 43 m^2 ; 2170 kg/h.
 5-22. It will increase 1.95 times.
 5-23. 910 kg/h.
 5-24. 1495 kg/h; 85 m^2 .

- 5-25. (a) 35 m³/h; (b) 32 m³/h.
 5-26. $G_{in} = 1640$ kg/h; $G_{fin} = 560$ kg/h.
 5-27. 35 °C.
 5-28. I. 11.8% (mass); II. 18.0% (mass); III. 43.0% (mass).
 5-29. 4 effects.
 5-30. 13.6 kg/h, i.e. 4.1% of the total amount of water evaporating in the second effect.
 5-31. 25.5%.
 5-32. (a) $\bar{x}_{in} = 11.4\%$; (b) $\Delta t = 21.3$ K.
 5-33. 86 kg/h.
 5-34. (a) 180 kg/h; (b) 14 kW.
 5-35. 15 °C.
 5-36. 1300 kg.
 5-37. $A = 20.3$ m²; $W = 7660$ kg/h.

Chapter 6. Mass Transfer. Absorption

- 6-1. $\rho_{mix} = 1050$ kg/m³; $\bar{X} = 1.333$ kg of nitrobenzene/kg of benzene; $c_x = 4.86$ kmol of nitrobenzene/m³ of mixture.
 6-2. 1143 kg/m³.
 6-3. 1.59 kg of alcohol/kg of air; $\rho_{mix} = 1.08$ kg/m³.
 6-4. \bar{c}_y (kg/m³): H₂—0.634; CH₄—11.59; C₂H₄—4.740.
 6-6. $\Delta c_y = 5.15 \times 10^{-3}$ kmol of C₂H₂/m³ of gas; $\Delta \bar{c}_y = 0.1340$ kg of C₂H₂/m³ of gas; $\Delta c_x = 5.26 \times 10^{-3}$ kmol of C₂H₂/m³ of liquid; $\Delta \bar{c}_x = 0.1369$ kg of C₂H₂/m³ of liquid.
 6-7. (a) chloroform—from liquid to vapour phase; benzene—from vapour to liquid phase; (b) $\Delta x = 0.08$ kmol of chloroform/kmol of mixture; $\Delta y = 0.10$ kmol of chloroform/kmol of mixture.
 6-8. 13.8% (mass); 42.5%.
 6-9. 40.5%; 85%.
 6-11. 0.001 22 kmol/m²·h·kPa.
 6-12. $\Delta Y_m = 0.02$ kmol of benzene/kmol of inert gas; $n_{0y} = 1.66$.
 6-13. (1) $L = 175\,000$ kg/h; (2) $\Delta P_m = 5.67$ mm Hg; (3) $n_{0y} = 6.02$.
 6-14. $L = 1475$ kg/h; $h_p = 7.2$.
 6-15. $L = 760$ kg/h; $n_{0y} = 4.68$; $h_p = 1.93$ m.
 6-17. 0.038 m/s.
 6-18. 2.16×10^{-6} m/s.
 6-19. 0.0285 m/s.
 6-20. $D = 2.15$ m; $h_{pi} = 5.4$ m.
 6-21. (1) $h_p = 5.1$ m; (2) $K_y = 0.0132$ kg of ammonia/m²·s (kg of ammonia/kg of air).
 6-22. 5.1 m.
 6-23. (1) $L = 12.3$ ton/h; (2) $X_b = 0.0611$ kmol of benzene/kmol of oil; (3) $D = 1.59$ m and $H_p = 7.02$ m; (4) $H_{pi} = 4.0$ m.
 6-24. 30 m³/h.
 6-25. $D = 1.03$ m; $X_b = 0.149$ kmol of benzene/kmol of absorbent.

Chapter 7. Distillation and Rectification

- 7-1. (a) 99 °C; 10.2% (mass) of cresol, 89.8% (mass) of water; 1.85% (volume) of cresol; 14 mm Hg; (b) 74 °C; 7.25% (mass) of cresol, 92.75% (mass) of water; 1.29% (volume) of cresol; 3.9 mm Hg.
 7-2. 94 kg; 31.3 kg.
 7-3. $x_b = 0.408$; $p = 962$ mm Hg = 128.3 kPa.
 7-4. (a) $x = 0.675$; $y^* = 0.90$. (b) at 50 °C the pressure cannot exceed 400 mm Hg.

- 7-6. 4.2% (mole) of water; 13.0% (mole) of acetic acid; 82.8% (mole) of acetone.
 7-7. 465 kg; 43.8% (mass) of benzene.
 7-8. $W = 22.6$ kg; $D = 2577.4$ kg; $\bar{x}_D = 73.6\%$.
 7-9. 608 kmol/h.
 7-11. 12.1% (mass).
 7-12. 0.897.
 7-13. 13 000 kg/h; 7.2% (mass).
 7-14. $G_D = 17\,150$ kg/h; $G_W = 64\,700$ kg/h; $G_V = 68\,400$ kg/h.
 7-15. (a) 25.4% (mass); (b) 4330 kg/h.
 7-16. 68.4% (mole).
 7-17. 74.4% (mole); 80.3% (mole).
 7-18. (a) 76.7% (mole); (b) 17.96% (mole).
 7-19. $n_s = 9$; 720 kg/h.
 7-20. (a) 7.1% (mass); (b) $G_V = 4400$ kg/h; (c) $G_{\text{wat}} = 97\,100$ kg/h.
 7-21. 59.2 m²; 2170 kg/h.
 7-22. In the reboiler—5787 kJ/kg; in the reflux condenser—4700 kJ/kg.
 7-23. 3.21; 83 ton/h.
 7-24. 9.4 m²; 10.75 ton/h.
 7-25. $n_{\text{pl}} = 17$.
 7-26. 190 kg/h.
 7-27. (a) $G_D = 1760$ kg/h; $G_W = 4240$ kg/h; (b) $p = 328$ mm Hg; (c) $n_{\text{pl}} = 13$; (d) $G_{\text{st}} = 1435$ kg/h; $G_{\text{wat}} = 46.5$ ton/h.

Chapter 8. Extraction and Leaching

- 8-2. $x_{A_1} = 62.2$; $x_{B_1} = 24.8$; $x_{A_2} = 18.9$; $x_{B_2} = 25.1\%$ (mass); 1.74 kg.
 8-3. $G_{E_1} = 1940$; $G'_{E_1} = 180$ kg; $x_{E_1} \approx 55\%$ (mass) (after separation of the solvent);
 $G_{E_2} = 1735$; $G'_{E_2} = 99$ kg; $x_{E_2} = 35\%$ (mass) (after separation of the solvent);
 $G_{R_2} = 915$; $G'_{R_2} = 850$ kg; $x_{R_2} = 4\%$ (mass) (after separation of the solvent).
 8-4. $G_s = 1334$ kg/h; $n_s = 4$.
 8-5. $n_s = 3$; $x_F = 4$ g/dm³; 1.2 g/dm³; 0.6 g/dm³.
 8-6. (a) $G_{s, \text{min}} = 99$ kg; (b) $n_s = 8$.
 8-7. 63% (mass); 70% (mass).
 8-8. 38.9 kg; 256 kg.
 8-9. 77.8 kg; 450 kg; $n_s = 7$.
 8-10. In extract 58% (mass); in raffinate about 0%; $n_{s, \text{min}} = 11$.
 8-11. $n_s = 18$; $G_{s, 0} = 1718$ kg; $G_s = 1734$ kg.
 8-12. (a) 7.7 kg; (b) 99.6%; (c) 24.9% (mass).
 8-13. $n_s = 2$.
 8-14. $n_s = 6$.
 8-15. (a) 0.1 ton or 2%; (b) 28.8 ton; (c) I—10%, II—7.9%, III—6.1%, IV—4.7%, V—3.6%.
 8-16. 565 kg; $n_s = 3$.

Chapter 9. Adsorption

- 9-1. 1430 kg; 2.55 m; 1 h 22 min.
 9-2. $\tau = 225$ min; $\tau_0 = 65$ min.
 9-4. $u = 0.08$ m/s; $H_0 = 0.4$ m.
 9-5. $q = 228$ kJ/kg of carbon; $Q = 1.21 \times 10^6$ kJ.
 9-6. 2.94×10^{-5} m/s.

Chapter 10. Drying

- 10-1. 33 times.
- 10-2. $x = 0.060$ kg/kg; $H = 209$ kJ/kg; $t_w = 43$ °C; $t_{\text{dew}} = 42$ °C.
- 10-3. $x = 0.020$ kg/kg; $H = 105$ kJ/kg; $t_{\text{dew}} = 24$ °C; $\varphi = 0.25$; $p_v = 23$ mm Hg.
- 10-4. $x = 0.069$ kg/kg; $\varphi = 0.8$.
- 10-5. (a) 0.0159 kg/kg; (b) 0.231 kg/kg; (c) 0.0154 kg/kg.
- 10-6. $g_{\text{win}} = 36.4$ kg/kg; $q_{\text{win}} = 4350$ kJ/kg; $g_{\text{sum}} = 47.6$ kg/kg; $q_{\text{sum}} = 3710$ kJ/kg.
- 10-7. $p_v = p_{\text{air}} = 372.5$ mm Hg; $x = 0.622$ kg/kg.
- 10-8. $p_{\text{air}} = 6.173$ at $= 6.06 \times 10^5$ Pa; $\rho = 5.68$ kg/m³; $x = 0.083$ kg/kg.
- 10-9. 43.5 kg/kg; 4.6 kg/h.
- 10-10. $p_v = 2.755$ at $= 270.3$ kPa; $\rho = 5.05$ kg/m³; $x = 0.404$ kg/kg.
- 10-11. 3230 m³ of humid air an hour.
- 10-12. 25 °C.
- 10-13. 9700 kg/h; 566 kW.
- 10-14. $47\,200$ kg/h; 910 kW.
- 10-15. 71.3% .
- 10-16. 35.8 K.
- 10-17. 7.4 kPa or 56 mm Hg; 7.4% .
- 10-18. 16.5 h.
- 10-19. 5.81 m².
- 10-20. 33 °C; 0.39 .
- 10-21. 38 °C.
- 10-22. $G = 20\,800$ kg/h; $G_{\text{st}} = 950$ kg/h.
- 10-23. 320 kg/h; 132 m².
- 10-24. $G = 1710$ kg/h; $p_v = 30$ mm Hg; 3.95% ; $G_w = 1315$ kg/h.
- 10-25. $t_2 = 53$ °C; $x_2 = 0.035$ kg/kg; $\theta = 38$ °C.
- 10-26. 127 °C.
- 10-27. $G_{\text{air}} = 7400$ kg/h; $G_{\text{st}} = 590$ kg/h; $p_{\text{abs}} = 8$ at.
- 10-28. $G_{\text{air}} = 16\,200$ kg/h; $G_{\text{st}} = 960$ kg/h.
- 10-29. 34 W/m²·K; 156 kg/h.
- 10-30. 868 m²; 1865 kg/h.
- 10-31. (a) 2545 kJ/kg; (b) 3685 kJ/kg.
- 10-32. 401 kg/h; 150 m².
- 10-33. $G_{\text{air}} = 6840$ kg/h; $G_{\text{st}} = 565$ kg/h; $p_{\text{abs}} = 10$ at; $A = 135$ m².
- 10-34. $G_{\text{fin}} = 423$ kg/h; $A = 70$ m²; the fraction of recycled spent air is 60.8% .

Chapter 11. Moderate and Deep Refrigeration

- 11-1. $\varepsilon = 8.22$; $P = 0.78$ kW.
- 11-2. $W = 1042$ kW; $L_w = 0.31$ kg/s.
- 11-3. (a) $q_0 = 1110$ kJ/kg; $\varepsilon = 4.84$; (b) $q_0 = 119.6$ kJ/kg; $\varepsilon = 1.96$; (c) $q_0 = 118.5$ kJ/kg; $\varepsilon = 4.87$.
- 11-4. 1.89 .
- 11-5. (a) 5.06 ; (b) 4.38 ; (c) 4.16 ; (d) 4.27 .
- 11-6. (a) 5.06 ; (b) 4.50 ; (c) 4.00 ; (d) 4.13 .
- 11-7. $Q_0 = 116.5$ kW; $\varepsilon = 4.96$.
- 11-8. 99.5 m³/h.
- 11-9. 5.82 kW.
- 11-10. 1098 kW.
- 11-11. 3.08 ; 37.8 kW.
- 11-12. $p_{\text{con}} = 1166.9$ kPa $= 11.895$ at; $p_e = 206.4$ kPa $= 2.41$ at; $\varepsilon = 4.94$; $V = 189$ m³/h; $P_{\text{th}} = 23.6$ kW; $P = 36.3$ kW; $t = 97$ °C; $V_w = 17$ m³/h.
- 11-13. $Q_0 = 85\,950$ W; $Q' = 53\,730$ W; $P = 23.2$ kW.
- 11-14. $p/p_0 = 4.75$; $L_w = 3125$ kg/h.
- 11-15. -82 °C.
- 11-16. (a) 11 °C; (b) 73 °C.

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- 11-17. (a) $4.92 \text{ kW} \cdot \text{h/kg}$; (b) $2.10 \text{ kW} \cdot \text{h/kg}$.
11-18. 0.0545 kg/kg , the consumption of energy per kg of liquid air is $4 \text{ kW} \cdot \text{h/kg}$.
11-19. $1.084 \text{ kW} \cdot \text{h/kg}$.
11-20. 0.066 kg/kg ; the consumption of energy per kg of liquid air is $1.31 \text{ kW} \cdot \text{h/kg}$.
11-21. $P = 46.7 \text{ kW}$; $L_{\text{air}} = 46.6 \text{ kg/h}$; the consumption of energy per kg of liquid air is about $1 \text{ kW} \cdot \text{h/kg}$.
11-22. $1.07 \text{ kW} \cdot \text{h/kg}$.
11-23. 0.69 .
11-24. $0.995 \text{ kW} \cdot \text{h/kg}$.
11-25. 1618 m^3 .
11-26. 113 h .
11-27. $2.2 \text{ kW} \cdot \text{h/kg}$.
11-28. $1.15 \text{ kW} \cdot \text{h/kg}$.
11-29. 0.67 .

BIBLIOGRAPHY

General

- 0-1. Adolphi, G. and Adolphi, H. *Grundzüge der Verfahrenstechnik für Chemiker, Ökonomen-Ingenieure der chemischen Industrie*. Leipzig, VEB, Deutscher Verlag für Grundstoff-Industrie (1970).
- 0-2. Ainberg, V. D. *Osnovy programmirovaniya na algoritmicheskom yazyke ALGOL-60* (Fundamentals of Programming in the Algorithmic Language ALGOL-60). Moscow, Mashinostroenie (1973).
- 0-3. Alabuzhev, P. M., Geronimus, V. B., Minkevich, L. M., and Shekhovtsov, B. A. *Teorii podobiya i razmernosti. Modelirovanie* (The Theories of Similarity and Dimensions. Modelling). Moscow, Vysshaya shkola (1968).
- 0-4. *Al'bom matematicheskikh opisaniy i algoritmov upravleniya tipovymi protsessami khimicheskoi tekhnologii* (An Album of Mathematical Descriptions and Algorithms of Control of Typical Chemical Engineering Processes). Vyp. 1, 2. Moscow, NIITEKhim (1965, 1967).
- 0-5. Averbukh, Ya. D., Zaostrovsky, F. P., and Matusevich, L. N. *Protsessy i apparaty khimicheskoi tekhnologii. Kurs lektsii* (Unit Operations of Chemical Engineering. A Course of Lectures). Part 1. Sverdlovsk, Ural Polytechnical Inst. (1969).
- 0-6. Batuner, L. M. *Protsessy i apparaty organicheskogo sinteza i biokhimicheskoi tekhnologii* (Unit Operations for Organic Synthesis and Biochemical Engineering). Moscow, Khimiya (1966).
- 0-7. Batuner, L. M. and Pozin, M. E. *Matematicheskie metody v khimicheskoi tekhnike* (Mathematical Methods in Chemical Engineering). 6th ed. Leningrad, Khimiya (1971).
- 0-8. Belyavsky, E. I., Degen, A. B., and Etin, Yu. B. *ALGOL-60*. Leningrad, Gidrometeoizdat (1966).
- 0-9. Benedek, P. and Laszlo, A. *Grundlagen des Chemieingenieurwesens*. Leipzig, VEB, Deutscher Verlag für Grundstoff-Industrie (1965).
- 0-10. Bennet, C. O. and Myers, J. E. *Momentum, Heat and Mass Transfer*. New York, McGraw-Hill (1962).
- 0-11. Beskov, S. D. *Tekhnokhimicheskie raschety* (Chemical Engineering Calculations). Moscow, Vysshaya shkola (1962).
- 0-12. Beveridge, G. S. G. and Schechter, R. S. *Optimization: Theory and Practice*. New York, McGraw-Hill (1970).
- 0-13. Bird, R. B., Stewart, W. E., and Lightfoot, E. N. *Transport Phenomena*. New York, Wiley (1961).
- 0-14. Bondar, A. G. *Matematicheskoe modelirovanie v khimicheskoi tekhnologii* (Mathematical Modelling in Chemical Engineering). Kiev, Vishcha shkola (1973).
- 0-15. Boyarinov, A. I. and Kafarov, V. V. *Metody optimizatsii v khimicheskoi tekhnologii* (Methods of Optimization in Chemical Engineering). Moscow, Khimiya (1969).

- 0-16. Braines, Ya. M. *Podobie i modelirovanie v khimicheskoi i neftekhimicheskoi tekhnologii* (Similarity and Modelling in Chemical and Petroleum Chemical Engineering). Moscow, Gostoptekhizdat (1961).
- 0-17. Bratu, Em. A. *Operatii si utilaje in industria chimica*. Vol. 1, Bucharest (1960).
- 0-18. Bretsznajder, S. *Wlasności gazów i cieczy*. Warsaw, WNT (1962).
- 0-19. Chernobylsky, I. I., Bondar, A. G., Gaevsky, B. A., Gorodinskaya, S. A., Ladiev, R. Ya., Tananaiko, Yu. M., and Mirgorodsky, V. T. *Mashiny i apparaty khimicheskikh proizvodstv* (Machines and Apparatuses for Chemical Plants). 3rd ed. Moscow, Mashinostroenie (1975).
- 0-20. Ciborowski, J. *Podstawy inzynierii chemicznej*. Warsaw, WNT (1965).
- 0-21. Clarke, L. and Davidson, J. F. *Manual for Process Engineering Calculations*. New York, McGraw-Hill (1962).
- 0-22. Coughanowr, D. R. and Koppel, L. B. *Process Systems Analysis and Control*. New York, McGraw-Hill (1965).
- 0-23. Coulson, J. M. and Richardson, J. F. *Chemical Engineering*, Vols. 1 and 2. London, Pergamon Press (1968).
- 0-24. Danckwerts, P. V. *Gas-Liquid Reactions*. New York.
- 0-25. Dolezalik, V. *Methody a pochody chemicke technologie*. Svazek 5. Podobnost a modelovani v chemicke technologii. Prague, SNTL (1959).
- 0-26. Elenkov, D. *Protsezi i aparati v khimicheskata promishlennost* (Unit Operations in the Chemical Industry). Sofia, Tekhnika (1962).
- 0-27. Floares, O. and Smigelski, O. *Calcule de operatii si utilaje din industria chimică*. Bucharest, Editora Technica (1966).
- 0-28. Gukhman, A. A. *Primenenie teorii podobiya k issledovaniyu protsessov teplo-massoobmena* (Application of the Similarity Theory to the Investigation of Heat and Mass Exchange Processes). Moscow, Vysshaya shkola (1967).
- 0-29. Gukhman, A. A. *Vvedenie v teoriyu podobiya* (Introduction to the Similarity Theory). 2nd. ed. Moscow, Vysshaya shkola (1973).
- 0-30. *International Critical Tables*. Vols. I-VII. New York, McGraw-Hill (1926-1930).
- 0-31. Johnstone, R. E. and Thring, M. W. *Pilot Plants, Models, and Scale-up Methods in Chemical Engineering*. New York, McGraw-Hill (1957).
- 0-32. Kafarov, V. V. *Metody kibernetiki v khimii i khimicheskoi tekhnologii* (Methods of Cybernetics in Chemistry and Chemical Engineering). 2nd ed. Moscow, Khimiya (1971).
- 0-33. Kafarov, V. V., Perov, V. L., Meshalkin, V. P. *Printsipy matematicheskogo modelirovaniya khimiko-tekhnologicheskikh sistem* (Principles of the Mathematical Modelling of Chemical Engineering Systems). Moscow, Khimiya (1974).
- 0-34. Kasatkin, A. G. *Osnovnye protsessy i apparaty khimicheskoi tekhnologii* (The Fundamental Processes and Apparatuses of Chemical Engineering). 9th. ed. Moscow, Khimiya (1973).
- 0-35. King, C. O. *Separation Processes*. New York, McGraw-Hill.
- 0-36. Kirpichev, M. V. *Teoriya podobiya* (The Similarity Theory). Moscow, Izd. AN SSSR (1953).
- 0-37. Knudsen, J. G. and Katz, D. L. *Fluid Dynamics and Heat Transfer*. New York, McGraw-Hill (1958).
- 0-38. Kozulin, N. A., Sokolov, V. N., and Shapiro, A. Ya. *Primery i zadachi po kursu oborudovaniya zavodov khimicheskoi promyshlennosti* (Examples and Problems in the Course of Chemical Plant Equipment). Moscow, Mashinostroenie (1966).
- 0-39. Kushkova, A. D. *Standartnye algoritmy rascheta protsessov khimicheskoi tekhnologii, realizuemye na ETsVM M-222, BESM-4, MIR-2. Metodicheskoe posobie*. (Standard Algorithms of Calculations of Chemical Engineering Processes Realized on the Electronic Digital Computers M-222,

- BESM-4, MIR-2. A Methodical Aid). Leningrad Technological Institute (1976).
- 0-40. Lashchinsky, A. A. and Tolchinsky, A. R. *Osnovy konstruirovaniya i rascheta khimicheskoi apparatury* (The Fundamentals of Design and Calculation of Chemical Apparatuses). Leningrad, Mashinostroenie (1970).
- 0-41. Levenspiel, O. *Chemical Reaction Engineering*. New York, Wiley (1965).
- 0-42. McCabe, W. L. and Smith, J. C. *Unit Operations of Chemical Engineering*. 2nd ed. New York, McGraw-Hill (1967).
- 0-43. Melnik, B. D. and Melnikov, E. B. *Kratkii Inzhenernyy spravochnik po tekhnologii neorganicheskikh veshchestv* (A Concise Engineers' Handbook on the Technology of Inorganic Substances). Moscow, Khimiya (1974).
- 0-44. Perry, R. H. and Chilton, C. H. (Eds.). *Chemical Engineers' Handbook*. 5th ed. New York, McGraw-Hill (1973).
- 0-45. Peters, M. S. *Elementary Chemical Engineering*. New York, McGraw-Hill.
- 0-46. Planovsky, A. N. and Nikolaev, P. I. *Protsessy i apparaty khimicheskoi i neftekhimicheskoi tekhnologii* (Processes and Apparatuses of Chemical and Petroleum Chemical Engineering). 2nd ed. Moscow, Khimiya (1972).
- 0-47. Planovsky, A. N., Ramm, V. M., and Kagan, S. Z. *Protsessy i apparaty khimicheskoi tekhnologii* (Processes and Apparatuses of Chemical Engineering). 5th ed. Moscow, Khimiya (1968).
- 0-48. Reid, R. C. and Sherwood, T. K. *The Properties of Gases and Liquids. Their Estimation and Correlation*. 2nd ed. New York, McGraw-Hill (1966).
- 0-49. Reikhsfeld, V. O. and Erkoval, L. N. *Oborudovanie proizvodstv osnovnogo organicheskogo sinteza i SK* (Equipment of Plants for Basic Organic Synthesis and Synthetic Rubber). 2nd ed. Leningrad, Khimiya (1974).
- 0-50. Romankov, P. G. (ed.). *Rukovodstvo k prakticheskim zanyatiyam v laboratorii protsessov i apparatov khimicheskoi tekhnologii* (Guide to Practical Lessons in a Chemical Engineering Laboratory), 4th ed. Leningrad, Khimiya (1975).
- 0-51. Romankov, P. G. and Kurochkina, M. I. *Gidromekhanicheskie protsessy khimicheskoi tekhnologii* (Hydromechanical Processes of Chemical Engineering). 2nd ed. Moscow, Khimiya (1974).
- 0-52. Romankov, P. G. and Noskov, A. A. *Sbornik raschetnykh diagramm po kursu protsessov i apparatov khimicheskoi tekhnologii* (A Collection of Calculation Diagrams to the Course of Unit Operations of Chemical Engineering). Moscow, Khimiya (1966).
- 0-53. Romankov, P. G., Rashkovskaya, N. B., and Frolov, V. F. *Massoobmenye protsessy khimicheskoi tekhnologii* (Mass-Exchange Processes of Chemical Engineering). Leningrad, Khimiya (1975).
- 0-54. Rosenbrock, H. H. and Storey, C. *Computational Techniques for Chemical Engineers*. Oxford, Pergamon Press (1966).
- 0-55. Sedov, L. I. *Metody podobiya i razmernosti v mekhanike* (Similarity and Dimension Methods in Mechanics). 5th ed. Moscow, Nauka (1965).
- 0-56. Sherwood, T. K., Pigford, R. L., and Wilke, C. R. *Mass Transfer*. New York.
- 0-57. Shreve, R. N. *Chemical Process Industries*. 3rd ed. New York, McGraw-Hill (1967).
- 0-58. Skelland, A. H. P. *Diffusional Mass Transfer*. New York, Wiley (1973).
- 0-59. Skoblo, A. I., Tregubova, I. A., and Egorov, N. N. *Protsessy i apparaty neftepererabatyvayushchei i neftekhimicheskoi promyshlennosti* (Processes and Apparatuses for the Petroleum Refining and Petroleum Chemical Industry). Moscow, Gostoptekhizdat (1962).
- 0-60. Slattery, G. H. *Momentum, Energy, and Mass Transfer in Continua*. New York, McGraw-Hill.
- 0-61. Smith, B. D. *Design of Equilibrium Stage Processes*. New York, McGraw-Hill (1962).

- 0-62. Smith, J. M. *Chemical Engineering Kinetics*. New York, McGraw-Hill (1956).
- 0-63. Smith, J. M. and Van Ness, H. C. *Introduction to Chemical Engineering Thermodynamics*. 2nd ed. New York, McGraw-Hill (1959).
- 0-64. Stabnikov, V. N., Popov, V. D., Redko, F. A., and Lysyansky, V. M. *Protsessy i apparaty pishchevykh proizvodstv* (Processes and Apparatuses for Food Enterprises), 3rd ed. Moscow, Pishchevaya promyshlennost' (1966).
- 0-65. Treybal, R. E. *Mass Transfer Operations*. 2nd ed. New York, McGraw-Hill (1968).
- 0-66. Van Winkle, M. *Distillation*. New York, McGraw-Hill (1967).
- 0-67. Venikov, V. A. *Teoriya podobiya—modelirovanie* (The Similarity Theory—Modelling). Moscow, Vysshaya shkola (1966).
- 0-68. Walas, S. M. *Reaction Kinetics for Chemical Engineers*. New York, McGraw-Hill (1959).
- 0-69. Whitwell, J. C. and Toner, R. K. *Conservation of Mass and Energy*. New York, McGraw-Hill.
- 0-70. Yablonsky, P. A. *Posobie po kursovomu proektirovaniyu* (An Aid for Course Projecting), 7th ed. Leningrad Technological Institute (1975).
- 0-71. Zakgeim, A. Yu. *Vvedenie v modelirovanie khimiko-tekhnologicheskikh protsessov* (Introduction to the Modelling of Chemical Engineering Processes). Moscow, Khimiya (1973).

To Chapter 1. Fundamentals of Applied Hydraulics

- 1-1. Altshul, A. D. *Gidravlicheskie soprotivleniya* (Hydraulic Resistances). Moscow, Nedra (1970).
- 1-2. Chugaev, R. R. *Gidravlika* (Hydraulics). Leningrad, Energiya (1971).
- 1-3. Frenkel, N. Z. *Gidravlika* (Hydraulics), 2nd ed. Moscow, Gosenergoizdat (1956).
- 1-4. Idelchik, I. E. *Spravochnik po gidravlicheskim soprotivleniyam* (Reference Book on Hydraulic Resistances). Moscow, Gosenergoizdat (1960).
- 1-5. Idelchik, I. E. *Aerodinamika promyshlennykh apparatov* (Aerodynamics of Industrial Apparatuses). Moscow, Energiya (1964).
- 1-6. Ivanov, E. A., Shepelev, A. V., and Lyalin, E. V. *Truboprovody v khimicheskoi promyshlennosti* (Piping in the Chemical Industry). Moscow, Mashgiz (1963).
- 1-7. Kutateladze, S. S. and Styrikovich, M. A. *Gidravlika gazo-zhidkostnykh sistem* (Hydraulics of Gas-Liquid Systems). Moscow, Gosenergoizdat (1958).
- 1-8. Levich, V. G. *Fiziko-khimicheskaya gidrodinamika* (Physicochemical Hydrodynamics), 2nd ed. Moscow, Fizmatgiz (1959).
- 1-9. Loitsyansky, L. G. *Mekhanika zhidkosti i gaza* (Fluid Mechanics), 4th ed. Moscow, Nauka (1973).
- 1-10. Millionshchikov, M. D. *Turbulentnye techeniya v pogranichnom sloe i v trubakh* (Turbulent Flows in a Boundary Layer and in Pipes). Moscow, Nauka (1969).
- 1-11. Prandtl, L. *Führer durch die Strömungslehre*, 3rd ed. Berlin (1949).
- 1-12. Romankov, P. G. and Kurochkina, M. I. *Gidromekhanicheskie protsessy khimicheskoi tekhnologii* (Hydromechanical Processes in Chemical Engineering), 2nd ed. Leningrad, Khimiya (1974).
- 1-13. Soo, S. L. *Fluid Dynamics of Multiphase Systems*. Waltham, Mass., Blaisdell Publ. Co. (1970).
- 1-14. *Spravochnik mashinostroitelya* (Machine Builders' Reference Book), Vol. 2, 3rd ed. Moscow, Mashgiz (1960).
- 1-15. Wallis, G. B. *One-Dimensional Two-Phase Flow*. New York, McGraw-Hill (1969).
- 1-16. Wilkinson, W. L. *Non-Newtonian Fluids. Fluid Mechanics, Mixing and Heat Transfer*. London, Pergamon Press (1960).

To Chapter 2. Pumps, Fans, Compressors

- 2-1. Cherkassky, V. M., Romanova, T. M., and Kaul, R. A. *Nasosy, kompressory, ventilyatory* (Pumps, Compressors, Fans), 2nd ed. Moscow, Energiya (1968).
- 2-2. Kalinushkin, M. P. *Ventilyatornye ustanovki* (Fan Installations), 6th ed. Moscow, Vysshaya shkola (1967).
- 2-3. Kiselev, V. I. *Nasosy, kompressory i ventilyatory* (Pumps, Compressors and Fans). Moscow, Metallurgizdat (1961).
- 2-4. Kristal, F. A. and Annett, F. A. *Pumps*, 2nd ed. New York, McGraw-Hill (1953).
- 2-5. Sidorov, M. D. *Spravochnik po vozdukhoduvnym i gazoduvnym mashinam* (Reference Book on Air and Gas Blowers). Moscow, Mashgiz (1962).
- 2-6. Stark, S. B. *Osnovy gidravliki, nasosy i vozdukhoduvnye mashiny. Sbornik zadach* (Fundamentals of Hydraulics, Pumps and Air Blowers. A Collection of Problems). Moscow, Metallurgizdat (1961).

To Chapter 3. Hydromechanical Separation Methods. Hydrodynamics of a Fluidized Bed. Agitation in a Liquid Medium

- 3-1. Aerov, M. E. and Todes, O. M. *Gidravlicheskie i teplovye osnovy raboty apparatov so statsionarnym i kipyashchim zernistym sloem* (The Hydraulic and Thermal Fundamentals of the Operation of Apparatuses with a Steady-State and Fluidized Granular Bed). Moscow, Khimiya (1968).
- 3-2. Chlenov, V. A. and Mikhailov, N. V. *Vibrokipyashchii sloi* (A Vibro-Fluidized Bed). Moscow, Nauka (1972).
- 3-3. Davidson, J. F. and Harrison, D. (Ed.). *Fluidization*. London, Academic Press (1971).
- 3-4. Gelperin, N. I., Ainstein, V. G., and Kvasha, V. B. *Osnovy tekhniki psevdoozhizheniya* (The Fundamentals of Fluidization Techniques). Moscow, Khimiya (1967).
- 3-5. Gordon, G. M. and Peisakhov, I. L. *Dust Collection and Gas Cleaning*. Moscow, Mir Publishers (1972).
- 3-6. Kouzov, P. B. *Ukazaniya po raschetu tsiklonov* (A6-52) [(Instructions on the Calculation of Cyclones (A6-52)]. Moscow (1971).
- 3-7. Lipatov, N. N. and Novikov, O. P. *Samorazgruzhayushchiesya separatory* (Self-Discharging Separators). Moscow, Mashinostroenie (1975).
- 3-8. Malinovskaya, T. A. *Razdelenie suspenzii v promyshlennosti organicheskogo sinteza* (Separation of Suspensions in the Industry of Organic Synthesis). Moscow, Khimiya (1971).
- 3-9. Montvid, A. E. *Mnogokamernye sgustiteli v proizvodstve glinozema* (Multiple-Chamber Thickeners in the Production of Alumina). Moscow, Metallurgizdat (1952).
- 3-10. Olevsky, V. A. *Konstruktsii i raschet mekhanicheskikh klassifikatorov i gidrotsiklonov* (The Designs and Calculation of Mechanical Classifiers and Hydrocyclones). Moscow, Gosgortekhnizdat (1960).
- 3-11. Povarov, A. I. *Gidrotsiklony* (Hydrocyclones). Moscow, Gosgortekhnizdat (1961).
- 3-12. Pozin, M. E., Mukhlenov, I. P., and Tarat E. Ya. *Pennye gazoochistiteli, teploobmenniki i absorbery. Rabota i raschet pennykh apparatov* (Foam Gas Scrubbers, Heat Exchangers and Absorbers. The Operation and Calculation of Foam Apparatuses). Leningrad, Goskhimizdat (1959).
- 3-13. Razumov, I. M. *Psevdoozhizhenie i pneumatotransport* (Fluidization and Pneumatic Transportation), 2nd ed. Moscow, Khimiya (1972).
- 3-14. Sokolov, V. I. *Sovremennye promyshlennyye tsentrifugi* (Modern Commercial Centrifuges), 2nd ed. Leningrad, Mashinostroenie (1967).

- 3-15. Štěrbáček, Z. and Tausk, P. *Micháni v chemickém průmyslu*. Prague, SNTL (1959).
- 3-16. Strek, F. *Mieszanie i mieszalniki*. Warsaw, WNT (1971).
- 3-17. Ushakov, S. G. and Zverev, N. I. *Inertsionnaya separatsiya pyli* (The Inertial Separation of Dust). Moscow, Energiya (1974).
- 3-18. Uzhov, V. N. *Bor'ba s pyl'yu v promyshlennosti* (Combatting Dust in Industry). Moscow, Goskhimizdat (1962).
- 3-19. Uzhov, V. N. and Valdberg, A. Yu. *Ochistka gazov mokrymi fil'trami* (Gas Purification with Wet Filters). Moscow, Khimiya (1972).
- 3-20. Zabrodsky, S. S. *Vysokotemperaturnye ustanovki s kipyashchim slozem* (High-Temperature Installations with a Fluidized Bed). Moscow, Energiya (1971).
- 3-21. Zhuzhikov, V. A. *Fil'trovanie* (Filtration), 3rd ed. Moscow, Khimiya (1971).

To Chapter 4. Heat Transfer in a Chemical Apparatus

- 4-1. Chapman, G. H. *Heat Transfer*. 2nd ed. New York, Macmillan (1967).
- 4-2. Chechetkin, A. V. *Vysokotemperaturnye teplonositeli* (High-Temperature Heat Carriers), 3rd ed. Moscow, Energiya (1971).
- 4-3. Chirkin, V. S. *Teploprovodnost' promyshlennykh materialov. Spravochnoe posobie* (The Thermal Conductivity of Industrial Materials. A Reference Aid), 2nd ed. Moscow, Mashgiz (1962).
- 4-4. Dolinin, N. P. *Ustanovki dlya nagreva khimicheskoi apparatury vysokotemperaturnymi organicheskimi teplonositelyami* (Installations for Heating Chemical Apparatuses with High-Temperature Heat Carriers). Moscow, Mashgiz (1963).
- 4-5. Gerasimov, S. G. (Ed.). *Teplotekhnicheskii spravochnik* (Heat Engineering Reference Book), Vols. I and II. Moscow, Gosenergoizdat (1957-1958).
- 4-6. Grigorev, V. A., Kolakh, T. A., Sokolovsky, V. S. and Temkin, R. M. *Kratkii spravochnik po teploobmennym apparatam* (Concise Reference Book on Heat-Exchange Apparatus). Ed. by prof. P. D. Lebedev. Moscow, Gosenergoizdat (1962).
- 4-7. Gröber, H., Erk, S. *Die Grundgesetze der Wärmeübertragung*, 3rd ed. revised by U. Grigull. Berlin, Springer (1955).
- 4-8. Hobler, T. *Ruch ciepła i wymienniki*, 2nd ed. Warsaw, WNT (1959).
- 4-9. Isachenko, I. P., Osipova, V. A., and Sukomel, A. S. *Heat Transfer*. Moscow, Mir Publishers (1974).
- 4-10. Jakob, M. *Heat Transfer*, Vols. 1 and 2. New York, Wiley (1949, 1957).
- 4-11. Kanevets, G. E. (Ed.). *Proektirovanie i optimizatsiya teploobmennyykh apparatov na ETsVM* (Design and Optimization of Heat Exchange Apparatuses on Electronic Digital Computers), Parts 1-3. Kiev, In-t kibernetiki AN UkSSR (1970).
- 4-12. Klimenko, A. P. and Kanevets, G. E. *Raschet teploobmennyykh apparatov na EVM* (Calculation of Heat Exchange Apparatuses on Electronic Computers). Moscow, Energiya (1966).
- 4-13. Koshkin, V. K., Kalinin, E. K., Dreitser, G. A. and Yarkho, S. A. *Nestatsionarnyi teploobmen* (Non-stationary Heat Exchange). Moscow, Mashinostroenie (1973).
- 4-14. Krasnoshchekov, E. A. and Sukomel, A. S. *Zadachnik po teploperedache* (Problems in Heat Transfer). Moscow, Gosenergoizdat (1963).
- 4-15. Kutateladze, S. S. *Osnovy teorii teploobmena* (Fundamentals of Heat Exchange Theory), 4th ed. Novosibirsk, Nauka (1970).
- 4-16. Kutateladze, S. S. and Borishansky, V. M. *Spravochnik po teploperedache* (Reference Book on Heat Transfer). Leningrad, Gosenergoizdat (1959).
- 4-17. Mankovsky, O. N., Tolchinsky, A. R., and Aleksandrov, M. V. *Teploobmennaya apparatura khimicheskikh proizvodstv* (Heat Exchange Apparatuses of Chemical Industries).

- tuses for Chemical Plants). Ed. by Associate Member of USSR Acad. of Sciences P. G. Romankov and Cand. Sc. M. I. Kurochkina. Leningrad, Khimiya (1976).
- 4-18. Mikheev, M. A. and Mikheeva, I. M. *Osnovy teploperedachi* (Fundamentals of Heat Transfer). Moscow, Energiya (1973).
 - 4-19. Petukhov, B. S. *Teploobmen i soprotivleniye pri laminarnom techenii zhidkosti v trubakh* (Heat Exchange and Resistance in the Laminar Flow of a Liquid in Pipes). Moscow, Energiya (1967).
 - 4-20. Steidl, H., Neužil, L., Fořt, I., and Vlček, J. *Úvod do proudění tekutin a sdílení tepla*. Prague, Academia (1975).
 - 4-21. Tsederberg, N. V. *Teploprovodnost' gazov i zhidkosti* (Thermal Conductivity of Gases and Liquids). Moscow, Gosenergoizdat (1963).
 - 4-22. Vargaftik, N. B. (Ed.) *Spravochnik po teplofizicheskim svoistvam gazov i zhidkosti* (Reference Book on the Thermophysical Properties of Fluids), 2nd ed. Moscow, Nauka (1972).
 - 4-23. Vorontsov, E. G. and Tananaiko, Yu. M. *Teploobmen v zhidkostnykh plenkakh* (Heat Exchange in Liquid Films). Kiev, Tekhnika (1972).

To Chapter 5. Evaporation. Crystallization

- 5-1. *Apparaty vyparnye. Katalog-spravochnik* (Evaporators. A Catalogue and Reference Book). Moscow, TsINTI-khimneftemash (1965).
- 5-2. Bamforth, A. W. *Industrial Crystallization*. London, Leonard Hill (1965).
- 5-3. Chernobytsky, I. I. *Vyparnye ustanovki* (Evaporators). Kiev, Vysshaya shkola (1970).
- 5-4. Kolach, T. A. and Radun, D. V. *Vyparnye stantsii* (Evaporating Plants). Moscow, Mashgiz (1963).
- 5-5. Matusevich, L. N. *Krystallizatsiya iz rastvorov khimicheskoi promyshlennosti* (Crystallization from Solutions in the Chemical Industry). Moscow, Khimiya (1968).
- 5-6. Nepenin, Yu. N. *Protssy i apparaty khimicheskoi tekhnologii. Rukovodstvo k kursovomu proektirovaniyu* (Unit Operations in Chemical Engineering. A Guide to Course Projecting). Leningrad, VZLTI (1956).
- 5-7. Planovsky, A. N. and Nikolaev, P. I. *Tipovoi raschet trekhkorpusnoi vyparnoi ustanovki* (Typical Calculation of a Three-Effect Evaporator). Moscow, MIKhM (1954).
- 5-8. Taubman, E. I. *Raschet i modelirovanie vyparnykh ustanovok* (Calculation and Modelling of Evaporators). Moscow, Khimiya (1970).
- 5-9. Tishchenko, I. A. *Raschet mnogokorpusnogo vyparnogo apparata* (Calculation of Multiple-Effect Evaporator). Moscow, ONTI (1938).
- 5-10. Udyrna, P. G. *Apparaty s pogruchnymi gorelkami* (Apparatuses with Submersible Burners). Moscow, Mashinostroenie (1965).

To Chapter 6. Mass Transfer. Absorption

- 6-1. Frank-Kamenetsky, D. A. *Diffuziya i teploperedacha v khimicheskoi kinetike* (Diffusion and Heat Transfer in Chemical Kinetics). Moscow, Nauka (1967).
- 6-2. Hobler, T. *Dyfuzyjny Ruch masy i Absorberiy*. Warsaw, WNT (1962).
- 6-3. Kafarov V. V. *Osnovy massoperedachi* (Fundamentals of Mass Transfer), 2nd ed. Moscow, Vysshaya shkola (1972).
- 6-4. Kogan, V. B., Friedman, V. M., and Kafarov, V. V. *Ravновесие mezhdu zhidkost'yu i parom* (Liquid-Vapour Equilibrium), Books 1-2. Moscow, Nauka (1966).

- 6-5. Ramm, V. M. *Absorbtsiya gazov* (Absorption of Gases). Moscow, Khimiya (1966).
- 6-6. Treybal, R. E. *Mass-Transfer Operations*, 2nd ed. New York, McGraw-Hill (1968).

To Chapter 7. Distillation and Rectification

- 7-1. Aleksandrov, I. A. *Rektifikatsionnye i absorbtsionnye apparaty* (Rectification and Absorption Apparatuses), 2nd ed. Moscow, Khimiya (1971).
- 7-2. Bagaturov, S. A. *Osnovy teorii i rascheta peregonki i rektifikatsii* (Fundamentals of the Theory and Calculations of Distillation and Rectification), 3rd. ed. Moscow, Khimiya (1974).
- 7-3. Kasatkin, A. G., Planovsky, A. N., and Chekhov, O. S. *Raschet tarel'chatykh rektifikatsionnykh i absorbtsionnykh apparatov* (Calculation of Plate Rectification and Absorption Apparatuses). Moscow, Standartgiz (1961).
- 7-4. Kirschbaum, E. *Destillier- und Rektifiziertchnik*, 4th ed. Berlin, Springer (1969).
- 7-5. Kogan, V. B. *Azeotropnaya i ekstraktivnaya rektifikatsiya* (Azeotropy and Extractive Rectification). Leningrad, Khimiya (1971).
- 7-6. *Kolonnye apparaty iz stali i chuguna. Otrasevye normaly* (Column Apparatuses of Steel and Iron. Standards for the Industry). Moscow, NIIKhim-mash (1964).
- 7-7. Platonov, V. M. and Bergo, B. G. *Razdelenie mnogokomponentnykh smesei* (Separation of Multicomponent Mixtures). Moscow, Khimiya (1965).
- 7-8. Siirde, E. K., Tearo, E. N., and Mikkal, V. Ya. *Distillyatsiya* (Distillation). Leningrad, Khimiya (1971).
- 7-9. Stabnikov, V. N. *Peregonka i rektifikatsiya spirta* (Distillation and Rectification of Alcohol). Moscow, Pishchepromizdat (1962).
- 7-10. Stabnikov, V. N. *Rektifikatsionnye apparaty* (Rectification Apparatuses). Moscow, Mashinostroenie (1965).
- 7-11. Stabnikov, V. N. *Raschet i konstruirovaniye kontaktnykh ustroystv rektifikatsionnykh i absorbtsionnykh apparatov* (Calculation and Design of Contact Equipment for Rectification and Absorption Apparatuses). Kiev, Tekhnika (1970).
- 7-12. Ziolkowski, Z. *Destylacja i rektyfikacja w przemyśle chemicznym*. Warsaw, WNT (1966).

To Chapter 8. Extraction and Leaching

- 8-1. Akselrud, G. A. and Lysyansky, V. M. *Ekstragirovanie. Sistema tverdoe telo-zhidkost'* (Leaching. System Solid-Liquid). Leningrad, Khimiya (1974).
- 8-2. Alders, L. *Liquid-Liquid Extraction Theory and Laboratory Practice*, 2nd ed. Amsterdam, Elsevier (1959).
- 8-3. Brounstein, B. I. and Zheleznyak, A. S. *Fiziko-khimicheskie osnovy zhidkostnoi ekstraktsii* (The Physicochemical Fundamentals of Liquid Extraction). Moscow, Khimiya (1966).
- 8-4. Hanson, C. (Ed.). *Recent Advances in Liquid-Liquid Extraction*. Oxford, Pergamon Press (1971).
- 8-5. Kasatkin, A. G. (Ed.). *Zhidkostnaya ekstraktsiya (teoriya i praktika)* [Liquid Extraction (Theory and Practice)], Collected Articles. Moscow, Goskhimizdat (1958).
- 8-6. Treybal, R. E. *Liquid Extraction*. New York, McGraw-Hill (1951).
- 8-7. Ziolkowski, Z. *Ekstrakcja cieczy w przemyśle chemicznym*. Warsaw, PWT (1961).

To Chapter 9. Adsorption

- 9-1. Korolkov, N. M. *Teoreticheskie osnovy ionobmennoi tekhnologii* (The Theoretical Fundamentals of Ion-Exchange Technology). Riga, Liesma (1968).
- 9-2. Nikolaevsky, K. M. *Proektirovanie rekuperatsii letuchikh rastvoritelei s adsorberami periodicheskogo deistviya* (Designing the Recuperation of Volatile Solvents with Batch Adsorbers). Moscow, Oborongiz (1961).
- 9-3. Romankov, P. G. and Lepilin, V. N. *Nepreryvnaya adsorptsiya parov i gazov* (The Continuous Adsorption of Gases and Vapours). Leningrad, Khimiya (1968).
- 9-4. Serpionova, E. N. *Promyshlennaya adsorptsiya gazov i parov* (The Industrial Adsorption of Gases and Vapours), 2nd ed. Moscow, Vysshaya shkola (1969).
- 9-5. Sokolov, V. A., Torocheshnikov, N. S., and Keltsev, N. V. *Molekulyarnye sita i ikh primenenie* (Molecular Sieves and Their Application). Moscow, Khimiya (1964).
- 9-6. Timofeev, D. P. *Kinetik adsorbtsii* (The Kinetics of Adsorption). Moscow, Izd. AN SSSR (1962).

To Chapter 10. Drying

- 10-1. Ginzburg, A. S. *Infrakrasnaya sushka v pishchevoi promyshlennosti* (Infrared Drying in the Food Industry). Moscow, Pishchevaya promyshlennost' (1966).
- 10-2. Ginzburg, A. S. *Osnovy teorii i tekhniki sushki pishchevykh produktov* (Fundamentals of Theory and Practice of Food Product Drying). Moscow, Pishchevaya promyshlennost' (1973).
- 10-3. Krasnikov, V. V. *Konvektivnaya sushka* (Convective Drying). Moscow, Energiya (1973).
- 10-4. Krischer, O. *Die wissenschaftlichen Grundlagen der Trocknungstechnik*. Berlin, Springer (1956).
- 10-5. Lebedev, P. D. *Raschet i proektirovanie sushil'nykh ustanovok* (Calculation and Design of Drying Installations). Moscow, Gosenergoizdat (1963).
- 10-6. Lurye, M. Yu. *Sushil'noe delo* (Drying), 3rd ed. Moscow, Gosenergoizdat (1948).
- 10-7. Lykov, A. V. *Teplo- i massoobmen v protsessakh sushki* (Heat and Mass Exchange in Drying Processes). Moscow, Gosenergoizdat (1956).
- 10-8. Lykov, M. V. *Sushka v khimicheskoi promyshlennosti* (Drying in the Chemical Industry). Moscow, Khimiya (1970).
- 10-9. Lykov, M. V. and Leonchik, B. I. *Raspylitel'nye sushilki* (Spray Dryers). Moscow, Mashinostroenie (1966).
- 10-10. Mushtaev, V. I., Efimov, M. G., and Ulyanov, V. M. *Teoriya i raschet sushil'nykh protsessov* (Theory and Calculation of Drying Processes). Moscow, MikhM (1974).
- 10-11. Romankov, P. G. and Rashkovskaya, N. B. *Sushka vo vzveshennom sostoyanii* (Drying in the Suspended State), 2nd ed. Leningrad, Khimiya (1968).

To Chapter 11. Moderate and Deep Refrigeration

- 11-1. Budnevich, S. F. *Protsessy glubokogo okhlazhdeniya* (Deep Refrigeration Processes). Moscow, Mashinostroenie (1966).
- 11-2. Fastovsky, V. G. *Razdelenie gazovykh smesei* (Separation of Gas Mixtures). Moscow, Gostekhizdat (1947).

-
- 11-3. Gelperin, N. I. (Ed.). *Spravochnik po razdeleniyu gazovykh smesei metodom glubokogo okhlazhdeniya* (Reference Book on the Separation of Gas Mixtures by the Deep Refrigeration Method), 2nd ed. Moscow, Goskhimizdat (1963).
 - 11-4. *Kholodil'naya tekhnika. Entsiklopedicheskiy spravochnik* (Refrigerating Equipment. An Encyclopedic Reference Book), Vols. I and II. Moscow, Gostorgizdat (1960, 1961).
 - 11-5. Kochetkov, N. D. *Kholodil'naya tekhnika* (Refrigerating Equipment). Moscow, Mashinostroenie (1966).
 - 11-6. Komarov, N. S. *Kholod* (Cold), 5th ed. Moscow, Gizlegpishcheprom (1953).
 - 11-7. Komarov, N. S. *Spravochnik kholodil'shchika* (Reference Book for Refrigeration Engineers), 2nd ed. Moscow, Mashgiz (1962).
 - 11-8. Koshkin, N. N. (Ed.) *Kholodil'nye mashiny* (Refrigerating Machines). Moscow, Pishchevaya promyshlennost' (1973).
 - 11-9. Malkov, M. P., Danilov, I. B., Zeldovich, A. G., and Fradkov, A. B. *Spravochnik po fiziko-tekhnicheskim osnovam glubokogo okhlazhdeniya* (Reference Book on the Physical and Technical Fundamentals of Deep Refrigeration). Moscow, Gosenergoizdat (1963).
 - 11-10. Rozenfeld, L. M., Tkachev, A. G., and Gurevich, E. S. *Primery i raschety kholodil'nykh mashin i apparatov* (Examples and Calculations of Refrigerating Machines and Apparatuses). Moscow, Gostorgizdat (1960).

APPENDICES

TABLES

TABLE A-1. ATOMIC MASSES OF ELEMENTS

Element	Symbol	Atomic mass	Element	Symbol	Atomic mass
Aluminium	Al	26.98	Lithium	Li	6.94
Antimony	Sb	121.75	Magnesium	Mg	24.31
Argon	Ar	39.95	Manganese	Mn	54.94
Arsenic	As	74.92	Mercury	Hg	200.6
Barium	Ba	137.34	Molybdenum	Mo	95.94
Beryllium	Be	9.01	Nickel	Ni	58.7
Bismuth	Bi	209	Nitrogen	N	14.007
Boron	B	10.81	Oxygen	O	16
Bromine	Br	79.9	Phosphorus	P	30.97
Cadmium	Cd	112.4	Platinum	Pt	195.1
Calcium	Ca	40.08	Potassium	K	39.10
Carbon	C	12.01	Radium	Ra	226
Chlorine	Cl	35.45	Silicon	Si	28.09
Chromium	Cr	52	Silver	Ag	107.87
Cobalt	Co	58.93	Sodium	Na	22.99
Copper	Cu	63.55	Strontium	Sr	87.62
Fluorine	F	19	Sulphur	S	32.06
Gold	Au	197	Tin	Sn	118.7
Helium	He	4.003	Titanium	Ti	47.90
Hydrogen	H	1.008	Uranium	U	238.03
Iodine	I	126.9	Vanadium	V	50.94
Iron	Fe	55.85	Xenon	Xe	131.3
Krypton	Kr	83.8	Zinc	Zn	65.38
Lead	Pb	207.2			

TABLE A-2. DENSITY OF SOLID MATERIALS

Material	Density, kg/m³	Bulk density, kg/m³	Material	Density, kg/m³	Bulk density, kg/m³
Non-Metals					
Alabaster	2500	—	Brick, ordinary	1500	—
Anthracite	1600	—	Caoutchouc	930	—
Apatite	3190	1850	Cement	2900	—
Asbestos	2600	—	Ceramics, acid-resistant	2600	—
Ash	2200	680	Chalk, in lumps	2200	1300

Material	Density, kg/m³	Bulk density, kg/m³	Material	Density, kg/m³	Bulk, density, kg/m³
Non-Metals			Quartz	2650	1500
Charcoal	1450	200	Rubber	1500	—
Clay, dry	—	1380	Salt, rock	2350	1020
Coal, pit	1350	800	Sand, dry	1500	1200
Coke	1300	500	Soda, crystalline	1450	800
Concrete	2300	—	Sodium nitre	2260	1200
Cork	240	—	Soil, dry	1800	1300
Enamel	2350	—	Stone castings	3000	—
Fabric, laminated	1380	—	Vinyl plastic	1380	—
Faolite	1730	—	Metals		
Glass	2500	—	Aluminium	2700	—
Granite	2700	—	Brass	8500	—
Gypsum, crystalline	2240	1300	Copper, rolled	8800	—
Kaolin	2200	—	Iron, gray	7250	—
Leather, dry	860	—	Lead	11400	—
Limestone	2650	1800	Steel	7850	—
Marble	2600	—	Refractories		
Paraffin	900	—	Chamotte	1900	—
Paronite	1200	—	Dinas brick	1900	—
Phosphorite	—	1600	Magnesite	2900	—
Pine	500	—			
Potash	2260	—			
Pyrite	5000	3300			

TABLE A-3. DENSITY OF LIQUIDS AT 0-20 °C

Liquid	Density, kg/m³	Liquid	Density, kg/m³
Acetic acid, 100%	1060	Ethyl alcohol, 70%	850
Ditto, 70%	1070	Ditto, 40%	920
Ditto, 30%	1040	Ditto, 10%	980
Acetone	802	Ethylene chloride	1280
Ammonia, 26%	910	Formic acid	1240
Aniline	1030	Fuel oil	890-950
Benzene	890	Glycerine, 100%	1263
Butyl alcohol	810	Ditto, 80%	1130
Calcium chloride, 25% solu- tion	1195	Hydrochloric acid, 30%	1150
Carbon disulphide	1290	Hydrochloric acid, fuming	1210
Carbon tetrachloride	1615	Kerosene	850
Chlorobenzene	1130	Mercury	13600
Chloroform	1530	Methyl alcohol, 100%	801
Dichloroethane	1250	Ditto, 90%	820
Diethyl ether	710	Ditto, 30%	950
Ethyl acetate	900	Naphthalene (molten)	1100
Ethyl alcohol, 100%	798	Nitric acid, 92%	1500
		Nitrobenzene	1200

TABLE A-3 (concluded)

Liquid	Density, kg/m³	Liquid	Density, kg/m³
Petrol	760	Sodium hydroxide, 30% solution	1330
Petroleum	790-950	Sulphuric acid, 98%	1830
Phenol (molten)	1060	Ditto, 60%	1500
Sodium chloride, 25% solution	1189	Ditto, 30%	1220
Sodium hydroxide, 10% solution	1110	Toluene	870
		Water	1000
		Xylene	880

TABLE A-4.

DENSITY OF ORGANIC LIQUIDS
(in kg/m³) DEPENDING ON
TEMPERATURE

Liquid	Temperature, °C									
	0	20	40	60	80	100	120	140	160	180
Acetic acid		1049	1028	1006	984	960	936	909	883	856
Acetone	813	791	767	745	721					
Aniline	1037	1023	1007	990	972	952	933	914	896	878
Benzene	900	879	858	836	815	793	769	744	719	691
Carbon tetrachloride	1634	1595	1555	1517	1477	1435	1391	1344	1297	1247
Ethyl alcohol	806	790	772	754	735	716	693	663	633	598
Ethyl ether	736	714	689	666	640	611	576	539	495	
Glycerine	1267	1259	1250	1238	1224	1208	1188	1163	1126	
n-Hexane	677	660	641	622	602	581	559	534	506	475
Methyl alcohol	810	792	774	756	736	714	690	664	634	598
Propyl alcohol	819	804	788	770	752	733	711	688	660	629
Toluene	885	866	847	829	810	791	773	754		
m-Xylene	882	865	847	831	813	796	777	759		

TABLE A-5. BASIC PHYSICAL PROPERTIES OF GASES

Gas	Formula	Density at 0 °C and 760 mm Hg, kg/m³	Molar mass	Gas constant, J/kg·K	$\gamma = \frac{c_p}{c_v}$	Boiling point at 760 mm Hg, °C	Specific heat of vaporiza- tion at 760 mm Hg, J	Critical points	
								Temperature, °C	Pressure (absolute), at
Acetylene	C ₂ H ₂	1.171	26.0	320	1.24	-83.7	830	+35.7	61.6
Air	—	1.293	(29.0)	287	1.40	-195	197	-140.7	37.2
Ammonia	NH ₃	0.77	17	488	1.29	-33.4	1374	+132.4	111.5
Argon	Ar	1.78	39.9	209	1.66	-185.9	163	-122.4	48.00
Benzene	C ₆ H ₆	—	78.1	106	1.1	+80.2 (subl.)	394	+288.5	47.7
Butane	C ₄ H ₁₀	2.673	58.1	143	1.08	-0.5	387	+152	37.5
Carbon dioxide	CO ₂	1.98	44.0	189	1.30	-78.2 (subl.)	574.0	+31.1	72.9
Carbon monoxide	CO	1.25	28.0	297	1.40	-191.5	212	-140.2	34.53
Chlorine	Cl ₂	3.22	70.9	117	1.36	-33.8	306	+144.0	76.1
Ethane	C ₂ H ₆	1.36	30.1	277	1.20	-88.50	486	+32.1	48.85
Ethylene	C ₂ H ₄	1.26	28.1	297	1.20	-103.7	482	+9.7	50.7
Helium	He	0.179	4.0	2080	1.66	-268.9	19.5	-268.0	2.26
Hydrogen	H ₂	0.0899	2.02	4130	1.407	-252.8	455	-239.9	12.80
Hydrogen sulphide	H ₂ S	1.54	34.1	244	1.30	-60.2	549	+100.4	188.9
Methane	CH ₄	0.72	16.0	519	1.31	-161.6	511	-82.15	45.6
Methyl chloride	CH ₃ Cl	2.3	50.5	165	1.28	-24.1	406	+148	66.0
Nitrogen	N ₂	1.25	28	297	1.40	-195.8	199.4	-147.1	33.49
Nitrogen dioxide	NO ₂	—	46.0	181	1.31	+21.2	712	+158.2	100.00
Oxygen	O ₂	1.429	32	260	1.40	-183.0	213	-118.8	49.71
Pentane	C ₅ H ₁₂	—	72.2	115	1.09	+36.1	360	+197.1	33.0
Propane	C ₃ H ₈	2.02	44.1	189	1.13	-42.1	427	+95.6	43
Propylene	C ₃ H ₆	1.91	42.1	198	1.17	-47.7	440	+91.4	45.4
Sulphur dioxide	SO ₂	2.93	64.1	130	1.25	-10.8	394	+157.5	77.78

TABLE A-6.

DYNAMIC VISCOSITIES OF WATER

Temperature, °C	Dynamic viscosity, mPa·s (cP)	Temperature, °C	Dynamic viscosity, mPa·s (cP)	Temperature, °C	Dynamic viscosity, mPa·s (cP)
0	1.792	33	0.7523	67	0.4233
1	1.731	34	0.7371	68	0.4174
2	1.673	35	0.7225	69	0.4117
3	1.619	36	0.7085	70	0.4061
4	1.567	37	0.6947	71	0.4006
5	1.519	38	0.6814	72	0.3952
6	1.473	39	0.6685	73	0.3900
7	1.428	40	0.6560	74	0.3849
8	1.386	41	0.6439	75	0.3799
9	1.346	42	0.6321	76	0.3750
10	1.308	43	0.6207	77	0.3702
11	1.271	44	0.6097	78	0.3655
12	1.236	45	0.5988	79	0.3610
13	1.203	46	0.5883	80	0.3565
14	1.171	47	0.5782	81	0.3521
15	1.140	48	0.5683	82	0.3478
16	1.111	49	0.5588	83	0.3436
17	1.083	50	0.5494	84	0.3395
18	1.056	51	0.5404	85	0.3355
19	1.030	52	0.5315	86	0.3315
20	1.005	53	0.5229	87	0.3276
20.2	1.000	54	0.5146	88	0.3239
21	0.9810	55	0.5064	89	0.3202
22	0.9579	56	0.4985	90	0.3165
23	0.9358	57	0.4907	91	0.3130
24	0.9142	58	0.4832	92	0.3095
25	0.8937	59	0.4759	93	0.3060
26	0.8737	60	0.4688	94	0.3027
27	0.8545	61	0.4618	95	0.2994
28	0.8360	62	0.4550	96	0.2962
29	0.8180	63	0.4483	97	0.2930
30	0.8007	64	0.4418	98	0.2899
31	0.7840	65	0.4355	99	0.2868
32	0.7679	66	0.4293	100	0.2838

TABLE A-7.

DYNAMIC VISCOSITIES OF GLYCERINE

Temperature, °C	Dynamic viscosity, mPa·s (cP)	Temperature, °C	Dynamic viscosity, mPa·s (cP)	Temperature, °C	Dynamic viscosity, mPa·s (cP)
0	12 100	40	330	100	13
5	7 050	50	180	120	5.2
10	3 950	60	102	140	1.8
15	2 350	70	59	160	1.0
20	1 480	80	35	180	0.45
30	600	90	21	200	0.22

TABLE A-8. DYNAMIC VISCOSITIES OF SELECTED AQUEOUS SOLUTIONS

Solute	Concentra- tion, % by mass	Dynamic viscosity $\mu \times 10^3$, Pa·s, at temperature, °C, of:				
		0	20	30	40	60
NaOH	5	—	1.3	1.05	0.85	—
	15	—	2.78	2.10	1.65	—
	25	—	7.42	5.25	3.86	—
NaCl *	5	1.86	1.07	0.87	0.71	0.51
	15	2.27	1.36	1.07	0.89	0.64
	25	3.31	1.89	—	—	—
NaNO ₃	10	—	1.07	0.88	0.72	0.54
	20	—	1.18	1.03	0.86	0.62
	30	—	1.33	1.3	1.07	0.79
Na ₂ CO ₃	10	—	1.74	1.38	1.1	—
	20	—	4.02	2.91	2.25	—
	30	—	—	8.35	5.6	—
KOH	10	—	1.23	1.0	0.83	—
	20	—	1.63	1.33	1.11	—
	30	—	2.36	1.93	1.57	—
KCl	5	1.7	0.99	0.8	0.66	0.48
	15	1.58	1.0	0.83	0.69	0.52
	20	—	1.02	0.85	0.72	0.54
KNO ₃	5	1.68	0.98	0.8	0.66	0.49
	15	—	0.98	0.8	0.69	0.51
	30	—	—	0.89	—	—
NH ₄ NO ₃	10	1.58	0.96	0.79	0.66	0.5
	30	1.51	1.0	0.84	0.73	0.57
	50	—	1.33	1.14	0.99	0.77
MgCl ₂	10	2.8	1.5	—	—	—
	20	5.3	2.7	—	—	—
	35	19.3	10.1	—	—	—
CaCl ₂ **	10	2.17	1.27	—	—	—
	20	3.14	1.89	—	—	—
	35	8.9	5.1	—	—	—

* See also Table A-50.
** See also Table A-51

TABLE A-9. ATOMIC CONSTANTS OF VISCOSITY

Atoms	H	O	N	Cl	Br	I	C
Atomic constants <i>n</i>	2.7	29.7	37.0	60.0	79.0	110.0	50.2

TABLE A-10.

CORRECTIONS *B* TO ATOMIC
CONSTANTS OF VISCOSITY

No.	Nature of bonds and groups	<i>B</i>
1	Double bond	−15.5
2	Five-membered ring	−24.0
3	Six-membered ring	−21.0
4	Side group of six-membered ring: molecular mass < 17	−9.0
	molecular mass > 16	−17.0
5	Ortho- and para-positions of second substituents	+3.0
6	Meta-position of second substituents	+1.0
7	<div><div><div><div>R</div><div>R</div></div><div>CH—CH</div><div><div>R</div><div>R</div></div></div></div>	+8.0
8	<div><div><div>R</div><div>R—C—R</div><div>R</div></div></div>	+13.0
9	<div><div><div><div>R</div><div>C</div></div><div><div>O</div><div>H</div></div></div></div>	+16.0
10	<div><div><div><div>R</div><div>C</div></div><div><div>O</div><div>CH₃</div></div></div></div>	+5.0
11	−CH=CHCH ₂ X (X is a negative group)	+4.0
12	<div><div><div><div>R</div><div>R</div></div><div>CH—X</div></div> (X is a negative group)</div>	+6.0
13	OH	+24.7
14	COO	−19.6
15	COOH	−7.9
16	NO ₂	−16.4

−*y*

μ, cP

0.2

0.19

0.18

0.17

0.16

0.15

0.14

0.13

0.12

0.11

0.10

0.09

0.08

0.07

0.06

0.05

0.04

0.03

0.02

0.01

0

0.45

0.50

0.55

0.60

0.65

0.70

0.75

0.80

0.85

0.90

0.95

1.0

+*y*

μ, cP

4.0

3.75

3.35

3.25

3.0

2.75

2.55

2.25

2.0

1.90

1.80

1.70

1.60

1.50

1.40

1.30

1.20

1.10

1.05

1.0

Fig. A-1.

determining
dynamic vis-
cosities of orga-
nic liquids

Conversion to SI
units: 1 cP=
= 1 mPa·s

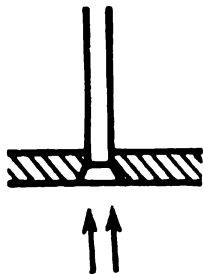
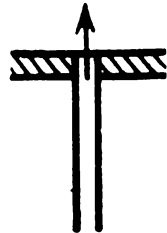
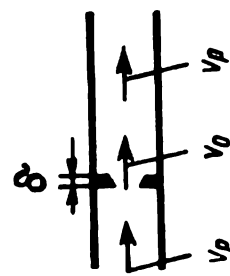
TABLE A-11. VALUES OF $\sqrt{MT_{cr}}$ FOR SELECTED GASES

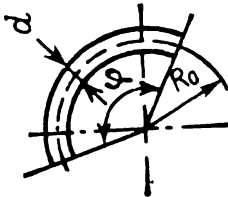
Gas	M	T _{cr} , K	$\sqrt{MT_{cr}}$
Air	29	132.7	61.9
Butane	58	426	157
Carbon dioxide	44	304	115.5
Carbon monoxide	28	134	61.4
Ethane	30	305	95.6
Ethylene	28	283	89.0
Hexane	86	508	209
Hydrogen	2	33	8.13
Methane	16	190	55.1
Nitrogen	28	126	59.5
Oxygen	32	154	70.2
Pentane	72	470	184
Propane	44	370	128
Water vapour	18	647	108

TABLE A-12. AVERAGE VALUES OF PIPE WALL ROUGHNESS

Pipes	e, mm
Steel pipes and tubes, seamless and welded, with insignificant corrosion	0.2
Old corroded steel pipes and tubes	0.67 and above
Pipes and tubes of roofing steel coated with varnish or drying oil	0.125
Iron water main pipes that have been in service	1.4
Aluminium technical smooth pipes and tubes	0.015-0.06
Finished seamless pipes and tubes of brass, copper and lead; glass pipes and tubes	0.0015-0.01
Concrete pipes; a smooth surface with float work	0.3-0.8
Concrete pipes; a rough surface	3-9
Petroleum pipes in medium conditions of service and pipes for saturated steam	0.2
Steam pipes functioning periodically	0.5
Air pipes for compressed air from a compressor	0.8
Condensate pipes functioning periodically	1.0

TABLE A-13. LOCAL RESISTANCE COEFFICIENTS

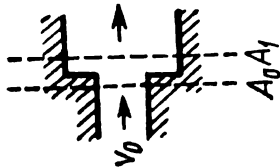
Kind of obstacle	Value of local resistance coefficient, ζ																																																
<div>Inlet to pipe</div> <div></div> <div>With sharp edges: $\zeta = 0.5$ With round edges: $\zeta = 0.2$</div>																																																	
<div>Outlet from pipe</div> <div></div> <div>$\zeta = 1$</div>																																																	
<div>Orifice plate (opening or orifice) with sharp edges in a straight pipe (or tube)</div> <div></div> <div>d_o = diameter of orifice, m δ = thickness of orifice plate, m v_o = mean velocity of flow in orifice, m/s v_p = mean velocity of flow in pipe, m/s f = pipe diameter, m</div>	<div>With $\delta/d_o = 0$ to 0.015, the loss of pressure $\Delta p = \zeta \frac{\rho v_p^2}{2}$ The value of ζ is determined from the table:</div> <table><tr><td>$(d_o/D)^2$</td><td>0.02</td><td>0.04</td><td>0.06</td><td>0.08</td><td>0.1</td><td>0.12</td><td>0.14</td><td>0.16</td><td>0.18</td><td>0.20</td><td>0.22</td></tr><tr><td>ζ</td><td>7000</td><td>1670</td><td>730</td><td>400</td><td>245</td><td>165</td><td>117</td><td>86.0</td><td>65.5</td><td>51.5</td><td>40.0</td></tr><tr><td>$(d_o/D)^2$</td><td>0.24</td><td>0.26</td><td>0.28</td><td>0.30</td><td>0.34</td><td>0.4</td><td>0.5</td><td>0.6</td><td>0.7</td><td>0.8</td><td>0.9</td></tr><tr><td>ζ</td><td>32.0</td><td>26.8</td><td>22.3</td><td>18.2</td><td>13.1</td><td>8.25</td><td>4.00</td><td>2.00</td><td>0.97</td><td>0.42</td><td>0.13</td></tr></table>	$(d_o/D)^2$	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.20	0.22	ζ	7000	1670	730	400	245	165	117	86.0	65.5	51.5	40.0	$(d_o/D)^2$	0.24	0.26	0.28	0.30	0.34	0.4	0.5	0.6	0.7	0.8	0.9	ζ	32.0	26.8	22.3	18.2	13.1	8.25	4.00	2.00	0.97	0.42	0.13
$(d_o/D)^2$	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.20	0.22																																						
ζ	7000	1670	730	400	245	165	117	86.0	65.5	51.5	40.0																																						
$(d_o/D)^2$	0.24	0.26	0.28	0.30	0.34	0.4	0.5	0.6	0.7	0.8	0.9																																						
ζ	32.0	26.8	22.3	18.2	13.1	8.25	4.00	2.00	0.97	0.42	0.13																																						

Pipe bends with round or square cross section  d = internal diameter of pipe, m R_0 = pipe bending radius, m	The resistance coefficient $\zeta = AB$ is determined from the tables:															
	Angle φ , deg.	20	30	45	60	90	110	130	150	180						
	A	0.31	0.45	0.6	0.78	1.0	1.13	1.20	1.28	1.40						
	R_0/d	1.0	2.0	4.0	6.0	15	30	50								
	B	0.21	0.15	0.11	0.09	0.06	0.04	0.03								
Right-angle standard iron elbow	Nominal opening, mm				12.5	25	37	50								
	ζ				2.2	2	1.6	1.1								
Ordinary valve	Values of ζ with valve completely open															
	D , mm	13	20	40	80	100	150	200	250	350						
	ζ	10.8	8.0	4.9	4.0	4.1	4.4	4.7	5.1	5.5						
Direct-flow valve	When $Re = \frac{vD}{\nu} \geq 3 \times 10^5$, the value of ζ is determined from the table:															
	D , mm	25	38	50	65	76	100	150	200	250						
		1.04	0.85	0.79	0.65	0.60	0.50	0.42	0.36	0.32						

T ABLE A-13 (continued).

Kind of obstacle	Value of local resistance coefficient ζ									
Direct-flow valve	When $Re < 3 \times 10^5$, the correction factor C given in the following table is applied to the value of ζ determined as above, i. e. $\zeta' = C\zeta$:									
	Re	5000	10 000	20 000	50 000	100 000	200 000	300 000		
	C	1.40	1.07	0.94	0.88	0.91	0.93	1		
Plug cock										
	Nominal, opening, mm	13	19	25	32	38	50 and above			
	ζ	4	2	2	2	2	2			
Gate valve										
	Nominal opening, mm	15-100	175-200	300 and above						
	ζ	0.5	0.25	0.15						

Sudden widening



A_0 = area of smaller cross section, m²
 v_0 = velocity of flow in smaller section, m/s
 A_1 = area of larger cross section, m²

$$Re = \frac{v_0 d_{eq}}{\nu}$$

$$\Delta p_{wid} = \zeta \frac{\rho v_0^2}{2}$$

Re	A_0/A_1					
	0.1	0.2	0.3	0.4	0.5	0.6
10	3.1	3.1	3.1	3.1	3.1	3.1
100	1.70	1.40	1.20	1.10	0.90	0.80
1000	2.0	1.60	1.30	1.05	0.90	0.60
3000	1.00	0.70	0.60	0.40	0.30	0.20
3500 and above	0.81	0.64	0.50	0.36	0.25	0.16

TABLE A-13 (concluded).

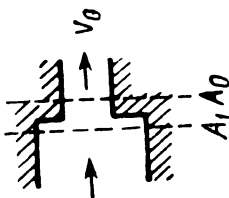
Kind of obstacle	Value of local resistance coefficient ζ						
<div>Sudden narrowing</div> <div></div> <div>A_0 = area of smaller cross section, m² v_0 = velocity of flow in smaller section, m/s A_1 = area of larger cross section, m² $Re = \frac{v_0 d_{eq}}{\nu}$ $\Delta p_{nar} = \zeta \frac{\rho v_0^2}{2}$</div>	A_0/A_1						
	Re	0.1	0.2	0.3	0.4	0.5	0.6
	10	5.0	5.0	5.0	5.0	5.0	5.0
	100	1.30	1.20	1.10	1.00	0.90	0.80
	1 000	0.64	0.50	0.44	0.35	0.30	0.24
	10 000	0.5	0.40	0.35	0.30	0.25	0.20
	> 10 000	0.45	0.40	0.35	0.30	0.25	0.20

TABLE A-14.

VALUES OF EQUIVALENT DIAMETER
 d_{eq} AND THE COEFFICIENT A FOR
SELECTED SECTIONS WITH LAMINAR
FLOW

Shape of section	d_{eq}	A
Circle with diameter d	d	64
Square with side a	a	57
Equilateral triangle with side a	$0.58a$	53
Ring of width a	$2a$	96
Rectangle with sides a and b :		
$a/b \approx 0$	$2a$	96
$a/b = 0.1$	$1.81a$	85
$a/b = 0.25$	$1.6a$	73
$a/b = 0.5$	$1.3a$	62
Ellipse (a =minor semi-axis, b =major semi-axis):		
$a/b = 0.1$	$1.55a$	78
$a/b = 0.3$	$1.4a$	73
$a/b = 0.5$	$1.3a$	68

TABLE A-15.

DISCHARGE COEFFICIENTS OF
ORIFICE PLATES

$Re = \frac{vd\rho}{\mu}$	Values of discharge coefficient C_d for $(d_o/d)^2$ equal to							
	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7
5 000	0.6032	0.6110	0.6341	—	—	—	—	—
10 000	0.6026	0.6092	0.6261	0.6530	0.6890	0.7367	0.7975	—
20 000	0.5996	0.6050	0.6212	0.6454	0.6765	0.7186	0.7753	0.8540
30 000	0.5990	0.6038	0.6187	0.6403	0.6719	0.7124	0.7650	0.8404
50 000	0.5984	0.6032	0.6168	0.6384	0.6666	0.7047	0.7553	0.8276
100 000	0.5980	0.6026	0.6162	0.6359	0.6626	0.6992	0.7472	0.8155
400 000	0.5978	0.6020	0.6150	0.6340	0.6600	0.6950	0.7398	0.8019

d =internal diameter of pipe, m; v =mean velocity of fluid in pipe, m/s;
 d_o =diameter of orifice of a normal orifice plate, m.

TABLE A-16.

VALUES OF THE ROUGHNESS
COEFFICIENT k

Pipe diameter, m	Values of coefficient k for $(d_o/d)^2$ equal to						
	0.1	0.2	0.3	0.4	0.5	0.6	0.7
0.05	1.0037	1.0063	1.0082	1.0118	1.0144	1.0172	1.02
0.10	1.0024	1.0045	1.0064	1.0065	1.0108	1.013	1.0148
0.20	1.0017	1.0023	1.0034	1.004	1.0052	1.006	1.007
0.30	1.0005	1.001	1.001	1.001	1.001	1.001	1.001

TABLE A-17.

CHARACTERISTICS OF RING AND
BROKEN SOLID SCRUBBER PACKINGS.

Type	Dimensions of packing element, mm	Number of elements in 1 m ³ of space filled with packing	Free volume, m ³ /m ³	Unit surface area, m ² /m ³	Mass of 1 m ³ of packing, kg
Porcelain rings	8×8×1.5	1 465 000	0.64	570	600
Ceramic rings	15×15×2	250 000	0.70	330	690
	25×25×3	53 200	0.74	204	532
	35×35×4	20 200	0.78	140	505
	50×50×5	6 000	0.785	87.5	530
	35×35×2.5	19 000	0.83	147	—
Steel rings	50×50×1	6 000	0.95	110	430
	42	14 400	0.388	80.5	—
Round gravel	43.2	12 600	0.565	68	1200
Broken andesite	42.6	14 000	0.56	77	455
	40.8	15 250	0.545	86	585
	28.6	27 700	0.535	110	660
	24.4	64 800	0.532	120	600
Catalyst of am- monia synthe- sis in small lumps	6.1	5 200 000	0.465	960	2420
Catalyst for CO conversion in tablets	$d=11.5$; $h=6$	1 085 000	0.38	460	1100
Sulphuric acid (vanadium)ca- talyt in tab- lets	$d=11$; $h=6.5$	1 000 000	0.43	415	614

TABLE A-18. CHARACTERISTICS OF WOOD GRID PACKING

Section of rod	Cross-sectional dimensions, mm	Rod spacing, mm	Rod row spacing, mm	Unit surface area, m ² /m ³
Rectangular	12.5×100	25.0	20	50
	12.5×100	12.5	20	75
	12.5×100	10.0	20	89
Triangular	30×30×30	30	12.5	78

TABLE A-19. DEPENDENCE OF ATMOSPHERIC PRESSURE ON ALTITUDE

Altitude, m	−600	0	+100	200	300	400	500	600	700	800	900	1000	1500
Atmospheric pressure p_{atm} , m H ₂ O	11.3	10.3	10.2	10.1	10.0	9.8	9.7	9.6	9.5	9.4	9.3	9.2	8.6

TABLE A-20. PERMISSIBLE SUCTION HEIGHT (in m) WHEN PUMPING WATER WITH PISTON PUMPS

Pump speed, rpm	Temperature of water, °C						
	0	20	30	40	50	60	70
50	7	6.5	6	5.5	4	2.5	0
60	6.5	6	5.5	5	3.5	2	0
90	5.5	5	4.5	4	2.5	1	0
120	4.5	4	3.5	3	1.5	0.5	0
150	3.5	3	2.5	2	0.5	0	0
180	2.5	2	1.5	1	0	0	0

TABLE A-21.

VALUES OF CONSTANTS *C* AND *m*
FOR VARIOUS TYPES OF MIXER

Type	Shape factor			Values of constants		Remarks
	<i>H</i> ₀ / <i>d</i>	<i>D</i> / <i>d</i>	<i>b</i> / <i>d</i>	<i>C</i>	<i>m</i>	
Two-bladed paddle	2	2	0.36	111.0 14.35	1.0 0.31	<i>Re</i> < 20 <i>Re</i> = 100 to 5 × 10 ⁴
	3	3	0.33	6.8	0.2	
Ditto with 45° pitched blades	3	3	0.33	4.05	0.2	
Four-bladed paddle	3	3	0.33	8.52	0.2	
Ditto with blades pitched 45° upward	3	3	0.33	5.05	0.2	
Ditto with blades pitched 60° upward	3	3	0.33	6.30	0.18	Round blades
Two-bladed anchor agitator	1.11	1.11	0.11	6.2	0.25	
Four-bladed anchor agitator	1.11	1.11	0.11	6.0	0.25	
Two-bladed propeller inclined at 22.5°	3	3	0.33	0.985	0.15	
Three-bladed propeller with inclination <i>d</i>	3.5	3.8	1	230 4.63 1.19	1.67 0.35 0.15	
Three-bladed turbine with 37-mm inlet opening	3	3	0.33	3.90	0.2	<i>Re</i> < 30 <i>Re</i> < 3 × 10 ³ <i>Re</i> > 3 × 10 ³
Six-bladed turbine with diffuser ring	1.78	2.4	0.25	5.98	0.15	

TABLE A-22.

SURFACE TENSION OF LIQUIDS

Liquid	Temperature, °C	Surface tension σ, dyn/cm	Formula giving temperature dependence of σ
Acetic acid	20	27.8	$\sigma_t = \sigma_0 - 0.146t$
Acetone	0	26.2	
	20	23.7	
	40	21.2	
	60	18.6	
Aniline	20	42.9	
Benzene	0	31.6	
	30	27.6	
	60	23.7	
Carbon disulphide	19	33.6	
	46	29.4	
Carbon tetrachloride	20	26.8	
Chloroform	10	28.5	
	60	21.7	

TABLE A-22 (concluded).

Liquid	Temperature, °C	Surface tension σ , dyn/cm	Formula giving temperature dependence of σ
Diethyl ether	20	17.0	$\sigma_t = \sigma_0 - 0.115t$
Ethyl acetate	20	23.9	
Ethyl alcohol	0	24.1	$\sigma_t = \sigma_0 - 0.092t$
	20	22.8	
	40	20.2	
	60	18.4	
Formic acid	17	37.5	
	80	30.8	
Methyl alcohol	20	22.6	
Nitrogen (liquid)	−196	8.5	
Olive oil	20	32.0	
Oxygen (liquid)	−183	13.2	
Paraffin oil	25	26.4	
Propyl alcohol	20	23.8	
Toluene	15	28.8	
Turpentine	15	27.3	
Water	0	75.6	
	20	72.8	

Note. Conversion to SI: 1 dyn/cm = 10^{−3} N/m.

TABLE A-23.

SURFACE TENSION OF AQUEOUS
SOLUTIONS (in dyn/cm)

Solute	Temperature, °C	Concentration, % by mass			
		5	10	20	50
H ₂ SO ₄	18	—	74.1	75.2	77.3
HNO ₃	20	—	72.7	71.1	65.4
NaOH	20	74.6	77.3	85.8	—
NaCl	18	74.0	75.5	—	—
Na ₂ SO ₄	18	73.8	75.2	—	—
NaNO ₃	30	72.1	72.8	74.4	79.8
KCl	18	73.6	74.8	77.3	—
KNO ₃	18	73.0	73.6	75.0	—
K ₂ CO ₃	10	75.8	77.0	79.2	106.4
NH ₄ OH	18	66.5	63.5	59.3	—
NH ₄ Cl	18	73.3	74.5	—	—
NH ₄ NO ₃	100	59.2	60.1	61.6	67.5
MgCl ₂	18	73.8	—	—	—
CaCl ₂	18	73.7	—	—	—

Note. Conversion to SI: 1 dyn/cm = 10^{−3} N/m.

TABLE A-24.

EMISSIONITIES ϵ FOR SELECTED MATERIALS

Material	ϵ	Material	ϵ
Aluminium	0.05-0.07	Iron, rough, oxidized	0.96
Asbestos	0.96	Lead	0.28
Brick masonry	0.93	Oil paint	0.78-0.96
Copper	0.57-0.87	Plaster	0.93
Glass	0.94	Varnish	0.8-0.98
Gypsum	0.78-0.9	Varnish, aluminium	0.4
Iron, galvanized	0.27	Water	0.93
Iron (steel), oxidized	0.74-0.96	Wood, dressed	0.9

TABLE A-25.

MEAN HEAT CAPACITIES OF SELECTED SOLIDS AT 0 TO 100 °C, kJ/kg·K

Aluminium	0.92	Iron	0.50
Asbestos	0.84	Kaolin	0.92
Brass	0.394	Lead	0.13
Brick, red	0.92	Limestone, lime	0.92
Brick, refractory	0.88-1.01	Magnesia	0.92
Bronze	0.385	Naphthalene	1.30
Cast iron	0.50	Paraffin	2.72
Cellulose	1.55	Quartz	0.80
Chalk	0.88	Rubber	1.68
Clay	0.92	Sand, dry	0.80
Coal	1.30	Slag	0.75
Coke	0.84	Steel	0.50
Concrete	1.13	Stone castings	0.84
Copper	0.385	Vinyl plastic	1.76
Cork	1.68	Wood (pine)	2.72
Fabric, laminated	1.47	Wool	1.63
Glass	0.42-0.84	Zinc	0.38
Ice	2.14		

TABLE A-26.

MEAN HEAT CAPACITIES OF SELECTED LIQUIDS (NOT GIVEN IN FIG. A-11), kJ/kg·K

Ammonia	4.19	Nitrogen, liquid	2.01
Hexane	2.51	Oxygen, liquid	1.68
Kerosene	2.10	Petrol	1.84
Machine oil	1.68	Phenol	2.35
Nitric acid	2.77	Sulphur dioxide	1.34
Nitrobenzene	1.38	Turpentine	1.76

TABLE A-27. MOLAR HEAT CAPACITIES OF GASES,
kJ/kmol·K (at $p_{abs} = 1$ atm) *

Gas	Temperature, °C			
	0	100	300	600
Air, carbon monoxide, nitrogen, oxygen	29.0	29.3	30.0	31.0
Ammonia	35.3	37.9	43.2	50.1
Carbon dioxide, sulphur dioxide	38.6	41.1	45.7	54.3
Chlorine	36.3	36.4	36.7	37.0
Hydrogen	29.1	29.3	29.7	30.4
Hydrogen sulphide	34.3	35.8	38.8	43.3
Methane	35.7	39.7	47.8	59.8
Water vapour	35.0	35.5	36.7	39.3

* The data of the table may be used with a permissible approximation for pressures of the order of magnitude of several atmospheres.

TABLE A-28. THERMAL CONDUCTIVITIES OF
SELECTED MATERIALS AT 0 TO
100 °C

Material	Density (bulk density for bulk materials), kg/m³	Thermal conductivity, W/m·k
Asbestos	600	0.151
Concrete	2300	1.28
Cork fines	160	0.047
Enamel	2350	0.872-1.163
Fabric, laminated	1380	0.244
Faolite	1730	0.419
Felt, woollen	300	0.047
Glass	2500	0.698-0.814
Glass wool	200	0.035-0.070
Ice	920	2.33
Masonry, common brick	1700	0.698-0.814
Masonry, insulating brick	600	0.116-0.209
Masonry, refractory brick	1840	1.05*
Magnesia, 85%, powder	216	0.070
Oil paint	—	0.233
Peat slabs	220	0.064
Plastic, foamed	30	0.047
Rust (scale)	—	1.16
Sand, dry	1500	0.349-0.814
Saw-dust	230	0.070-0.093
Scale, boiling stone	—	1.163-3.49
Slag wool	250	0.076
Sovelite	450	0.098
Stone castings	3000	0.698
Vinyl plastics	1380	0.163

TABLE A-28 (concluded).

Material	Density (bulk density for bulk materials), kg/m³	Thermal conductivity, W/m·K
Wood (pine), across grain	600	0.140-0.174
Wood (pine), along grain	600	0.384
Metals		
Aluminium	2700	203.5
Brass	8500	93.0
Bronze	8000	64.0
Cast iron	7500	46.5-93.0
Copper	8800	384
Lead	11 400	34.9
Steel	7850	46.5
Steel, stainless	7900	17.5

* At temperature of from 800 to 1100 °C.

TABLE A-29.

THERMAL CONDUCTIVITIES OF AQUEOUS SOLUTIONS

Solute	Concentration, % by mass	Temperature, °C	Thermal conductivity, W/m·K
NH ₃	26	18	0.45
BaCl ₂	21	32	0.58
KBr	40	32	0.50
KOH	21	32	0.58
	42	32	0.55
K ₂ SO ₄	10	32	0.60
KCl	15	32	0.58
	30	32	0.56
MgSO ₄	22	32	0.59
MgCl ₂	11	32	0.58
	29	32	0.52
CuSO ₄	18	32	0.58
NaBr	20	32	0.57
	40	32	0.54
Na ₂ CO ₃	10	32	0.58
NaCl	12.5	32	0.58
	25	32	0.48
H ₂ SO ₄	30	32	0.52
	60	32	0.44
	90	32	0.35
HCl	12.5	32	0.52
	25	32	0.48
	38	32	0.44
CH ₃ COOH	50	25	0.36
CH ₃ ·CH ₂ OH	10	12	0.52
	30	12	0.42
	50	13	0.32
	70	14	0.24
	90	15	0.19

TABLE A-30.

THERMAL CONDUCTIVITIES OF
GASES (at $p_{\text{abs}}=1 \text{ atm}^*$), W/m·K

Gas	Temperature, °C			
	0	50	100	200
Air	0.0244	0.0279	0.0326	0.0395
Ammonia	0.0209	0.0256	0.0314	—
Carbon dioxide	0.0140	0.0186	0.0233	0.0314
Carbon monoxide	0.0221	0.0244	—	—
Ethane	0.0174	0.0233	0.0314	—
Ethylene	0.0163	0.0209	0.0267	—
Hydrogen	0.1628	0.1861	0.2210	0.2559
Methane	0.0302	0.0361	0.0465	—
Nitrogen	0.0233	0.0267	0.0314	0.0384
Oxygen	0.0244	0.0291	0.0326	0.0407
Water vapour	0.0163	0.0198	0.0244	0.0326

* See note to Table A-27

TABLE A-31.

MEAN VALUES OF THERMAL
CONDUCTANCE OF WALL FOULING
DEPOSITS

Heat-carrying fluid	Thermal conductance of wall fouling deposits $\frac{1}{r_{\text{foul}}} \times 1.163^{-1} \text{ W/m}^2 \cdot \text{K}$
Water, polluted	1200-1600*
Water, medium quality	1600-2500*
Water, good quality	2500-5000*
Water, purified	2500-5000*
Water, distilled	10 000
Petroleum products, refined; oils; cool- ant vapours	2500
Petroleum products, crude	1000
Organic liquids; brines; liquid cool- ants	5000
Water vapour (containing oil)	5000
Organic vapours	10 000
Air	2400

* The lower values of the thermal conductance of the fouling deposits for water corres-
pond to higher temperatures.

TABLE A-32.

COEFFICIENTS OF VOLUME
EXPANSION OF WATER (in K⁻¹)

<i>t</i> , °C	<i>β</i> × 10 ⁵	<i>t</i> , °C	<i>β</i> × 10 ⁵	<i>t</i> , °C	<i>β</i> × 10 ⁵
5	1.5	50	46	100	75
10	9	60	52	120	85
20	21	70	58	140	97
30	30	80	64	160	110
40	39	90	70		

TABLE A-33.

COEFFICIENTS OF VOLUME
EXPANSION OF LIQUIDS AT
ABOUT 20 °C (in K⁻¹)

Liquid	<i>β</i> × 10 ³	Liquid	<i>β</i> × 10 ³
Acetic acid	107	Olive oil	70
Amyl alcohol	93	Paraffin oil	90
Aniline	85	Pentane	159
Benzene	124	Petrol	125
Carbon disulphide	121	Solution of CaCl ₂ , 6%*	25
Chloroform	126	Ditto, 41%	46
Diethyl ether	163	Solution of NaCl, 26%*	44
Ethyl alcohol	110	Sulphuric acid	57
Glycerine	53	Toluene	109
Kerosene	100	Turpentine	94
Methyl alcohol	122	<i>m</i> -Xylene	101

* For the values of *β* for brines see Danilov, G. N. et al *Sbornik zadach i raschetov po teploperedache* (Collection of Problems and Calculations in Heat Transfer) Moscow, Gostorg-izdat (1961)

TABLE A-34. BASIC CHARACTERISTICS OF SHELL AND TUBE HEAT EXCHANGERS WITH TUBES HAVING A DIAMETER OF 25×2 mm. TUBE PITCH IS 32 mm

Diameter of shell (inner), mm	Number of tubes	Length of tubes, mm							
		1000	1500	2000	2500	3000	4000	5000	6000
		Heat exchange surface area *, m²							
Single-Pass Exchangers									
150	13	1.0	1.5		2.5		4.0		6.0
259	43			6.5	8.0	10	13	17	20
400	121	9.0	14	18	23	28	37	47	56
600	283		33	43	54	65	87	109	131
800	511			77	97	117	157	197	237
1000	823			124	156	189	253	318	382
Double-Pass Exchangers									
400	110		12	17	21	25	34	42	51
600	266		30	40	51	61	82	102	123
800	488			74	93	112		189	
1000	792			119		181	244	305	
1200	1152			174	220		355	445	
1400	1596			240	301	366	490		
Four-Pass Exchangers									
400	90				17	21			42
600	232				44	53	72	90	108
800	446			67	85	102	137	172	207
1000	736			111	140		226	284	342
1200	1088				207	250	335	420	506
1400	1518				288	348	466	585	705
1600	2022							780	940
1800	2594								1200
Six-Pass Exchangers									
600	204			31	39	47	63	79	95
800	404			61	77	93	124	156	188
1000	686				130	157	211	265	320
1200	1024							395	476
1400	1446							557	672
1600	1936								900
1800	2500								1160
2000	3118								1442
2200	3876								1800

* Calculated along outer diameter of the tubes.

TABLE A-35.

NUMBER OF TUBES ARRANGED IN
TUBE SHEET ALONG HEXAGONS AND
ALONG CONCENTRIC CIRCLES

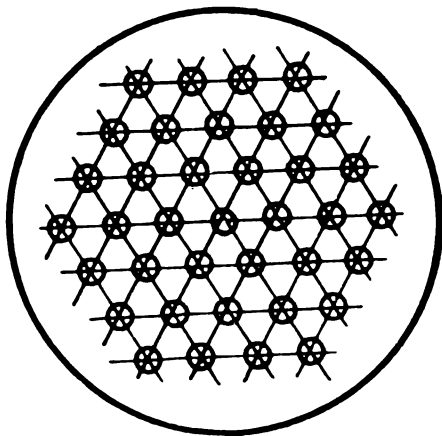


Fig. A-2. Arrangement of tubes along hexagons

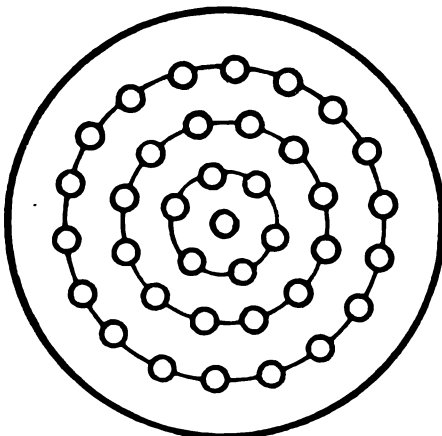


Fig. A-3. Arrangement of tubes along concentric circles

Number of hexagons or circles	Layout along hexagons — number of tubes							Layout along circles — number of tubes	
	along hexagon diagonal	total (without account of segments)	in 1st row of segment	in 2nd row of segment	in 3rd row of segment	in all segments	total	along outer circle	total
1	3	7	—	—	—	—	7	6	7
2	5	19	—	—	—	—	19	12	19
3	7	37	—	—	—	—	37	18	37
4	9	61	—	—	—	—	61	25	62
5	11	91	—	—	—	—	91	31	93
6	13	127	—	—	—	—	127	37	130
7	15	169	3	—	—	18	187	43	173
8	17	217	4	—	—	24	241	50	223
9	19	271	5	—	—	30	301	56	279
10	21	331	6	—	—	36	367	62	341
11	23	397	7	—	—	42	439	69	410
12	25	469	8	—	—	48	517	75	485
13	27	547	9	2	—	66	613	81	566
14	29	631	10	5	—	90	721	87	653
15	31	721	11	6	—	102	823	94	747
16	33	817	12	7	—	114	931	100	847
17	35	919	13	8	—	126	1045	106	953
18	37	1027	14	9	—	138	1165	113	1066
19	39	1141	15	12	—	162	1303	119	1185
20	41	1261	16	13	4	198	1459	125	1310
21	43	1387	17	14	7	228	1615	131	1441
22	45	1519	18	15	8	246	1765	138	1579
23	47	1657	19	16	9	264	1921	144	1723

TABLE A-37.

SPECIFIC HEAT OF SOLUTION L_{cr} OF
SELECTED SALTS IN WATER (1 kmol
OF SALT IN n kmol OF WATER)

Formula of salt	Molar mass, kg/kmol	L_{cr} , kJ/kmol	n
NaCl	58.5	+ 4 944	100
Na ₂ SO ₄	142	− 1 927	400
Na ₂ SO ₄ ·10H ₂ O	322	+ 78 600	400
NaNO ₃	85	+ 21 080	200
K ₂ CO ₃ ·1.5H ₂ O	165	+ 1 590	400
KCl	74.6	+ 17 560	100
KNO ₃	101	+ 35 700	200
KOH·2H ₂ O	92	+ 126	170 + 30
(NH ₄) ₂ SO ₄	132	+ 9 930	400
CaCl ₂ ·6H ₂ O	219	+ 18 060	400
MgCl ₂ ·6H ₂ O	203	− 12 360	400

Note. The plus and minus signs stand for solution with the absorption and liberation of heat, respectively

TABLE A-38.

PRESSURE OF SATURATED WATER
VAPOUR AT TEMPERATURES FROM
− 20 TO 100 °C

t , °C	p , mm Hg	t , °C	p , mm Hg	t , °C	p , mm Hg	t , °C	p , mm Hg	t , °C	p , mm Hg
−20	0.772	5	6.54	30	31.82	55	118.0	80	355.1
19	0.850	6	7.01	31	33.70	56	123.8	81	369.7
18	0.935	7	7.51	32	35.66	57	129.8	82	384.9
17	1.027	8	8.05	33	37.73	58	136.1	83	400.6
16	1.128	9	8.61	34	39.90	59	142.6	84	416.8
15	1.238	10	9.21	35	42.18	60	149.4	85	433.6
14	1.357	11	9.84	36	44.56	61	156.4	86	450.9
13	1.486	12	10.52	37	47.07	62	163.8	87	468.7
12	1.627	13	11.23	38	49.65	63	171.4	88	487.1
11	1.780	14	11.99	39	52.44	64	179.3	89	506.1
10	1.946	15	12.79	40	55.32	65	187.5	90	525.8
9	2.125	16	13.63	41	58.34	66	196.1	91	546.1
8	2.321	17	14.53	42	61.50	67	205.0	92	567.0
7	2.532	18	15.48	43	64.80	68	214.2	93	588.6
6	2.761	19	16.48	44	68.26	69	223.7	94	610.9
5	3.008	20	17.54	45	71.88	70	233.7	95	633.9
4	3.276	21	18.65	46	75.65	71	243.9	96	657.6
3	3.566	22	19.83	47	79.60	72	254.6	97	682.1
2	3.879	23	21.07	48	83.71	73	265.7	98	707.3
−1	4.216	24	22.38	49	88.02	74	277.2	99	733.2
0	4.579	25	23.76	50	92.51	75	289.1	100	760.0
+1	4.93	26	25.21	51	97.20	76	301.4		
2	5.29	27	26.74	52	102.1	77	314.1		
3	5.69	28	28.35	53	107.2	78	327.3		
4	6.10	29	30.04	54	112.5	79	341.0		

Note. Conversion to SI: 1 mm Hg = 133.3 Pa.

TABLE A-39. PHYSICAL PROPERTIES OF WATER
(ON SATURATION LINE)

p , at	t , °C	ρ , $\frac{\text{kg}}{\text{m}^3}$	H , $\frac{\text{kJ}}{\text{kg}}$	c , $\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$	$\lambda\times 10^2$, $\frac{\text{W}}{\text{m}\cdot\text{K}}$	$\alpha\times 10^7$, $\frac{\text{m}^2}{\text{s}}$	$\mu\times 10^6$, $\frac{\text{Pa}\cdot\text{s}}{\text{Pa}}$	$\nu\times 10^6$, $\frac{\text{m}^2}{\text{s}}$	$\beta\times 10^4$, K^{-1}	$\sigma\times 10^4$, $\frac{\text{kg}}{\text{s}^2}$	ρ_r
1	0	1000	0	4.23	55.1	1.31	1790	1.79	−0.63	756	13.7
1	10	1000	41.9	4.19	57.5	1.37	1310	1.31	+0.70	762	9.52
1	20	998	83.8	4.19	59.9	1.43	1000	1.01	1.82	727	7.02
1	30	996	126	4.18	61.8	1.49	804	0.81	3.21	712	5.42
1	40	992	168	4.18	63.4	1.53	657	0.66	3.87	697	4.31
1	50	988	210	4.18	64.8	1.57	549	0.556	4.49	677	3.54
1	60	983	251	4.18	65.9	1.61	470	0.478	5.11	662	2.98
1	70	978	293	4.19	66.8	1.63	406	0.415	5.70	643	2.55
1	80	972	335	4.19	67.5	1.66	355	0.365	6.32	626	2.21
1	90	965	377	4.19	68.0	1.68	315	0.326	6.95	607	1.95
1.03	100	958	419	4.23	68.3	1.69	282	0.295	7.5	589	1.75
1.46	110	951	461	4.23	68.5	1.69	256	0.268	8.0	569	1.58
2.02	120	943	503	4.23	68.6	1.72	231	0.244	8.6	549	1.43
2.75	130	935	545	4.27	68.6	1.72	212	0.226	9.2	529	1.32
3.68	140	926	587	4.27	68.5	1.72	196	0.212	9.7	507	1.23
4.85	150	917	629	4.32	68.4	1.72	185	0.202	10.3	487	1.17
6.30	160	907	671	4.36	68.3	1.72	174	0.191	10.8	466	1.10
8.08	170	897	713	4.40	67.9	1.72	163	0.181	11.5	444	1.05
10.23	180	887	755	4.44	67.5	1.72	153	0.173	12.2	424	1.01

TABLE A-40. OUTDOOR DESIGN TEMPERATURES*

City	Winter	Summer	
	Dry-bulb, °C	Dry-bulb, °C	Wet-bulb, °C
Europe			
Athens	−1	37	22
Berlin	−15	32	21
Brussels	−10	30	21
Bucharest	−20	32	22
Budapest	−12	33	21
Copenhagen	−13	28	20
Hamburg	−15	28	19
Helsinki	−24	27	19
Istanbul	−4	34	23
Leningrad	−7.7	17.5	14
Lisbon	3	34	22
London	−1	28	19
Madrid	−4	36	22
Marseilles	−6	33	22
Moscow	−10.8 (Jan)	18 (July)	—
Naples	—	35	24
Oslo	−17	27	19
Paris	−10	32	21

TABLE A-40 (continued)

City	Winter	Summer	
	Dry-bulb, °C	Dry-bulb, °C	Wet-bulb, °C
Prague	−16	32	19
Rome	−1	36	23
Sevastopol	−12	34	—
Stockholm	−19	27	19
Vienna	−15	33	21
Africa			
Addis Ababa	−3	27	19
Alexandria	5	38	24
Algiers	3	37	26
Cairo	4	40	22
Dakar	15	36	23
Tangier	—	33	24
Tripoli	4	39	27
Tunis	—	42	27
Asia			
Ankara	−14	35	20
Bagdad	4	45	23
Bangkok	16	36	28
Beirut	4	33	26
Bombay	16	34	28
Calcutta	10	38	28
Delhi	—	40	24
Djakarta	20	33	26
Hanoi	—	38	30
Hong Kong	6	33	28
Jerusalem	−2	35	21
Shanghai	−1	36	28
Singapore	18	32	28
Teheran	−5	35	22
Tokyo	−9	33	26
Australia			
Melbourne	0	35	21
Sydney	5	35	23
North America			
Boston	−18	33	24
Chicago	−23	35	24
Houston	−7	35	26
Los Angeles	2	32	21
Montreal	−23	30	23
New York	−18	35	24
Ottawa	−23	31	23
San Francisco	2	29	18

TABLE A-40 (concluded)

City	Winter	Summer	
	Dry-bulb, °C	Dry-bulb, °C	Wet-bulb, °C
Toronto	—23	31	26
Washington	—18	35	26
Central and South America			
Bogota	—1	21	18
Buenos Aires	—1	35	24
Guatemala	—	31	23
Havana (Cuba)	15	32	26
Lima	15	31	24
Mexico City	2	26	16
Montevideo	2	33	23
Panama	—	31	26
Rio de Janeiro	13	32	26
Santiago	2	32	20

* Heizung, Lüftung, *Klimatechnik Taschenbuch*. Berlin, VEB Verlag für Bauwesen (1964).

TABLE A-41. VALUES OF HENRY'S COEFFICIENT k FOR AQUEOUS SOLUTIONS OF SELECTED GASES (THE TABLE GIVES THE VALUES OF $k \times 10^{-6}$ IN mm Hg)

Gas	Temperature, °C										
	0	5	10	15	20	25	30	40	60	80	100
Acetylene	0.55	0.64	0.73	0.82	0.92	1.01	1.11	—	—	—	—
Air	32.8	37.1	41.7	46.1	50.4	54.7	58.6	66.1	76.5	81.7	81.6
Bromine	0.0162	0.0209	0.0278	0.0354	0.0451	0.056	0.0688	0.101	0.191	0.307	—
Carbon dioxide	0.553	0.666	0.792	0.93	1.08	1.24	1.41	1.77	2.59	—	—
Carbon monoxide	26.7	30	33.6	37.2	40.7	44	47.1	52.9	62.5	64.3	64.3
Chlorine	0.204	0.25	0.297	0.346	0.402	0.454	0.502	0.6	0.731	0.73	—
Ethane	9.55	11.8	14.4	17.2	20	23	26	32.2	42.9	50.2	52.6
Ethylene	4.19	4.96	5.84	6.8	7.74	8.67	9.62	—	—	—	—
Hydrogen	44	46.2	48.3	50.2	51.9	53.7	55.4	57.1	58.1	57.4	56.6
Hydrogen sulphide	0.203	0.239	0.278	0.321	0.367	0.414	0.463	0.566	0.782	1.03	1.12
Methane	17	19.7	22.6	25.6	28.5	31.4	34.1	39.5	47.6	51.8	53.3
Nitrogen	40.2	45.4	50.8	56.1	61.1	65.7	70.2	79.2	90.9	95.9	95.4
Oxygen	19.3	22.1	24.9	27.7	30.4	33.3	36.1	40.7	47.8	52.2	53.3

Note. Conversion to SI: 1 mm Hg=133.3 Pa

TABLE A-42.

DIFFUSION COEFFICIENTS OF GASES AND
VAPOURS IN AIR (IN STANDARD
CONDITIONS) *

Gas	$D^{\circ} \times 10^6,$ m^2/s	$D^{\circ},$ m^2/h
Ammonia	17.0	0.0612
Benzene	7.7	0.0277
Carbon dioxide	13.8	0.0497
Carbon disulphide	8.9	0.0321
Diethyl ether	7.8	0.028
Ethyl alcohol	10.2	0.0367
Hydrogen	61.1	0.22
Hydrogen chloride	13.0	0.0467
Methyl alcohol	13.3	0.0478
Nitrogen	13.2	0.0475
Oxygen	17.8	0.064
Sulphur dioxide	10.3	0.037
Sulphur trioxide	9.4	0.034
Water vapour	21.9	0.079

* For other temperatures and pressures, by Eq. (6-24)

$$D = D^{\circ} \frac{p^{\circ}}{p} \left(\frac{T}{T^{\circ}} \right)^{3/2}$$

TABLE A-43.

DIFFUSION COEFFICIENTS OF SELECTED
GASES IN WATER AT 20 °C*

Gas	$D \times 10^9,$ m^2/s	$D \times 10^6,$ m^2/h
Ammonia	1.8	6.6
Carbon dioxide, nitrous oxide	1.8	6.4
Chlorine, hydrogen sulphide	1.6	5.8
Hydrogen	5.3	19.1
Hydrogen chloride (at 12 °C)	2.3	8.3
Nitrogen	1.9	6.9
Oxygen	2.1	7.5

* For other temperatures: $D_t = D_{20} [1 + 0.02 (t - 20)]$

TABLE A-44. PHYSICAL PROPERTIES OF SELECTED ORGANIC LIQUIDS

Liquid	Chemical formula	Molar mass, kg/kmol	Density, kg/m³	Boiling point, °C	Saturated vapour pressure at 20 °C, mm Hg	Melting point, °C
Acetone	CH ₃ COCH ₃	58.08	810	56	186	−94.3
Benzene	C ₆ H ₆	78.11	900	80.2	75	5.5
Butyl alcohol	C ₄ H ₉ OH	74.12	810	117.7	4.7	−90
Carbon disulphide	CS ₂	76.13	1290	46.3	298	−112
Carbon tetrachloride	CCl ₄	153.84	1630	76.7	90.7	−22.8
Chloroform	CHCl ₃	119.38	1530	61.2	160	—
Dichloroethane	CH ₂ Cl—CH ₂ Cl	98.97	1250	83.7	65	—
Diethyl ether	C ₂ H ₅ OC ₂ H ₅	74.12	710	34.5	442	−116.3
Ethyl acetate	CH ₃ COOC ₂ H ₅	88.10	900	77.15	73	−83.6
Ethyl alcohol	C ₂ H ₅ OH	46.07	790	78.3	44	−114.5
Isoamyl alcohol	C ₅ H ₁₁ OH	88.15	810	132	2.2	−117
Isobutyl alcohol	C ₄ H ₉ OH	74.12	800	108	8.8	−108
Isopropyl acetate	CH ₃ COOC ₃ H ₇	130.18	870	142.5	6	—
Isopropyl alcohol	C ₃ H ₇ OH	60.09	785	82.4	32.4	−89
Methyl acetate	CH ₃ COOCH ₃	74.08	930	57.5	170	—
Methyl alcohol	CH ₃ OH	32.04	800	64.7	95.7	−98
Petrol	—	—	650-760	70-120	—	—
Propyl acetate	CH ₃ COOC ₃ H ₇	102.13	850	101.6	25	—
Propyl alcohol	C ₃ H ₇ OH	60.09	800	97.2	14.5	−126
Toluene	C ₆ H ₅ CH ₃	92.13	870	110.8	22.3	−95
Turpentine	C ₁₀ H ₁₆	136.1	870-880	155-190	4	—
Xylenes (mixture)	C ₆ H ₄ (CH ₃) ₂	106.16	860	136-145	10	−13 to −48

TABLE A-45. SPECIFIC HEATS OF VAPORIZATION OF SELECTED SUBSTANCES (in kJ/kg)

Substance	Temperature, °C				
	0	20	60	100	140
Acetic acid	—	—	—	406.4 (at 118°C)	395.5
Acetone	565.7	553.1	519.6	473.5	—
Ammonia	1265.4	1190.0	—	—	—
Aniline	—	—	—	—	435.8 (at 184 °C)
Benzene	448.3	435.8	408.5	379.2	346.1
Butyl alcohol	703.9	687.2	653.6	611.7	561.5
Carbon dioxide	235.1	155.4	—	—	—
Carbon disulphide	374.6	367.0	344.4	316.4	282.4
Carbon tetrachloride	218.3	213.7	201.9	185.6	168.0
Chlorine	266.5	253.1	222.0	176.8	71.23
Chlorobenzene	375.8	369.5	354.4	338.1	320.5
Chloroform	271.5	263.1	247.6	231.3	—
Diethyl ether	387.6	366.6	326.4	282.4	228.4
Ethyl acetate	427.4	411.5	385.9	355.7	317.2
Ethyl alcohol	921.8	913.4	879.9	812.9	712.3
Freon-12	155.0	144.9	132.4	—	—
Isopropyl alcohol	775.2	750.0	699.7	636.9	557.3
Methyl alcohol	1198.3	1173.2	1110.4	1013.9	892.6
Nitrobenzene	—	—	—	—	331.9 (at 211 °C)
Propyl alcohol	812.9	791.9	745.8	683.0	595.0
Toluene	414.8	407.7	388.8	368.7	344.0
Water	2493.1	2446.9	2359.0	2258.4	2149.5

TABLE A-46. BOILING POINTS (in °C) OF SELECTED ORGANIC LIQUIDS AT PRESSURES UNDER 1 atm

Pressure (abs.), mm Hg	Aniline	Glycerine	Nitrobenzene	Octane	Toluene	m-Xylene
10	69.4	167.2	84.9	19.2	6.4	28.3
20	82.0	182.2	99.3	31.5	18.4	41.1
40	96.7	198.0	115.4	45.1	31.8	55.3
60	106.0	208.0	125.8	53.8	40.3	64.4
100	119.9	220.1	139.9	65.7	51.9	76.8
200	140.1	240.0	161.2	83.6	69.5	95.5
400	161.9	263.0	185.8	104.0	89.5	116.7
760	184.4	290.0	210.6	125.6	110.6	139.1

TABLE A-47.

EQUILIBRIUM COMPOSITIONS OF
LIQUID AND VAPOUR FOR SELECTED
BINARY SYSTEMS WITH $p_{\text{tot}}=1\text{ atm}$
(abs.)

Methyl alcohol-water			Chloroform-benzene		
$t, ^\circ\text{C}$	% (mole) of methyl alcohol		$t, ^\circ\text{C}$	% (mole) of chloroform	
	in liquid	in vapour		in liquid	in vapour
100.0	0	0	80.6	0	0
96.4	2	13.4	79.8	8	10
93.5	4	23.0	79.0	15	20
91.2	6	30.4	78.2	22	30
87.7	10	41.8	77.3	29	40
81.7	20	57.9	76.4	36	50
78.0	30	66.5	75.3	44	60
75.3	40	72.9	74.0	54	70
73.1	50	77.9	71.9	66	80
71.2	60	82.5	68.9	79	90
69.3	70	87.0	61.4	100	100
67.5	80	91.5			
66.0	90	95.8			
64.5	100	100			

Water-acetic acid			Nitrogen-oxygen		
$t, ^\circ\text{C}$	% (mole) of water		T, K	% (mole) of nitrogen	
	in liquid	in vapour		in liquid	in vapour
118.1	0	0	90.1	0	0
115.4	5	9.2	89.5	3.5	13.0
113.8	10	16.7	89	6.2	20.2
110.1	20	30.2	88	11.5	30.4
107.5	30	42.5	87	17.1	39.7
105.8	40	53.0	86	22.2	47.8
104.4	50	62.6	85	27.7	55.7
103.2	60	71.6	84	33.8	63.1
102.1	70	79.5	83	40.5	70.1
101.3	80	86.4	82	47.8	76.4
100.6	90	93.0	81	56.6	82.3
100.0	100	100	80	66.6	88.0
			79	78.4	93.2
			78	91.9	97.8
			77.3	100	100

TABLE A-48.

PHYSICAL PROPERTIES
OF SATURATED AMMONIA VAPOUR

Tempera- ture, °C	Pressure (abs.), at	Specific volume		Density		Specific heat of vaporization, $L=H''-H'$, kJ/kg
		liquid v' , dm ³ /kg	vapour v'' , m ³ /kg	liquid ρ' , kg/dm ³	vapour ρ'' , kg/m ³	
−50	0.4168	1.4245	2.6170	0.7020	0.382	1416
−45	0.5562	1.4367	2.0015	0.6960	0.500	1402
−40	0.7318	1.4493	1.5503	0.6900	0.645	1388
−35	0.9503	1.4623	1.2151	0.6839	0.823	1374
−30	1.219	1.4757	0.9630	0.6777	1.038	1360
−25	1.546	1.4895	0.7712	0.6714	1.297	1345
−20	1.940	1.5037	0.6236	0.6650	1.604	1329
−15	2.410	1.5185	0.5087	0.6585	1.966	1314
−10	2.966	1.5338	0.4184	0.6520	2.390	1297
−5	3.619	1.5496	0.3469	0.6453	2.883	1281
0	4.379	1.5660	0.2897	0.6386	3.452	1263
+5	5.259	1.5831	0.2435	0.6317	4.108	1246
+10	6.271	1.6008	0.2058	0.6247	4.859	1227
+15	7.431	1.6193	0.1740	0.6175	5.718	1210
+20	8.741	1.6386	0.1494	0.6103	6.694	1188
+25	10.225	1.6588	0.1283	0.6028	7.795	1168
+30	11.895	1.6800	0.1107	0.5952	9.034	1146
+35	13.765	1.7023	0.0959	0.5875	10.431	1124
+40	15.850	1.7257	0.0833	0.5795	12.005	1101
+45	18.165	1.7504	0.0726	0.5713	12.774	1078
+50	20.727	1.7766	0.0635	0.5629	15.756	1053

TABLE A-49.

PHYSICAL PROPERTIES
OF SATURATED DIFLUORODICHLORO-
METHANE(FREON-12) VAPOUR

Temperature, °C	Pressure (abs.), at	Density		Specific heat o vaporization L , kJ/kg
		liquid, kg/dm ³	vapour, kg/m ³	
+40	9.78	1.25	53.1	132.4
+30	7.59	1.29	41.2	138.7
+25	6.63	1.31	36.1	142.0
+20	5.79	1.33	31.5	145.0
+10	4.32	1.36	23.8	150.0
0	3.15	1.39	17.7	155.0
−10	2.24	1.43	12.8	160.0
−15	1.86	1.44	10.8	161.7
−20	1.54	1.46	9.04	163.8

TABLE A-49 (concluded).

Temperature, °C	Pressure (abs.), at	Density		Specific heat of vaporization <i>L</i> , kJ/kg
		liquid, kg/dm³	vapour, kg/m³	
—25	1.26	1.47	7.52	165.5
—30	1.03	1.49	6.2	167.6
—35	0.824	1.5	5.07	169.3
—40	0.655	1.52	4.1	171.0
—50	0.399	1.54	2.6	175.0
—60	0.231	1.57	1.56	178.0
—70	0.125	1.6	0.888	182.0
—80	0.063	1.63	0.47	185.0

TABLE A-50.

PHYSICAL PROPERTIES OF AQUEOUS
SODIUM CHLORIDE SOLUTIONS AT
LOW TEMPERATURES

Salt con- centration, in solution, % (mass)	Density at 15 °C, kg/m³	Freez- ing point, °C	Dynamic viscosity $\mu \times 10^4$, Pa·s					Thermal conduc- tivity, λ , W/m·K		
			0 °C	—5 °C	—10 °C	—15 °C	—20 °C	0 °C	—10 °C	—20 °C
0.1	1000	0.0	17.66	—	—	—	—	0.5815	—	—
1.5	1010	—0.9	17.85	—	—	—	—	0.5780	—	—
2.9	1020	—1.8	18.05	—	—	—	—	0.5757	—	—
4.3	1030	—2.6	18.25	—	—	—	—	0.5734	—	—
5.6	1040	—3.5	18.44	—	—	—	—	0.5710	—	—
7.0	1050	—4.4	18.74	—	—	—	—	0.5687	—	—
8.3	1060	—5.4	19.13	23.05	—	—	—	0.5664	—	—
9.6	1070	—6.4	19.62	23.74	—	—	—	0.5641	—	—
11.0	1080	—7.5	20.21	24.43	—	—	—	0.5606	—	—
12.3	1090	—8.6	20.80	25.21	—	—	—	0.5582	—	—
13.6	1100	—9.8	21.48	26.09	—	—	—	0.5559	—	—
14.9	1110	—11.0	22.37	27.17	33.45	—	—	0.5536	0.5187	—
16.2	1120	—12.2	23.25	28.35	34.92	—	—	0.5513	0.5164	—
17.5	1130	—13.6	24.33	29.72	36.79	—	—	0.5489	0.5140	—
18.8	1140	—15.1	25.60	31.20	38.75	47.77	—	0.5466	0.5117	—
20.0	1150	—16.6	26.88	32.77	40.81	50.13	—	0.5443	0.5094	—
21.2	1160	—18.2	28.25	34.43	43.07	52.78	—	0.5420	0.5071	—
22.4	1170	—20.0	29.63	36.40	45.62	55.82	68.67	0.5408	0.5059	0.4768
23.1	1175	—21.2	30.41	37.47	47.09	57.49	70.44	0.5396	0.5047	0.4757
23.7	1180	—17.2	31.39	38.55	48.66	59.35	—	0.5385	0.5035	—
24.9	1190	—9.5	32.96	40.71	—	—	—	0.5361	—	—
26.1	1200	—1.7	34.73	—	—	—	—	0.5338	—	—
26.3	1203	0.0	35.02	—	—	—	—	0.5338	—	—

TABLE A-51.

PHYSICAL PROPERTIES OF AQUEOUS
CALCIUM CHLORIDE SOLUTIONS AT
LOW TEMPERATURES

Salt con- centration in solu- tion, % (mass)	Density at 15 °C, kg/m³	Freez- ing point, °C	Dynamic viscosity $\mu \times 10^4$, Pa·s				Thermal conductivity λ , W/m·K			
			0 °C	−10 °C	−20 °C	−30 °C	0 °C	−10 °C	−20 °C	−30 °C
0.1	1000	0.0	17.76	—	—	—	0.5815	—	—	—
5.9	1050	−3.0	19.82	—	—	—	0.5675	—	—	—
11.5	1100	−7.1	22.96	—	—	—	0.5524	—	—	—
16.8	1150	−12.7	27.66	43.65	—	—	0.5350	0.504	—	—
17.8	1160	−14.2	28.74	45.13	—	—	0.5303	0.500	—	—
18.9	1170	−15.7	29.92	46.70	—	—	0.5257	0.497	—	—
19.9	1180	−17.4	31.20	48.46	—	—	0.5210	0.493	—	—
20.9	1190	−19.2	32.77	50.72	—	—	0.5164	0.490	—	—
21.9	1200	−21.2	34.43	53.27	86.13	—	0.5117	0.486	0.465	—
22.8	1210	−23.3	36.20	56.11	90.15	—	0.5071	0.484	0.463	—
23.8	1220	−25.7	38.16	59.25	94.76	—	0.5024	0.480	0.459	—
24.7	1230	−28.3	40.22	62.69	99.96	—	0.4978	0.477	0.457	—
25.7	1240	−31.2	42.58	66.81	105.7	148.1	0.4931	0.473	0.455	0.437
26.6	1250	−34.6	45.22	70.83	111.7	158.9	0.4885	0.470	0.452	0.436
27.5	1260	−38.6	48.07	75.24	118.5	171.7	0.4838	0.464	0.449	0.435
28.4	1270	−43.6	51.21	80.25	126.9	188.4	0.4792	0.463	0.446	0.434
29.4	1280	−50.1	54.94	86.33	137.9	212.9	0.4745	0.459	0.444	0.433
29.9	1286	−55.0	56.90	90.45	143.9	225.6	0.4722	0.457	0.443	0.431
30.3	1290	−50.6	58.86	93.29	149.6	238.4	0.4699	0.456	0.442	0.430
31.2	1300	−41.6	63.37	100.6	161.9	265.9	0.4652	0.452	0.438	0.429
32.1	1310	−33.9	68.28	108.7	176.3	307.1	0.4605	0.449	0.436	0.428
33.0	1320	−27.1	73.87	117.3	191.9	—	0.4571	0.444	0.434	—
33.9	1330	−21.2	80.15	127.2	210.0	—	0.4524	0.441	0.431	—
34.7	1340	−15.6	86.52	138.1	—	—	0.4478	0.438	—	—
35.6	1350	−10.2	93.20	151.9	—	—	0.4431	0.433	—	—
36.4	1360	−5.1	100.9	—	—	—	0.4396	—	—	—
37.3	1370	0.0	109.2	—	—	—	0.4350	—	—	—

TABLE A-52.

SPECIFIC HEAT CAPACITY OF AQUE-
OUS SODIUM CHLORIDE AND CALCIUM
CHLORIDE SOLUTIONS (in kJ/kg·K)

Density at 15 °C, kg/m³	Sodium chloride			Density at 15 °C, kg/m³	Calcium chloride			
	0 °C	−10 °C	−20 °C		0 °C	−10 °C	−20 °C	−30 °C
1010	4.077	—	—	1100	3.503	—	—	—
1020	4.006	—	—	1110	3.444	—	—	—
1030	3.943	—	—	1120	3.385	—	—	—
1040	3.884	—	—	1130	3.331	3.306	—	—
1050	3.830	—	—	1140	3.276	3.251	—	—
1060	3.775	—	—	1150	3.226	3.201	—	—
1070	3.725	—	—	1160	3.176	3.155	—	—
1080	3.679	—	—	1170	3.130	3.109	—	—

TABLE A-52 (concluded)

Density at 15 °C, kg/m³	Sodium chloride			Density at 15 °C, kg/m³	Calcium chloride			
	0 °C	−10 °C	−20 °C		0 °C	−10 °C	−20 °C	−30 °C
1090	3.633	—	—	1180	3.088	3.063	—	—
1100	3.591	3.582	—	1190	3.046	3.021	—	—
1110	3.553	3.541	—	1200	3.004	2.979	2.954	—
1120	3.515	3.503	—	1210	2.967	2.941	2.916	—
1130	3.478	3.469	—	1220	2.933	2.908	2.883	—
1140	3.444	3.432	—	1230	2.899	2.874	2.849	—
1150	3.411	3.398	—	1240	2.870	2.845	2.819	2.795
1160	3.377	3.365	—	1250	2.841	2.816	2.791	2.765
1170	3.344	3.335	3.323	1260	2.812	2.786	2.761	2.736
1175	3.331	3.323	3.310	1270	2.782	2.757	2.732	2.707
1203	3.251	—	—	1280	2.757	2.732	2.707	2.682
				1286	2.740	2.715	2.690	2.665
				1370	2.531	—	—	—

TABLE A-53.

SPECIFIC HEATS OF ADSORPTION OF
SELECTED ORGANIC SUBSTANCES BY
CARBON

Substance	Formula	Heat of adsorption	
		kJ/kmol	kJ/kg
Benzene*	C_6H_6	61 590	789.8
Butyl chloride	$CH_3(CH_2)_3Cl$	65 360	706.4
sec-Butyl chloride	$CH_3CHClC_2H_5$	60 340	652.4
tert-Butyl chloride	$(CH_3)_3CCl$	56 980	615.9
Carbon disulphide*	CS_2	52 380	689.3
Carbon tetrachloride*	CCl_4	64 110	415.2
Chloroform*	$CHCl_3$	60 760	508.2
Dichloromethane	CH_2Cl_2	51 960	611.3
Diethyl ether*	$(C_2H_5)_2O$	64 950	877.8
Ethyl alcohol*	C_2H_5OH	62 850	1366
Ethyl bromide*	C_2H_5Br	58 240	534.6
Ethyl chloride*	C_2H_5Cl	50 280	779.3
Ethyl formate*	$HCOOC_2H_5$	60 760	820.8
Ethyl iodide*	C_2H_5I	58 660	376.3
Isopropyl chloride	$CH_3CHClCH_3$	54 810	699.3
Methane	CH_4	18 860	1230
Methyl alcohol	CH_3OH	54 890	1715
Methyl chloride	CH_3Cl	38 550	763.4
Petrol	—	50 280	628.5
Propyl alcohol	C_3H_7OH	68 720	1145
Propyl chloride	$CH_3(CH_2)_2Cl$	61 170	779.3

Notes. 1. The values of the heat of adsorption are given for the conditions of adsorption of 1 kmol of vapour by 500 kg of carbon at 0 °C (except for petrol, for which the ratio between the amounts of adsorbed substance and carbon is not indicated).
2. An asterisk indicates substances for which the heats of adsorption have been calculated by Eq. (9-8): $q=ma^n$ (in J/kg of carbon).

TABLE A-54.

EFFICIENCIES IN EXTRACTION WITH
SUCCESSIVE STAGE CONTACTS (in % by
Mass of Extracted Solute)

Ratio of flows a	Contact Stages							
	1		2		3		4	
	effici- ency	total volume of solvent	effici- ency	total volume of solvent	effici- ency	total volume of solvent	effici- ency	total volume of solvent
1	50.00	1	75.00	2	87.50	3	93.75	4
2	66.67	2	88.89	4	96.30	6	98.76	8
3	75.00	3	93.75	6	98.44	9	99.61	12
4	80.00	4	96.00	8	99.20	12	99.84	16
5	83.33	5	97.22	10	99.54	15	99.92	20
6	85.71	6	97.96	12	99.71	18	99.96	24
7	87.50	7	98.44	14	99.81	21	99.98	28
8	88.89	8	98.76	16	99.86	24	99.98	32
9	90.00	9	99.00	18	99.90	27	99.99	36
10	90.90	10	99.17	20	99.92	30	99.99	40

TABLE A-55.

EFFICIENCIES IN COUNTERCURRENT LEACHING (in % by Mass of Extracted Solute)

Ratio of flows a	Extraction Stages n_s					
	1	2	3	4	5	6
1	50.00	66.67	75.00	80.00	83.33	85.71
2	66.67	85.71	93.00	96.77	98.42	99.21
3	75.00	92.31	97.50	99.17	99.73	99.91
4	80.00	95.24	98.82	99.71	99.93	—
5	83.33	96.77	99.36	99.87	99.97	—
6	85.71	97.67	99.61	99.94	—	—
7	87.50	98.24	99.75	99.96	—	—
8	88.89	98.63	99.83	99.98	—	—
9	90.00	99.90	99.88	99.99	—	—
10	90.90	99.10	99.91	—	—	—

TABLE A-56.

PROPERTIES OF SATURATED WATER VAPOUR AND STEAM DEPENDING ON TEMPERATURE

Tempera- ture, °C	Pressure (abs.), at	Specific volume, m³/kg	Density, kg/m³	Specific enthalpy, kJ/kg		Specific heat of vaporization L , kJ/kg
				liquid, H'	vapour, H''	
0	0.0062	206.5	0.00484	0	2493.1	2493.1
5	0.0089	147.1	0.00680	20.95	2502.7	2481.7
10	0.0125	106.4	0.00940	41.90	2512.3	2470.4
15	0.0174	77.9	0.01283	62.85	2522.4	2459.5
20	0.0238	57.8	0.01729	83.80	2532.0	2448.2
25	0.0323	43.40	0.02304	104.75	2541.7	2436.9
30	0.0433	32.93	0.03036	125.70	2551.3	2425.6
35	0.0573	25.25	0.03960	146.65	2561.0	2414.3
40	0.0752	19.55	0.05114	167.60	2570.6	2403.0
45	0.0977	15.28	0.06543	188.55	2579.8	2391.3
50	0.1258	12.054	0.0830	209.50	2589.5	2380.0
55	0.1605	9.589	0.1043	230.45	2598.7	2368.2
60	0.2031	7.687	0.1301	251.40	2608.3	2356.9
65	0.2550	6.209	0.1611	272.35	2617.5	2345.2
70	0.3177	5.052	0.1979	293.30	2626.3	2333.0
75	0.393	4.139	0.2416	314.3	2636	2321
80	0.483	3.414	0.2929	335.2	2644	2310
85	0.590	2.832	0.3531	356.2	2653	2297
90	0.715	2.365	0.4229	377.1	2662	2285
95	0.862	1.985	0.5039	398.1	2671	2273
100	1.033	1.675	0.5970	419.0	2679	2260
105	1.232	1.421	0.7036	440.4	2687	2248
110	1.461	1.212	0.8254	461.3	2696	2234
115	1.724	1.038	0.9635	482.7	2704	2221
120	2.025	0.893	1.1199	504.1	2711	2207
125	2.367	0.7715	1.296	525.4	2718	2194
130	2.755	0.6693	1.494	546.8	2726	2179

TABLE A-56 (concluded)

Tempera- ture, °C	Pressure (abs.), at	Specific volume, m³/kg	Density, kg/m³	Specific enthalpy, kJ/kg		Specific heat of vaporization L, kJ/kg
				liquid, H'	vapour, H''	
135	3.192	0.5831	1.715	568.2	2733	2165
140	3.685	0.5096	1.962	589.5	2740	2150
145	4.238	0.4469	2.238	611.3	2747	2125
150	4.855	0.3933	2.543	632.7	2753	2120
160	6.303	0.3075	3.252	654.1	2765	2089
170	8.080	0.2431	4.113	719.8	2776	2056
180	10.23	0.1944	5.145	763.8	2785	2021
190	12.80	0.1568	6.378	808.3	2792	1984
200	15.85	0.1276	7.840	852.7	2798	1945
210	19.55	0.1045	9.567	897.9	2801	1904
220	23.66	0.0862	11.600	943.2	2803	1860
230	28.53	0.07155	13.98	989.3	2802	1813
240	34.13	0.05967	16.76	1035	2799	1763
250	40.55	0.04998	20.01	1082	2792	1710
260	47.85	0.04199	23.82	1130	2783	1653
270	56.11	0.03538	28.27	1178	2770	1593
280	65.42	0.02988	33.47	1226	2754	1528
290	75.88	0.02525	39.60	1275	2734	1459
300	87.6	0.02131	46.93	1327	2710	1384
310	100.7	0.01799	55.59	1380	2682	1302
320	115.2	0.01516	65.95	1437	2650	1213
330	131.3	0.01273	78.53	1498	2613	1117
340	149.0	0.01064	93.98	1564	2571	1009
350	168.6	0.00884	113.2	1638	2519	881.2
360	190.3	0.00716	139.6	1730	2444	713.6
370	214.5	0.00585	171.0	1890	2304	411.5
374	225	0.00310	322.6	2100	2100	0

TABLE A-57.

PROPERTIES OF SATURATED WATER
VAPOUR AND STEAM DEPENDING ON
PRESSURE

Pressure (abs.), at	Tempera- ture, °C	Specific, volume, m³/kg	Density, kg/m³	Specific enthalpy, kJ/kg		Specific heat of vaporization L, kJ/kg
				liquid, H'	vapour, H''	
0.01	6.6	131.60	0.00760	27.7	2506	2478
0.015	12.7	89.64	0.01116	53.2	2518	2465
0.02	17.1	68.27	0.01465	71.6	2526	2455
0.025	20.7	55.28	0.01809	86.7	2533	2447
0.03	23.7	46.53	0.02149	99.3	2539	2440
0.04	28.6	35.46	0.02820	119.8	2548	2429
0.05	32.5	28.73	0.03481	136.2	2556	2420
0.06	35.8	24.19	0.04133	150.0	2562	2413
0.08	41.1	18.45	0.05420	172.2	2573	2400

TABLE A-57 (concluded)

Pressure (abs.), at	Tempera- ture, °C	Specific, volume, m³/kg	Density, kg/m³	Specific enthalpy, kJ/kg		Specific heat of vaporization L, kJ/kg
				liquid, H'	vapour, H''	
0.10	45.4	14.96	0.06686	190.2	2581	2390
0.12	49.0	12.60	0.07937	205.3	2588	2382
0.15	53.6	10.22	0.09789	224.6	2596	2372
0.20	59.7	7.977	0.1283	250.1	2607	2358
0.30	68.7	5.331	0.1876	287.9	2620	2336
0.40	75.4	4.072	0.2456	315.9	2632	2320
0.50	80.9	3.304	0.3027	339.0	2642	2307
0.60	85.5	2.785	0.3590	358.2	2650	2296
0.70	89.3	2.411	0.4147	375.0	2657	2286
0.80	93.0	2.128	0.4699	389.7	2663	2278
0.90	96.2	1.906	0.5246	403.1	2668	2270
1.0	99.1	1.727	0.5790	415.2	2677	2264
1.2	104.2	1.457	0.6865	437.0	2686	2249
1.4	108.7	1.261	0.7931	456.3	2693	2237
1.6	112.7	1.113	0.898	473.1	2703	2227
1.8	116.3	0.997	1.003	483.6	2709	2217
2.0	119.6	0.903	1.107	502.4	2710	2208
3.0	132.9	0.6180	1.618	558.9	2730	2171
4.0	142.9	0.4718	2.120	601.1	2744	2141
5.0	151.1	0.3825	2.614	637.7	2754	2117
6.0	158.1	0.3222	3.104	667.9	2768	2095
7.0	164.2	0.2785	3.591	694.3	2769	2075
8.0	169.6	0.2454	4.075	718.4	2776	2057
9.0	174.5	0.2195	4.536	740.0	2780	2040
10	179.0	0.1985	5.037	759.6	2784	2024
11	183.2	0.1813	5.516	778.1	2787	2009
12	187.1	0.1668	5.996	795.3	2790	1995
13	190.7	0.1545	6.474	811.2	2793	1984
14	194.1	0.1438	6.952	826.7	2795	1968
15	197.4	0.1346	7.431	840.9	2796	1956
16	200.4	0.1264	7.909	854.8	2798	1943
17	203.4	0.1192	8.389	867.7	2799	1931
18	206.2	0.1128	8.868	880.3	2800	1920
19	208.8	0.1070	9.349	892.5	2801	1909
20	211.4	0.1017	9.83	904.2	2802	1898
30	232.8	0.06802	14.70	1002	2801	1800
40	249.2	0.05069	19.73	1079	2793	1715
50	262.7	0.04007	24.96	1143	2780	1637
60	274.3	0.03289	30.41	1199	2763	1565
70	284.5	0.02769	36.12	1249	2746	1497
80	293.6	0.02374	42.13	1294	2726	1432
90	301.9	0.02064	48.45	1337	2705	1369
100	309.5	0.01815	55.11	1377	2684	1306
120	323.1	0.01437	69.60	1455	2638	1183
140	335.0	0.01164	85.91	1531	2592	1061
160	345.7	0.00956	104.6	1606	2540	934
180	355.4	0.00782	128.0	1684	2483	799
200	364.2	0.00614	162.9	1783	2400	617
225	374.0	0.00310	322.6	2100	2100	0

TABLE A-58. SI UNITS AND CONVERSION FACTORS

Quantity	SI unit	Conversion factors for most frequently encountered units of other systems and non-system units
Acceleration, linear	m/s ²	1 in/s ² = 0.0254 m/s ² 1 ft/s ² = 0.3048 m/s ²
Angle, plane*	rad	1° (deg) = $\frac{\pi}{180}$ rad 1' = $\frac{\pi}{10\,800}$ rad 1 revolution = 2 π rad = 6.28 rad
Area	m ²	1 ft ² = 0.0929 m ² 1 in ² = 6.451 × 10 ⁻⁴ m ²
Density	kg/m ³	1 ton/m ³ = 1 kg/dm ³ = 1 g/cm ³ = 10 ³ kg/m ³ 1 kgf · s ² /m ⁴ = 9.81 kg/m ³ 1 lb/ft ³ = 16.02 kg/m ³ 1 lb/in ³ = 27.68 × 10 ³ kg/m ³
Density of heat flux (heat stress, unit heat load)	W/m ²	1 kcal/m ² · h = 1.163 W/m ²
Diffusion coefficient	m ² /s	1 ft ² /s = 0.0929 m ² /s
Enthalpy, specific	J/kg	1 kcal/kg = 1 cal/g = 4190 J/kg 1 BTU/lb = 2326 J/kg
Entropy, specific	J/kg · K	1 kcal/kg · K = 4190 J/kg · K 1 BTU/lb · °F = 4190 J/kg · K
Force (weight)	N	1 kgf = 9.81 N 1 dyn = 10 ⁻⁵ N 1 sn = 10 ³ N 1 lbf = 4.45 N
Frequency	Hz	1 s ⁻¹ = 1 Hz 1 rps = 1 Hz 1 rpm = $\frac{1}{60}$ Hz
Heat capacity, specific	J/kg · K	1 kcal/kg · K = 4190 J/kg · K 1 erg/g · K = 10 ⁻⁴ J/kg · K 1 BTU/lb · °F = 4190 J/kg · K
Heat transfer coefficient, individual and overall	W/m ² · K	1 kcal/m ² · h · K = 1.163 W/m ² · K 1 BTU/ft ² · h · °F = 5.6 W/m ² · K
Length*	m	1 μm (micron) = 10 ⁻⁶ m 1 Å = 10 ⁻¹⁰ m 1 ft (') = 0.3048 m 1 in (") = 0.0254 m
Mass*	kg	1 ton (metric) = 1000 kg 1 lb = 0.454 kg

TABLE A-58 (continued)

Quantity	SI unit	Conversion factors for most frequently encountered units of other systems and non-system units
Power	W	1 kgf·m/s = 9.81 W 1 erg/s = 10 ⁻⁷ W 1 kcal/h = 1.163 W 1 lbf·ft/s = 1.356 W
Pressure	Pa	1 bar = 10 ⁵ Pa 1 mbar = 100 Pa 1 dyn/cm ² = 1 μbar = 0.1 Pa 1 kgf/cm ² = 1 at = 735 mm Hg = 9.81 × 10 ⁴ Pa 1 atm = 760 mm Hg = 101 325 Pa 1 kgf/m ² = 9.81 Pa 1 mm H ₂ O = 9.81 Pa 1 mm Hg = 133.3 Pa 1 lbf/in ² (psi) = 6894.76 Pa 1 lbf/ft ² = 47.88 Pa
Rate of flow, mass	kg/s	1 lb/s = 0.454 kg/s 1 lb/h = 1.26 × 10 ⁻⁴ kg/s
Rate of flow, volumetric	m ³ /s	1 l/min = 16.67 × 10 ⁻⁶ m ³ /s 1 ft ³ /s = 28.3 × 10 ⁻³ m ³ /s 1 in ³ /s = 16.4 × 10 ⁻⁶ m ³ /s
Specific heat (of phase transition)	J/kg	1 kcal/kg = 1 cal/g = 4190 J/kg 1 BTU/lb = 2326 J/kg
Surface tension	N/m	1 kgf/m = 9.81 J/m ² = 9.81 N/m 1 erg/cm ² = 1 dyn/cm = 10 ⁻³ N/m
Thermal conductivity	W/m·K	1 kcal/m·h·K = 1.163 W/m·K 1 BTU/ft·h·°F = 1.73 W/m·K
Time*	s	1 h = 3600 s 1 min = 60 s
Temperature*	K	t °C = (t + 273.15) K t °F = $\left[\frac{5}{9} (t - 32) + 273.15 \right]$ K
Velocity, angular	rad/s	1 rpm = $\frac{\pi}{30}$ rad/s 1 rps = 2π rad/s = 6.28 rad/s
Velocity, linear	m/s	1 ft/s = 0.3048 m/s
Viscosity, dynamic	Pa·s	1 P = 1 dyn·s/cm ² = 0.1 Pa·s 1 cP = $\frac{1}{9180}$ kgf·s/m ² = 10 ⁻³ Pa·s
Viscosity, kinematic	m ² /s	1 St = 1 cm ² /s = 10 ⁻⁴ m ² /s 1 ft ² /s = 0.093 m ² /s 1 ft ² /h = 25.81 m ² /s

TABLE A-58 (concluded).

Quantity	SI unit	Conversion factors for most frequently encountered units of other systems and non-system units
Volume	m ³	1 l = 10 ⁻³ m ³ 1 ft ³ = 28.3 dm ³ = 0.0283 m ³ 1 in ³ = 16.387 cm ³ = 16.39 × 10 ⁻⁶ m ³
Volume, specific	m ³ /kg	1 m ³ /ton = 10 ⁻³ m ³ /kg 1 l/kg = 1 cm ³ /g = 10 ⁻³ m ³ /kg
Work, energy, quantity of heat	J	1 kgf·m = 9.81 J 1 erg = 10 ⁻⁷ J 1 kW·h = 3.6 × 10 ⁶ J 1 kcal = 4.19 kJ = 4.1868 × 10 ³ J 1 lbf·ft = 1.356 J 1 lbf·in = 0.113 J 1 BTU = 1055.1 J

Notes. 1. The values of the conversion factors are given with an accuracy that is sufficient for engineering calculations.
2. An asterisk indicates quantities that are basic ones in the SI system.

DIAGRAMS AND NOMOGRAMS*

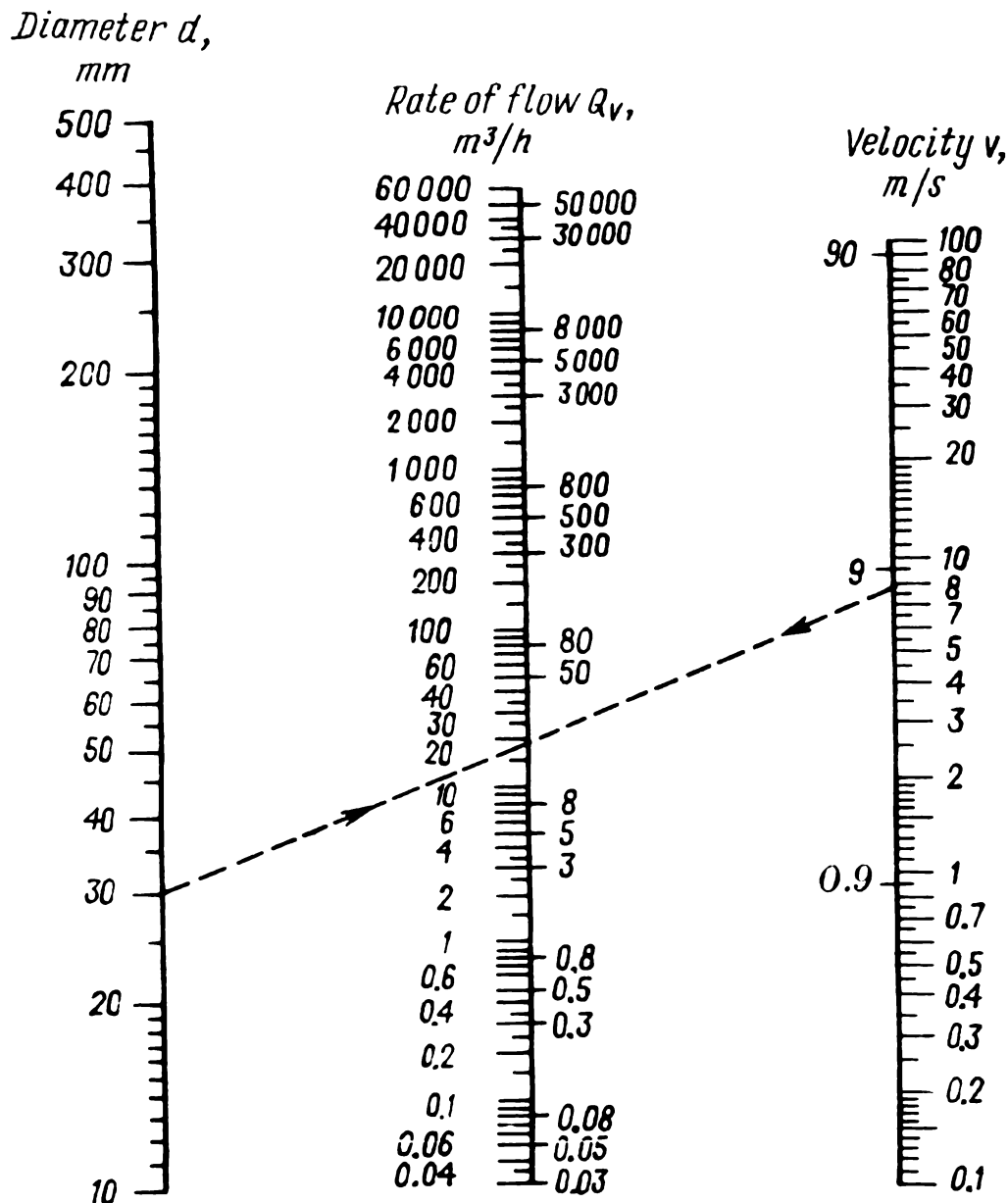


Fig. A-4. Nomogram for determining rate of flow of a fluid in round pipes

* For Fig. A-1 see p. 550, for Figs. A-2 and A-3 see p. 568.

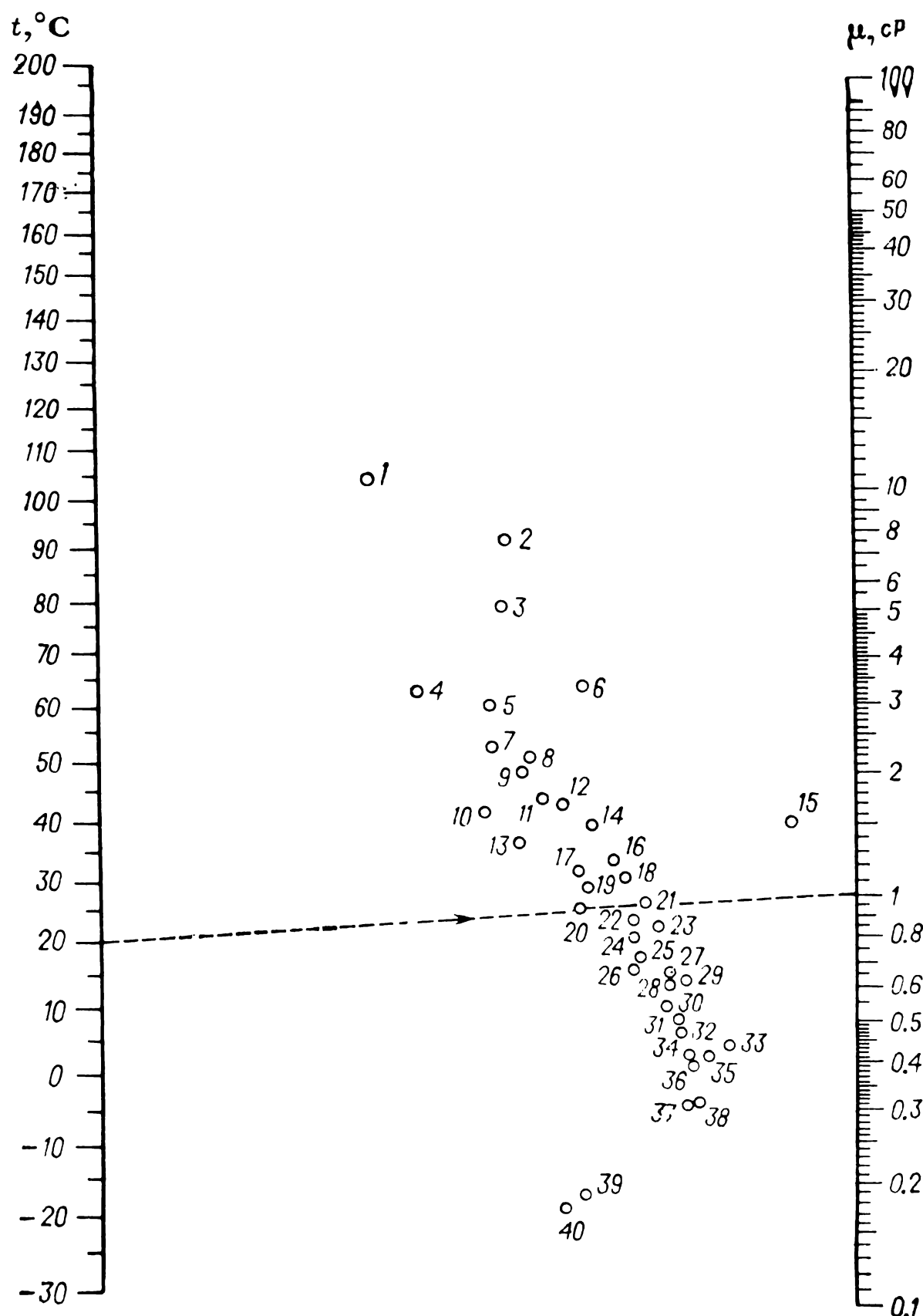


Fig. A-5. Nomogram for determining the dynamic viscosities of liquids at various temperatures

Conversion to SI: $1 \text{ cP} = 1 \times 10^{-3} \text{ Pa}\cdot\text{s}$

Liquid	Point No.	Liquid	Point No.	Liquid	Point No.
Acetic acid, 100%	18	Ethyl alcohol, 100%	19	Methyl alcohol, 30%	13
Acetic acid, 70%	12	Ethyl alcohol, 40%	10	Naphthalene	9
Acetone	34	Ethylene chloride	23	Nitrobenzene	14
Ammonia	39	Ethylene glycol	4	Octane	28
Amyl alcohol	17	Glycerine, 100%	1	Pentane	38
Aniline	8	Glycerine, 50%	7	Phenol	5
Benzene	25	Heptane	31	Sulphur dioxide	35
Butyl alcohol	11	Hexane	36	Sulphuric acid, 111%	2
Carbon dioxide	40	Mercury	15	Sulphuric acid, 98%	3
Carbon disulphide	33	Methyl acetate	32	Sulphuric acid, 60%	6
Carbon tetrachloride	21	Methyl alcohol, 100%	26	Toluene	27
Chlorobenzene	22	Methyl alcohol, 90%	24	Turpentine	16
Chloroform	29			Water	20
Diethyl ether	37				
Ethyl acetate	30				

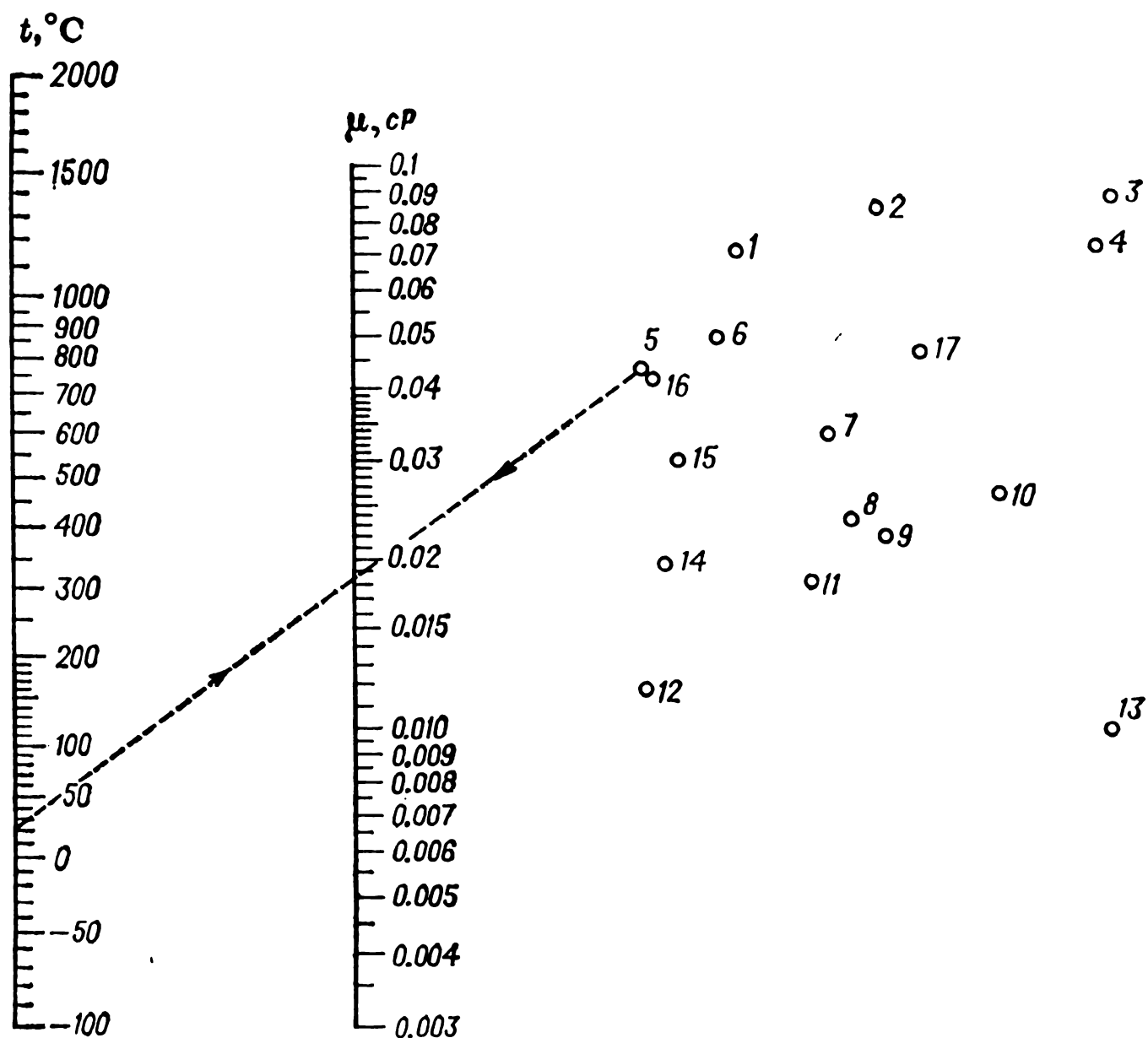
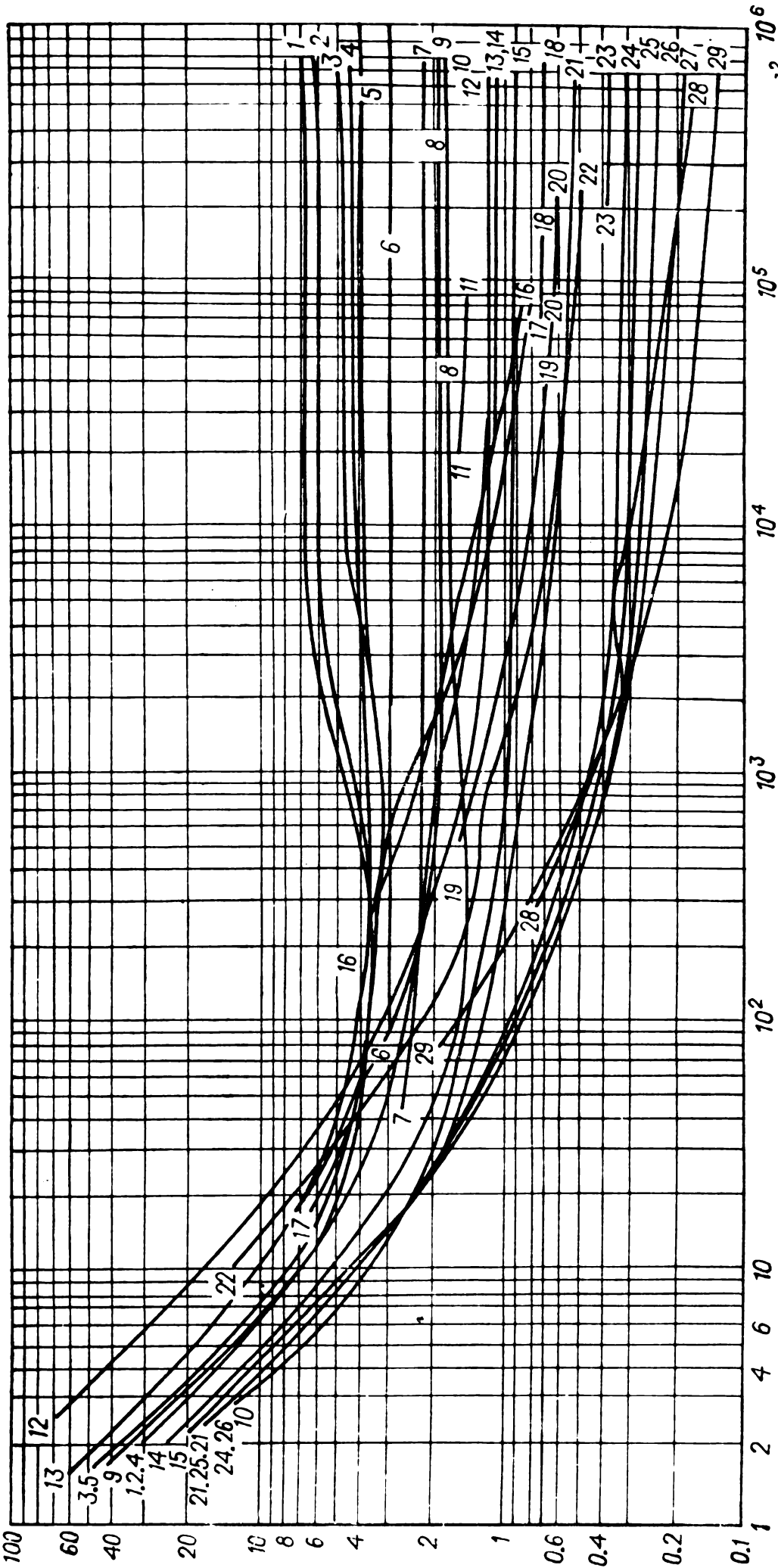


Fig. A-6. Nomogram for determining the dynamic viscosities of gases at $p = 1$ atm (the data may also be used with a permissible approximation for pressures of several atmospheres):

1—O₂; 2—NO; 3—CO₂; 4—HCl; 5—Air; 6—N₂; 7—SO₂; 8—CH₄; 9—H₂O; 10—NH₃;
 11—C₂H₆; 12—H₂; 13—C₆H₆; 14—9H₂+N₂; 15—3H₂+N₂; 16—CO; 17—Cl₂.
 Conversion to SI: 1 cP = 1×10^{-3} Pa·s

$$N_p = \frac{P_{steady}}{\rho n^3 d^5}$$



$$Re_c = \frac{\rho n d^2 \omega}{\mu}$$

Fig. A-7. Dependence of power number on Reynolds number:

1 — open turbine agitator with six straight vertical blades ($b=0.20d_{ag}$, $L=0.25d_{ag}$) and $D/d_{ag}=3$ in a vessel (tank) with four baffles ($B/d_{ag}=0.17$); 2 — turbine agitator of type 1 with $B/d_{ag}=0.10$; 3 — open turbine agitator with six curved vertical blades ($b=0.20d_{ag}$, $L=0.25d_{ag}$) and $D/d_{ag}=3$ in a vessel with four baffles ($B/d_{ag}=0.10$); 4 — turbine agitator of type 1 with $B/d_{ag}=0.04$; 5 — open turbine agitator with six swept (arrow-shaped) blades ($b=0.20d_{ag}$, $L=0.25d_{ag}$) and $D/d_{ag}=3$ in a vessel with four baffles ($B/d_{ag}=0.10$); 6 — one-sided radial disk agitator with six straight vertical blades ($b=0.10d_{ag}$, $L=0.35d_{ag}$) at the bottom of the disk and $D/d_{ag}=2.5$ in a vessel with four baffles ($B/d_{ag}=0.25$); 7 — radial turbine agitator with sixteen blades with a stator in a vessel without baffles; 8 — two-bladed paddle agitator with straight vertical blades ($b=0.25d_{ag}$) and $D/d_{ag}=4.35$ in a vessel with three baffles ($B/d_{ag}=0.11$); 9 — eight-bladed paddle agitator with straight blades ($b=0.25d_{ag}$) pitched at 45° and $D/d_{ag}=3$ in a vessel with four baffles ($B/d_{ag}=0.10$); 10 — two-bladed paddle agitator of type 8 with $D/d_{ag}=3$ in a vessel with four baffles ($B/d_{ag}=0.10$); 11 — shrouded turbine agitator with six blades with a stator and $D/d_{ag}=2.4$ in a vessel without baffles; 12 — turbine agitator similar to type 11 with $D/d_{ag}=3$ in a vessel without baffles; 13 — turbine agitator of type 12 without a stator with $D/d_{ag}=3$ in a vessel with four baffles ($B/d_{ag}=0.10$); 14 — turbine agitator of type 1 in a vessel without baffles; 15 — three-bladed propeller agitator, $s=2d_{ag}$ with $D/d_{ag}=3$ in a vessel with four baffles ($B/d_{ag}=0.10$); 16 — four-bladed paddle agitator of type 8 with $D/d_{ag}=3$ in a vessel without baffles; 17 — four-bladed paddle agitator with blades ($b=0.25d_{ag}$) pitched at 60° and $D/d_{ag}=3$ in a vessel without baffles; 18 — three-bladed propeller agitator of type 15 but with $s=1.33d_{ag}$ and $D/d_{ag}=16$ in a vessel with three baffles ($B/d_{ag}=0.06$); 19 — four-bladed paddle mixer of type 9 with $D/d_{ag}=5.2$ in a vessel without baffles; 20 — two-bladed agitator of type 8 with $D/d_{ag}=3$ in a vessel without baffles; 21 — three-bladed propeller agitator of type 15 with $D/d_{ag}=3.3$ in a vessel without baffles; 22 — four-bladed paddle agitator of type 9 (the same as 19) with $D/d_{ag}=2.4$ to 3.0 in a vessel without baffles; 23 — three-bladed propeller agitator of type 15 with $s=1.09d_{ag}$ and $D/d_{ag}=9.6$ in a vessel with three baffles ($B/d_{ag}=0.06$); 24 — ditto with $s=d_{ag}$ and $D/d_{ag}=3$ in a vessel with four baffles ($B/d_{ag}=0.10$); 25 — ditto with $s=1.04d_{ag}$ and $D/d_{ag}=4.5$ in a vessel without baffles; 26 — ditto with $s=d_{ag}$ and $D/d_{ag}=3$ in a vessel without baffles; 27 — ditto with $s=1.05d_{ag}$ and $D/d_{ag}=2.7$ in a vessel without baffles; 28 — ditto with $s=d_{ag}$ and $D/d_{ag}=3.8$ in a vessel without baffles; 29 — two-bladed paddle agitator of type 8 with narrow blades ($b=0.13d_{ag}$ to $0.17d_{ag}$) and $D/d_{ag}=1.1$ in a vessel without baffles.

The symbols used to characterize the agitators are as follows: D = diameter of the vessel; d_{ag} = diameter of the agitator; b = width of an agitator blade; L = length of a blade; B = width of a baffle; s = pitch of a propeller agitator

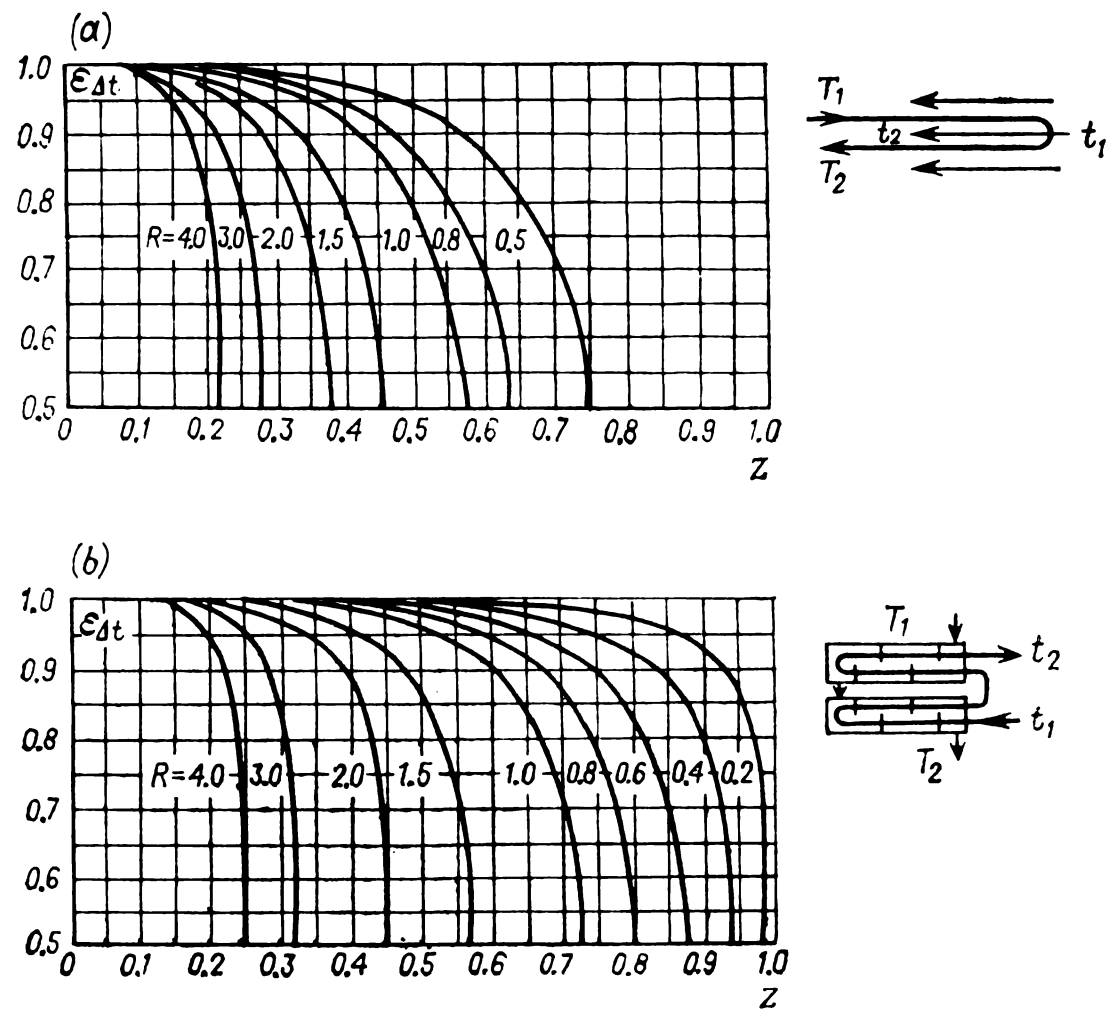


Fig. A-8. Correction factors $\epsilon_{\Delta t}$ for combined (parallel-countercurrent) flow in multipass shell-and-tube heat exchangers:
a—with one pass on the shell side and two, four, six and more passes on the tube side;
b—with two passes on the shell side with baffles and four passes on the tube side. The dimensionless ratios are:

$$Z = \frac{t_2 - t_1}{T_1 - t_1} \quad \text{and} \quad R = \frac{T_1 - T_2}{t_2 - t_1}$$

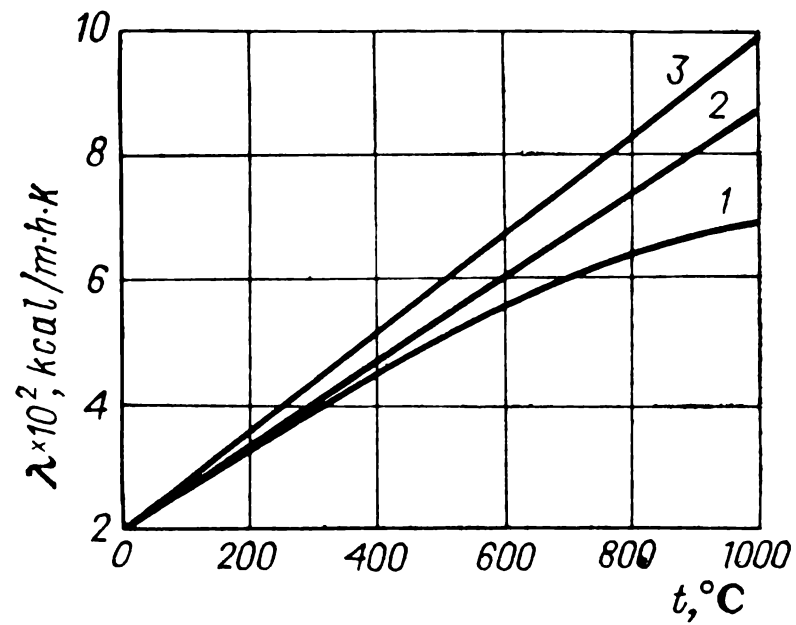


Fig. A-9. Thermal conductivities of flue gases. Composition of flue gases, % (volume): CO₂—13, O₂—5; (N₂ + H₂O)—82:
 1—content of water vapour 0; 2—ditto, 15%; 3—ditto, 20%.
 Conversion to SI: 1 kcal/m·h·K=1.163 W/m·K.

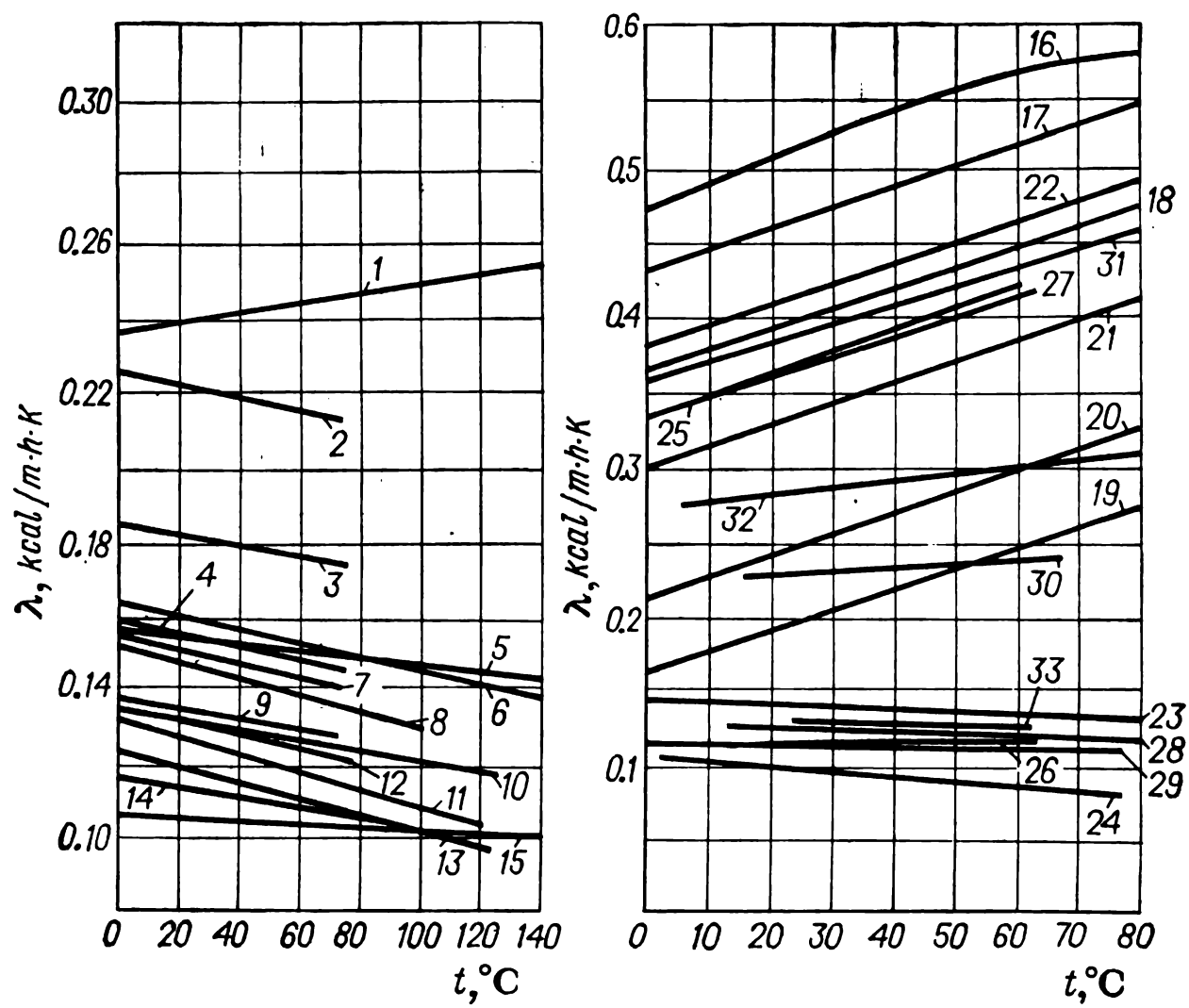


Fig. A-10. Thermal conductivities of selected liquids

Conversion to SI: 1 kcal/m·h·K = 1.163 W/m·K

Substance	Curve No	Substance	Curve No.	Substance	Curve No.
Acetic acid	7	Ethyl alcohol, 80%	19	Kerosene	28
Acetone	8	Ethyl alcohol, 60%	20	Methyl alcohol, 100%	3
Ammonia, 26%	31	Ethyl alcohol, 40%	21	Methyl alcohol, 40%	32
Aniline	6	Ethyl alcohol, 20%	22	Nitrobenzene	10
Benzene	11	Formic acid	2	Octane	33
Butyl alcohol	9	Glycerine, anhydrous	1	Sodium chloride, 25%	18
Calcium chloride, 25%	17	Glycerine, 50%	25	Sulphuric acid, 98%	30
Carbon disulphide	23	Hexane	26	Toluene	13
Carbon tetrachloride	24	Hydrochloric acid, 30%	27	Vaseline oil	15
Castor oil	5	Isopropyl alcohol	12	Water	16
Diethyl ether	29			Xylene	14
Ethyl alcohol, 100%	4				

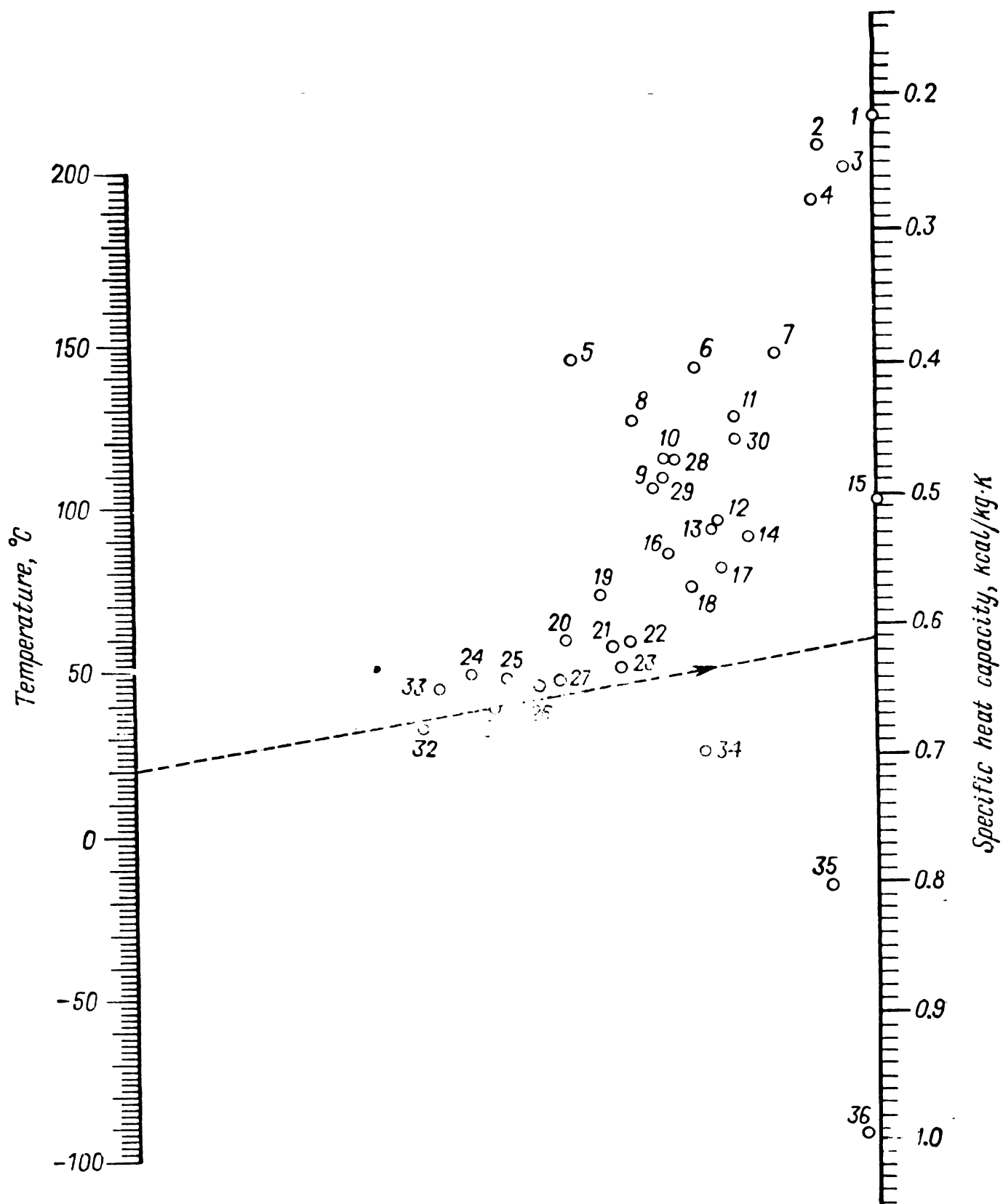


Fig. A-11. Nomogram for determining heat capacities of liquids

Conversion to SI: $1 \text{ kcal/kg}\cdot\text{K} = 4.19 \times 10^3 \text{ J/kg}\cdot\text{K}$

Substance	Point No.	Substance	Point No.	Substance	Point No.
Acetic acid, 100%	16	Ethyl alcohol	31	Methyl alcohol	23
Acetone	18	Ethyl bromide	1	Octane	15
Amyl acetate	12	Ethyl chloride	11	Propyl alcohol	25
Aniline	14	Ethyl iodide	5	Sodium chloride, 25%	35
Benzene	29	Ethylene glycol	22	Sulphuric acid, 100%	7
Butyl alcohol	24	Glycerine	21	Toluene (from -60 to $+40^\circ\text{C}$)	28
Calcium chloride, 25%	34	Heptane	19	Toluene (from 40 to 100°C)	30
Carbon disulphide	4	Hydrochloric acid, 30%	26	Water	36
Carbon tetrachloride	2	Isobutyl alcohol	33	<i>o</i> - and <i>m</i> -Xylene	9
Chlorobenzene	6	Isopentane	20	<i>p</i> -Xylene	10
Chloroform	3	Isopropyl alcohol (from 0 to 50°C)	32		
Diethyl ether	17	Isopropyl alcohol (from -50 to 0°C)	27		
Diphenyl	8				
Ethyl acetate	13				

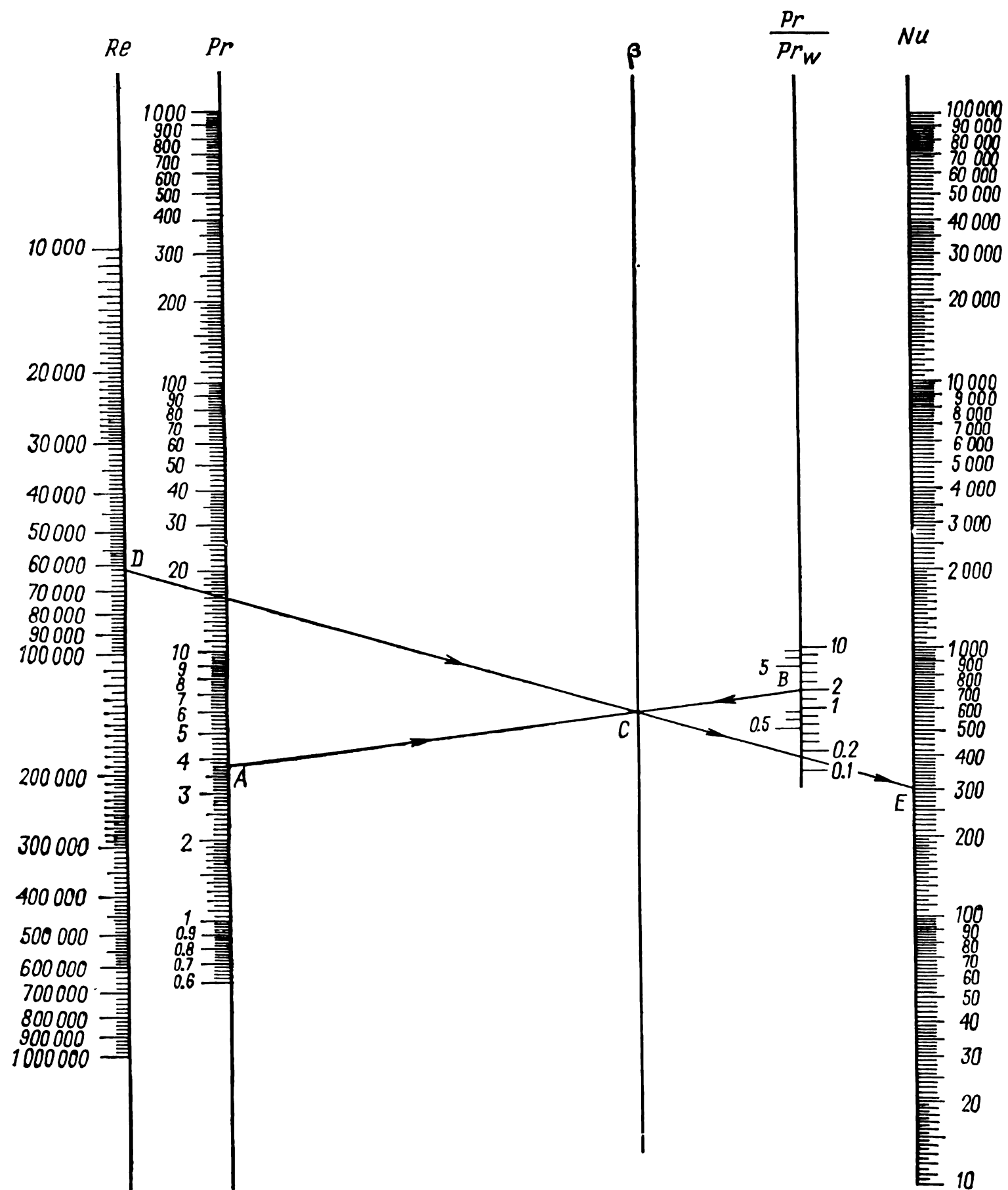


Fig. A-12. Nomogram for determining the individual heat transfer coefficient in straight pipes at $Re > 10\,000$ and $\epsilon_l = 1$:
First step: $AB \rightarrow C$; second step: $DC \rightarrow E$

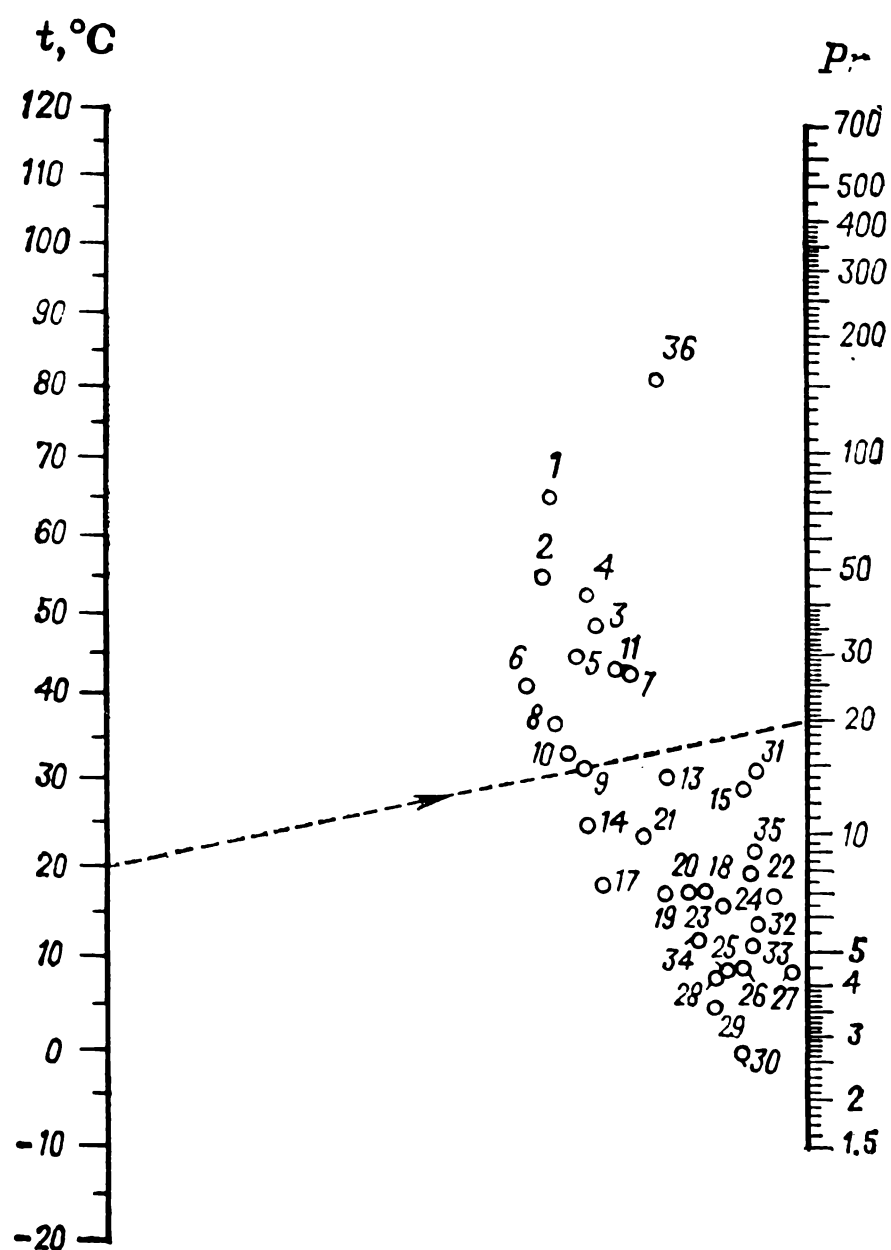


Fig. A-13. Values of Prandtl number for liquids

Substance	Point No.	Substance	Point No.	Substance	Point No.
Acetic acid, 100%	15	Ethyl acetate	24	Methyl alcohol, 100%	20
Acetic acid, 50%	9	Ethyl alcohol, 100%	13	Methyl alcohol, 40%	10
Acetone	25	Ethyl alcohol, 50%	8	Octane	33
Ammonia, 26%	14	Ethyl bromide	29	Pentane	26
Amyl acetate	31	Ethyl iodide	27	Sulphuric acid, 111%	1
Aniline	5	Ethylene glycol	36	Sulphuric acid, 98%	2
Benzene	22	Glycerine, 50%	6	Sulphuric acid, 60%	4
Butyl alcohol	11	Heptane	32	Toluene	23
Carbon disulphide	30	Hydrochloric acid, 30%	21	Water	17
Carbon tetrachloride	18	Isoamyl alcohol	3	Xylene	19
Chlorobenzene	35	Isopropyl alcohol	7		
Chloroform	34				
Diethyl ether	28				

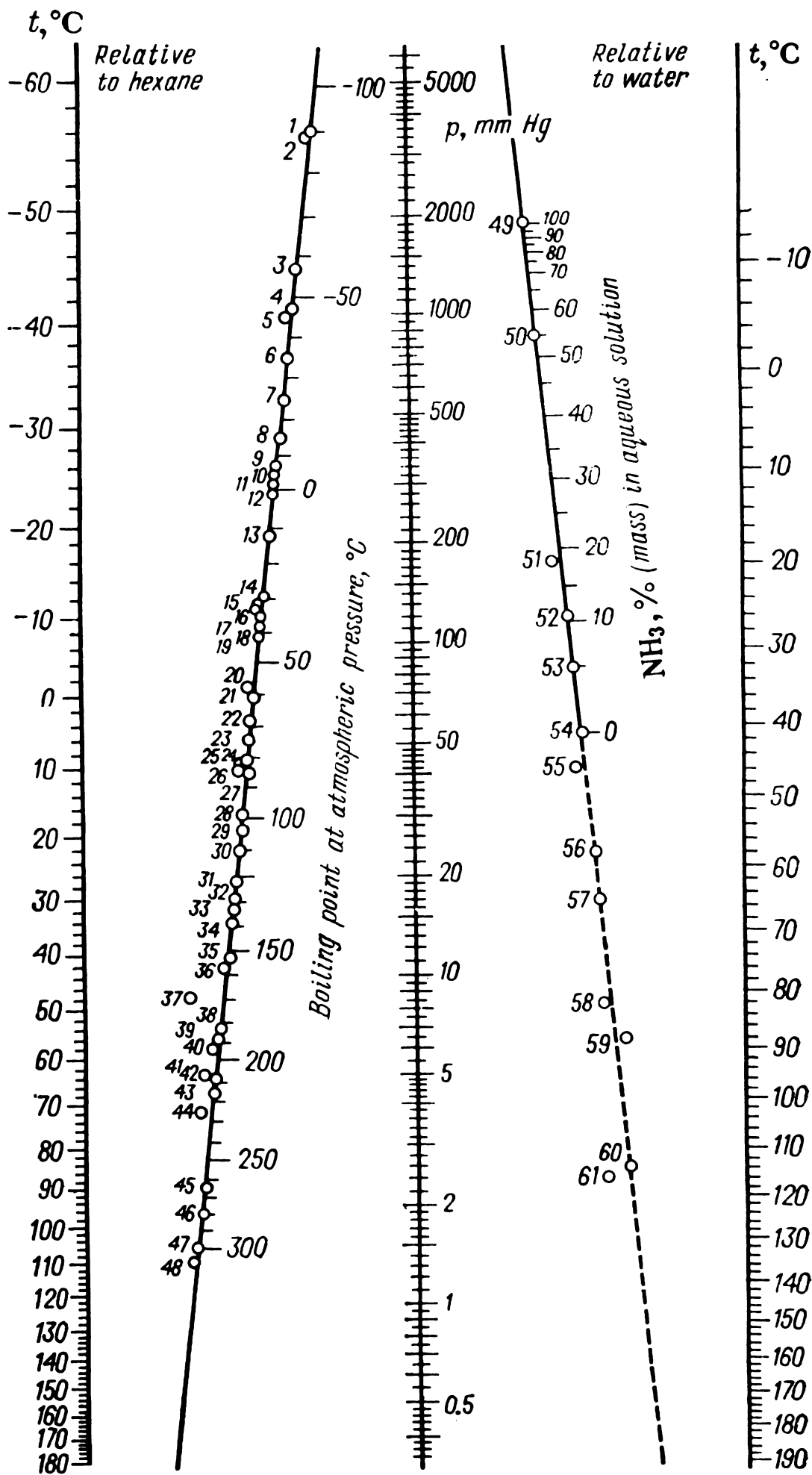


Fig. A-14. (for explanations see following page)

Fig. A-14. Nomogram for determining the saturated vapour pressure and boiling point of selected liquids

Conversion to SI: 1 mm Hg=133.3 Pa

Substance	Point No.	Substance	Point No.	Substance	Point No.
Acetic acid	55	1,2-Dichloroethane	26	Methyl formate	16
Acetone	51	Diethyl ether	15	Methyl monosilane	3
Acetylene	2	Dioxan	29	Methylene chloride	19
Allene	6	Diphenyl	45	Naphthalene	43
Ammonia	49	Ethane	1	α -Naphthalene bro-	
Aniline	40	Ethyl acetate	25	mide	46
Benzene	24	Ethyl alcohol	53	α -Naphthol	47
Bromobenzene	35	Ethyl bromide	18	β -Naphthol	48
1,3-Butadiene	10	Ethyl chloride	13	Nitrobenzene	37
Butane	11	Ethyl formate	20	Octane	31
α -Butylene	9	Ethylene glycol	59		32*
β -Butylene	12	Fluorobenzene	27	Pentane	17
Butylene glycol	58	Glycerine	60	Propane	5
Carbon tetrachloride	23	Heptane	28	Propionic acid	56
Chlorobenzene	33	Hexane	22	Propylene	4
Chloroform	21	Iodobenzene	39	Tetralin	42
<i>m</i> -Cresol	44	Isobutyric acid	57	Toluene	30
<i>o</i> -Cresol	41	Isoprene	14	Vinyl chloride	8
Decalin	38	Mercury	61	Water	54
Decane	36	Methyl alcohol	52	<i>m</i> -Xylene	34
		Methyl amine	50		
		Methyl chloride	7		

* According to the data of different authors.

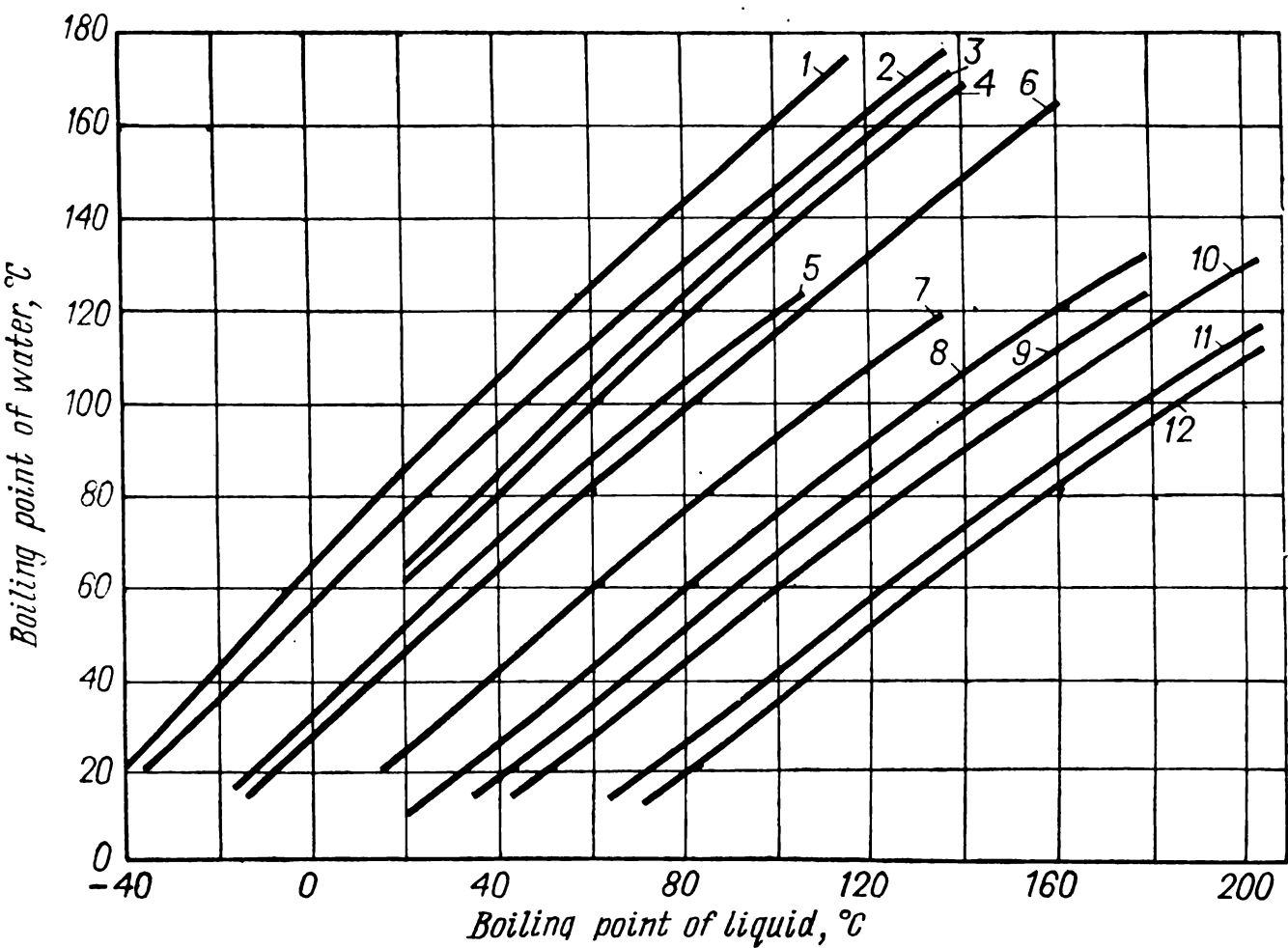


Fig. A-15. Linearity diagram (Dühring lines) for determining the boiling point (relative to water):

1—diethyl ether; 2—carbon disulphide; 3—acetone; 4—chloroform; 5—carbon tetrachloride; 6—benzene; 7—toluene; 8—chlorobenzene; 9—o-xylene; 10—bromobenzene; 11—benzaldehyde; 12—aniline

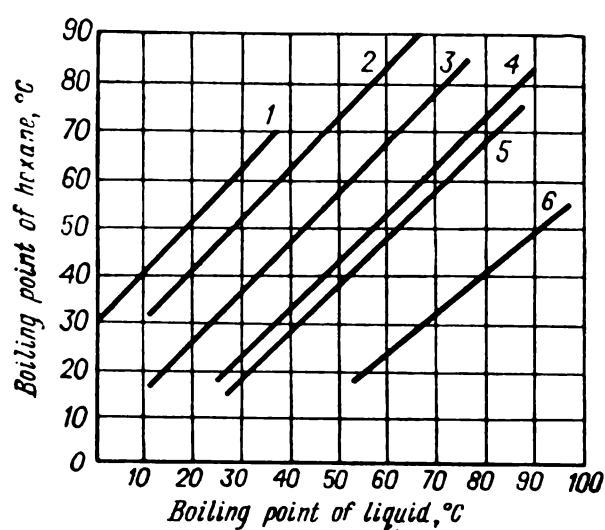


Fig. A-16. Linearity diagram (Dühring lines) for determining the boiling point (relative to hexane):
1—diethyl ether; 2—carbon disulphide; 3—chloroform; 4—carbon tetrachloride; 5—benzene; 6—toluene

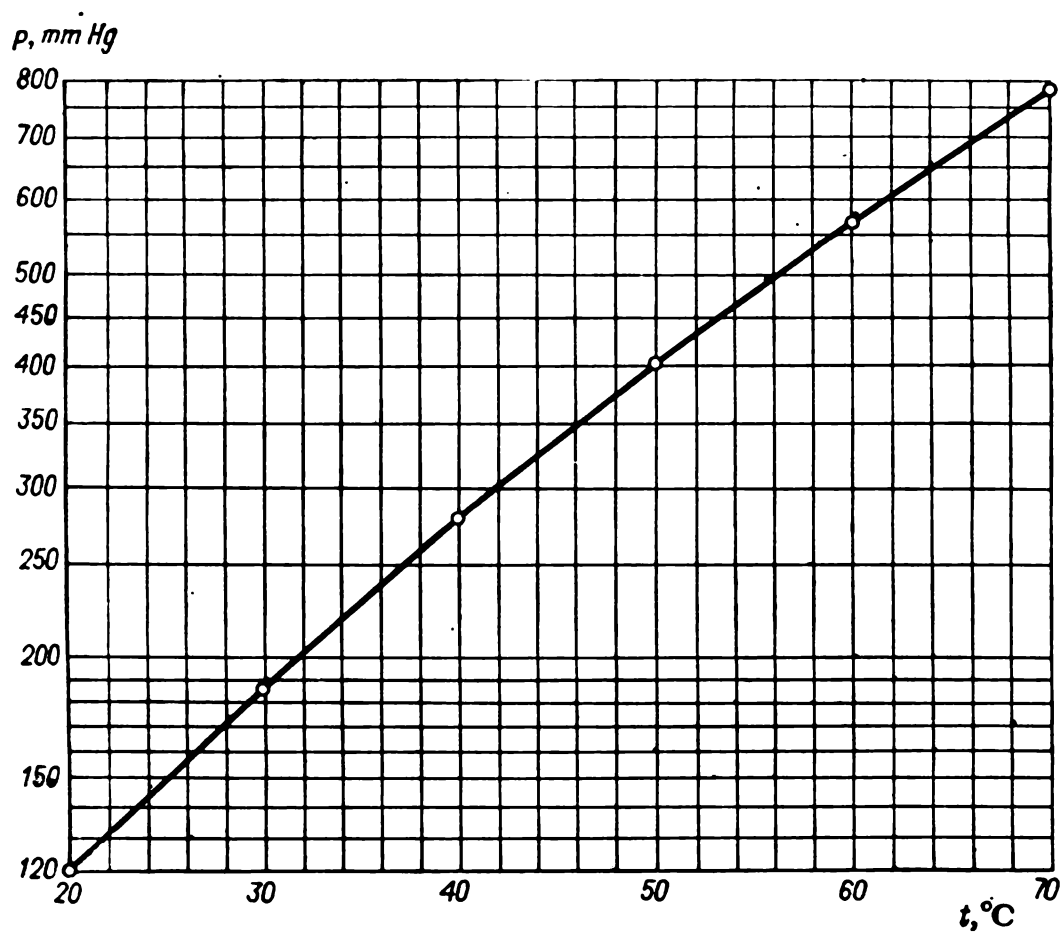


Fig. A-17. Temperature dependence of saturated vapour pressure of hexane
Conversion to SI: 1 mm Hg=133.3 Pa

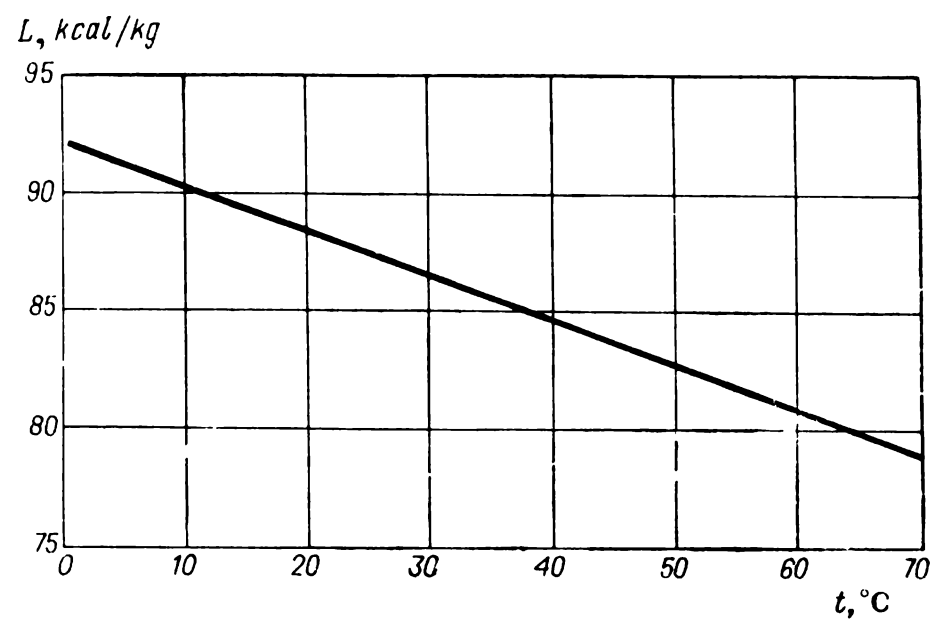


Fig. A-18. Temperature dependence of specific heat of vaporization of hexane
Conversion to SI: 1 kcal/kg=4.19 kJ/kg

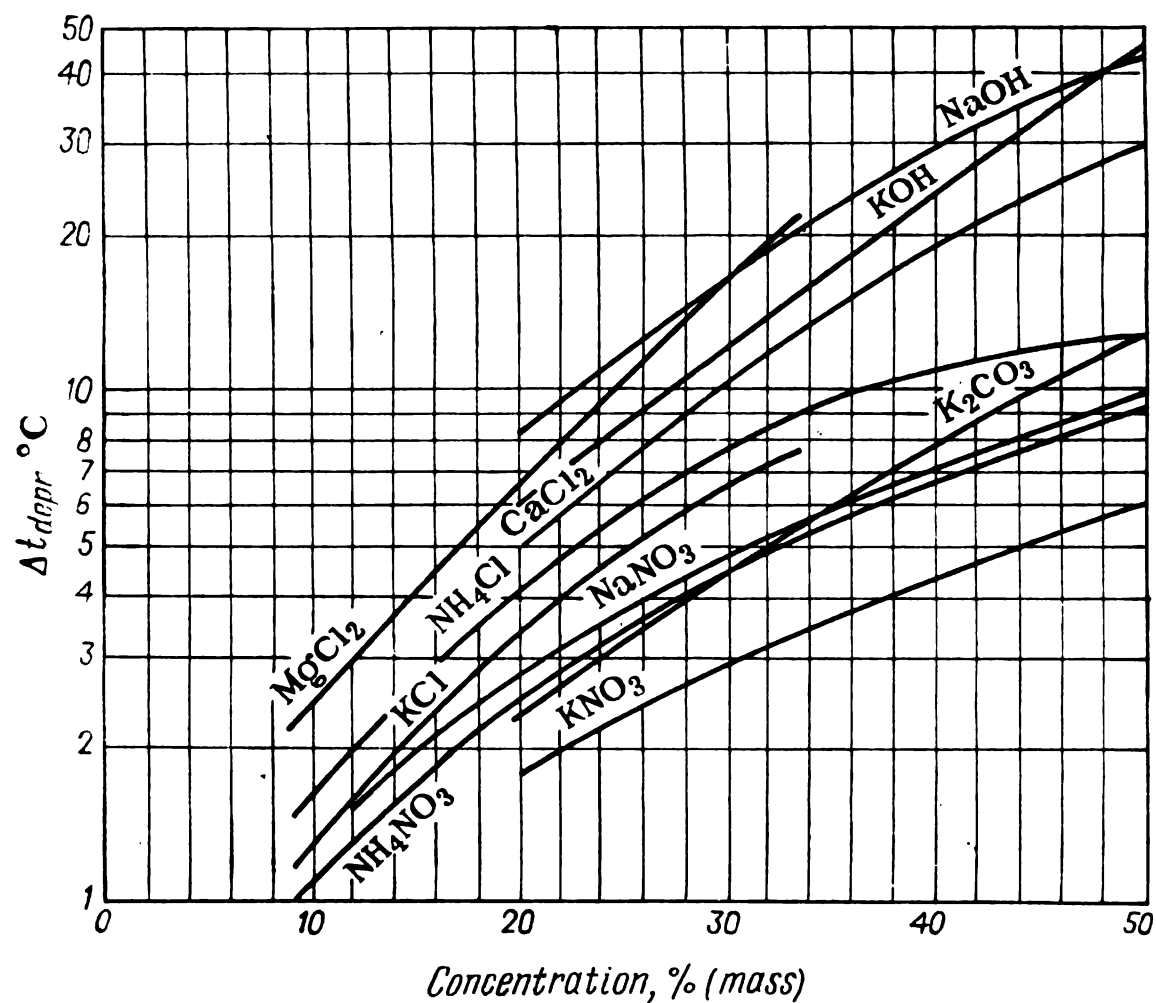


Fig. A-19. Elevation of boiling point of aqueous solutions (at atmospheric pressure) depending on the concentration

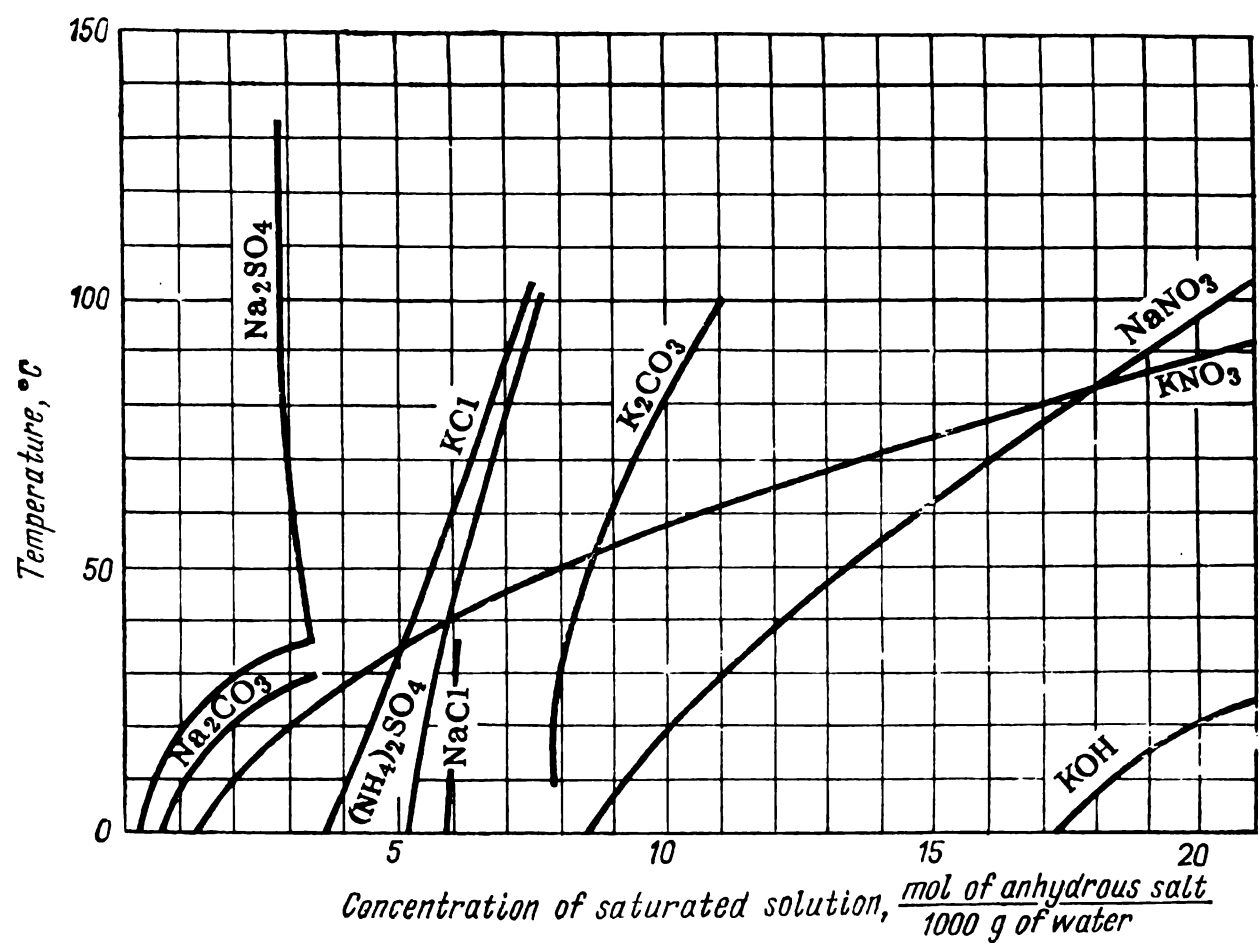


Fig. A-20. Solubility of selected salts in water depending on the temperature

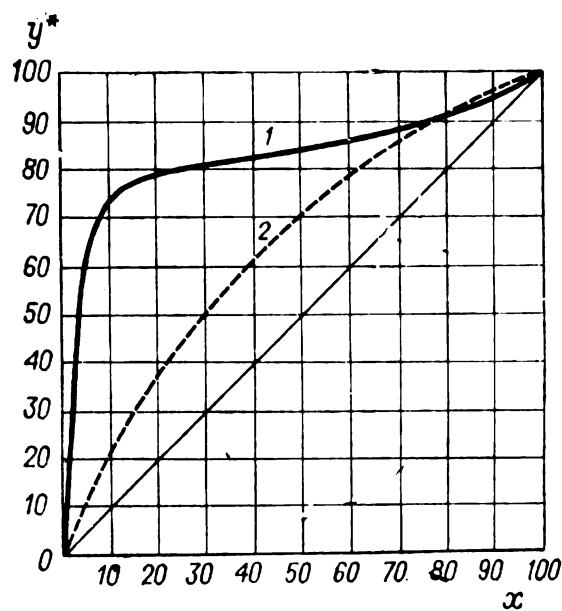


Fig. A-21. Equilibrium curves for $p_{\text{tot}} = 1$ atm:
1 — acetone-water; 2 — carbon tetrachloride-toluene

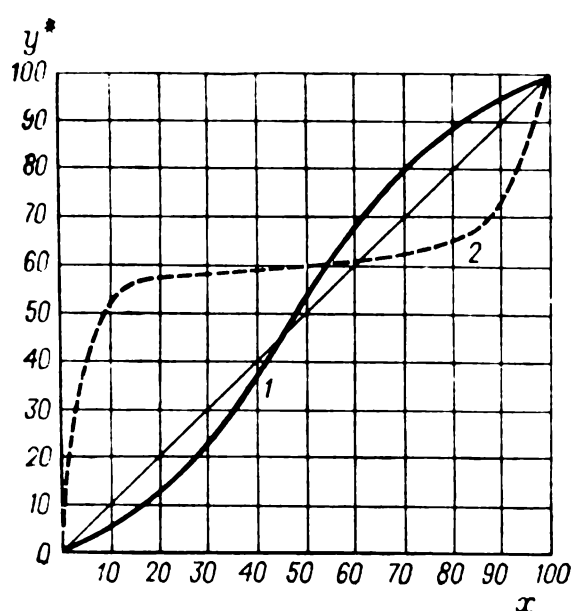


Fig. A-22. Equilibrium curves (with azeotropic point) for $p_{\text{tot}} = 1 \text{ atm}$:
1 — water-formic acid; 2 — methyl alcohol-benzene

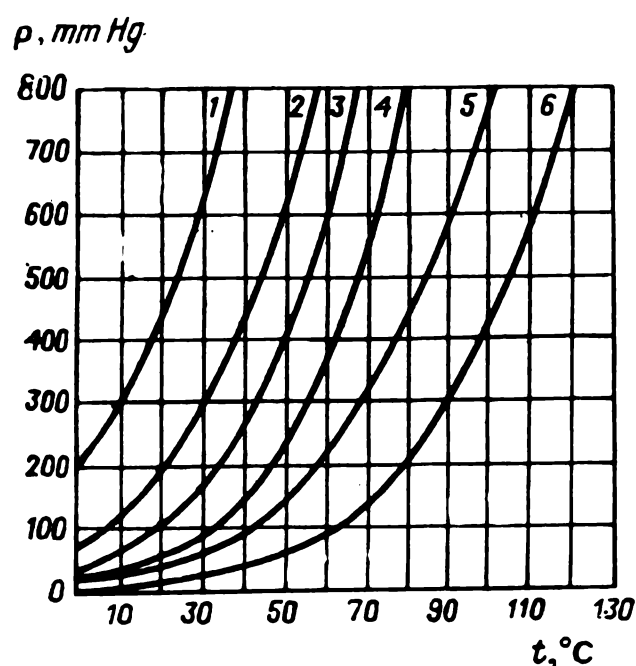


Fig. A-23. Temperature dependence of saturated vapour pressure of selected organic liquids:

1 — diethyl ether; 2 — acetone; 3 — methyl alcohol;
4 — ethyl alcohol; 5 — formic acid; 6 — acetic acid
Conversion to SI: 1 mm Hg = 133.3 Pa

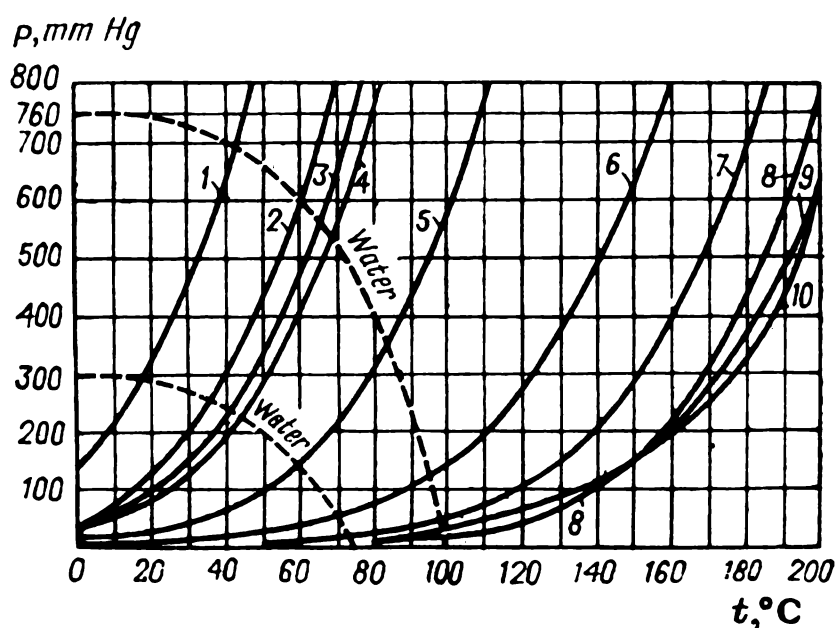


Fig. A-24. Temperature dependence of saturated vapour pressure of organic liquids immiscible with water:

1 — carbon disulphide; 2 — hexane; 3 — carbon tetrachloride; 4 — benzene; 5 — toluene; 6 — turpentine; 7 — aniline; 8 — cresol; 9 — nitrobenzene; 10 — nitrotoluene
Conversion to SI: 1 mm Hg = 133.3 Pa

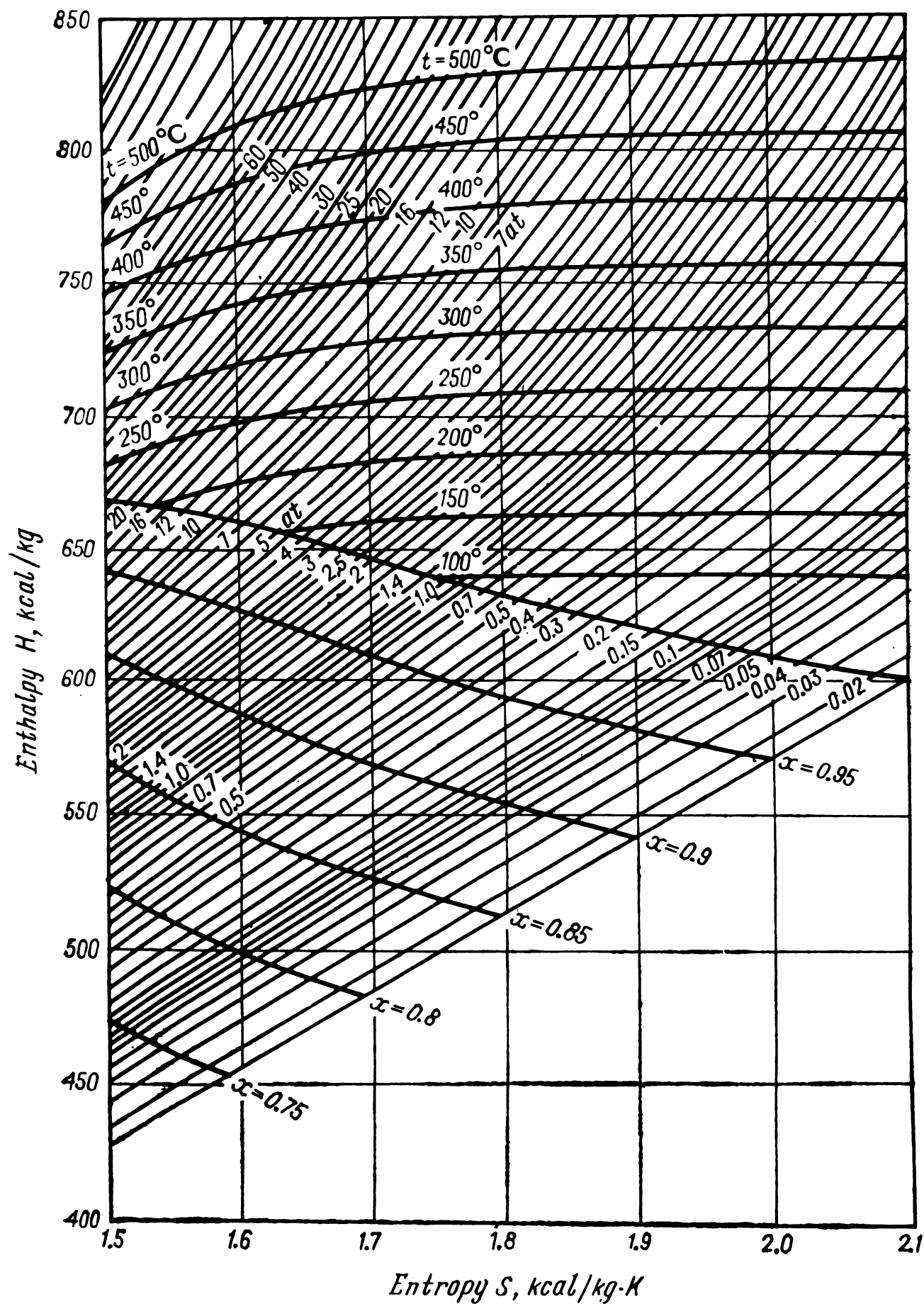


Fig. A-25. H - S diagram for water vapour and steam
Conversion to SI: 1 kcal/kg=4.19 kJ/kg 1 at= 9.81×10^4 Pa

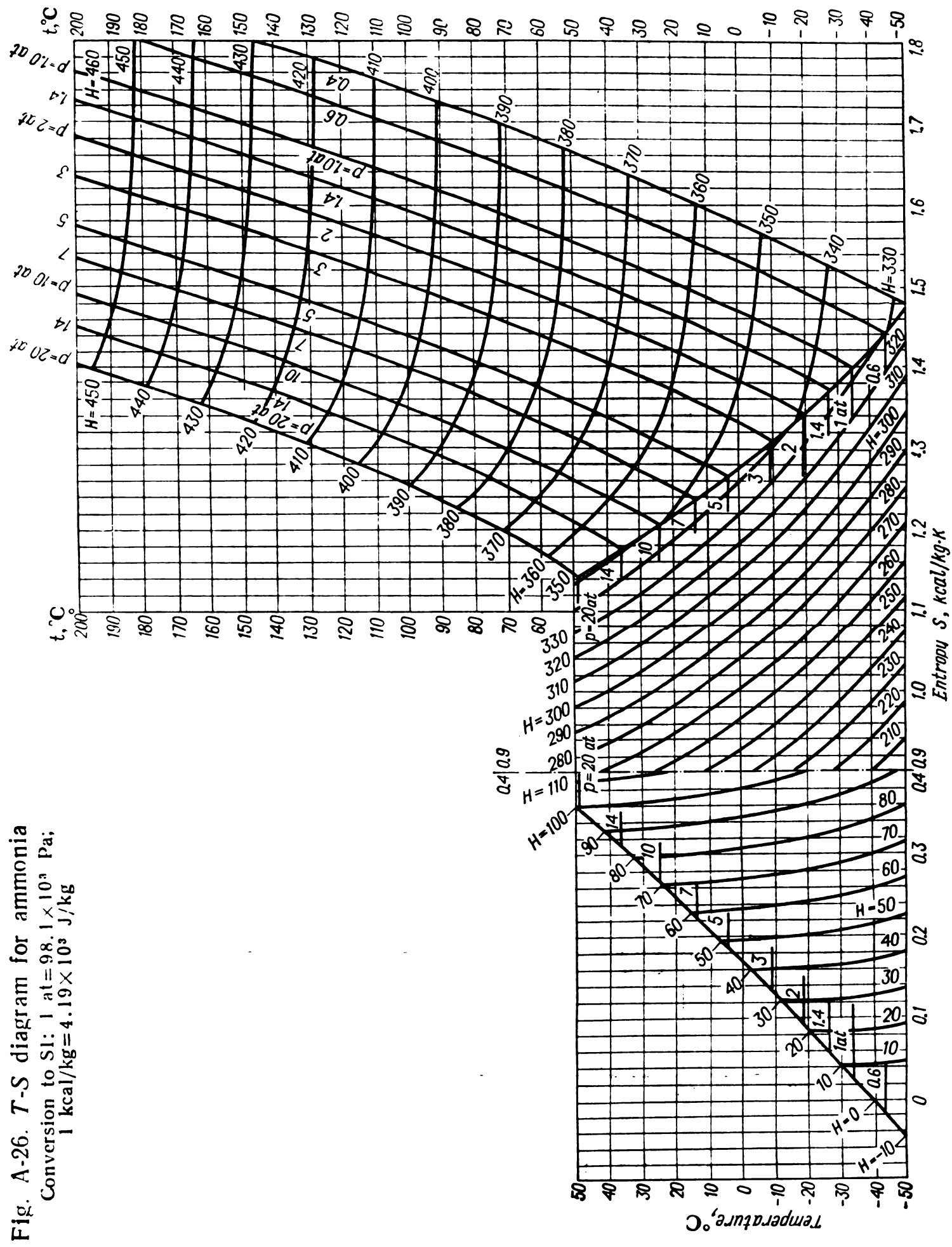


Fig. A-26. T-S diagram for ammonia
Conversion to SI: 1 at=98.1 × 10³ Pa;
1 kcal/kg=4.19 × 10³ J/kg

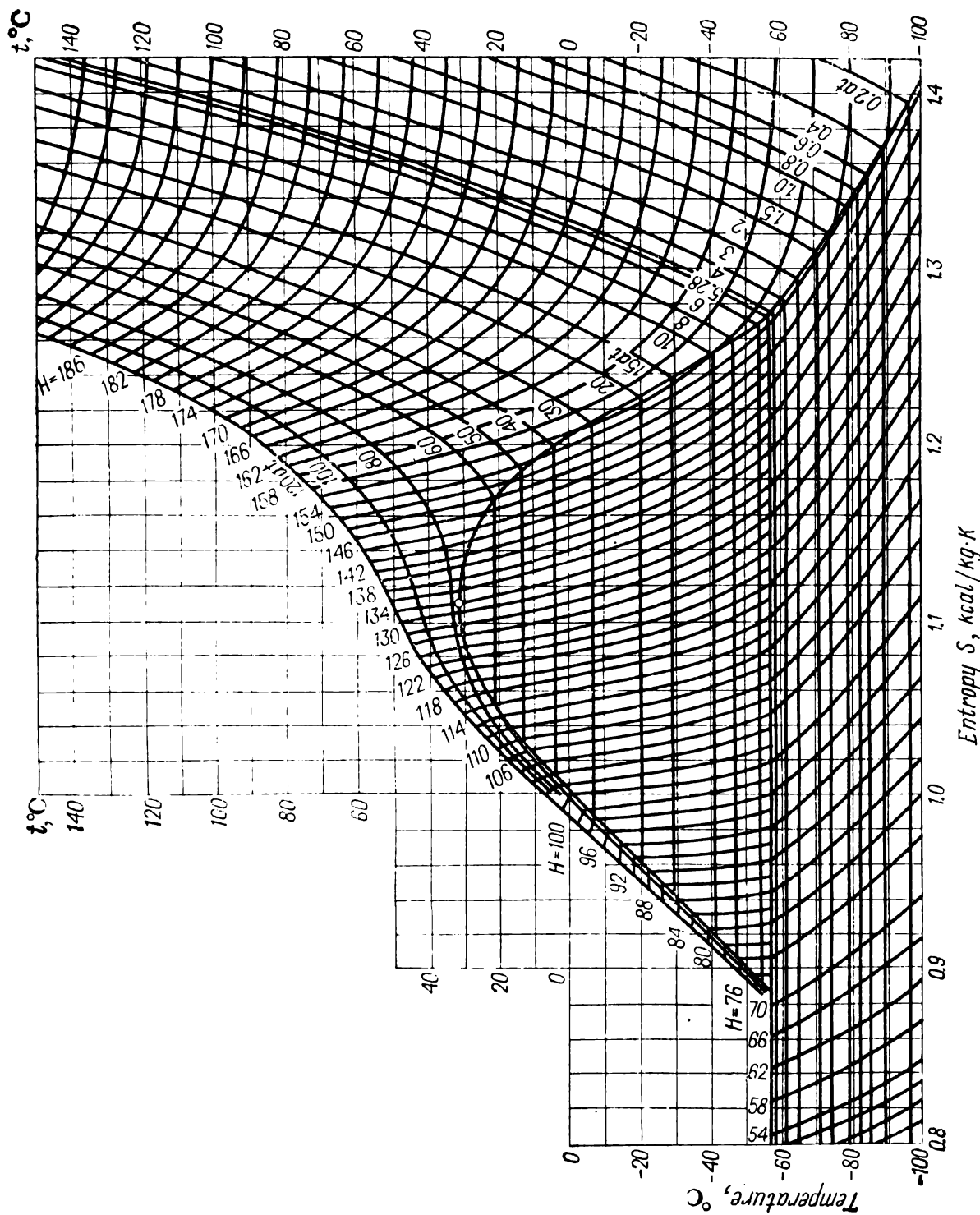


Fig. A-27. T-S diagram for carbon dioxide
Conversion to SI: 1 at=98.1×10³ Pa; 1 kcal/kg=4.19×10³ J/kg

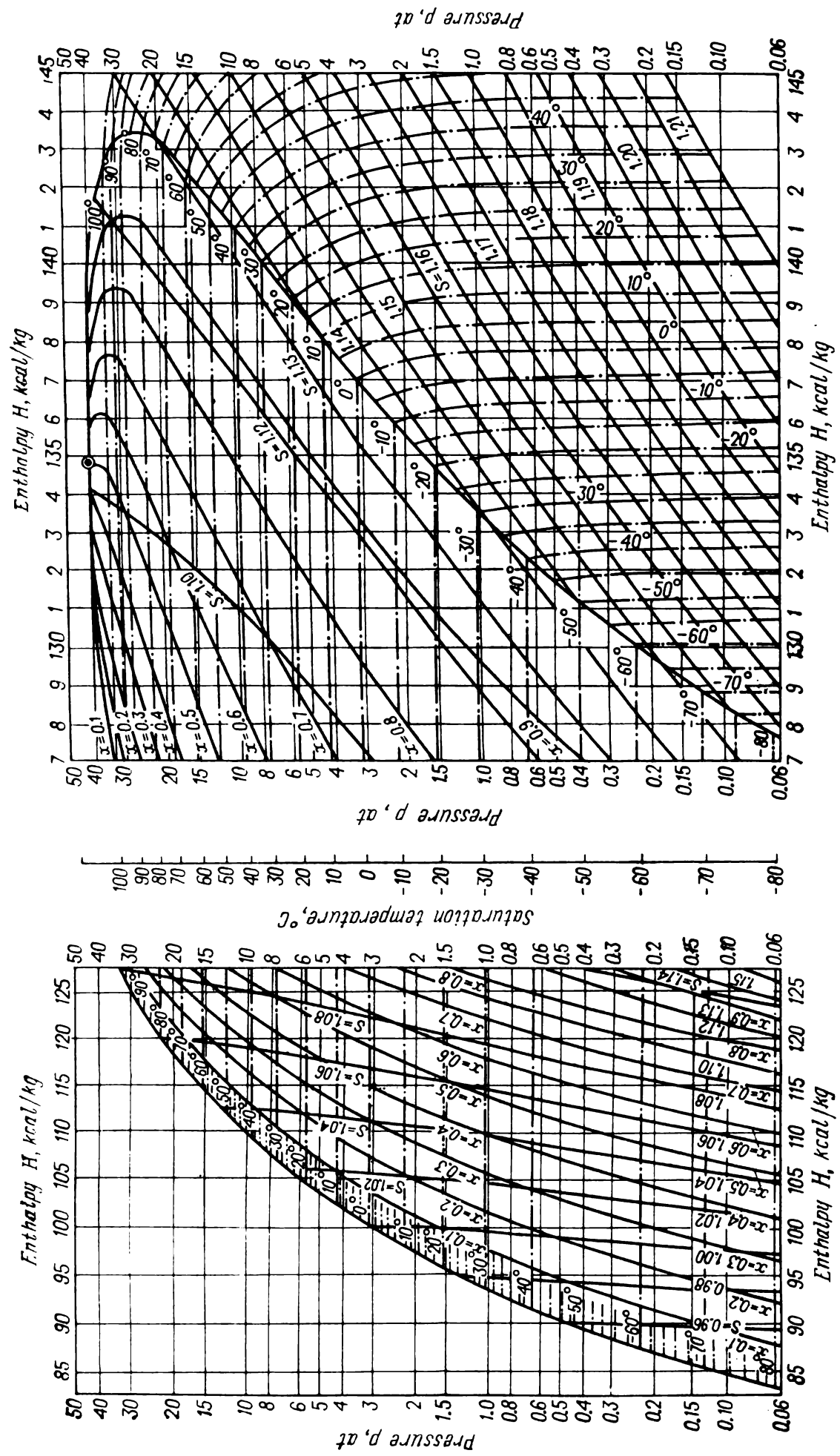


Fig. A-28. Diagram of H against $\log p$ for difluorodichloromethane (Freon-12)

Conversion to SI: 1 at= 98.1×10^3 Pa; 1 kcal/kg= 4.19×10^3 J/kg

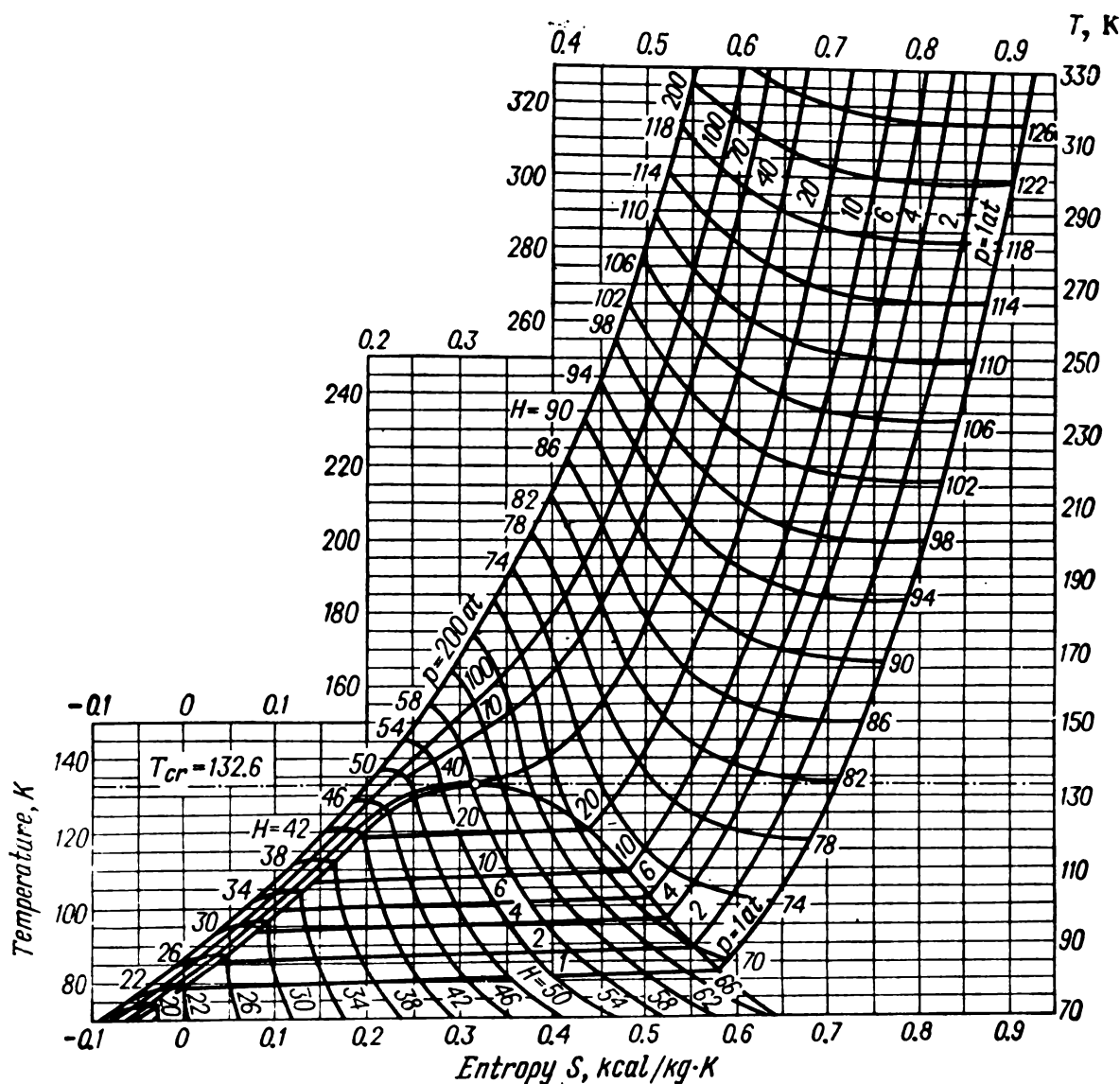


Fig. A-29. T-S diagram for air
Conversion to SI: 1 at=98.1×10³ Pa; 1 kcal/kg=4.19×10³ J/kg

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COMMON LOGARITHMS (mantissas)

<i>N</i>	0	1	2	3	4	5	6	7	8	9
0	—∞	0000	3010	4771	6020	6990	7782	8451	9031	9542
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3576	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
<i>N</i>	0	1	2	3	4	5	6	7	8	9

COMMON LOGARITHMS (mantissas)

N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8707	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9966	9974	9978	9983	9987	9991	9996
N	0	1	2	3	4	5	6	7	8	9